Prova

calculo de area para frente

4) Calcular o comprimento do arco formada pela função $y: x^{2/3}-1$, $[1 \leq x \leq 2]$

$$L: \sqrt{1+(f'(x))^2} \cdot dx$$

$$y': \frac{2}{3} x^{-1/3}$$

$$\int_1^2 \sqrt{1+\left(\frac{2}{3x^{1/3}}\right)^2}$$

$$\int_1^2 \sqrt{1+\left(\frac{4}{9x^{2/3}}\right)} \cdot dx$$

$$\int_1^2 \cancel{\sqrt{\frac{9+4}{9x^{2/3}}}} \cdot dx$$

$$\int_1^2 \sqrt{\frac{9x^{2/3}+4}{9x^{2/3}}} \cdot dx$$

$$\int_1^2 \frac{\sqrt{9x^{2/3}+4}}{\sqrt{9x^{2/3}}}$$

$$\int_1^2 \frac{\sqrt{9x^{2/3}+4}}{3 \cdot (x^{2/3})^{1/2}} \cdot dx$$

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$$L = \frac{1}{3}\int_1^2 \frac{\sqrt{9x^{2/3}+4}}{x^{1/3}} \cdot dx$$

$u = x^{2/3}$

$du = \frac{2}{3} x^{-1/3} \cdot dx$

$$L = \frac{1}{3}\int_1^2 \sqrt{9x^{2/3}+4} \cdot \frac{1}{x^{1/3}} \cdot dx$$

$du = \frac{2}{3x^{1/3}} \, dx$

$\frac{3}{2} du = \frac{1}{x^{1/3}} \cdot dx$

$\sqrt{9u+4} = du$

$$\int_1^2 \frac{1}{3} \sqrt{9 \cdot u + 4} \cdot \frac{3}{2} du$$

$$\int_1^2 \frac{1}{3} \cdot \frac{3}{2} \sqrt{9u+4} \cdot du$$

$$\int_1^2 \frac{1}{2} \sqrt{\underbrace{9u+4}_{v}} \cdot du$$

$v = 9u + 4$

$dv = 9$

$$\int_1^2 \frac{1}{2} v^{1/2} du$$

$\frac{dv}{9} = du$

$$\int_1^2 \frac{1}{2} v^{1/2} \cdot \frac{1}{9} \cdot dv$$

$$\left[\int_1^2 \frac{1}{18} \cdot \frac{v^{1/2+1}}{\frac{1}{2}+1} \right]_1^2$$

$$\int_1^2 \frac{1}{2} \frac{1}{9} \cdot v^{1/2} \cdot dv$$

$$\left[\frac{1}{18} \cdot \frac{v^{3/2}}{3/2} \right]_1^2$$

$$\int_1^2 \frac{1}{18} v^{1/2} \cdot dv$$

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$$\frac{1}{18} \cdot \frac{2}{3} \left[v^{3/2} \right]_1^2$$

$$\frac{1}{27} \left[(9u + 4)^{3/2} \right]_1^2$$

$$\frac{1}{27} \left[(9(2) + 4)^{3/2} \right] - \left[(9 \cdot (1) + 4)^{3/2} \right]$$

$$\frac{1}{27} (103,19 - 46,87) = \boxed{2,085}$$

5) Calcule o comprimento do arco da equação

$$y = \frac{1}{3} \cdot (\underbrace{2 + x}_{u})^{3/2}, \quad [0, 3]$$

$$y' = \frac{1}{3} \cdot u^{3/2} \qquad u = 2 + x^2$$

$$u' = 2x$$

$$y' = \frac{1}{3} \cdot \frac{3}{2} \cdot u^{1/2} \cdot u'$$

$$y' = \frac{1}{2} \cdot (u)^{1/2} \cdot u'$$

$$y' = \frac{1}{2} \cdot (2 + x^2)^{1/2} \cdot 2x$$

$$y' = \frac{2x}{2} \cdot (2 + x^2)^{1/2}$$

$$y' = x \cdot \sqrt{2 + x^2}$$

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$$L = \int_0^3 \sqrt{1 + [x \cdot \sqrt{2 + x^2}]^2} \cdot dx$$

$$\int_0^3 \sqrt{1 + \sqrt{x^2(2 + x^2)}} \cdot dx$$

$$\int_0^3 \sqrt{1 + [\sqrt{2x^2 + x^4}]^2} \cdot dx$$

$$\int_0^3 \sqrt{1 + 2x^2 + x^4} \cdot dx$$

$$L = \int_0^3 \sqrt{(1 + x^2)^2} \, dx$$

$$\int_0^3 (1 + x^2) \cdot dx$$

$$\int_0^3 x + \frac{x^3}{3}$$

$$\left[x + \frac{x^3}{3} \right]_0^3$$

$$\left[\frac{3}{1} + \frac{3^3}{3} \right] - [0]$$

$$[3 + 9] = \boxed{12}$$