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"The journey of a thousand miles begins with one step."
— Lao Tzu

Resumo

A teoria dos grafos, um campo fundamental da matemática e da ciência da computação, oferece estruturas robustas para modelar relacionamentos e percorrer estruturas de grafos. Esta pesquisa se aprofunda na exploração de grafos, crucial para aplicações como roteamento de redes, robótica e geração procedural. Como as aplicações do mundo real frequentemente envolvem ambientes complexos com restrições de tempo, como operações de busca e resgate com drones, este estudo foca em sistemas multiagente para a exploração de grafos, onde múltiplos agentes autônomos colaboram para otimizar a distribuição de tarefas.

Este trabalho apresenta uma estrutura modular estruturada em classes extensíveis, facilitando a adição de novos algoritmos e conjuntos de dados. Ao ir além das limitações da exploração de labirintos perfeitos (NAEEM, 2021), empregamos o NetworkX (HAGBERG *et al.*, 2008)—uma biblioteca de grafos geral—que permite a análise de estruturas de grafos mais diversas. Essa flexibilidade possibilitou comparações em diversos conjuntos de dados para revelar padrões no desempenho dos algoritmos sob diferentes características dos grafos.

O objetivo deste estudo é propor um algoritmo multiagente eficiente para exploração de grafos sem comunicação, uma necessidade crucial em cenários onde a comunicação é impraticável ou impossível. Baseando-se no método de exploração de labirintos perfeitos detalhado por Rodrigues (2024), este estudo estende a abordagem para grafos gerais, incluindo aqueles com ciclos, e avalia o desempenho do algoritmo em quatro conjuntos de dados de complexidade variada. Ao estabelecer um benchmark para a eficiência da exploração, esta pesquisa permite uma avaliação dos custos e benefícios da incorporação da comunicação em estratégias multiagente, proporcionando uma base comparativa para o desenvolvimento futuro de algoritmos de exploração.

Além disso, esta pesquisa incorporou heurísticas e estratégias alternativas para aprimorar o algoritmo de Tarry, que tradicionalmente depende da seleção aleatória de vizinhos. As adaptações resultantes demonstraram melhorias de desempenho em conjuntos de dados específicos, ilustrando como processos de seleção direcionados podem otimizar a eficiência da exploração com base nas propriedades dos conjuntos de dados.

Abstract

Graph theory, a pivotal field in mathematics and computer science, offers robust frameworks for modeling relationships and traversing graph structures. This research delves into graph exploration, crucial for applications like network routing, robotics, and procedural generation. As real-world applications often involve complex environments with time constraints, such as search-and-rescue operations with drones, this study focuses on multi-agent systems for graph exploration, where multiple autonomous agents collaborate to optimize task distribution.

This work presents a modular framework structured into extensible classes, facilitating the addition of new algorithms and datasets. By moving beyond the constraints of perfect maze exploration (NAEEM, 2021), we employed NetworkX (HAGBERG *et al.*, 2008)—a general graph library—which allows for analyzing more diverse graph structures. This flexibility enabled comparisons across various datasets to reveal patterns in algorithm performance under different graph characteristics.

This study's objective is to propose an efficient multi-agent algorithm for graph exploration without communication, a crucial need in scenarios where communication is impractical or impossible. Building on the method for perfect maze exploration detailed by Rodrigues (2024), this study extends the approach to general graphs, including those with cycles, and evaluates the algorithm's performance across four datasets of varying complexity. By establishing a benchmark for exploration efficiency, this research enables an assessment of the costs and benefits of incorporating communication in multi-agent strategies, providing a comparative baseline for future exploration algorithm development.

Additionally, this research incorporated heuristics and alternative strategies to enhance Tarry's algorithm, which traditionally relies on random neighbor selection. The resulting adaptations demonstrated performance improvements on specific datasets, illustrating how targeted selection processes can optimize exploration efficiency based on dataset properties. These insights pave the way for future research into more effective graph exploration strategies in autonomous systems.

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1 Introduction

1.1 Motivation

Graph theory is a fundamental field of mathematics with significant relevance in computer science due to its ability to model relationships between objects. One of the key challenges in computer science is graph traversal and exploration, which involves systematically navigating through the nodes of a graph with a specific purpose. This topic has practical applications across diverse fields, such as network routing, robotics, procedural generation, electronics design, and more.

When considering the complexities and time constraints imposed by real-world environments, graph exploration must be extended to consider multi-agent systems. In such systems, multiple autonomous agents collaborate to explore graphs, aiming to distribute tasks to achieve optimal efficiency. However, communication strategies on these systems are challenging, as they need to balance the trade-off between the amount of information shared among agents versus the quality of the solution. In other words, while communication can allow agents to speed up their exploration, it may cost time and/or energy.

Various versions of this problem have been explored in research. One such approach, discussed by Kivelevitch and Cohen (2010), proposed a generalization of Tarry's Algorithm with significant emphasis on minimizing data transfer. However, the authors did not experiment with this solution on general graphs.

Despite the previously mentioned research, the concept of zero-communication exploration has not been concretely established in the literature. We believe this gap is significant, as practical situations might involve scenarios where communication is limited or impossible due to bandwidth limitations or energy consumption, such as deep-sea exploration, search with energy-limited agents, or high-efficiency state space search.

1.2 Objective

This study aims to develop an efficient algorithm for graph exploration by multi-agents, specifically to find a goal node. A previous paper by Rodrigues (2024) discussed an approach for perfect maze exploration (NAEEM, 2021), which can be modeled as a tree, a connected acyclic undirected graph. Expanding the proposed algorithm to connected cyclic graphs enables the definition of a lower bound for exploration algorithms. This provides a benchmark for evaluating and comparing exploration strategies, allowing researchers to quantify the impact of communication on exploration efficiency relative to a baseline no-communication algorithm. This comparison is crucial for assessing the cost of implementation and the value brought by communication in contrast to a simpler, zero-communication strategy.

Alongside this main objective, this research seeks to incorporate heuristics and alternative strategies proven effective in zero-communication scenarios to enhance Tarry's algorithm. As outlined by Kivelevitch and Cohen (2010), Tarry's algorithm relies on a random selection of neighbors for exploration, which may not always yield optimal results for different graph structures. Our goal was not only to analyze how these choices are made but also to investigate whether a secondary algorithm or heuristic could guide the selection process, thereby improving the overall efficiency of exploration.

1.3 Definitions

This section aims to define key concepts in graph theory and multi-agent exploration that are essential for understanding the context and methodology of our research, ensuring clarity and consistency in the subsequent discussions.

1.3.1 Graph

Graphs are structures in mathematics and computer science used to represent relationships between pairs of objects. According to Manber (1989), a graph $G = (V, E)$ is defined as a set V of vertices (or nodes) and a set E of edges, that connect pairs of vertices.

1.3.1.1 Common Nomenclature

In graph theory, the basic terminology includes the following components:

- Node (Vertex): The fundamental unit of a graph, representing an object or a point.

- Edge: A connection between a pair of vertices, representing a relationship between them.
- Adjacency: Two vertices v and w of a graph G are adjacent if there is an edge vw joining them, and the vertices v and w are then incident with such an edge. (WILSON, 1996)
- Degree: Degree of a vertex v of G is the number of edges incident with v. (WILSON, 1996)
- Path: A path in a graph G, as defined by Wilson (1996), is a finite sequence of edges of the form v_0v_1, v_1v_2, ... in which any two consecutive edges are adjacent, all edges are distinct and all vertices v_0, v_1, ..., v_m are distinct (except, possibly, $v_0 = v_m$).
- Cycle: A path where $v_0 = v_m$ and with at least one edge. (WILSON, 1996)

1.3.1.2 Key Properties

A few key properties of graphs that interest us are:

- Connectivity: A graph is connected if and only if there is a path between each pair of vertices. (WILSON, 1996)
- Acyclicity: A graph is acyclic if it has no cycles. A tree is an acyclic connected graph.

Additionally, we differentiate between "perfect" and "imperfect" mazes:

- Perfect Maze: A maze that contains no loops, meaning it can be represented as a tree (an acyclic connected graph).
- Imperfect Maze: A maze that includes loops, making it representable as a general graph but not as a tree.

1.3.1.3 Traversal

Graph traversal is the process of visiting all the vertices in a graph in a systematic manner. Two fundamental algorithms for graph traversal are:

- Breadth-first search (BFS): Expands the frontier between discovered and undiscovered vertices uniformly across the breadth of the frontier. (MANBER, 1989)

- Depth-first search (DFS): Explores edges out of the most recently discovered vertex v that still has unexplored edges leaving it. (MANBER, 1989)

- Tarry's Algorithm: Tarry's Algorithm is used to traverse the entire maze, or graph, in a depth first fashion, where each edge is travelled twice, and the agent finishes the motion at the same initial vertex it started from.(KIVELEVITCH; COHEN, 2010) This algorithm is described in the context of a physical maze, where the agent must physically move between adjacent nodes. It was first described over botanical mazes built as amusements for the XIX century aristocracy.

1.3.2 Agent

An agent in maze exploration, as defined by Kivelevitch and Cohen (2010), is an entity able to move within the maze in any direction as long as it does not encounter obstacles.

In the context of graph exploration, this concept extends to an agent's ability to traverse edges between vertices within the graph structure. Each agent acts independently, making decisions based on its individual memory and the local information available at its current vertex and its adjacent edges.

1.3.3 Mixed Radix

A mixed radix system is a positional numerical system in which the numerical base for each digit varies. This allows for more flexible representation of numbers, accommodating various scales within the same numerical framework.

One mathematical approach for representing mixed radix numbers can be found in Arndt (2011), which establishes a comprehensive arithmetical method for manipulating them. However, for the specific subset of real numbers within the interval [0, 1], a more specialized approach is utilized, as described by Rodrigues (2024).

In this context, the mixed radix representation $A = [a_0, a_1, a_2, ..., a_{n-1}]$ of a number x with respect to a radix vector $M = [m_0, m_1, m_2, ..., m_{n-1}]$, where $x \in [0, 1]$, is given by:

$$x = \sum_{k=0}^{n-1} a_k \prod_{j=0}^{k} \frac{1}{m_j} \tag{1.1}$$

where a_j are non-negative integers, m_j are integers such that $m_j \geq 2$, and $0 \leq a_j \leq m_j$. (RODRIGUES, 2024)

For instance, if x is represented by $0.2_3 1_2 1_4 0_3 1_5$, and considering that a_i is on the left of a_{i+1}, x might be transformed to the decimal base by the following steps:

$$x = 2_3 1_2 1_4 0_3 1_5 \tag{1.2}$$

$$A = [2, 1, 1, 0, 1] \tag{1.3}$$

$$M = [3, 2, 4, 3, 5] \tag{1.4}$$

$$n = 5 \tag{1.5}$$

$$x = \sum_{k=0}^{4} a_k \prod_{j=0}^{k} \frac{1}{m_j} \approx 0.8777778 \tag{1.6}$$

1.3.3.1 Extending Mixed Radix with Unary Digits

In our study, we extended the mixed radix system to include unary digits, which are used to represent steps in a sequence where no meaningful decisions are made. These unary digits do not contribute to the numerical value and can be omitted in the conversion to standard numerical forms.

Within our framework, the mixed radix representation $A = [a_0, a_1, a_2, ..., a_{n-1}]$ of a number x with respect to a radix vector $M = [m_0, m_1, m_2, ..., m_{n-1}]$, where $x \in [0, 1]$, is given by:

Let $U = \{k \mid m_k \neq 1\}$ be the set of indices where the base m_k is not unary.

$$x = \sum_{k \in U} u_k \prod_{j=0}^{k} \frac{1}{m_j} \tag{1.7}$$

where a_k are non-negative integers, m_j are integers such that $m_j \geq 1$, and $0 \leq a_j \leq m_j$.

Consider the mixed radix representation $x = 0.2_3 I_1 1_2 I_1 I_1 1_4 0_3 1_5$, where I_1 represents the unary digit and each a_i is on the left of a_{i+1}. We can transform x to the decimal base using the following steps:

$$x = 2_3 I_1 1_2 I_1 I_1 1_4 0_3 1_5 \tag{1.8}$$

$$A = [2, I, 1, I, I, 1, 0, 1] \tag{1.9}$$

$$M = [3, 1, 2, 1, 1, 4, 3, 5] \tag{1.10}$$

$$U = [0, 2, 5, 6, 7] \tag{1.11}$$

$$x = \sum_{k \in U} a_k \prod_{j=0}^{k} \frac{1}{m_j} \approx 0.8777778 \tag{1.12}$$

2 Methodology

This section intends to discuss specific design choices that support our multi-agent search algorithm, as detailed in our GitHub repository (PIERO, 2024). Section 2.1 introduces the models and classes developed to improve our algorithm. Section 2.2 describes the core concepts of our algorithm and provides pseudocode to guide its implementation. Section 2.3 presents a mixed radix numerical representation used for the agent's path and examples to demonstrate its functionality. Section 2.4 presents alternative exploration strategies, including Backward Interval Filling and four Tarry algorithm variations to improve efficiency and coordination. Section 2.5 highlights the algorithm's capability to handle complex graphs, as shown through an imperfect maze exploration. Finally, Section 2.6 visualizes individual exploration trees for each agent, demonstrating their paths and handling of graph structures.

2.1 Modeling

In this section, we detail the modeling approach chosen to structure our problem, emphasizing the use of classes to achieve greater flexibility in our solution. The main classes include **Simulation**, **Graph** and **Agent**, each of which will be defined in subsequent sections. Figure 2.1 presents a simplified class diagram that illustrates the overall structure of our model.

2.1.1 Simulation

The **Simulation** class acts as a central controller in our multi-agent graph exploration problem. It is primarily responsible for managing shared information, including the graph structure. Additionally, it orchestrates the agents and the overall execution of the exploration process.

In more detail, its responsibilities include:

- Shared Information: Storing and managing all information that is common between

FIGURE 2.1 – Simplified Class Diagram of the Multi-Agent Maze Exploration Model

agents.

- Orchestration: Coordination agent activities and possible interactions during the exploration.
- Visualization: Providing tools for tracking the progress and visualizing the result of the exploration.
- Metrics: Collecting and analyzing performance metrics of the exploration.

As shown by Figure 2.1, the core method of the **Simulation** is *simulate* that is responsible for starting the exploration and managing each agent in parallel, until all agents have reached the goal or have no more moves available. The pseudocode in Algorithm 1 provides a breakdown of the function's steps and logic.

Algorithm 1 Simulation - simulate()

```
/* Graph to be explored */
graph ← Graph()
/* Agents */
agents ← [Agent(0), Agent(1), Agent(2), ...]

/* Agents that stopped already */
agents_stopped ← [ ]
for all agent in agents do
    append(agents_stopped, FALSE)
end for

/* Exploration */
while False in agents_stopped do
    for all agent in agents do
        /* Moving each agent in parallel */
        agent.move_one_step()
        if agent.status == "FINISHED" or agent.status == "STOPPED" then
            agents_stopped[agent.id] ← TRUE
        end if
    end for
end while

/* Calculating Metrics */
for all agent in agents do
    /* Simplified function for calculating metrics */
    calculate_metrics(agent)
end for

/* Simulation is complete */
/* Showing graphical result */
/* Simplified function for displaying */
display_demo_exploration(agents)
```

2.1.2 Graph

The **Graph** class, as introduced in Section 1.3.1, serves as a basic representation of the graph structure for our multi-agent exploration. Our implementation is built over the graph library NetworkX (HAGBERG *et al.*, 2008), allowing for the efficient management of nodes and edges and facilitating the agents' traversal of the graph.

A key method within this class is *get_neighbors*. This method returns a list of neighboring nodes in a specific order, aiding in the consistent traversal of the graph.

2.1.3 Agent

The **Agent** class represents an agent, as defined in Section 1.3.2 and illustrated in Figure 2.1.

An **Agent** is uniquely identified by its *id* field and possesses the *status* and *algorithm* fields, which define its behavior during each step of the exploration process.

Additionally, an **Agent** records its path using its own graph, defined by the *known_graph* field, and a mixed radix representation of it in the *visited_path* field.

Furthermore, the **Agent** possesses the *current_node* field, which stores its current position in the graph, and the *interval* field, specific to our graph exploration algorithm, aiding in its traversal strategy.

The core method of the **Agent** class is *move_one_step*, which is responsible for determining the agent's next action during the exploration, as demonstrated in Algorithm 1. While the detailed logic of how the agent decides on its actions will be discussed in Section 2.2, we provide a pseudocode representation of this method in Algorithm 2.

Algorithm 2 Agent - move_one_step()

```
procedure MOVE_ONE_STEP(self, graph):
    /* Checking if the agent has already stopped or finished */
    if self.status == "FINISHED" or self.status == "STOPPED" then
        return
    end if

    /* Get current position */
    current_position ← graph[self.current_node]

    /* Taking new step */
    /* Neighbors are returned following a specific ordering */
    neighbors ← graph.get_neighbors(current_position)
    /* Get only non visited neighbors */
    non_visited_neighbors ← [ ]
    for all neighbor in neighbors do
        if self.known_tree[neighbor].visited == FALSE then
            append(non_visited_neighbors, neighbor)
        end if
    end for
    /* This decision will be explained in Section 2.2 */
    next_step ← self.define_agent_next_step(non_visited_neighbors)
    /* If invalid next_step, just move back */
    if next_step == "-1" then
        self.step_back()
        return
    end if

    /* Updating after step */
    self.current_node ← next_step
    self.known_tree[self.current_node].visited ← TRUE

    /* Checking if in goal */
    if current_position.goal == TRUE then
        self.status ← "FINISHED"
        return
    end if
end procedure
```

2.2 Graph Exploration Algorithm

As mentioned in Section 1.2, the aim of this study is to develop an effective algorithm for graph exploration using multiple agents without communication between them. A significant challenge in this context is avoiding the redundant exploration of nodes already visited by other agents, as it consumes resources without contributing new information to the overall exploration.

To address this problem, we adopt the solution proposed by Rodrigues (2024), that consists on dividing the graph into distinct intervals, which are then split among the agents. This approach ensures that each agent explores a unique section of the graph, thereby minimizing overlap and maximizing efficiency. The intervals are determined based on the number of agents, ensuring a non-overlapping distribution of the graph's nodes. Figure 2.2 illustrates this concept, showing a graph with intervals assigned to different colored agents.

FIGURE 2.2 – Three agents (Red, Blue, and Yellow) disperse from each other in a graph. Source: Rodrigues (2024)

It is important to note that while the numerical intervals are non-overlapping, the dispersion may not always be fair or uniform. This is evident in Figure 2.3, which illustrates three agents dispersing in an unbalanced tree, contrasting with the more uniform dispersion shown in Figure 2.2. The graph shows the nodes colored based on the agent intervals they fall into. If a node has more than one color, it indicates that the node is within the intervals of multiple agents.

Mathematically, this dispersion can be expressed by a set of equations. Assume there are k agents, denoted as $a_1, a_2, a_3, ..., a_k$. The distribution of intervals can be represented as shown in Equation 2.1, based on the approach proposed by Rodrigues (2024).

FIGURE 2.3 – Three agents (Red, Blue, and Yellow) disperse in a non-uniform graph.

$$
\begin{aligned}
d &= 1/k \\
a_1&: \ [0, d[\\
a_2&: \ [d, 2d[\\
a_3&: \ [2d, 3d[\\
&\ldots \\
a_k&: \ [(k-1)d, 1]
\end{aligned}
\tag{2.1}
$$

With the division of intervals explained, we now describe the step-by-step process of our algorithm for multi-agent graph exploration. This algorithm extends the work of Rodrigues (2024), incorporating modifications to handle previously visited nodes and potential cycles.

1. **Initialization**

   - Each agent is assigned a unique identifier and a specific interval based on Equation 2.1.
   - Each agent begins with an empty tree representation of the graph, representing the knowledge it has accumulated during the exploration. Since each agent independently explores the graph, their trees may differ, reflecting the order in which nodes are visited.
   - All agents start at the same node, although the specific starting node is arbitrary.

- The starting node is assigned the interval $[0, 1]$ and is added to the agent's tree.

2. **Exploration Strategy**

   - At each step, an agent examines the adjacent nodes that have not been previously visited by it. To maintain consistency in exploration, these nodes are sorted by their unique identifiers.
   - The agent dynamically calculates convergence intervals for each unvisited adjacent node. The calculation method is as follows:
     - **Single Child**: If there is only one child node, it inherits the convergence interval of the current node.
     - **Multiple Children**: If there are multiple children, the current node's convergence interval is uniformly divided among them.
   - Each adjacent node is then added to the agent's tree along with the corresponding edge and its assigned interval.
   - When adding a node, called the target node, to the tree in generic graphs, a cycle may be detected if the target node has not been visited yet but already appears in the tree. In this case, the new edge would form a return edge, as identified by Tarjan's Algorithm. To prevent cycles, the previous occurrence of the target node is removed from the tree, and it is then re-added with its updated edge and interval, ensuring a valid acyclic structure for the exploration tree. In Figure 2.11, an example of this situation is shown. The target node was correctly removed from the agent's tree, but for demonstration purposes, a copy of the tree was created, and the target node was re-added with a modified identifier (appending "_0" to its ID) to illustrate the cycle occurrence. That figure is based on this modified tree, not the original agent's tree.
   - The agent then chooses the first node whose convergence interval intersects with its own. If no such node exists or if all adjacent nodes have been visited, the current node is marked as explored, and the agent backtracks to the parent node.

3. **Completion Criteria**

   - The agent continues the exploration process until it either finds the goal or fills its assigned interval. If the goal is not found within its interval, it must be within the interval of another agent.
   - After finishing its interval, the agent may stop or adopt an alternative strategy to accelerate the search. The default behavior implemented is to do a depth-first search (DFS), ignoring nodes already explored in the initial attempt.

The steps described in the **Initialization** topic are implemented in the initialization procedures of the **Simulation** and **Agent** classes. The pseudocode presented in Algorithm 3 illustrates the *define_agent_next_step* method, which implements the step-by-step process described in the **Exploration Strategy** topic.

Algorithm 3 Agent - define_agent_next_step()

```
procedure DEFINE_AGENT_NEXT_STEP(self, unvisited_neighbors):
    /* Calculating the convergence intervals */
    /* This will be explained in Section 2.3 */
    intervals ← self.calculate_neighbors_intervals(unvisited_neighbors)

    /* Checking for backtracking or cycle */
    for all neighbor n_j in unvisited_neighbors do
        if self.known_tree[n_j].interval then
            if self.current_node in self.known_tree[n_j].get_edges() then
                /* Backtrack */
                intervals[n_j] ← self.known_tree[n_j].interval
            else
                /* Cycle */
                /* The agent removes the previous node from its tree
                /* so it doesn't cause a cycle */
                self.known_tree.remove_node(n_j)
            end if
        end if
    end for

    /* Adding neighbors to tree */
    self.add_neighbors_to_tree(unvisited_neighbors, intervals)
```

To better understand the proposed algorithm, Section A.1 in the Appendix provides a step-by-step example of its execution on the graph shown in Figure 2.8. It explains how the trees in Figures 2.9, 2.10, and 2.11 are generated.

Algorithm 3 Agent - define_agent_next_step()

```
    /* Checking for intersecting intervals */
    /* Is important to take notice that as the neighbors are ordered
    /* And the interval of a neighbor is proportional to its position
    /* As we will see in Algorithm 4, the neighbor are visited in
    /* incresing order of intervals */
    for all neighbor n_j in unvisited_neighbors do
        max_neighbor ← intervals[n_j][1]
        min_neighbor ← intervals[n_j][0]
        max_agent ← self.interval[1]
        min_agent ← self.interval[0]

        if max_agent < min_neighbor then

            /* If the node's interval is on the right of the agent's interval, surely the
            agent has finished its interval, since the nodes are in increasing order of
            intervals as previously mentioned */
            finished_interval ← TRUE
            break
        else min_agent < max_neighbor and max_agent > min_neighbor
            /* Found intersecting intervals */
            return n_j
        end if
    end for

    /* If got here, didn't find any intersecting intervals  */
    /* If back on root, the agent filled its interval  */
    if self.current_node == self.starting_node then
        self.filled_interval ← TRUE
    end if

    /* Just step back if interval isn't filled */
    if self.filled_interval == FALSE then
        return "-1"
    end if

    /* If interval is filled, change strategies */
    self.status ← "DFS_SEARCH"
    return
end procedure
```

2.3 Mixed Radix Path Representation

This section details how the mixed radix representation described in Section 1.3.3.1 is used to represent the path taken by the agent, including the decision taken in each node as used in Algorithm 3.

Our approach utilizes the same mixed radix representation technique as described by Rodrigues (2024). This method allows an agent to record each decision made along the path without the need to retrospectively analyze previous choices. Instead, it only requires the current node's convergence interval and its edges to make decisions, reducing computational overhead. This is based on the following logic:

- The path starts at "0.", and the starting node is assigned the interval $[0, 1]$.
- If the current node has a single child, the agent appends the unary digit I_1 to the path. This indicates that there is only one possible path to take from this node.
- If the current node has multiple children (e.g., n children), the agent chooses the ith child and appends $(i-1)_n$ to the path. This notation denotes the specific choice among the available children, with i representing the child's position in an ordered list and the subscript n denotes the number of children or the length of the ordered list.
- When the agent needs to return to a parent node, it removes the last value appended to the mixed radix representation, effectively backtracking to the previous decision point.

To illustrate this concept, we present Figures 2.4, 2.5, 2.6, and 2.7, which depict the progression of the Blue agent from the starting node to the goal in Figure 2.3. Figure 2.4 shows the agent at the starting node before taking the first step. Figures 2.5, 2.6, and 2.7 display the first three steps taken by the agent. Each figure explicitly details the path at that step and the intervals calculated for each child of the current node. Additionally, the mixed radix path representation between the *Starting Point* and *Goal* for the same exploration is provided, along with an example of how it can be converted into a numerical value to calculate the corresponding interval.

FIGURE 2.4 – Blue agent at the starting node of graph from Figure 2.3.

FIGURE 2.5 – Blue agent taking its first step in the graph from Figure 2.3.

FIGURE 2.6 – Blue agent taking its second step in the graph from Figure 2.3.

FIGURE 2.7 – Blue agent taking its third step in the graph from Figure 2.3.

$$x = 0_2 I_1 2_3 2_3 \tag{2.2}$$

Following the conversion as described by Equation 1.7, we can transform this mixed radix notation into a numerical value:

$$x = 0_2 I_1 2_3 2_3 \tag{2.3}$$

$$A = [0, I, 2, 2] \tag{2.4}$$

$$M = [2, 1, 3, 3] \tag{2.5}$$

$$U = [0, 2, 3] \tag{2.6}$$

$$x = \sum_{k \in U} a_k \prod_{j=0}^{k} \frac{1}{m_j} = \frac{4}{9} \approx 0.4444444 \tag{2.7}$$

The function *calculate_neighbors_intervals* used in Algorithm 3 is defined by this logic and the Equation 1.7 as can be seen in Algorithm 4

Algorithm 4 Agent - calculate_neighbors_intervals()

```
procedure CALCULATE_NEIGHBORS_INTERVALS(self, unvisited_neighbors):
    /* Fetching current node interval */
    current_node_interval ← self.known_tree[self.current_node].interval
    current_interval_size ← (current_node_interval[1]-current_node_interval[0])
    interval_chunk ← current_interval_size / unvisited_neighbors.length()

    /* Calculating interval for each neighbor */
    intervals ← [ ]
    for all neighbor n_j in unvisited_neighbors do
        interval_start ← current_node_interval[0] + interval_chunk * j
        interval_end ← interval_start + interval_chunk
        intervals.append((interval_start, interval_end))
    end for
    return intervals
end procedure
```

To ensure precision in interval calculations, we use the BigNumber library gmpy2 (MARTELLI, 2015) instead of standard double-precision, which is limited to about 15 decimal places. For deeper datasets, as shown in Table 3.1, this approach is essential, as interval scales decrease by factors of $(1/2)^{depth}$. Using gmpy2 prevents rounding errors that could otherwise lead to inaccuracies in the path representation and each node's interval.

2.4 Alternative Exploration Algorithms

In our implementation, modifying and experimenting with variations of the exploration algorithm is simple. By simply altering the $define_agent_next_step$ function in Algorithm 2, we can easily incorporate different strategies based on the agent's current status or algorithm.

In this work, we have explored six alternatives: our core algorithmm, one variation of it and four adaptations of Tarry's algorithm.

2.4.1 Exploration Algorithm without Communication

2.4.1.1 Backward Interval Filling

The **Backward Interval Filling** algorithm extends the primary zero-communication strategy developed in this work. Instead of initiating a new Depth-First Search (DFS) traversal after completing a designated interval, agents continue exploring by traversing the next interval in reverse order. "Next interval" refers to the interval initially assigned to the next agent in the list from Equation 2.1. Note that each agent is assigned both intervals from the start, so no communication is needed. For the last agent, whose interval ends at 1, its second interval is assigned as the first agent's interval, which starts at 0. Thus, once the agent finishes exploring its original interval, it moves up on its tree to the nearest common ancestor shared with the end of the next interval, then proceeds to move downwards to the end leaf of its second interval, which it will traverse in reverse order. This variation ensures that agents remain active even after completing their primary interval, enhancing overall graph coverage and reducing idle time. It takes advantage of the fact that the tree may be unbalanced; when one interval contains significantly fewer nodes than the next, the agent assigned to the smaller interval can finish quickly and then explore the larger interval from the opposite direction. For exemplo, in Figure 2.3, the yellow agent, after traversing its interval, will traverse the red nodes in reverse order.

2.4.2 Exploration Strategies with Communication: Tarry Algorithm Variations

This section presents several adaptations of Tarry's algorithm that utilize communication between agents. Each variation builds upon the foundational logic of Tarry's approach but introduces different heuristics and optimizations aimed at improving performance and accommodating various graph structures.

2.4.2.1 Extended Tarry Algorith

The **Extended Tarry Algorithm**, proposed by Kivelevitch and Cohen (2010), is an multi-agent adaptation of Tarry's algorithm. In this approach, agents share information about visited nodes, enabling them to identify unvisited cells and determine the Last Common Location (LCL) — the most recent shared position among agents before diverging paths.The algorithm consists of two phases:

- **Exploration Phase**: Each agent moves to unvisited cells, or to cells unvisited by itself, choosing arbitrarily among multiple options. If all cells have been visited, agents retreat to the last unvisited cell, marking cells as dead ends.
- **Cooperative Backtracking Phase**: Once an agent finds the goal and is defined as the pioneer, other agents backtrack to the Last Common Location (LCL) with it and follow the pioneer's path to the goal.

The functions *define_next_first_phase_tarry_step* and

define_next_second_phase_tarry_step, as shown in Algorithms 5 and 6, implement this logic.

Algorithm 5 Agent - define_next_first_phase_tarry_step()

```
procedure DEFINE_NEXT_FIRST_PHASE_TARRY_STEP(non_visited_neighbors):
    new_neighbors ← []
    already_visited_neighbors ← []
    /* Splitting Neighbors */
    for all neighbor in non_visited_neighbors do:
        if neighbor not visited and not deadEnd then:
            new_neighbors.append(neighbor)
        else if neighbor visited then:
            already_visited_neighbors.append(neighbor)
        end if
    end for
    if new_neighbors is not empty then:
        /* Choosing a random non-visited neighbor */
        return random_choice(new_neighbors)
    else if already_visited_neighbors is not empty then:
        /* Choosing a random non-visited by this agent neighbor */
        return random_choice(already_visited_neighbors)
    end if
    return "-1"
end procedure
```

2.4.2.2 Tarry Tie-Breaker Variation

In this variation, agents employ Tarry's algorithm but modify the selection process described in Algorithm 5. Instead of randomly choosing among valid neighbors, agents prioritize their choices based on interval assignments derived from mixed radix path intervals. This enhancement aims to better disperse the agents by having each one prioritize a specific section of the graph.

Algorithm 6 Agent - define_next_second_phase_tarry_step()

```
procedure DEFINE_NEXT_SECOND_PHASE_TARRY_STEP(non_visited_neighbors,
    pioneer_id, pioneer_effective_path, lcl)
    if self.status == "TARRY_FIRST_PHASE" then
        self.status ← "TARRY_SECOND_PHASE"
    end if
    if self.status == "TARRY_SECOND_PHASE" then
        if self.current_node_id != lcl[pioneer_id][self.id] then
            /* Step back until the last common location */
            return "-1"
        else
            self.status ← "TARRY_FOLLOW_PIONEER"
        end if
    end if
    current_node_index ← pioneer_effective_path.index(self.current_node_id)
    /* Follow pioneer's path */
    return pioneer_effective_path[current_node_index + 1]
end procedure
```

2.4.2.3 Tarry Interval Priority Variation

Each agent traverses its assigned interval as described in Algorithm 3, but rather than using depth-first search (DFS) or backward filling once the interval is complete, it begins Tarry's algorithm, as described in Algorithms 5 and 6. This strategy seeks to leverage the additional information gathered during the interval search to enhance exploration efficiency and reduce redundant backtracking.

2.4.2.4 Delayed Tarry Variation

In this approach, agents begin by navigating their assigned intervals for a predetermined number of steps to ensure an initial spread across distinct branches of the graph. After this dispersal phase, they switch to Tarry's algorithm to continue exploration. This method aims to reduce collisions by increasing the probability that agents explore separate regions early on, leading to improved coverage and efficiency in the overall exploration.

2.5 Graph Visualization

One significant enhancement is the capability to visualize graphs beyond perfect mazes, expanding our algorithm's applicability to general graphs.

To demonstrate this capability, we applied our algorithm to explore an imperfect maze, which is a cyclic graph, using three agents. The overall exploration of the maze is depicted

in Figure 2.8.

FIGURE 2.8 – Exploration of an imperfect maze(graph) by three agents.

This visualization showcases how our algorithm can handle graphs with cycles, illustrating the adaptability of our approach.

2.6 Agent Tree Visualization

In addition to overall graph exploration, we developed a method to visualize the individual exploration trees constructed by each agent during the process. These trees represent the paths traversed by the agents and provide insight into the structure of their independent exploration, including how cycles are managed and intervals are assigned to each node.

The Figures 2.9, 2.10 and 2.11 show the trees constructed by the 3 agents for the exploration displayed in Figure 2.8.

FIGURE 2.9 – Trees constructed by agent post-exploration: **Agent 1**.

FIGURE 2.10 – Trees constructed by agent post-exploration: **Agent 2**.

FIGURE 2.11 – Trees constructed by agent post-exploration: **Agent 3**.

These visualizations illustrate how each agent independently builds its representation of the graph, leading to varied exploration trees. In Figure 2.11, the highlighted nodes, $(3, 4)$ and $(3, 4)_0$, demonstrate how cycles can occur during exploration.

Initially, node $(3, 4)$ was added to the tree when the agent discovered it while ex-

ploring the neighbors of node $(4, 4)$, even though it hadn't been visited yet. Later, as the exploration progressed, the agent reached node $(3, 4)$ while checking the neighbors of node $(3, 3)$, revealing a cycle. According to the method outlined in Section 2.2, the original node $(3, 4)$ was removed from the agent's tree to maintain an acyclic structure. For illustration purposes, a copy of the tree was created, and node $(3, 4)$ was re-added with a modified identifier (appending "_0" to its ID) to visualize this occurrence.

Finally, when node $(3, 4)$ was reached again while exploring the neighbors of node $(2, 4)$, the agent again removed it from its tree and added it as a child of node $(2, 4)$. This step-by-step process clarifies the presence of node $(3, 4)$_0 in the figure, along with the edges connecting it to nodes $(4, 4)$ and $(3, 3)$, and the edge from node $(2, 4)$ indicating when node $(3, 4)$ was finally explored.

3 Results and Discussion

This chapter presents the results of comparing the proposed no-communication algorithm with Tarry's algorithm, as well as analyzing Tarry's performance against its variants. We evaluate these approaches on different datasets, detailed in Section 3.1, to observe how distinct graph properties influence the efficiency of the solutions. Section 3.2 discusses the performance of the no-communication algorithms. Section 3.3 then presents the analysis of Tarry's algorithm and its variants.

3.1 Datasets for Algorithm Evaluation

This section describes the datasets used to evaluate the performance of the algorithms. The datasets can be accessed on GitHub (PIERO, 2024) and consist of four classes of graphs:

- Perfect Mazes: The dataset of perfect mazes (NAEEM, 2021) is the same as the one used by Rodrigues (2024) and includes 250 mazes for each of four sizes: 10x10, 20x20, 30x30, and 40x40.
- Random Trees: The dataset of random trees was generated using the *random_unlabeled_tree* method from NetworkX (HAGBERG *et al.*, 2008). This function generates an unlabeled tree uniformly at random using Wilf's algorithm (WILF, 1981), resulting in 250 trees for sizes: 100, 250, 500, 1000, and 1500 nodes. The goal node was chosen by randomly selecting one of the leaf nodes. Compared to the perfect mazes, these trees tend to be less deep and broader.
- Barabási-Albert Graphs: The dataset of Barabási-Albert graphs was generated using the dual Barabási-Albert preferential attachment method (MOSHIRI, 2018) from NetworkX, specifically *dual_barabasi_albert_graph* function. This method constructs a random graph by incrementally adding new nodes that connect to existing nodes with a higher degree (BARABáSI; ALBERT, 1999). For this dual approach, at each step of edge addition for a new node, it can be chosen whether the node will connect with either n_1 or n_2 edges, determined by a probability p for n_1 and $1 - p$

for n_2. In our case, n_1 is 2, n_2 is 3, and p is set to 0.2. This approach results in networks that exhibit a power-law degree distribution, distinguishing them from the other datasets. The dataset includes graphs of sizes 100, 250, 500, 1000, and 1500 nodes.

- Small-World Network: The Small-World Network dataset was generated using NetworkX's *navigable_small_world_graph* function (HAGBERG *et al.*, 2008). Nodes are organized in an $n \times n$ grid, where each node connects to its local neighbors. Additionally, each node has one long-range connection assigned with a probability inversely proportional to the manhattan distance, given by $d((i,j),(k,l)) = |k-i| + |l-j|$, raised to the power $-r$ (KLEINBERG, 2000). In this dataset, q is set to 1, meaning each node has one additional long-range connection, and r is set to 2.5. This configuration creates graphs of sizes 100, 250, 500, 1000 and 1500 nodes.

The sizes were chosen to approximately match the maze sizes used in Rodrigues (2024) and to ensure that the computational demands for running all experiments remained manageable. For example, a 40x40 maze has 1600 cells, so graph sizes such as 1500 nodes were selected to provide a comparable level of complexity while keeping the analysis feasible across multiple runs.

For all datasets, after generating the graphs, nodes that did not contribute new decisions— specifically, those with a degree of 2 —were contracted. This contraction involves adding an edge between the node's two neighbors and removing the node itself. In the case of a perfect maze, this effectively eliminates every corridor. This removal process conserves memory and processing time, thus enhancing exploration efficiency and analysis, which particularly relevant for larger datasets. However, this process was not applied to the perfect mazes dataset used by Rodrigues (2024), as doing so would alter the structure of the mazes and affect the calculation of metrics, hindering valid comparisons.

Each dataset has distinct characteristics that can significantly influence the performance of the evaluated algorithms. To illustrate these effects, we present two tables: Table 3.1 shows the average depth, counting only meaningful choices, of a Depth-First Search (DFS) exploration for each generated graph, excluding cycles, while Table 3.2 displays the average number of leaf nodes resulting from the same DFS exploration.

Additionally, we include Table 3.3, which shows the average number of nodes remaining after the contraction process for each dataset. This table provides further insight into the structure of the datasets post-contraction.

TABLE 3.1 – Average Depth of DFS Exploration for Each Dataset

Dataset	**10x10/100**	**250**	**20x20/500**	**30x30/1000**	**40x40/1500**
Perfect Mazes	6.79	-	18.31	31.29	48.98
Random Trees	6.78	10.52	15.20	22.24	27.04
Barabási-Albert	10.69	21.17	33.65	57.04	66.10
Small-World	0.004	0.22	0.002	0.23	0.001

TABLE 3.2 – Average Number of Leaf Nodes for Each Dataset

Dataset	**10x10/100**	**250**	**20x20/500**	**30x30/1000**	**40x40/1500**
Perfect Mazes	11.58	-	41.88	92.58	162.14
Random Trees	34.76	85.64	172.22	345.41	516.53
Barabási-Albert	21.17	46.70	81.21	149.26	191.41
Small-World	1.01	1.22	1.00	1.23	1.00

TABLE 3.3 – Average Number of Nodes After Contraction for Each Dataset

Dataset	**10x10/100**	**250**	**20x20/500**	**30x30/1000**	**40x40/1500**
Perfect Mazes	100	-	500	1000	1500
Random Trees	51.26	124.56	251.03	502.02	752.89
Barabási-Albert	59.89	125.15	207.66	366.44	444.65
Small-World	98.26	223.41	482.296	959.528	1442.376

The results presented in the tables allow for a clear comparison of the datasets. The Barabási-Albert graphs exhibited the greatest average depth in the DFS exploration and the second highest number of leaf nodes, indicating their complexity. Notably, they were also the most affected by the contraction process, resulting in a significant reduction in node count. In contrast, the Random Trees dataset had the least depth and the highest average number of leaf nodes, showcasing its broader structure. Perfect Mazes, while exhibiting moderate depth and leaf nodes, did not undergo contraction, preserving their original node counts for valid metric comparisons.

The Small-World dataset exhibits unique characteristics in Tables 3.1 and 3.2, where average depth approaches zero and the number of leaf nodes remains near one. This occurs due to the DFS traversal's fixed search order (North, East, South, West, followed by any other connected node), producing a nearly linear, degenerate tree structure where each parent node has only one child. Corridor contraction further simplifies this structure, reducing the tree to a single node, hence the minimal depth and single leaf node. Although this pattern implies a single agent almost always can traverse the grid without

backtracking, additional agents can still enhance performance by distributing across the grid while following different search orders, making this dataset suitable for our analysis.

3.2 No-Communication Algorithms Performance

3.2.1 Metrics for Analysis

This section focuses on the results for the zero-communication algorithms, which include the basic distributed search algorithm and its Backward Interval Filling variant. Tarry's algorithm is also included for comparison, despite involving communication. The reason for this choice is that zero-communication algorithms provide a lower limit for assessing the efficiency of any distributed search algorithm employing communication, and we aim to compare them with a communication-based approach like Tarry's.

The metrics used in this comparison are the same as those from Kivelevitch and Cohen (2010):

- Steps Taken by the Pioneer: This measures the number of steps needed by the first agent to find the goal, serving as a key indicator of exploration efficiency.
- Fraction Explored Before the Pioneer Found the Goal: This indicates the proportion of the graph explored by the agents when the goal is reached, providing insight into how well the agents dispersed during exploration.

These metrics together give a clear view of the algorithm's efficiency and how exploration patterns vary across different graph structures and sizes.

3.2.2 Perfect Maze Results

The results for all maze sizes (10x10, 20x20, 30x30, and 40x40) are presented in Figures 3.1 and 3.2, showing how the different no-communication algorithms perform across various maze dimensions. Figure 3.1 represents the steps taken by the pioneer, while Figure 3.2 shows the explored fraction before the goal was found, allowing for a detailed comparison of exploration efficiency and coverage.

(a) 10x10 Maze

(b) 20x20 Maze

(c) 30x30 Maze

(d) 40x40 Maze

FIGURE 3.1 – Comparison of the pioneer's average steps between our no-communication algorithms and Tarry's algorithm across different sizes of perfect mazes. The subfigures illustrate how algorithm performance scales with maze size.

(a) 10x10 Maze

(b) 20x20 Maze

(c) 30x30 Maze

(d) 40x40 Maze

FIGURE 3.2 – Comparison of the explored fraction achieved by our no-communication algorithms and Tarry's algorithm across different sizes of perfect mazes. The subfigures illustrate how coverage efficiency scales with maze size.

As this dataset was also used by Rodrigues (2024), and the results shown in Figures 3.1 and 3.2 match his, they confirm the correctness of our new implementation.

The results show that adding more agents helps the pioneer find the goal cell faster, as the average number of steps taken decreases with more agents. This trend eventually levels off at a certain point when there are many agents. The explored fraction behaves similarly, with more agents spreading out quickly and covering more of the maze, though it also stabilizes after a certain number of agents. This outcome is expected, as more agents lead to quicker exploration and better coverage.

In addition to the expected results, the Backward Interval Filling algorithm shows a strange pattern when the number of agents is a power of two, starting from the 20x20 maze size. This matches earlier findings in (RODRIGUES, 2024), but it hasn't been fully explored in previous studies. While we recognize the theory suggested by Rodrigues (2024), we did not investigate this behavior in detail in our research.

Building on these findings, we compared the performance of our no-communication algorithms with Tarry's algorithm. As expected, both Tarry's algorithm and the Backward Interval Filling variant perform better than the basic version of our algorithm. However, the level of improvement depends on the maze size. In larger mazes, Tarry's algorithm shows a bigger advantage. We conjecture that in smaller mazes, it's possible to quickly find the optimal solution by simply adding more agents. But in larger mazes, communication is crucial for avoiding repeated steps, which we assume significantly enhances Tarry's performance.

3.2.3 Random Tree Results

The results for the random trees of sizes 100, 500, 1000 and 1500 nodes are presented in Figure 3.3. These sizes were chosen because they effectively represent the trends observed for the metrics without needing to include every possible size. Figure 3.3 shows the steps taken by the pioneer, while the Figure 3.4 shows the explored fraction.

(a) 100-node Tree

(b) 500-node Tree

(c) 1000-node Tree

(d) 1500-node Tree

FIGURE 3.3 – Comparison of the average steps taken by the pioneer across different no-communication algorithms and Tarry's algorithm for various sizes of random trees. The subfigures illustrate how algorithm performance varies with tree sizes.

(a) 100-node Tree

(b) 500-node Tree

(c) 1000-node Tree

(d) 1500-node Tree

FIGURE 3.4 – Comparison of the explored fraction achieved by the pioneer across different no-communication algorithms and Tarry's algorithm for various sizes of random trees. The subfigures illustrate how coverage efficiency varies with tree sizes.

The results for the random trees confirm that the conclusions from Section 3.2.2 still apply. Performance improves with more agents, Tarry's algorithm remains the most efficient, and its advantages increase with size.

Even though the trends hold, it's important to note that Tarry outperforms the Back Filling Interval Variation by approximately 39.6% and our proposed algorithm by 47.3% in the 40x40 maze. In contrast, in the 1500-node trees, Tarry shows a substantial improvement, being about 81.7% better than our proposed algorithm and 80.8% better than the Back Filling Interval Variation. This demonstrates the significant impact of dataset characteristics on the relative performance of our no-communication algorithms.

Referring to Figure 3.4, the results differ from expectations, as the plots are noisy and don't show a clear pattern where Tarry's algorithm consistently explores a larger fraction. Previously, the explored fraction increased with the number of agents, but that trend isn't evident here, as all the algorithms are around the same values with little difference

regardless of the number of agents.

This difference arises from the dataset characteristics. In deep trees like Perfect Mazes, agents must explore down to the leaf nodes to find the goal, setting a lower limit on the number of steps. In shallower trees like Random Trees, agents can quickly reach leaf nodes, potentially finding the goal earlier regardless of agent count.

As a result, some instances show a very small explored fraction, while others have a higher one, leading to noisy averages. For example, considering Tarry's with 1500 nodes and 20 agents, the explored fraction averaged 49.21% with a standard deviation of 30.07%, highlighting this variability. This large spread means this metric is not very relevant for this kind of dataset. On the other hand, zero-communication algorithms do not incur on any loss with respect to this metric.

FIGURE 3.5 – Histogram of explored fractions for random trees with 1500 nodes and 20 agents using Tarry's Algorithm.

As expected, Figure 3.5 shows a fairly uniform distribution in the explored fractions, reflecting the characteristics of the random trees.

3.2.4 Barabási-Albert Results

The results for the Barabási-Albert graphs with 100, 250, 500, and 1000 nodes are shown in Figures 3.6 and 3.7. These sizes were selected to capture the overall trends in exploration efficiency while accounting for computational constraints on larger Barabási-Albert graphs. Notably, for the 1000-node graph, the maximum number of agents was limited to 30 instead of the standard 40 due to performance considerations. Figure 3.6 illustrates the average steps taken by the pioneer, while Figure 3.7 displays the explored fraction achieved.

(a) 100-node Barabási-Albert

(b) 250-node Barabási-Albert

(c) 500-node Barabási-Albert

(d) 1000-node Barabási-Albert

FIGURE 3.6 – Comparison of the average steps taken by the pioneer across different no-communication algorithms and Tarry's algorithm for various sizes of Barabási-Albert graphs. The subfigures illustrate how algorithm performance varies with graph sizes.

(a) 100-node Barabási-Albert

(b) 250-node Barabási-Albert

(c) 500-node Barabási-Albert

(d) 1000-node Barabási-Albert

FIGURE 3.7 – Comparison of the explored fraction achieved by the pioneer across different no-communication algorithms and Tarry's algorithm for various sizes of Barabási-Albert graphs. The subfigures illustrate how coverage efficiency varies with graph sizes.

As expected, performance improves with more agents, and Tarry's algorithm remains the most effective approach. However, for this dataset, the Backward Interval Filling algorithm shows no improvement over the basic exploration algorithm. We suspect this is because the Barabási-Albert graphs are highly connected and contain many cycles, leading each agent to form a distinct exploration tree. The same nodes often have significantly different intervals based on the paths taken by each agent. Consequently, each agent completes its initial interval and often reaches the goal before starting a second one.

Unlike the trend observed in Section 3.2.2, the explored fraction decreases with additional agents. This likely occurs because the multiple cycles lead to frequent intersections between agents' exploration trees, adding redundant information. As agents reach the goal quickly, there is less time to explore other parts of the graph.

3.2.5 Small-World Results

The results for Small-World graphs with 100, 250, 500, and 1000 nodes are shown in Figures 3.8 and 3.9. These sizes were chosen to capture overall trends in exploration efficiency within computational limits for larger graphs. For the 100-node and 250-node graphs, up to 40 agents were used, while for the 500-node and 1000-node graphs, the maximum was limited to 30 agents due to performance constraints.

(a) 100-node Small-World

(b) 250-node Small-World

(c) 500-node Small-World

(d) 1000-node Small-World

FIGURE 3.8 – Comparison of the average steps taken by the pioneer across different no-communication algorithms and Tarry's algorithm for various sizes of Small-World graphs. The subfigures illustrate how algorithm performance varies with graph sizes.

(a) 100-node Small-World

(b) 250-node Small-World

(c) 500-node Small-World

(d) 1000-node Small-World

FIGURE 3.9 – Comparison of the explored fraction achieved by the pioneer across different no-communication algorithms and Tarry's algorithm for various sizes of Small-World graphs. The subfigures illustrate how coverage efficiency varies with graph sizes.

In Figure 3.8, we see a similar trend to Section 3.2.4: Tarry's algorithm consistently outperforms the other methods, with performance improving as the number of agents increases, while the Backward Interval Filling algorithm shows no gains over the basic exploration algorithm. This is likely for the same reason as with Barabási-Albert graphs; Small-World graphs are highly connected, and searches often complete before agents finish their assigned intervals. For both graph types, Tarry's algorithm outperforms the no-communication methods by 80.86% and 84.11%, respectively, with 30 agents on 1000-node graphs. While Small-World graphs' grid structure could potentially aid dispersion for non-communication algorithms, it seems to enhance Tarry's performance even more, resulting in this observed discrepancy.

In Figure 3.9, which shows the explored fraction before the pioneer reaches the goal, we see similarly noisy results as in Section 3.2.3. Tarry's algorithm generally explores a smaller portion of the graph, though the difference is not substantial. This variability

likely stems from inconsistent exploration across instances; in some cases, agents must explore a significant portion of the graph to locate the goal, while in others, they explore very little. Tarry's average explored fraction for 1000 nodes with 20 agents is 46.17%, with a standard deviation of 28.81%, confirming this inconsistency. Figure 3.10 further illustrates the spread of these exploration results.

FIGURE 3.10 – Histogram of explored fractions for Small World Graphs with 1000 nodes and 20 agents using Tarry's Algorithm.

3.3 Tarry's Variants Performance

3.3.1 Metrics for Analysis

This section examines the results for Tarry's algorithm and its variants, specifically Tarry Tie-Breaker, Tarry Interval Priority, and Delayed Tarry.

In addition to the previously mentioned metrics in Section 3.2.1, this section presents a relative difference in performance compared to the base Tarry algorithm. This approach is particularly useful for interpreting results when the performances of the algorithms are very close on some datasets, as relative values help to better visualize the differences when absolute values are too similar.

3.3.2 Perfect Maze Results

The results for all maze sizes (10x10, 20x20, 30x30, and 40x40) are shown in Figures 3.11, 3.13, and 3.12, demonstrating the performance of Tarry's algorithm and its variations across different maze dimensions. Figure 3.11 illustrates the number of steps taken by the algorithms, while Figure 3.13 displays the fraction of the maze explored before reaching

the goal, providing insights into exploration efficiency and coverage across maze sizes. Additionally, Figure 3.12 presents the relative difference in pioneer steps compared to the original Tarry's algorithm, helping to better compare the variations, which may show similar behaviors.

(a) 10x10 Maze

(b) 20x20 Maze

(c) 30x30 Maze

(d) 40x40 Maze

FIGURE 3.11 – Comparison of the pioneer's average steps across Tarry's algorithm and its variations for different sizes of perfect mazes. The subfigures illustrate how algorithm performance changes with maze size.

(a) 10x10 Maze

(b) 20x20 Maze

(c) 30x30 Maze

(d) 40x40 Maze

FIGURE 3.12 – Relative difference in pioneer steps between Tarry's algorithm variants and the original Tarry's algorithm, across different sizes of perfect mazes. The subfigures illustrate how the relative performance changes with maze size.

(a) 10x10 Maze

(b) 20x20 Maze

(c) 30x30 Maze

(d) 40x40 Maze

FIGURE 3.13 – Comparison of the explored fraction across Tarry's algorithm and its variations for different sizes of perfect mazes. The subfigures illustrate how the explored fraction changes with maze size.

When analyzing Figures 3.11 and 3.12, it becomes evident that there is no uniform trend across all maze sizes. The Tarry Tie-Breaker and Delayed Tarry variations consistently outperform the standard Tarry algorithm, except in cases with a small number of agents. In those cases, early variations in exploration paths can significantly affect the outcome due to the extensive search required by each agent. However, a more intriguing behavior is observed with the Tarry Interval Priority variation. Although it performs similarly but slightly worse for smaller maze sizes, its performance improves for the largest maze analyzed. We conjecture this behavior can be attributed to the benefits of dispersion and the additional information obtained through interval filling, which only becomes significant during more extensive exploration. In mazes with considerable depth and narrow width, we believe interval filling initially causes conflicts, as agents may search for the start of their intervals in branches that have already been visited by others. How-

ever, in larger mazes, we assume these initial setbacks are offset by the advantages gained through the extra information during deeper exploration, which cannot be easily matched by Tarry's random choices.

An interesting observation can also be made regarding the explored fraction, as shown in Figure 3.13. The results indicate that the Tarry Interval Priority algorithm consistently explores a smaller portion of the graph compared to the other variations. Since mazes have few leaf nodes and communication prevents agents from overlapping in explored branches, the use of intervals likely directs agents more efficiently toward unexplored areas, potentially allowing them to head directly to leaf nodes before finding the actual solution.

3.3.3 Random Tree Results

The results for the random trees, using the same sizes as in Section 3.2.3, are presented in Figures 3.14, 3.16, and 3.15.

(a) 100-node Tree

(b) 500-node Tree

(c) 1000-node Tree

(d) 1500-node Tree

FIGURE 3.14 – Comparison of the pioneer's average steps across Tarry's algorithm and its variations for different sizes of random trees. The subfigures illustrate how algorithm performance changes with tree size.

(a) 100-node Tree

(b) 500-node Tree

(c) 1000-node Tree

(d) 1500-node Tree

FIGURE 3.15 – Relative difference in pioneer steps between Tarry's algorithm variants and the original Tarry's algorithm across different sizes of random trees. The subfigures illustrate how the relative performance changes with tree size.

(a) 100-node Tree

(b) 500-node Tree

(c) 1000-node Tree

(d) 1500-node Tree

FIGURE 3.16 – Comparison of the explored fraction across Tarry's algorithm and its variations for different sizes of random trees. The subfigures illustrate how the explored fraction changes with tree size.

According to Figures 3.14 and 3.15, the results show a distinct pattern compared to those observed in Section 3.3.2. The Tarry Interval Priority variation consistently underperforms across all tree sizes, with the disparity becoming even more pronounced when the number of agents is small. This outcome is likely due to the wide structure of the trees, where each agent in this variation needs to explore many leaf nodes within its assigned interval, resulting in inefficiencies. In contrast, only the Tarry Tie-Breaker shows small but consistent improvement for smaller number of nodes, although this advantage tends to disappear for larger trees.

When examining Figure 3.16, a similar behavior to what was previously discussed in Figure 3.4 can be observed. The explored fraction displays a pattern that is largely independent of the strategy used, due to the high variability in the fraction of nodes explored across different tree instances.

3.3.4 Barabási-Albert Results

The results for the Barabási-Albert graphs, using the same sizes as in Section 3.2.4, are presented in Figures 3.17, 3.19, and 3.18.

(a) 100-node Barabási-Albert

(b) 250-node Barabási-Albert

(c) 500-node Barabási-Albert

(d) 1000-node Barabási-Albert

FIGURE 3.17 – Comparison of the pioneer's average steps across Tarry's algorithm and its variations for different sizes of Barabási-Albert graphs. The subfigures illustrate how algorithm performance changes with graph size.

(a) 100-node Barabási-Albert

(b) 250-node Barabási-Albert

(c) 500-node Barabási-Albert

(d) 1000-node Barabási-Albert

FIGURE 3.18 – Relative difference in pioneer steps between Tarry's algorithm variants and the original Tarry's algorithm across different sizes of Barabási-Albert graphs. The subfigures illustrate how the relative performance changes with graph size.

(a) 100-node Barabási-Albert

(b) 250-node Barabási-Albert

(c) 500-node Barabási-Albert

(d) 1000-node Barabási-Albert

FIGURE 3.19 – Comparison of the explored fraction across Tarry's algorithm and its variations for different sizes of Barabási-Albert graphs. The subfigures illustrate how the explored fraction changes with graph size.

Examining Figures 3.17 and 3.18, the poorer performance of the Tarry Interval Priority variation stands out. The results closely resemble those from the no-communication algorithms in 3.2.4, likely due to the high connectivity in Barabási-Albert graphs. This structure accelerates exploration, often completing before an agent exhausts its interval, thus yielding similar outcomes.

Additionally, while the Tie-Breaker and Delayed Tarry variations perform worse than base Tarry on smaller datasets, they tend to improve, becoming consistently comparable or slightly superior as graph size increases.

As expected, the explored fraction of the graph before the pioneer reaches the goal shows the same downward trend observed in 3.2.4, with a smaller explored fraction as more agents are added. For larger Barabási-Albert graphs, the choice of algorithm has

little impact on the explored fraction; however, for the smaller 100-node graph, Tarry's and the Tarry Tie-Breaker variation tend to explore less.

3.3.5 Small-World Results

The results for Small-World graphs, using the same sizes as in Section 3.2.5 are presented in Figures 3.20, 3.21, and 3.22.

(a) 100-node Small-World

(b) 250-node Small-World

(c) 500-node Small-World

(d) 1000-node Small-World

FIGURE 3.20 – Comparison of the pioneer's average steps across Tarry's algorithm and its variations for different sizes of Small-World graphs. The subfigures illustrate how algorithm performance changes with graph size.

(a) 100-node Small-World

(b) 250-node Small-World

(c) 500-node Small-World

(d) 1000-node Small-World

FIGURE 3.21 – Relative difference in pioneer steps between Tarry's algorithm variants and the original Tarry's algorithm across different sizes of Small-World graphs. The subfigures illustrate how the relative performance changes with graph size.

(a) 100-node Small-World

(b) 250-node Small-World

(c) 500-node Small-World

(d) 1000-node Small-World

FIGURE 3.22 – Comparison of the explored fraction across Tarry's algorithm and its variations for different sizes of Small-World graphs. The subfigures illustrate how the explored fraction changes with graph size.

In analyzing the results in Figures 3.20 and 3.21, we find that the Tarry Interval Priority variation performs poorly. This is likely due to agents not completing their intervals, similar to the Backward Interval Filling results, which yield outcomes akin to the base exploration algorithm in Figure 3.8. Furthermore, the other variations show reduced performance on smaller datasets, while their influence is negligible in the largest dataset, where all methods yield comparable results. These findings suggest that the added complexity does not enhance performance; Tarry's algorithm disperses effectively enough that heuristics are unnecessary for finding the goal quickly. For instance, the average number of steps for the pioneer in the 1000-node case is around 27, which is just 2.7% of the total number of nodes.

Examining the explored fraction in Figure 3.22, we observe a similarly noisy behavior to that seen in Figure 3.9. The explored fraction demonstrates considerable variability across

instances, regardless of the variation applied. This indicates that there is no dominant solution for this metric, reflecting the inconsistency in exploration outcomes.

4 Conclusions and Future Works

This work presented an algorithm for multi-agent graph exploration without communication, expanding on the solution proposed by Rodrigues (2024). Given the flexibility of our implementation, we analyzed three different datasets beyond the original one by Arthur, validating our approach with previously obtained results and enabling a broader comparison across various datasets. This allowed us to investigate how different graph properties impact the performance of no-communication algorithms, establishing a baseline for exploration algorithms and providing a foundation to assess the impact of communication when evaluating distinct metrics.

Alongside defining the impact of communication on performance, we implemented various heuristics and enhancements to Tarry's algorithm, which yielded significant performance improvements in only a few specific conditions. These findings suggest that, while additional information can help guide the selection process, its effectiveness may be limited to particular scenarios that warrant further investigation.

Given the modular design of our framework, it can easily expand to new algorithms and datasets. This flexibility enables future evaluations across a wider range of datasets, helping determine if the performance patterns hold in different contexts and providing insight into effective strategies for specific exploration tasks.

In future research, introducing new metrics and test cases could deepen our understanding of the results, providing stronger support for our conjectures. This would help pinpoint the key properties that influence algorithm performance, offering a clearer basis for selecting the most suitable exploration strategy for each scenario, beyond relying solely on empirical analysis.

In addition to implementing new algorithms, future work could explore various communication models among agents. For instance, agents might only communicate when they occupy the same cell, which could provide a more cost-efficient strategy compared to maintaining a shared map. By measuring how different communication strategies impact performance, we can gain insights into their effectiveness in various applications, while also considering the costs associated with each communication method.

Our current experiments did not address collisions between agents, allowing multiple

agents to occupy the same node at the same time. This simplified assumption enabled a clearer focus on evaluating exploration strategies. However, future studies could address agent collisions to simulate more realistic scenarios and analyze how such interactions affect the exploration results.

Another area of interest lies in identifying particular cases where these algorithms achieve notably higher performance. By understanding the specific conditions under which each exploration strategy is most effective, we can target their practical applications in relevant fields more precisely. One straightforward application of zero-communication distributed search is the deployment of a swarm of drones over a topological map, allowing them to operate without expending energy on communication. This approach is viable even if the map is initially unknown, as long as it can be constructed consistently by all drones independently.

In conclusion, this research advances multi-agent graph exploration without communication, establishing benchmarks to evaluate the impact and performance of exploration algorithms and developing heuristics that significantly enhance established methods.

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Appendix A - Practical Example of Graph Exploration

A.1 Detailed Step-by-Step Execution

This section provides a detailed step-by-step exploration of the graph shown in Figure 2.8. The process is illustrated for all three agents, with each step specifying the intervals calculated for each child node. Nodes that would create cycles are marked in red, and the corresponding edges are represented as dotted lines. To maintain clarity and conciseness, some straightforward steps, such as those involving simple backtracking or non-decision points, are omitted. This ensures a focused understanding of how the agents navigate and construct their respective exploration trees.

The following figures present the step-by-step exploration process for each agent, highlighting their decisions and progress throughout the graph.

FIGURE A.1 – Agent 1 - Starting Node for graph from Figure 2.8.

FIGURE A.2 – Agent 2 - Starting Node for graph from Figure 2.8.

FIGURE A.3 – Agent 3 - Starting Node for graph from Figure 2.8.

FIGURE A.4 – Agent 1 - First Step exploring the graph.

FIGURE A.5 – Agent 2 - First Step exploring the graph.

FIGURE A.6 – Agent 3 - First Step exploring the graph.

FIGURE A.7 – Agent 1 - Second Step exploring the graph.

FIGURE A.8 – Agent 2 - Second Step exploring the graph.

FIGURE A.9 – Agent 3 - Second Step exploring the graph.

FIGURE A.10 – Agent 1 - Multiple Steps of exploration.

FIGURE A.11 – Agent 2 - Multiple Steps of exploration.

FIGURE A.12 – Agent 3 - Multiple Steps of exploration.

FIGURE A.13 – Agent 1 - Final exploration steps.

FIGURE A.14 – Agent 2 - Final exploration steps.

FIGURE A.15 – Agent 3 - Final exploration steps.

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11. RESUMO:

Graph theory, a pivotal field in mathematics and computer science, offers robust frameworks for modeling relationships and traversing graph structures. This research delves into graph exploration, crucial for applications like network routing, robotics, and procedural generation. As real-world applications often involve complex environments with time constraints, such as search-and-rescue operations with drones, this study focuses on multi-agent systems for graph exploration, where multiple autonomous agents collaborate to optimize task distribution.

This work presents a modular framework structured into extensible classes, facilitating the addition of new algorithms and datasets. By moving beyond the constraints of perfect maze exploration (NAEEM, 2021), we employed NetworkX (HAGBERG *et al.*, 2008)—a general graph library—which allows for analyzing more diverse graph structures. This flexibility enabled comparisons across various datasets to reveal patterns in algorithm performance under different graph characteristics.

This study's objective is to propose an efficient multi-agent algorithm for graph exploration without communication, a crucial need in scenarios where communication is impractical or impossible. Building on the method for perfect maze exploration detailed by Rodrigues (2024), this study extends the approach to general graphs, including those with cycles, and evaluates the algorithm's performance across four datasets of varying complexity. By establishing a benchmark for exploration efficiency, this research enables an assessment of the costs and benefits of incorporating communication in multi-agent strategies, providing a comparative baseline for future exploration algorithm development.

Additionally, this research incorporated heuristics and alternative strategies to enhance Tarry's algorithm, which traditionally relies on random neighbor selection. The resulting adaptations demonstrated performance improvements on specific datasets, illustrating how targeted selection processes can optimize exploration efficiency based on dataset properties. These insights pave the way for future research into more effective graph exploration strategies in autonomous systems.

12. GRAU DE SIGILO:
(**X**) **OSTENSIVO** ( ) **RESERVADO** ( ) **SECRETO**