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MULTI-AGENT GRAPH EXPLORATION WITHOUT COMMUNICATION

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To my family, for their love and constant support. To my friends, for standing by me through all moments.

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"The journey of a thousand miles begins with one step."

— Lao Tzu

Resumo

A teoria dos grafos, um campo fundamental da matemática e da ciência da computação, oferece estruturas robustas para modelar relacionamentos e percorrer estruturas de grafos. Esta pesquisa se aprofunda na exploração de grafos, cruciais para aplicações como roteamento de redes, robótica e geração procedural. Como as aplicações no mundo real frequentemente envolvem ambientes complexos com restrições de tempo, este estudo foca em sistemas multiagentes para a exploração de grafos, onde múltiplos agentes autônomos colaboram para otimizar a distribuição de tarefas.

O objetivo deste estudo é propor um algoritmo eficiente de exploração de grafos por múltiplos agentes sem comunicação, uma necessidade crucial em cenários onde a comunicação é impraticável ou impossível, como exploração em águas profundas ou ambientes com restrição de energia. Com base no método para exploração de labirintos perfeitos detalhado por Rodrigues (2024), este estudo estende a abordagem para grafos mais gerais, que podem incluir ciclos. Além disso, os algoritmos foram reimplementados utilizando uma biblioteca geral de grafos e não uma biblioteca específica para labirintos perfeitos (NAEEM, 2021). Ao estruturar a implementação em classes bem definidas e extensíveis, a pesquisa oferece uma estrutura versátil para aplicações mais amplas em sistemas multiagentes, abrindo caminho para novos avanços na exploração autônoma de grafos.

Abstract

Graph theory, a pivotal field in mathematics and computer science, offers robust frameworks for modeling relationships and traversing graph structures. This research delves into graph exploration, crucial for applications like network routing, robotics, and procedural generation. As real-world applications often involve complex environments with time constraints, this study focuses on multi-agent systems for graph exploration, where multiple autonomous agents collaborate to optimize task distribution.

This study's objective is to propose an efficient multi-agent algorithm for graph exploration without communication, a crucial need in scenarios where communication is impractical or impossible, such as deep-sea exploration or energy-constrained environments. Building on the method for perfect maze exploration detailed by Rodrigues (2024), this study extends the approach to general graphs, that may include cycles. Several algorithmic variations, combining the proposed approach and Tarry's variation (KIVELEVITCH; COHEN, 2010), were implemented in an effort to compare how different algorithms perform in various scenarios and to identify efficient use cases. Furthermore, the algorithms were reimplemented using a generic graph library instead of a specialized perfect maze library (NAEEM, 2021).

By structuring the exploration into well-defined and extensible classes, the research offers a versatile framework for broader applications in multi-agent systems. This framework was used to evaluate different datasets, containing graphs with varying properties, paving the way for further advancements in autonomous graph exploration.

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List of Abbreviations and Acronyms

DFS Depth First Search
BFS Breath First Search
LCL Last Common Location

List of Symbols

G Graph

v_n Vertex

$v_n v_m$ Edge

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1 Introduction

1.1 Motivation

Graph theory is a fundamental field of mathematics with significant relevance in computer science due to its ability to model relationships between objects. One of the key challenges in computer science is graph traversal and exploration, which involves systematically navigating through the nodes of a graph with a specific purpose. This topic has practical applications across diverse fields, such as network routing, robotics, procedural generation, electronics design, and more.

When considering the complexities and time constraints imposed by real-world environments, graph exploration must be extended to consider multi-agent systems. In such systems, multiple autonomous agents collaborate to explore graphs, aiming to distribute tasks to achieve optimal efficiency. However, communication strategies on these systems are challenging, as they need to balance the trade-off between the amount of information shared among agents versus the quality of the solution. In other words, while communication can allow agents to speed up their exploration, it may cost time and/or energy.

Various versions of this problem have been explored in research. One such approach, discussed by Kivelevitch and Cohen (2010), proposed a generalization of Tarry's Algorithm with significant emphasis on minimizing data transfer. However, the authors did not experiment with this solution on general graphs.

Despite the previously mentioned research, the concept of zero-communication exploration has not been concretely established in the literature. We believe this gap is significant, as practical situations might involve scenarios where communication is limited or impossible due to bandwidth limitations or energy consumption, such as deep-sea exploration, search with energy-limited agents, or high-efficiency state space search.

1.2 Objective

This study aims to develop an efficient algorithm for graph exploration by multi-agents, specifically for finding a goal node. A previous paper by Rodrigues (2024) discussed an approach for perfect maze exploration (NAEEM, 2021), which can be modeled as a tree, a connected acyclic undirected graph. Expanding the proposed algorithm to connected cyclic graphs enables the definition of a lower bound for exploration algorithms. This provides a benchmark for evaluating and comparing exploration strategies, allowing researchers to quantify the impact of communication on exploration efficiency relative to a baseline no-communication algorithm. This comparison is crucial for assessing the cost of implementation and the value brought by communication in contrast to a simpler, zero-communication strategy.

Alongside this main objective, this research seeks to incorporate heuristics and alternative strategies proven effective in zero-communication scenarios to enhance Tarry's algorithm. As outlined by Kivelevitch and Cohen (2010), Tarry's algorithm relies on a random selection of neighbors for exploration, which may not always yield optimal results for different graph structures. Our goal was not only to analyze how these choices are made but also to investigate whether a secondary algorithm or heuristic could guide the selection process, thereby improving the overall efficiency of exploration.

1.3 Definitions

This section aims to define key concepts in graph theory and multi-agent exploration that are essential for understanding the context and methodology of our research, ensuring clarity and consistency in the subsequent discussions.

1.3.1 Graph

Graphs are structures in mathematics and computer science used to represent relationships between pairs of objects. According to Manber (1989), a graph $G = (V, E)$ is defined as a set V of vertices (or nodes) and a set E of edges, that connect pairs of vertices.

1.3.1.1 Common Nomenclature

In graph theory, the basic terminology includes the following components:

- Node (Vertex): The fundamental unit of a graph, representing an object or a point.

- Edge: A connection between a pair of vertices, representing a relationship between them.
- Adjacency: Two vertices v and w of a graph G are adjacent if there is an edge vw joining them, and the vertices v and w are then incident with such an edge. (WILSON, 1996)
- Degree: Degree of a vertex v of G is the number of edges incident with v. (WILSON, 1996)
- Path: A path in a graph G, as defined by Wilson (1996), is a finite sequence of edges of the form v_0v_1, v_1v_2, ... in which any two consecutive edges are adjacent, all edges are distinct and all vertices v_0, v_1, ..., v_m are distinct (except, possibly, $v_0 = v_m$).
- Cycle: A path where $v_0 = v_m$ and with at least one edge. (WILSON, 1996)

1.3.1.2 Key Properties

A few key properties of graphs that interest us are:

- Connectivity: A graph is connected if and only if there is a path between each pair of vertices. (WILSON, 1996)
- Acyclicity: A graph is acyclic if it has no cycles. A tree is an acyclic connected graph.

Additionally, we differentiate between ”perfect” and ”imperfect” mazes:

- Perfect Maze: A maze that contains no loops, meaning it can be represented as a tree (an acyclic connected graph).
- Imperfect Maze: A maze that includes loops, making it representable as a general graph but not as a tree.

1.3.1.3 Traversal

Graph traversal is the process of visiting all the vertices in a graph in a systematic manner. Two fundamental algorithms for graph traversal are:

- Breadth-first search (BFS): Expands the frontier between discovered and undiscovered vertices uniformly across the breadth of the frontier. (MANBER, 1989)

- Depth-first search (DFS): Explores edges out of the most recently discovered vertex v that still has unexplored edges leaving it. (MANBER, 1989)

- Tarry's Algorithm: Tarry's Algorithm is used to traverse the entire maze, or graph, in a depth first fashion, where each edge is travelled twice, and the agent finishes the motion at the same initial vertex it started from.(KIVELEVITCH; COHEN, 2010) This algorithm is described in the context of a physical maze, where the agent must physically move between adjacent nodes. It was first described over botanical mazes built as amusements for the XIX century aristocracy.

1.3.2 Agent

An agent in maze exploration, as defined by Kivelevitch and Cohen (2010), is an entity able to move within the maze in any direction as long as it does not encounter obstacles.

In the context of graph exploration, this concept extends to an agent's ability to traverse edges between vertices within the graph structure. Each agent acts independently, making decisions based on its individual memory and the local information available at its current vertex and its adjacent edges.

1.3.3 Mixed Radix

A mixed radix system is a positional numerical system in which the numerical base for each digit varies. This allows for more flexible representation of numbers, accommodating various scales within the same numerical framework.

One mathematical approach for representing mixed radix numbers can be found in Arndt (2011), which establishes a comprehensive arithmetical method for manipulating them. However, for the specific subset of real numbers within the interval [0, 1], a more specialized approach is utilized, as described by Rodrigues (2024).

In this context, the mixed radix representation $A = [a_0, a_1, a_2, ..., a_{n-1}]$ of a number x with respect to a radix vector $M = [m_0, m_1, m_2, ..., m_{n-1}]$, where $x \in [0, 1]$, is given by:

$$x = \sum_{k=0}^{n-1} a_k \prod_{j=0}^{k} \frac{1}{m_j} \tag{1.1}$$

where a_j are non-negative integers, m_j are integers such that $m_j \geq 2$, and $0 \leq a_j \leq m_j$. (RODRIGUES, 2024)

For instance, if x is represented by $0.2_3 1_2 1_4 0_3 1_5$, and considering that a_i is on the left of a_{i+1}, x might be transformed to the decimal base by the following steps:

$$x = 2_3 1_2 1_4 0_3 1_5 \tag{1.2}$$
$$A = [2, 1, 1, 0, 1] \tag{1.3}$$
$$M = [3, 2, 4, 3, 5] \tag{1.4}$$
$$n = 5 \tag{1.5}$$
$$x = \sum_{k=0}^{4} a_k \prod_{j=0}^{k} \frac{1}{m_j} \approx 0.8777778 \tag{1.6}$$

1.3.3.1 Extending Mixed Radix with Unary Digits

In our study, we extended the mixed radix system to include unary digits, which are used to represent steps in a sequence where no meaningful decisions are made. These unary digits do not contribute to the numerical value and can be omitted in the conversion to standard numerical forms.

Within our framework, the mixed radix representation $A = [a_0, a_1, a_2, ..., a_{n-1}]$ of a number x with respect to a radix vector $M = [m_0, m_1, m_2, ..., m_{n-1}]$, where $x \in [0, 1]$, is given by:

Let $U = \{k \mid m_k \neq 1\}$ be the set of indices where the base m_k is not unary.

$$x = \sum_{k \in U} u_k \prod_{j=0}^{k} \frac{1}{m_j} \tag{1.7}$$

where a_k are non-negative integers, m_j are integers such that $m_j \geq 1$, and $0 \leq a_j \leq m_j$.

Consider the mixed radix representation $x = 0.2_3 I_1 1_2 I_1 I_1 1_4 0_3 1_5$, where I_1 represents the unary digit and each a_i is on the left of a_{i+1}. We can transform x to the decimal base using the following steps:

$$x = 2_3 I_1 1_2 I_1 I_1 1_4 0_3 1_5 \tag{1.8}$$
$$A = [2, I, 1, I, I, 1, 0, 1] \tag{1.9}$$
$$M = [3, 1, 2, 1, 1, 4, 3, 5] \tag{1.10}$$
$$U = [0, 2, 5, 6, 7] \tag{1.11}$$
$$x = \sum_{k \in U} a_k \prod_{j=0}^{k} \frac{1}{m_j} \approx 0.8777778 \tag{1.12}$$

2 Methodology

This section intends to discuss specific design choices that support our multi-agent search algorithm. Section 2.1 introduces the models and classes developed to improve our algorithm. Section 2.2 describes the core concepts of our algorithm and provides pseudocode to guide its implementation. Section 2.3 presents a mixed radix numerical representation used for the agent's path and examples to demonstrate its functionality. Section 2.4 presents alternative exploration strategies, including Backward Interval Filling and four Tarry algorithm variations to improve efficiency and coordination. Section 2.5 highlights the algorithm's capability to handle complex graphs, as shown through an imperfect maze exploration. Finally, Section 2.6 visualizes individual exploration trees for each agent, demonstrating their paths and handling of graph structures.

2.1 Modeling

In this section, we detail the modeling approach chosen to structure our problem, emphasizing the use of classes to achieve greater flexibility in our solution. The main classes include **Simulation**,**Graph** and **Agent**, each of which will be defined in subsequent sections. Figure 2.1 presents a simplified class diagram that illustrates the overall structure of our model.

2.1.1 Simulation

The **Simulation** class acts as a central controller in our multi-agent graph exploration problem. It is primarily responsible for managing shared information, including the graph structure. Additionally, it orchestrates the agents and the overall execution of the exploration process.

In more detail, its responsibilities include:

- Shared Information: Storing and managing all information that is common between agents.

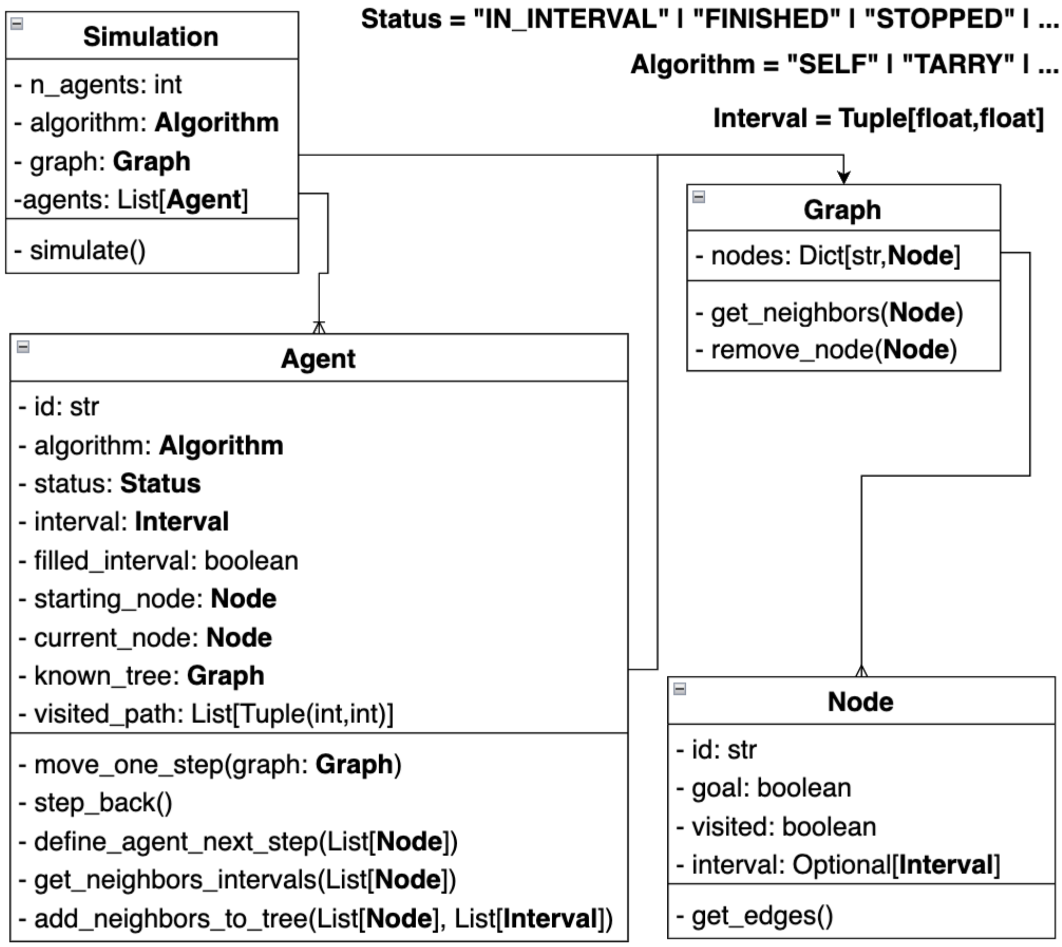


FIGURE 2.1 – Simplified Class Diagram of the Multi-Agent Maze Exploration Model

- Orchestration: Coordination agent activities and possible interactions during the exploration.
- Visualization: Providing tools for tracking the progress and visualizing the result of the exploration.
- Metrics: Collecting and analyzing performance metrics of the exploration.

As shown by Figure 2.1, the core method of the **Simulation** is *simulate* that is responsible for starting the exploration and managing each agent in parallel, until all agents have reached the goal or have no more moves available. The pseudocode in Algorithm 1 provides a breakdown of the function's steps and logic.

2.1.2 Graph

The **Graph** class, as introduced in Section 1.3.1, serves as a basic representation of the graph structure for our multi-agent exploration. Our implementation is built over the

Algorithm 1 Simulation - simulate()

```
/* Graph to be explored */
graph ← Graph()
/* Agents */
agents ← [Agent(0), Agent(1), Agent(2), ...]

/* Agents that stopped already */
agents_stopped ← [ ]
for all agent in agents do
    append(agents_stopped, FALSE)
end for

/* Exploration */
while False in agents_stopped do
    for all agent in agents do
        /* Moving each agent in parallel */
        agent.move_one_step()
        if agent.status == "FINISHED" or agent.status == "STOPPED" then
            agents_stopped[agent.id] ← TRUE
        end if
    end for
end while

/* Calculating Metrics */
for all agent in agents do
    /* Simplified function for calculating metrics */
    calculate_metrics(agent)
end for

/* Simulation is complete */
/* Showing graphical result */
/* Simplified function for displaying */
display_demo_exploration(agents)
```

graph library NetworkX (HAGBERG *et al.*, 2008), allowing for the efficient management of nodes and edges and facilitating the agents' traversal of the graph.

A key method within this class is *get_neighbors*. This method returns a list of neighboring nodes in a specific order, aiding in the consistent traversal of the graph.

2.1.3 Agent

The **Agent** class represents an agent, as defined in Section 1.3.2 and illustrated in Figure 2.1.

An **Agent** is uniquely identified by its *id* field and possesses the *status* and *algorithm* fields, which define its behavior during each step of the exploration process.

Additionally, an **Agent** records its path using its own graph, defined by the *known_graph* field, and a mixed radix representation of it in the *visited_path* field.

Furthermore, the **Agent** possesses the *current_node* field, which stores its current position in the graph, and the *interval* field, specific to our graph exploration algorithm, aiding in its traversal strategy.

The core method of the **Agent** class is *move_one_step*, which is responsible for determining the agent's next action during the exploration, as demonstrated in Algorithm 1. While the detailed logic of how the agent decides on its actions will be discussed in Section 2.2, we provide a pseudocode representation of this method in Algorithm 2.

Algorithm 2 Agent - move_one_step()

```
procedure MOVE_ONE_STEP(self, graph):
    /* Checking if the agent has already stopped or finished */
    if self.status == "FINISHED" or self.status == "STOPPED" then
        return
    end if

    /* Get current position */
    current_position ← graph[self.current_node]

    /* Taking new step */
    /* Neighbors are returned following a specific ordering */
    neighbors ← graph.get_neighbors(current_position)
    /* Get only non visited neighbors */
    non_visited_neighbors ← [ ]
    for all neighbor in neighbors do
        if self.known_tree[neighbor].visited == FALSE then
            append(non_visited_neighbors, neighbor)
        end if
    end for
    /* This decision will be explained in Section 2.2 */
    next_step ← self.define_agent_next_step(non_visited_neighbors)
    /* If invalid next_step, just move back */
    if next_step == "-1" then
        self.step_back()
        return
    end if

    /* Updating after step */
    self.current_node ← next_step
    self.known_tree[self.current_node].visited ← TRUE

    /* Checking if in goal */
    if current_position.goal == TRUE then
        self.status ← "FINISHED"
        return
    end if
end procedure
```

2.2 Graph Exploration Algorithm

As mentioned in Section 1.2, the aim of this study is to develop an effective algorithm for graph exploration using multiple agents without communication between them. A significant challenge in this context is avoiding the redundant exploration of nodes already visited by other agents, as it consumes resources without contributing new information to the overall exploration.

To address this problem, we adopt the solution proposed by Rodrigues (2024), that consists on dividing the graph into distinct intervals, which are then split among the agents. This approach ensures that each agent explores a unique section of the graph, thereby minimizing overlap and maximizing efficiency. The intervals are determined based on the number of agents, ensuring a non-overlapping distribution of the graph's nodes. Figure 2.2 illustrates this concept, showing a graph with intervals assigned to different colored agents.

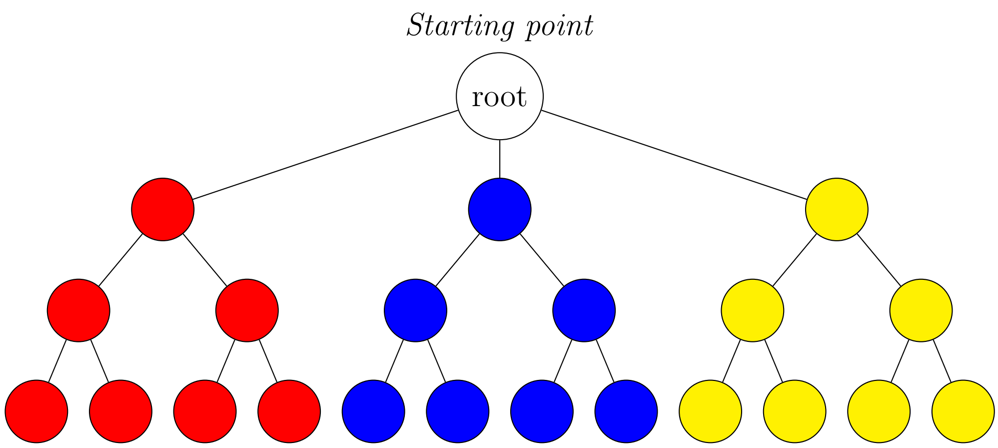


FIGURE 2.2 – Three agents (Red, Blue, and Yellow) disperse from each other in a graph. Source: Rodrigues (2024)

It is important to note that while the numerical intervals are non-overlapping, the dispersion may not always be fair or uniform. This is evident in Figure 2.3, which illustrates three agents dispersing in an unbalanced tree, contrasting with the more uniform dispersion shown in Figure 2.2. The graph shows the nodes colored based on the agent intervals they fall into. If a node has more than one color, it indicates that the node is within the intervals of multiple agents.

Mathematically, this dispersion can be expressed by a set of equations. Assume there are k agents, denoted as $a_1, a_2, a_3, ..., a_k$. The distribution of intervals can be represented as shown in Equation 2.1, based on the approach proposed by Rodrigues (2024).

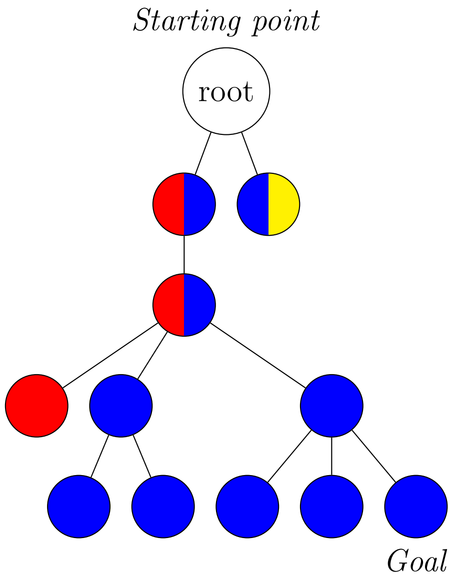


FIGURE 2.3 – Three agents (Red, Blue, and Yellow) disperse in a non-uniform graph.

$$
\begin{aligned}
d &= 1/k \\
a_1&\colon\ [0, d[\\
a_2&\colon\ [d, 2d[\\
a_3&\colon\ [2d, 3d[\\
&\ldots \\
a_k&\colon\ [(k-1)d, 1]
\end{aligned}
\tag{2.1}
$$

With the division of intervals explained, we now describe the step-by-step process of our algorithm for multi-agent graph exploration. This algorithm extends the work of Rodrigues (2024), incorporating modifications to handle previously visited nodes and potential cycles.

1. **Initialization**

    - Each agent is assigned a unique identifier and a specific interval based on the Equation Set 2.1.
    - Each agent begins with an empty tree representation of the graph, representing the knowledge it has accumulated during the exploration. Since each agent independently explores the graph, their trees may differ, reflecting the order in which nodes are visited.
    - All agents start at the same node, although the specific starting node is arbitrary.

- The starting node is assigned the interval $[0, 1]$ and is added to the agent's tree.

2. **Exploration Strategy**

- At each step, an agent examines the adjacent nodes that have not been previously visited by it. To maintain consistency in exploration, these nodes are sorted by their unique identifiers.
- The agent dynamically calculates convergence intervals for each unvisited adjacent node. The calculation method is as follows:
  - **Single Child**: If there is only one child node, it inherits the convergence interval of the current node.
  - **Multiple Children**: If there are multiple children, the current node's convergence interval is uniformly divided among them.
- Each adjacent node is then added to the agent's tree along with the corresponding edge and its assigned interval.
- When adding a node, called the target node, to the tree in generic graphs, a cycle may be detected if the target node has not been visited yet but already appears in the tree. In this case, the new edge would form a return edge, as identified by Tarjan's Algorithm. To prevent cycles, the previous occurrence of the target node is removed from the tree, and it is then re-added with its updated edge and interval, ensuring a valid acyclic structure for the exploration tree. In Figure 2.7, an example of this situation is shown, where the target node was not actually removed but was instead renamed to allow visualization of the cycle occurrence.
- The agent then chooses the first node whose convergence interval intersects with its own. If no such node exists or if all adjacent nodes have been visited, the current node is marked as explored, and the agent backtracks to the parent node.

3. **Completion Criteria**

- The agent continues the exploration process until it either finds the goal or fills its assigned interval. If the goal is not found within its interval, it must be within the interval of another agent.
- After finishing its interval, the agent may stop or adopt an alternative strategy to accelerate the search. The default behavior implemented is to do a depth-first search (DFS), ignoring nodes already explored in the initial attempt.

The steps described in the **Initialization** topic are implemented in the initialization procedures of the **Simulation** and **Agent** classes. The pseudocode presented in Algorithm

3 illustrates the *define_agent_next_step* method, which implements the step-by-step process described in the **Exploration Strategy** topic.

Algorithm 3 Agent - define_agent_next_step()

```
procedure DEFINE_AGENT_NEXT_STEP(self, unvisited_neighbors):
    /* Calculating the convergence intervals */
    /* This will be explained in Section 2.3 */
    intervals ← self.get_neighbors_intervals(unvisited_neighbors)

    /* Checking for backtracking or cycle */
    for all neighbor n_j in unvisited_neighbors do
        if self.known_tree[n_j].interval then
            if self.current_node in self.known_tree[n_j].get_edges() then
                /* Backtrack */
                intervals[n_j] ← self.known_tree[n_j].interval
            else
                /* Cycle */
                /* The agent removes the previous node from its tree
                /* so it doesn't cause a cycle */
                self.known_tree.remove_node(n_j)
            end if
        end if
    end for

    /* Adding neighbors to tree */
    self.add_neighbors_to_tree(unvisited_neighbors, intervals)
```

Algorithm 3 Agent - define_agent_next_step()

```
    /* Checking for intersecting intervals */
    /* Is important to take notice that as the neighbors are ordered
    /* And the interval of a neighbor is proportional to its position
    /* As we will see in Algorithm 4, the neighbor are visited in
    /* incresing order of intervals */
    for all neighbor n_j in unvisited_neighbors do
        max_neighbor ← intervals[n_j][1]
        min_neighbor ← intervals[n_j][0]
        max_agent ← self.interval[1]
        min_agent ← self.interval[0]

        if max_agent < min_neighbor then

            /* If the node's interval is on the right of the agent's interval, surely the
            agent has finished its interval, since the nodes are in increasing order of
            intervals as previously mentioned */
            finished_interval ← TRUE
            break
        else min_agent < max_neighbor and max_agent > min_neighbor
            /* Found intersecting intervals */
            return n_j
        end if
    end for

    /* If got here, didn't find any intersecting intervals */
    /* If back on root, the agent filled its interval */
    if self.current_node == self.starting_node then
        self.filled_interval ← TRUE
    end if

    /* Just step back if interval isn't filled */
    if self.filled_interval == FALSE then
        return "-1"
    end if

    /* If interval is filled, change strategies */
    self.status ← "DFS_SEARCH"
    return
end procedure
```

2.3 Mixed Radix Path Representation

This section details how the mixed radix representation described in Section 1.3.3.1 is used to represent the path taken by the agent, including the decision taken in each node as used in Algorithm 3.

Our approach utilizes the same mixed radix representation technique as described by Rodrigues (2024). This method allows an agent to record each decision made along the path without the need to retrospectively analyze previous choices. Instead, it only requires the current node's convergence interval and its edges to make decisions, reducing computational overhead. This is based on the following logic:

- The path starts at "0.", and the starting node is assigned the interval $[0, 1]$.
- If the current node has a single child, the agent appends the unary digit I_1 to the path. This indicates that there is only one possible path to take from this node.
- If the current node has multiple children (e.g., n children), the agent chooses the ith child and appends $(i-1)_n$ to the path. This notation denotes the specific choice among the available children, with i representing the child's position in an ordered list and the subscript n denotes the number of children or the length of the ordered list.
- When the agent needs to return to a parent node, it removes the last value appended to the mixed radix representation, effectively backtracking to the previous decision point.

To illustrate this concept, we present the mixed radix path representation between the *Starting Point* and *Goal* from the Figure 2.3.

$$x = 0_2 I_1 2_3 2_3 \tag{2.2}$$

Following the conversion as described by Equation 1.7, we can transform this mixed radix notation into a numerical value:

$$x = 0_2 I_1 2_3 2_3 \tag{2.3}$$

$$A = [0, I, 2, 2] \tag{2.4}$$

$$M = [2, 1, 3, 3] \tag{2.5}$$

$$U = [0, 2, 3] \tag{2.6}$$

$$x = \sum_{k \in U} a_k \prod_{j=0}^{k} \frac{1}{m_j} \approx 0.4444444 \tag{2.7}$$

The function *get_neighbors_intervals* used in Algorithm 3 is defined by this logic and the Equation 1.7 as can be seen in Algorithm 4

Algorithm 4 Agent - get_neighbors_intervals()

```
procedure GET_NEIGHBORS_INTERVALS(self, unvisited_neighbors):
    /* Fetching current node interval */
    current_node_interval ← self.known_tree[self.current_node].interval
    current_interval_size ← (current_node_interval[1]-current_node_interval[0])
    interval_chunk ← current_interval_size / unvisited_neighbors.length()

    /* Calculating interval for each neighbor */
    intervals ← [ ]
    for all neighbor n_j in unvisited_neighbors do
        interval_start ← current_node_interval[0] + interval_chunk * j
        interval_end ← interval_start + interval_chunk
        intervals.append((interval_start, interval_end))
    end for
    return intervals
end procedure
```

2.4 Alternative Exploration Algorithms

In our implementation, modifying and experimenting with variations of the exploration algorithm is simple. By simply altering the *define_agent_next_step* function in Algorithm 2, we can easily incorporate different strategies based on the agent's current status or algorithm.

In this work, we have explored five alternatives: one variation of our core algorithm and four adaptations of Tarry's algorithm.

2.4.1 Exploration Algorithm without Communication

2.4.1.1 Backward Interval Filling

The **Backward Interval Filling** algorithm extends the primary zero-communication strategy developed in this work. Instead of initiating a new Depth-First Search (DFS) traversal after completing a designated interval, agents continue exploring by filling the next interval in reverse order. Once an interval is finished, the agent moves up to the nearest common ancestor shared with the start of the next interval, then proceeds to explore downwards. This modification ensures that agents remain active, even after finishing their primary interval, enhancing the overall graph coverage and reducing idle time.

2.4.2 Exploration Strategies with Communication: Tarry Algorithm Variations

This section presents several adaptations of Tarry's algorithm that utilize communication between agents. Each variation builds upon the foundational logic of Tarry's approach but introduces different heuristics and optimizations aimed at improving performance and accommodating various graph structures.

2.4.2.1 Extended Tarry Algorith

The **Extended Tarry Algorithm**, proposed by Kivelevitch and Cohen (2010), is an adaptation of Tarry's algorithm. In this approach, agents share information about visited nodes, enabling them to identify unvisited cells and determine the Last Common Location (LCL)—the most recent shared position among agents before diverging paths.The algorithm consists of two phases:

- **Exploration Phase**: Each agent moves to unvisited cells, or to cells unvisited by itself, choosing arbitrarily among multiple options. If all cells have been visited, agents retreat to the last unvisited cell, marking cells as dead ends.
- **Cooperative Backtracking Phase**:Once an agent finds the goal and is defined as the pioneer, other agents backtrack to the Last Common Location (LCL) with it and follow the pioneer's path to the goal.

The functions $define\_next\_first\_phase\_tarry\_step$ and $define\_next$ $\_second\_phase\_tarry\_step$, as shown in Algorithms 5 and 6, implement this logic.

Algorithm 5 Agent - define_next_first_phase_tarry_step()

```
procedure DEFINE_NEXT_FIRST_PHASE_TARRY_STEP(non_visited_neighbors):
    new_neighbors ← []
    already_visited_neighbors ← []
    /* Splitting Neighbors */
    for all neighbor in non_visited_neighbors do:
        if neighbor not visited and not deadEnd then:
            new_neighbors.append(neighbor)
        else if neighbor visited then:
            already_visited_neighbors.append(neighbor)
        end if
    end for
    if new_neighbors is not empty then:
        /* Choosing a random non-visited neighbor */
        return random_choice(new_neighbors)
    else if already_visited_neighbors is not empty then:
        /* Choosing a random non-visited by this agent neighbor */
        return random_choice(already_visited_neighbors)
    end if
    return "-1"
end procedure
```

Algorithm 6 Agent - define_next_second_phase_tarry_step()

```
procedure DEFINE_NEXT_SECOND_PHASE_TARRY_STEP(non_visited_neighbors,
      pioneer_id, pioneer_effective_path, lcl)
    if self.status == "TARRY_FIRST_PHASE" then
        self.status ← "TARRY_SECOND_PHASE"
    end if
    if self.status == "TARRY_SECOND_PHASE" then
        if self.current_node_id != lcl[pioneer_id][self.id] then
            /* Step back until the last common location */
            return "-1"
        else
            self.status ← "TARRY_FOLLOW_PIONEER"
        end if
    end if
    current_node_index ← pioneer_effective_path.index(self.current_node_id)
    /* Follow pioneer's path */
    return pioneer_effective_path[current_node_index + 1]
end procedure
```

2.4.2.2 Tarry Tie-Breaker Variation

In this variation, agents employ Tarry's algorithm but modify the selection process described in Algorithm 5. Instead of randomly choosing among valid neighbors, agents prioritize their choices based on interval assignments derived from mixed radix path intervals. This enhancement aims to improve exploration efficiency by leveraging structured knowledge of the graph's structure to better disperse the agents.

2.4.2.3 Tarry Interval Priority Variation

Each agent attempts to fill its assigned interval as described in Algorithm 3, but rather than using depth-first search (DFS) or backward filling once the interval is complete, it begins with Tarry's algorithm, as described in Algorithms 5 and 6. This strategy seeks to leverage the additional information gathered during the interval search to enhance exploration efficiency and reduce redundant backtracking.

2.4.2.4 Delayed Tarry Variation

In this approach, agents begin by navigating their assigned intervals for a predetermined number of steps to ensure an initial spread across distinct branches of the graph. After this dispersal phase, they switch to Tarry's algorithm to continue exploration. This method aims to reduce collisions by increasing the probability that agents explore separate regions early on, leading to improved coverage and efficiency in the overall exploration.

2.5 Graph Visualization

One significant enhancement is the capability to visualize graphs beyond perfect mazes, expanding our algorithm's applicability to general graphs.

To demonstrate this capability, we applied our algorithm to explore an imperfect maze, which is a cyclic graph, using three agents. The overall exploration of the maze is depicted in Figure 2.4.

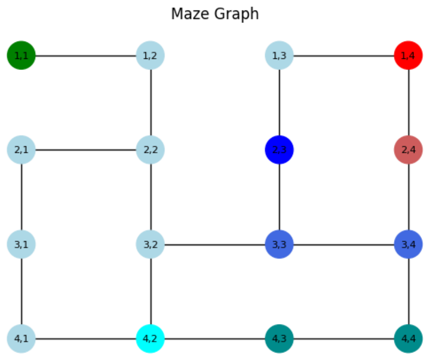


FIGURE 2.4 – Exploration of an imperfect maze(graph) by three agents.

This visualization showcases how our algorithm can handle graphs with cycles, illustrating the adaptability of our approach.

2.6 Agent Tree Visualization

In addition to overall graph exploration, we developed a method to visualize the individual exploration trees constructed by each agent during the process. These trees represent the paths traversed by the agents and provide insight into the structure of their independent exploration, including how cycles are managed and intervals are assigned to each node.

The Figures 2.5, 2.6 and 2.7 show the trees constructed by the 3 agents for the exploration displayed in Figure 2.4.

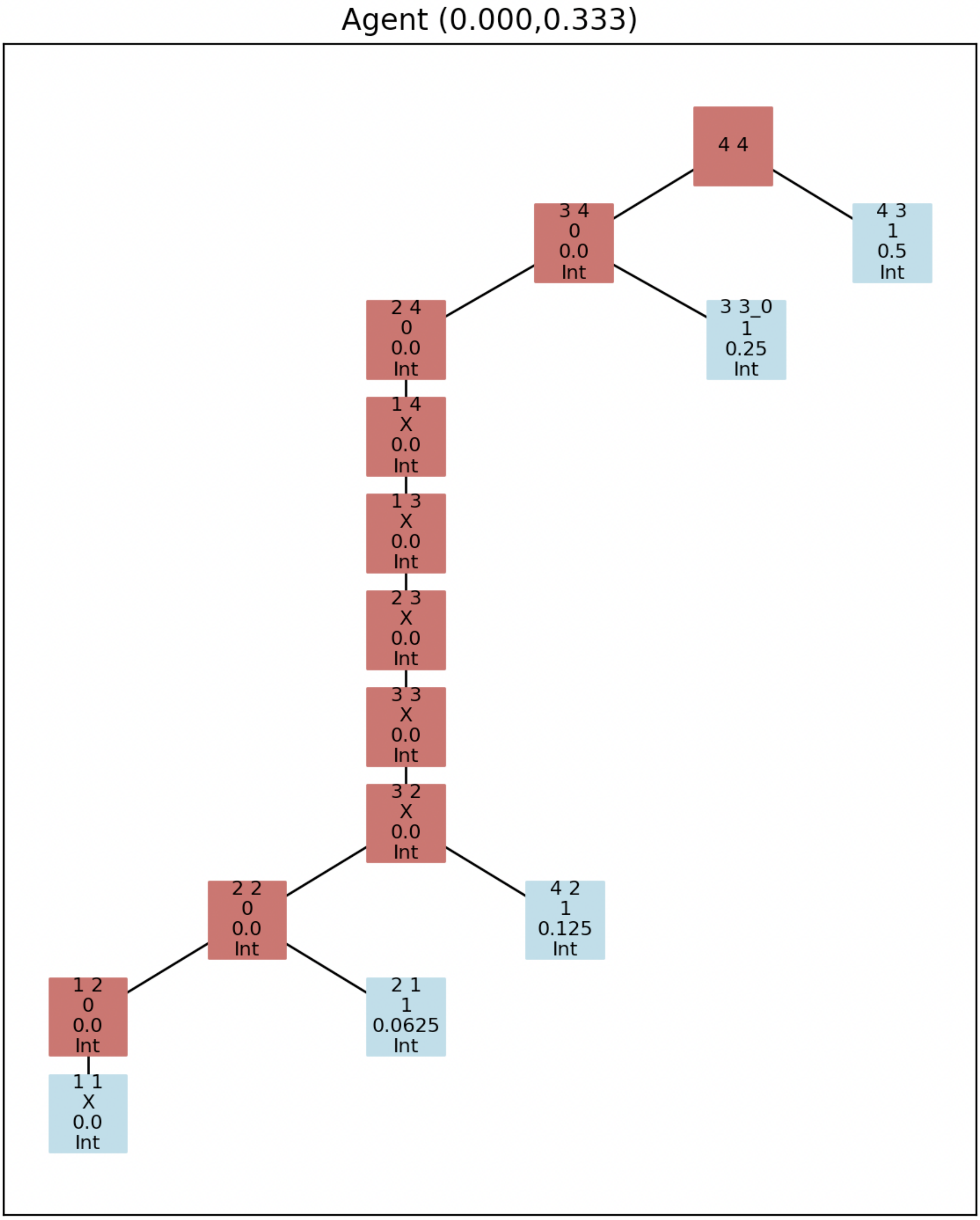


FIGURE 2.5 – Trees constructed by agent post-exploration: **Agent 1**.

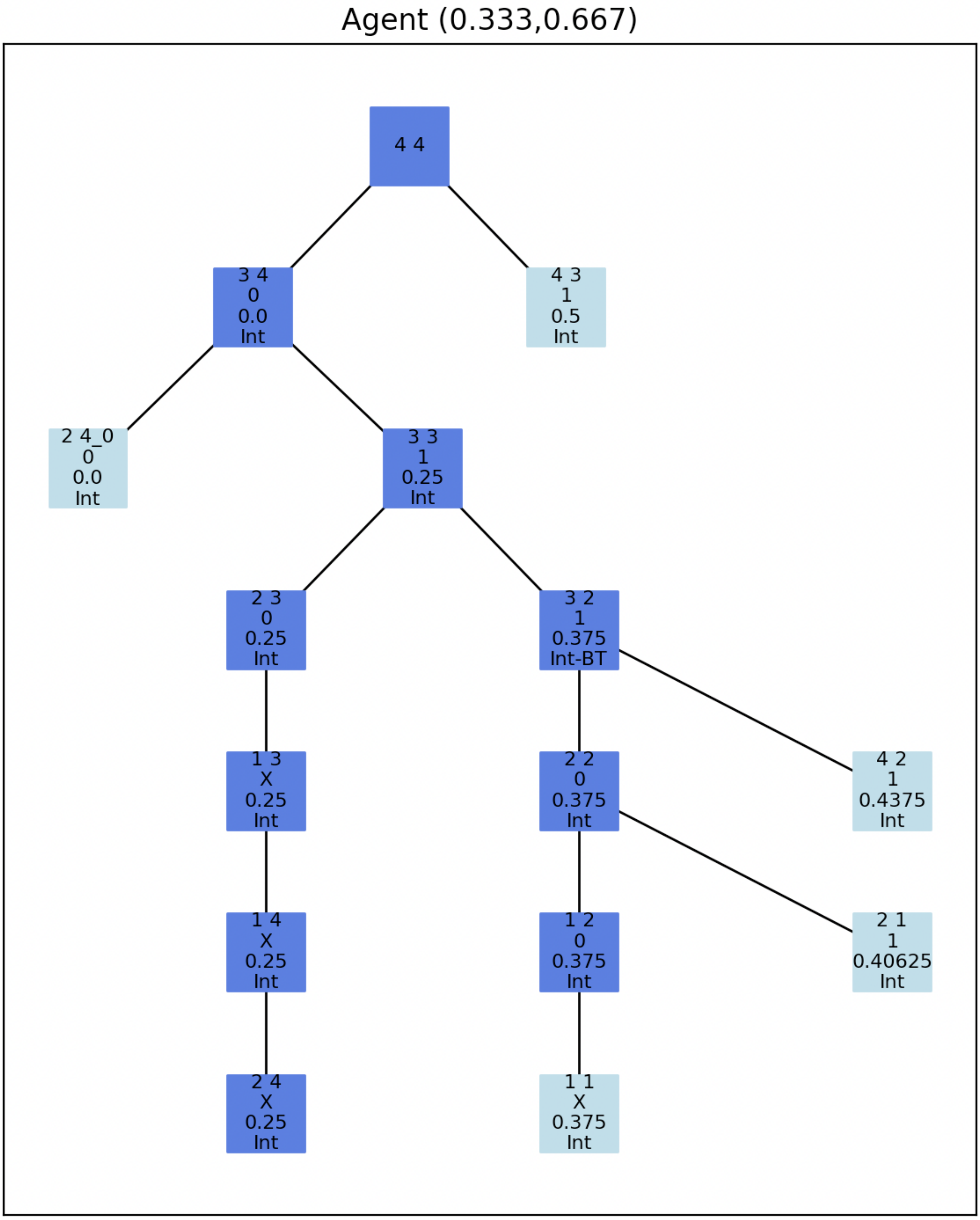


FIGURE 2.6 – Trees constructed by agent post-exploration: **Agent 2**.

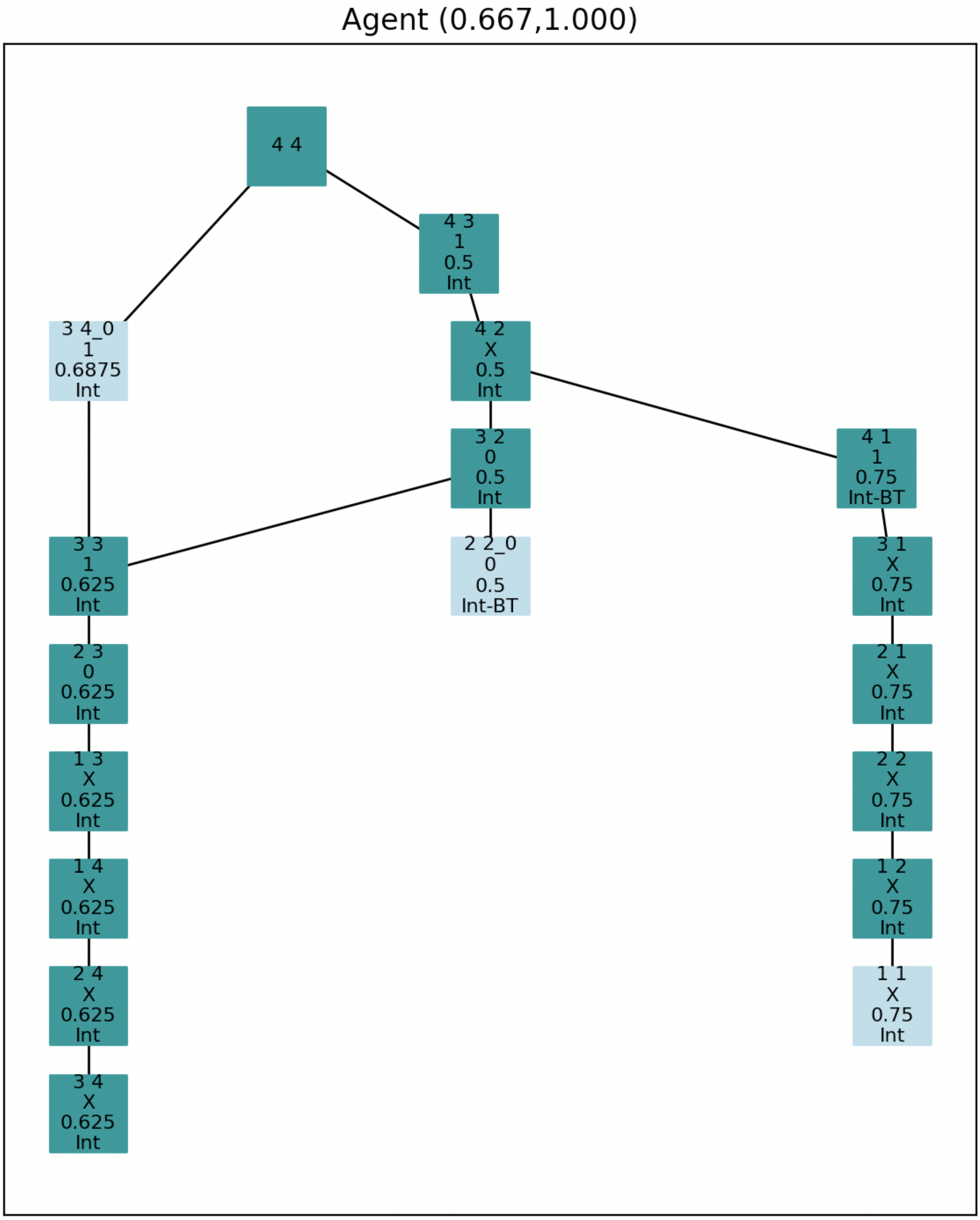


FIGURE 2.7 – Trees constructed by agent post-exploration: **Agent 3**.

These visualizations reveal how each agent independently constructs its own representation of the graph, adapting to imperfections and resulting in different exploration paths. For instance, Figure 2.7 shows a cycle occurrence, demonstrating how cycles can appear during exploration. Although cycles are detected, they are disregarded in the actual traversal.

3 Results and Discussion

This section presents the results of comparing the proposed no-communication algorithm with Tarry's algorithm, as well as analyzing Tarry's performance against its variants. We evaluate these approaches on different datasets, detailed in Section 3.1, to observe how distinct graph properties influence the efficiency of the solutions. Section 3.2 discusses the performance of the no-communication algorithms. Section 3.3 then presents the analysis of Tarry's algorithm and its variants.

3.1 Datasets for Algorithm Evaluation

This section describes the datasets used to evaluate the performance of the algorithms. The datasets consist of two types of graphs:

- Perfect Mazes: The dataset of perfect mazes (NAEEM, 2021) is the same as the one used by Rodrigues (2024) and includes 250 mazes for each of four sizes: 10x10, 20x20, 30x30, and 40x40.

- Random Trees: The dataset of random trees was generated using the *random_unlabeled_tree* method from NetworkX (HAGBERG *et al.*, 2008). This dataset includes 250 trees for each of the following sizes: 100, 250, 500, 1000, and 1500 nodes. After generating the graphs, nodes that did not contribute any new decisions to the tree (specifically, nodes with only two edges) were contracted by adding an edge between their two neighbors and then removed.

The analysis focuses on datasets and sizes that yield significant results, while all generated graphs can be found in the appendix.

3.2 No-Communication Algorithms Performance

3.2.1 Metrics for Analysis

This section focuses on the results for the no-communication algorithms, which include the proposed algorithm, its Backward Interval Filling variant, and Tarry's algorithm for comparison. Although Tarry's algorithm involves communication, it serves as a baseline to assess the efficiency of the no-communication strategies.

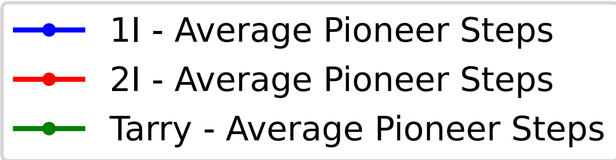
The metrics used in this comparison are:

- Steps Taken by the Pioneer: This measures the number of steps needed by the first agent to find the goal, serving as a key indicator of exploration efficiency.
- Fraction Explored Before the Pioneer Found the Goal: This indicates the proportion of the graph covered by the agents before the goal was reached, providing insight into how well the agents dispersed during exploration.

These metrics together give a clear view of the algorithm's efficiency and how exploration patterns vary across different graph structures and sizes.

3.2.2 Perfect Maze Results

The results for all maze sizes (10x10, 20x20, 30x30, and 40x40) are presented in Figures 3.5 and 3.10, showing how the different no-communication algorithms perform across various maze dimensions. Figure 3.5 represents the steps taken by the pioneer, while Figure 3.10 shows the explored fraction before the goal was found, allowing for a detailed comparison of exploration efficiency and coverage.



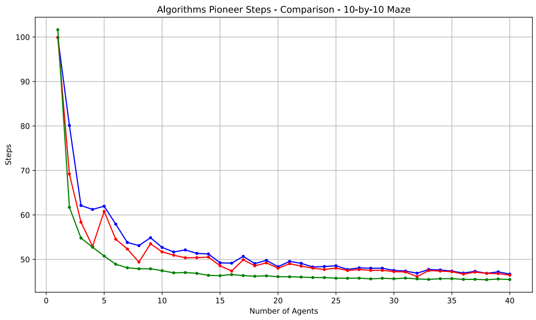


FIGURE 3.1 – (a) 10x10 Maze

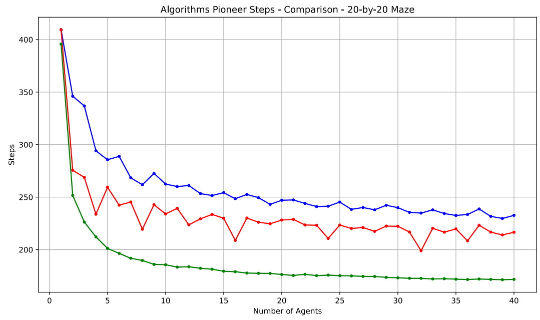


FIGURE 3.2 – (b) 20x20 Maze

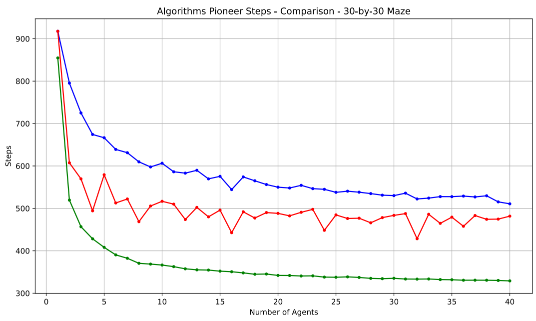


FIGURE 3.3 – (c) 30x30 Maze

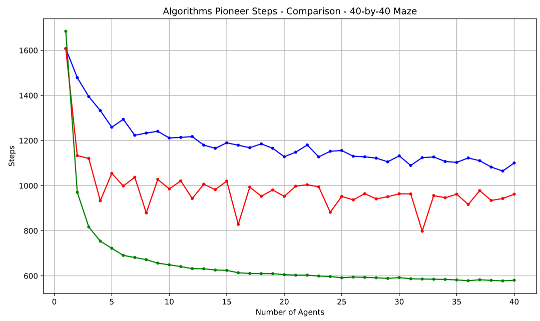
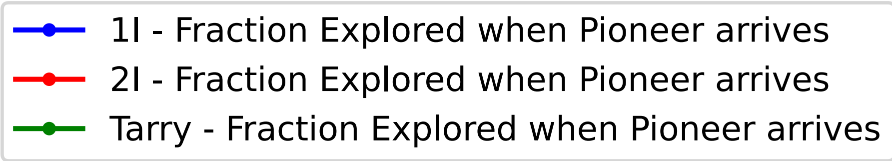


FIGURE 3.4 – (d) 40x40 Maze

FIGURE 3.5 – Comparison of the pioneer's average steps between our no-communication algorithms and the Tarry's algorithm across different sizes of perfect mazes. The subfigures illustrate how algorithm performance scales with maze size: (a) 10x10, (b) 20x20, (c) 30x30, and (d) 40x40.



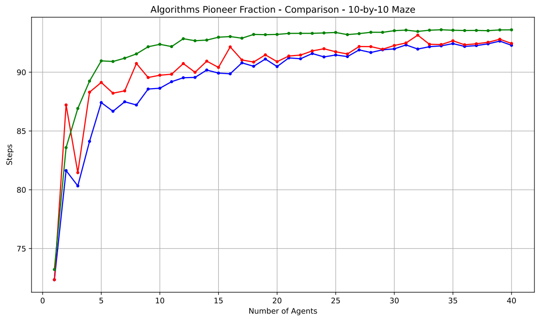


FIGURE 3.6 – (a) 10x10 Maze

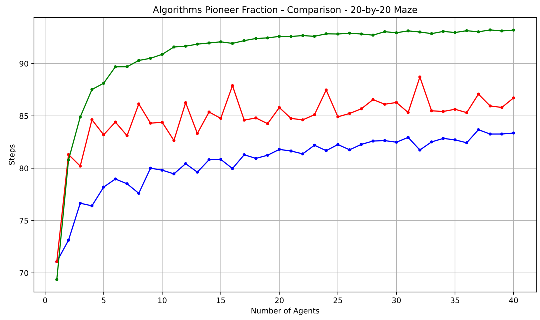


FIGURE 3.7 – (b) 20x20 Maze

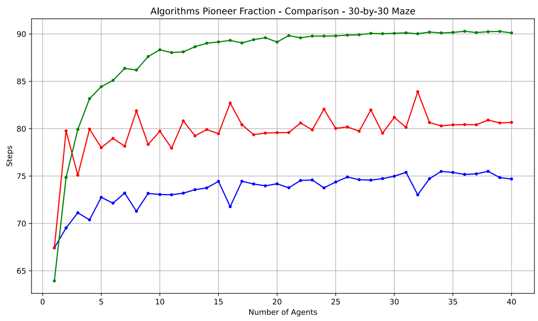


FIGURE 3.8 – (c) 30x30 Maze

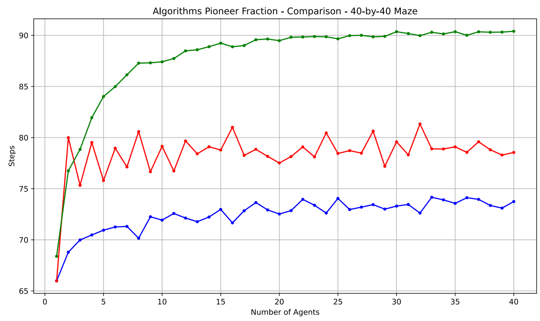


FIGURE 3.9 – (d) 40x40 Maze

FIGURE 3.10 – Comparison of the explored fraction achieved by our no-communication algorithms and Tarry's algorithm across different sizes of perfect mazes. The subfigures illustrate how coverage efficiency scales with maze size: (a) 10x10, (b) 20x20, (c) 30x30, and (d) 40x40.

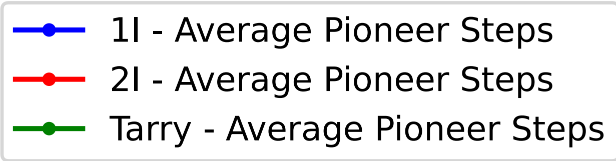
The results show that adding more agents helps the pioneer find the goal cell faster, as the average number of steps taken decreases with more agents. This trend eventually levels off at a certain point when there are many agents. The explored fraction behaves similarly, with more agents spreading out quickly and covering more of the maze, though it also stabilizes over time. This outcome is expected, as more agents lead to quicker exploration and better coverage.

In addition to the expected results, the Backward Interval Filling algorithm shows a strange pattern when the number of agents is a power of two, starting from the 20x20 maze size. This matches earlier findings in (RODRIGUES, 2024), but it hasn't been fully explored in previous studies. While we recognize the theory suggested by Rodrigues (2024), we did not investigate this behavior in detail in our research.

Building on these findings, we compared the performance of our no-communication algorithms with Tarry's algorithm. As expected, both Tarry's algorithm and the Backward Interval Filling variant perform better than the basic version of our algorithm. However, the level of improvement depends on the maze size. In larger mazes, Tarry's algorithm shows a bigger advantage. This is because, in smaller mazes, it's possible to quickly find the optimal solution by simply adding more agents. But in larger mazes, communication is crucial for avoiding repeated steps, which significantly enhances Tarry's performance.

3.2.3 Random Tree Results

The results for the random trees of sizes 100, 500, and 1500 nodes are presented in Figure 3.14. These sizes were chosen because they effectively represent the trends observed across all tree dimensions without the need to include every possible size. Figure 3.14 shows the steps taken by the pioneer, while the explored fraction is not displayed due to its high variability in trees, which could lead to misleading conclusions.



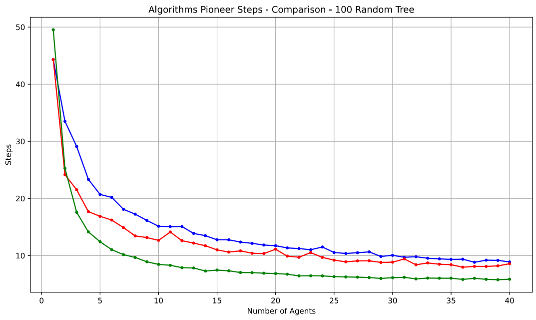


FIGURE 3.11 – (a) 100-node Tree

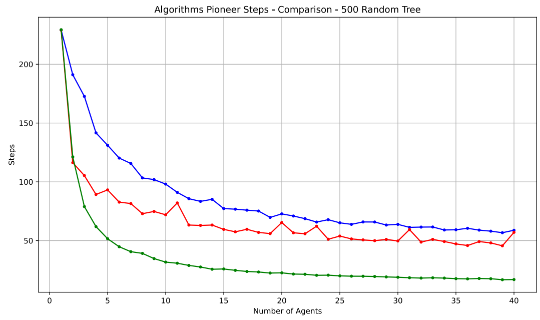


FIGURE 3.12 – (b) 500-node Tree

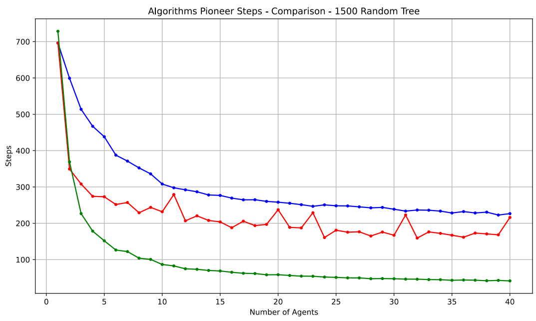


FIGURE 3.13 – (c) 1500-node Tree

FIGURE 3.14 – Comparison of the average steps taken by the pioneer across different no-communication algorithms and Tarry's algorithm for various sizes of random trees. The subfigures illustrate how algorithm performance varies with tree size: (a) 100 nodes, (b) 500 nodes, and (c) 1500 nodes.

The results for the random trees confirm that the conclusions from Section 3.2.2 still apply. Performance improves with more agents, Tarry's algorithm remains the most efficient, and its advantages increase with size.

Even though the trends hold, it's important to note that Tarry outperforms the Back Filling Interval Variation by approximately 39.6% and our proposed algorithm by 47.3% in the 40x40 maze. In contrast, in the 1500-node trees, Tarry shows a substantial improvement, being about 81.7% better than our proposed algorithm and 80.8% better than the Back Filling Interval Variation. This demonstrates the significant impact of dataset characteristics on the relative performance of our no-communication algorithms.

3.3 Tarry's Variants Performance

3.3.1 Metrics for Analysis

This section examines the results for Tarry's algorithm and its variants, specifically Tarry Tie-Breaker, Tarry Interval Priority, and Delayed Tarry.

In addition to the previously mentioned metrics in Section 3.2.1, this section introduces a metric for the relative difference in performance compared to the base Tarry algorithm. This measure is particularly useful for interpreting results when the algorithms' performances are very close on some datasets, providing a clearer picture of the improvements or regressions.

3.3.2 Perfect Maze Results

3.3.3 Random Tree Results

4 Conclusions and Future Works

This work presented an algorithm for graph exploration in a multi-agent system without communication, expanding on the solution proposed by Rodrigues (2024). The flexibility provided by our framework, which no longer relies on the open-source perfect maze generator by Naeem (2021), allows for expansion to various exploration policies and algorithmic variations. Additionally, the modular design of our system enables integration of new exploration strategies, paving the way for future advancements.

We conducted tests on 100 randomly generated 40x40 perfect mazes and compared the results across three algorithms: our own, a backward interval variation, and an extended version of Tarry's algorithm. The results were validated against previous work (RODRIGUES, 2024), confirming the correctness and reliability of our approach.

Looking forward, we aim to validate our algorithm against established graph generation algorithms, assessing its efficiency on a larger scale. Exploring the implementation of more advanced algorithms and optimizing the agents' decision-making processes will be essential for enhancing the applicability of our approach.

By advancing multi-agent graph exploration without communication, this research paves the way for innovations in autonomous systems, with potential applications across various domains to be explored in future research.

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11. RESUMO:

Graph theory, a pivotal field in mathematics and computer science, offers robust frameworks for modeling relationships and traversing graph structures. This research delves into graph exploration, crucial for applications like network routing, robotics, and procedural generation. As real-world applications often involve complex environments with time constraints, this study focuses on multi-agent systems for graph exploration, where multiple autonomous agents collaborate to optimize task distribution.

This study's objective is to propose an efficient multi-agent algorithm for graph exploration without communication, a crucial need in scenarios where communication is impractical or impossible, such as deep-sea exploration or energy-constrained environments. Building on the method for perfect maze exploration detailed by Rodrigues (2024), this study extends the approach to general graphs, that may include cycles. Several algorithmic variations, combining the proposed approach and Tarry's variation (KIVELEVITCH; COHEN, 2010), were implemented in an effort to compare how different algorithms perform in various scenarios and to identify efficient use cases. Furthermore, the algorithms were reimplemented using a generic graph library instead of a specialized perfect maze library (NAEEM, 2021).

By structuring the exploration into well-defined and extensible classes, the research offers a versatile framework for broader applications in multi-agent systems. This framework was used to evaluate different datasets, containing graphs with varying properties, paving the way for further advancements in autonomous graph exploration.

12. GRAU DE SIGILO:

(X) OSTENSIVO ( ) **RESERVADO** ( ) **SECRETO**