INSTITUTO TECNOLÓGICO DE AERONÁUTICA

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MULTI-AGENT GRAPH EXPLORATION WITHOUT COMMUNICATION

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MULTI-AGENT GRAPH EXPLORATION WITHOUT COMMUNICATION

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To my family, for their love and constant support. To my friends, for standing by me through all moments.

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"The journey of a thousand miles begins with one step."

— Lao Tzu

Resumo

Teste

Abstract

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List of Abbreviations and Acronyms

DFS Depth First Search
BFS Breath First Search
LCL Last Common Location

List of Symbols

G Graph

v_n Vertex

$v_n v_m$ Edge

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1 Introduction

1.1 Motivation

Graph theory is a fundamental field of mathematics with significant relevance in computer science due to its ability to model relationships between objects. One of the key challenges in computer science is graph traversal and exploration, which involves systematically navigating through the nodes of a graph with a specific purpose. This topic has practical applications across diverse fields, such as network routing, robotics, procedural generation, electronics design, and more.

When considering the complexities and time constraints imposed by real-world environments, graph exploration must be expanded to consider multi-agent systems. In such systems, multiple autonomous agents collaborate to explore graphs, aiming to distribute tasks to achieve optimal efficiency. However, communication strategies on these systems are challenging, as they need to balance the trade-off between the amount of information shared among agents versus the quality of the solution. In other words, while communication can allow agents to speed up their exploration, it may cost time and/or energy.

Various versions of this problem have been explored in research. One such approach, discussed by Kivelevitch and Cohen (2010), proposed a generalization of Tarry's Algorithm with significant emphasis on minimizing data transfer. Nevertheless, this solution is limited to mazes and does not extend to general graphs.

Despite the previously mentioned research, the concept of zero-communication exploration has not been concretely established in the literature. We believe this gap is significant, as practical situations might involve scenarios where communication is limited or impossible due to bandwidth limitations or energy consumption, such as deep-sea exploration, search with energy-limited agents, or high-efficiency state space search.

This work aims to address this gap by presenting a continuation and expansion of a method initially developed for perfect maze exploration (NAEEM, 2021) as discussed by Rodrigues (2024). Building upon this groundwork, the research extends the method to accommodate diverse graph structures by removing dependencies on specific projects.

General improvements, such as structuring the exploration into well-defined and extensible classes and incorporating built-in testing, strive to make the development process more versatile and robust, paving the way for broader applications in multi-agent systems.

1.2 Objective

This study aims to propose a viable algorithm for graph exploration by multi-agents, specifically targeting the identification of a goal node. A previous paper by Rodrigues (2024) discussed an approach for perfect maze exploration, which can be modeled as a tree, a connected acyclic undirected graph. Expanding the proposed algorithm to connected cyclic graphs could allow further discussion and contribute to research in real-world applications such as robotics, as well as in the theoretical analysis of parallelism and communication between agents.

1.3 Definitions

This section aims to define key concepts in graph theory and multi-agent exploration that are essential for understanding the context and methodology of our research, ensuring clarity and consistency in the subsequent discussions.

1.3.1 Graph

Graphs are structures in mathematics and computer science used to represent relationships between pairs of objects. According to Manber (1989), a graph $G = (V, E)$ is defined as a set V of vertices (or nodes) and a set E of edges, that connect pairs of vertices.

1.3.1.1 Common Nomenclature

In graph theory, the basic terminology includes the following components:

- Vertex (Node): The fundamental unit of a graph, representing an object or a point.
- Edge: A connection between a pair of vertices, representing a relationship between them.
- Adjacency: Two vertices v and w of a graph G are adjacent if there is an edge vw joining them, and the vertices v and w are then incident with such an edge. (WILSON, 1996)

- Degree: Degree of a vertex v of G is the number of edges incident with v. (WILSON, 1996)
- Path: A path in a graph G, as defined by Wilson (1996), is a finite sequence of edges of the form v_0v_1, v_1v_2, ... in which any two consecutive edges are adjacent, all edges are distinct and all vertices v_0, v_1, ..., v_m are distinct (except, possibly, $v_0 = v_m$).
- Cycle: A path where $v_0 = v_m$ and with at least one edge. (WILSON, 1996)

1.3.1.2 Key Properties

A few key properties of graphs that interest us are:

- Connectivity: A graph is connected if and only if there is a path between each pair of vertices. (WILSON, 1996)
- Acyclicity: A graph is acyclic if it has no cycles. A tree is an acyclic connected graph.

1.3.1.3 Traversal

Graph traversal is the process of visiting all the vertices in a graph in a systematic manner. Two fundamental algorithms for graph traversal are:

- Breadth-first search (BFS): Expands the frontier between discovered and undiscovered vertices uniformly across the breadth of the frontier. (MANBER, 1989)
- Depth-first search (DFS): Explores edges out of the most recently discovered vertex v that still has unexplored edges leaving it. (MANBER, 1989)
- Tarry's Algorithm: Tarry's Algorithm is used traverse the entire maze, or graph, in a depth first fashion, where each edge is travelled twice, and the agent finishes the motion at the same initial vertex it started from.(KIVELEVITCH; COHEN, 2010) This algorithm is described in the context of a physical maze, where the agent must physically move between adjacent nodes. It was first described over botanical mazes built as amusements for the XIX century aristocracy.

1.3.2 Agent

An agent in maze exploration, as defined by Kivelevitch and Cohen (2010), is an entity able to move within the maze in any direction as long as it does not encounter obstacles.

In the context of graph exploration, this concept extends to an agent's ability to traverse edges between vertices within the graph structure. Each agent acts independently, making decisions based on its individual memory and the local information available at its current vertex and its adjacent edges.

1.3.3 Mixed Radix

A mixed radix system is a positional numerical system in which the numerical base for each digit varies. This allows for more flexible representation of numbers, accommodating various scales within the same numerical framework.

One mathematical approach for representing mixed radix numbers can be found in Arndt (2011), which establishes a comprehensive arithmetical method for manipulating them. However, for the specific subset of real numbers within the interval [0, 1], a more specialized approach is utilized, as described by Rodrigues (2024).

In this context, the mixed radix representation $A = [a_0, a_1, a_2, ..., a_{n-1}]$ of a number x with respect to a radix vector $M = [m_0, m_1, m_2, ..., m_{n-1}]$, where $x \in [0, 1]$, is given by:

$$x = \sum_{k=0}^{n-1} a_k \prod_{j=0}^{k} \frac{1}{m_j} \tag{1.1}$$

where a_j are non-negative integers, m_j are integers such that $m_j \geq 2$, and $0 \leq a_j \leq m_j$. (RODRIGUES, 2024)

For instance, if x is represented by $2_3 1_2 1_4 0_3 1_5$, and considering that a_i is on the left of a_{i+1}, x might be transformed to the decimal base by the following steps:

$$x = 2_3 1_2 1_4 0_3 1_5 \tag{1.2}$$

$$A = [2, 1, 1, 0, 1] \tag{1.3}$$

$$M = [3, 2, 4, 3, 5] \tag{1.4}$$

$$n = 5 \tag{1.5}$$

$$x = \sum_{k=0}^{4} a_k \prod_{j=0}^{k} \frac{1}{m_j} \approx 0.8777778 \tag{1.6}$$

1.3.3.1 Extending Mixed Radix with Unary Digits

In our study, we extended the mixed radix system to include unary digits, which are used to represent steps in a sequence where no meaningful decisions are made. These unary digits do not contribute to the numerical value and can be omitted in the conversion to standard numerical forms.

Within our framework, the mixed radix representation $A = [a_0, a_1, a_2, ..., a_{n-1}]$ of a number x with respect to a radix vector $M = [m_0, m_1, m_2, ..., m_{n-1}]$, where $x \in [0, 1]$, is given by:

Let $U = \{k \mid m_k \neq 1\}$ be the set of indices where the base m_k is not unary.

$$x = \sum_{k \in U} u_k \prod_{j=0}^{k} \frac{1}{m_j} \tag{1.7}$$

where a_k are non-negative integers, m_j are integers such that $m_j \geq 1$, and $0 \leq a_j \leq m_j$.

Consider the mixed radix representation $x = 2_3 X_1 1_2 X_1 X_1 1_4 0_3 1_5$, where X_1 represents the unary digit and each a_i is on the left of a_{i+1}. We can transform x to the decimal base using the following steps:

$$x = 2_3 X_1 1_2 X_1 X_1 1_4 0_3 1_5 \tag{1.8}$$

$$A = [2, X, 1, X, X, 1, 0, 1] \tag{1.9}$$

$$M = [3, 1, 2, 1, 1, 4, 3, 5] \tag{1.10}$$

$$U = [0, 2, 5, 6, 7] \tag{1.11}$$

$$x = \sum_{k \in U} a_k \prod_{j=0}^{k} \frac{1}{m_j} \approx 0.8777778 \tag{1.12}$$

2 Methodology

2.1 Modeling

2.1.1 Simulation

2.1.2 Graph

2.1.3 Agent

2.2 Mixed Radix Representation

2.3 Graph Exploration Algorithm

3 Results and Discussion

3.1 Performance Evaluation Methodology

3.2 Baseline Algorithm Performance

3.3 Incremental Algorithm Performance

3.4 Comparison with Tarry's Algorithm

4 Conclusions and Future Works

Bibliography

ARNDT, J. Mixed radix numbers. In: **Matters Computational**. Berlin, Heidelberg: Springer, 2011. p. 183–209.

KIVELEVITCH, E.; COHEN, K. Multi-agent maze exploration. **Journal of Aerospace Computing, Information, and Communication**, v. 7, p. 391–405, 2010.

MANBER, U. **Introduction to Algorithms: A Creative Approach**. 1st. ed. New York: Addison-Wesley, 1989.

NAEEM, M. A. **pyamaze**. 2021. `https://www.github.com/MAN1986/pyamaze`. Accessed on: June, 2024.

RODRIGUES, A. J. D. S. **Multi-agent graph exploration without communication**. Bachelor's Thesis — Instituto Tecnológico de Aeronáutica,, São José dos Campos, 2024.

WILSON, R. J. **Introduction to Graph Theory**. 4th. ed. Boston, MA: Longman Group Ltd, 1996.

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