teambrbr002
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1 Matematica

1.1 2-SAT

```
// solve() retorna um par, o first fala se eh possivel
// atribuir, o second fala se cada variavel eh verdadeira
//
// O(|V|+|E|) = O(#variaveis + #restricoes)

138 struct sat {
e6c     int n, tot;
789     vector<vector<int>> g;
0ca     vector<int> vis, comp, id, ans;
4ce     stack<int> s;

141     sat() {}
172     sat(int n_) : n(n_), tot(n), g(2*n) {}

f32     int dfs(int i, int& t) {
cf0         int lo = id[i] = t++;
efc         s.push(i), vis[i] = 2;
48e         for (int j : g[i]) {
740             if (!vis[j]) lo = min(lo, dfs(j, t));
994             else if (vis[j] == 2) lo = min(lo, id[j]);
d64         }
3de         if (lo == id[i]) while (1) {
3c3             int u = s.top(); s.pop();
9c5             vis[u] = 1, comp[u] = i;
91d             if ((u>>1) < n and ans[u>>1] == -1) ans[u>>1] = ~u&1;
2ef             if (u == i) break;
60d         }
253         return lo;
dec     }

74a     void add_impl(int x, int y) { // x -> y = !x ou y
26a         x = x >= 0 ? 2*x : -2*x-1;
2b8         y = y >= 0 ? 2*y : -2*y-1;
a1e         g[x].push_back(y);
1e2         g[y^1].push_back(x^1);
ef0     }
e85     void add_cl(int x, int y) { // x ou y
0b5         add_impl(~x, y);
254     }
487     void add_xor(int x, int y) { // x xor y
0b7         add_cl(x, y), add_cl(~x, ~y);
9a1     }
978     void add_eq(int x, int y) { // x = y
c86         add_xor(~x, y);
b91     }
b10     void add_true(int x) { // x = T
18b         add_impl(~x, x);
9e2     }
d14     void at_most_one(vector<int> v) { // no max um verdadeiro
54d         g.resize(2*(tot+v.size()));
f14         for (int i = 0; i < v.size(); i++) {
8c9             add_impl(tot+i, ~v[i]);
a8f             if (i) {
b6a                 add_impl(tot+i, tot+i-1);
3d3                 add_impl(v[i], tot+i-1);
0f7             }
084         }
258         tot += v.size();
b00     }

a8e     pair<bool, vector<int>> solve() {
27b         ans = vector<int>(n, -1);
6bb         int t = 0;
0de         vis = comp = id = vector<int>(2*tot, 0);
53c         for (int i = 0; i < 2*tot; i++) if (!vis[i]) dfs(i, t);
f88         for (int i = 0; i < tot; i++)
4c9             if (comp[2*i] == comp[2*i+1]) return {false, {}};
997         return {true, ans};
7b3     }
ef6 };
```

1.2 Avaliacao de Interpolacao

```
// Dado 'n' pontos (i, y[i]), i \in [0, n),
// avalia o polinomio de grau n-1 que passa
// por esses pontos em 'x'
// Tudo modular, precisa do mint
//
// O(n)

ee8 mint evaluate_interpolation(int x, vector<mint> y) {
80e     int n = y.size();

184     vector<mint> sulf(n+1, 1), fat(n, 1), ifat(n);
6fa     for (int i = n-1; i >= 0; i--) sulf[i] = sulf[i+1] * (x - i);
29b     for (int i = 1; i < n; i++) fat[i] = fat[i-1] * i;
0da     ifat[n-1] = 1/fat[n-1];
3db     for (int i = n-2; i >= 0; i--) ifat[i] = ifat[i+1] * (i + 1);

ca1     mint pref = 1, ans = 0;
5ea     for (int i = 0; i < n; pref *= (x - i++)) {
42f         mint num = pref * sulf[i+1];

b4e         mint den = ifat[i] * ifat[n-1 - i];
0bd         if ((n-1 - i)%2) den *= -1;

03f         ans += y[i] * num * den;
ce6     }
ba7     return ans;
4fe }
```

1.3 Berlekamp-Massey

```
// guess_kth(s, k) chuta o k-esimo (0-based) termo
// de uma recorrencia linear que gera s
// Para uma rec. lin. de ordem x, se passar 2x termos
// vai gerar a certa
// Usar aritmetica modular
//
// Pro fast_evaluate, precisa de ntt e divmod (powerSeries.cpp)
//
// Complexidades: (n = |s|)
// evaluate: O(n^2 log k)
// fast_evaluate: O(n log n log k)
// berlekampMassey: O(n^2 + O(evaluate))
```

```
b7c template<typename T> T evaluate(vector<T> c, vector<T> s, ll k) {
ff2     int n = c.size();
9ee     assert(c.size() <= s.size());

d09     auto mul = [&](const vector<T> &a, const vector<T> &b) {
564         vector<T> ret(a.size() + b.size() - 1);
d75         for (int i = 0; i < a.size(); i++) for (int j = 0; j <
    b.size(); j++)
cff             ret[i+j] += a[i] * b[j];
83d         for (int i = ret.size()-1; i >= n; i--) for (int j = n-1;
    j >= 0; j--)
112             ret[i-j-1] += ret[i] * c[j];
16d         ret.resize(min<int>(ret.size(), n));
edf         return ret;
3b9     };

1a6     vector<T> a = n == 1 ? vector<T>({c[0]}) : vector<T>({0, 1}),
    x = {1};
95f     while (k) {
7f1         if (k&1) x = mul(x, a);
b28         a = mul(a, a), k >>= 1;
8ea     }
dd6     x.resize(n);

ce8     T ret = 0;
e72     for (int i = 0; i < n; i++) ret += x[i] * s[i];
edf     return ret;
7e2 }

83e mint fast_evaluate(poly c, poly s, ll k) {
fb0     if (k < s.size()) return s[k];
ff2     int n = c.size();
9ee     assert(c.size() <= s.size());

79f     auto f = poly(n + 1, 1);
bcc     for (int i = 0; i < n; i++) f[i] = -c[n-i-1];

7ce     poly a = n == 1 ? poly({c[0]}) : poly({0, 1}), x = {1};
95f     while (k) {
3df         if (k&1) x = divmod(convolution(x, a), f).second;
cac         a = divmod(convolution(a, a), f).second, k >>= 1;
11b     }

eb1     mint ret = 0;
e72     for (int i = 0; i < n; i++) ret += x[i] * s[i];
edf     return ret;
95d }
```

```
192 template<typename T> vector<T> berlekamp_massey(vector<T> s) {
ce8     int n = s.size(), l = 0, m = 1;
222     vector<T> b(n), c(n);
46e     T ld = b[0] = c[0] = 1;
620     for (int i = 0; i < n; i++, m++) {
793         T d = s[i];
ab6         for (int j = 1; j <= l; j++) d += c[j] * s[i-j];
5f0         if (d == 0) continue;
8b4         vector<T> temp = c;
369         T coef = d / ld;
ba6         for (int j = m; j < n; j++) c[j] -= coef * b[j-m];
88f         if (2 * l <= i) l = i + 1 - l, b = temp, ld = d, m = 0;
76a     }
90c     c.resize(l + 1);
844     c.erase(c.begin());
0dc     for (T& x : c) x = -x;
807     return c;
4d9 }

2cf template<typename T> T guess_kth(const vector<T>& s, ll k) {
cc3     auto c = berlekamp_massey(s);
96a     return evaluate(c, s, k);
697 }
```

1.4 Binomial Distribution

```
// binom(n, k, p) retorna a probabilidade de k sucessos
// numa binomial(n, p)

361 double logfact[MAX];

9e4 void calc() {
7a0     logfact[0] = 0;
152     for (int i = 1; i < MAX; i++) logfact[i] = logfact[i-1] +
    log(i);
67a }

94c double binom(int n, int k, double p) {
271     return exp(logfact[n] - logfact[k] - logfact[n-k] + k * log(p)
    + (n-k) * log(1 - p));
587 }
```

1.5 Convolucao de GCD / LCM

```
// O(n log(n))

// multiple_transform(a)[i] = \sum_d a[d * i]
bbe template<typename T> void multiple_transform(vector<T>& v, bool
    inv = false) {
64a     vector<int> I(v.size()-1);
847     iota(I.begin(), I.end(), 1);
674     if (inv) reverse(I.begin(), I.end());
dad     for (int i : I) for (int j = 2; i*j < v.size(); j++)
a8a         v[i] += (inv ? -1 : 1) * v[i*j];
338 }

// gcd_convolution(a, b)[k] = \sum_{gcd(i, j) = k} a_i * b_j
fe2 template<typename T> vector<T> gcd_convolution(vector<T> a,
    vector<T> b) {
bdf     multiple_transform(a), multiple_transform(b);
799     for (int i = 0; i < a.size(); i++) a[i] *= b[i];
dea     multiple_transform(a, true);
3f5     return a;
984 }

// divisor_transform(a)[i] = \sum_{d|i} a[i/d]
be7 template<typename T> void divisor_transform(vector<T>& v, bool inv
    = false) {
64a     vector<int> I(v.size()-1);
847     iota(I.begin(), I.end(), 1);
5ea     if (!inv) reverse(I.begin(), I.end());
dad     for (int i : I) for (int j = 2; i*j < v.size(); j++)
14f         v[i*j] += (inv ? -1 : 1) * v[i];
aa7 }

// lcm_convolution(a, b)[k] = \sum_{lcm(i, j) = k} a_i * b_j
b1b template<typename T> vector<T> lcm_convolution(vector<T> a,
    vector<T> b) {
3af     divisor_transform(a), divisor_transform(b);
799     for (int i = 0; i < a.size(); i++) a[i] *= b[i];
d8f     divisor_transform(a, true);
3f5     return a;
f5a }
```

1.6 Coprime Basis

```
// Dado um conjunto de elementos A constroi uma base B
// de fatores coprimos tal que todo elemento A[i]
// pode ser fatorado como A[i] = \prod B[j]^p_ij
//
// Sendo n o numero de inserts, a complexidade esperada fica
// O(n*(n*loglog(MAX) + log(MAX)^2))
//
// No pior caso, podemos trocar n*loglog(MAX) por
//  8n, se MAX <= 1e6
// 10n, se MAX <= 1e9
// 16n, se MAX <= 1e18
// 26n, se MAX <= 1e36

ebc template <typename T> struct coprime_basis {
a00     vector<T> basis;

60e     coprime_basis() {}
055     coprime_basis(vector<T> v) { for (T i : v) insert(i); }

845     void insert(T z) {
c3c         int n = basis.size();
efe         basis.push_back(z);
43c         for (int i = n; i < basis.size(); i++) {
21c             for (int j = (i != n) ? i+1 : 0; j < basis.size();
    j++) {
4ce                 if (i == j) continue;
024                 T &x = basis[i];
c91                 if (x == 1) {
fac                     j = INF;
5e2                     continue;
6e0                 }
544                 T &y = basis[j];
3c9                 T g = gcd(x, y);
e10                 if (g == 1) continue;
15b                 y /= g, x /= g;
8c6                 basis.push_back(g);
069             }
422         }
fe8         basis.erase(remove(basis.begin(), basis.end(), 1),
    basis.end());
1a5     }

4ba     vector<int> factor(T x) {
21d         vector<int> fat(basis.size());
6fd         for (int i = 0; i < basis.size(); i++) {
25c             while (x % basis[i] == 0) x /= basis[i], fat[i]++;
8de         }
6a7         return fat;
b5d     }
671 };
```

1.7 Crivo de Eratosthenes

```
// "O" crivo
//
// Encontra maior divisor primo
// Um numero eh primo sse divi[x] == x
// fact fatora um numero <= lim
// A fatoracao sai ordenada
//
// crivo - O(n log(log(n)))
// fact - O(log(n))

f12 int divi[MAX];

fb9 void crivo(int lim) {
f53     for (int i = 1; i <= lim; i++) divi[i] = 1;

d46     for (int i = 2; i <= lim; i++) if (divi[i] == 1)
018         for (int j = i; j <= lim; j += i) divi[j] = i;
349 }

d41 #warning A funcao fact ira adicionar o 1 no vetor se voce tentar
    fatorar especificamente o numero 1
470 void fact(vector<int>& v, int n) {
ac8     if (n != divi[n]) fact(v, n/divi[n]);
ab4     v.push_back(divi[n]);
1db }

// Crivo linear
//
// Mesma coisa que o de cima, mas tambem
// calcula a lista de primos
//
// O(n)

f12 int divi[MAX];
fd3 vector<int> primes;

fb9 void crivo(int lim) {
d5a     divi[1] = 1;
f70     for (int i = 2; i <= lim; i++) {
3eb         if (divi[i] == 0) divi[i] = i, primes.push_back(i);
3ba         for (int j : primes) {
```

```
522             if (j > divi[i] or i*j > lim) break;
00b             divi[i*j] = j;
491         }
85a     }
519 }

// Crivo de divisores
//
// Encontra numero de divisores
// ou soma dos divisores
//
// O(n log(n))

f12 int divi[MAX];

fb9 void crivo(int lim) {
f53     for (int i = 1; i <= lim; i++) divi[i] = 1;

424     for (int i = 2; i <= lim; i++)
594         for (int j = i; j <= lim; j += i) {
                // para numero de divisores
9e0             divi[j]++;
                // para soma dos divisores
278             divi[j] += i;
c58         }
fc1 }

// Crivo de totiente
//
// Encontra o valor da funcao
// totiente de Euler
//
// O(n log(log(n)))

5f4 int tot[MAX];

fb9 void crivo(int lim) {
a27     for (int i = 1; i <= lim; i++) {
bc9         tot[i] += i;
feb         for (int j = 2*i; j <= lim; j += i)
837             tot[j] -= tot[i];
678     }
212 }

// Crivo de funcao de mobius
//
// O(n log(log(n)))
```

```
4e1 char meb[MAX];

fb9 void crivo(int lim) {
649     for (int i = 2; i <= lim; i++) meb[i] = 2;
ace     meb[1] = 1;
842     for (int i = 2; i <= lim; i++) if (meb[i] == 2)
8d8         for (int j = i; j <= lim; j += i) if (meb[j]) {
686             if (meb[j] == 2) meb[j] = 1;
ae1             meb[j] *= j/i%i ? -1 : 0;
97f         }
9bc }

// Crivo linear de funcao multiplicativa
//
// Computa f(i) para todo 1 <= i <= n, sendo f
// uma funcao multiplicativa (se gcd(a,b) = 1,
// entao f(a*b) = f(a)*f(b))
// f_prime tem que computar f de um primo, e
// add_prime tem que computar f(p^(k+1)) dado f(p^k) e p
// Se quiser computar f(p^k) dado p e k, usar os comentarios
//
// O(n)

fd3 vector<int> primes;
623 int f[MAX], pot[MAX];
//int expo[MAX];

5c4 void sieve(int lim) {
        // Funcoes para soma dos divisores:
fc9     auto f_prime = [](int p) { return p+1; };
31c     auto add_prime = [](int fpak, int p) { return fpak*p+1; };
        //auto f_pak = [](int p, int k) {};

02d     f[1] = 1;
f70     for (int i = 2; i <= lim; i++) {
e6b         if (!pot[i]) {
e74             primes.push_back(i);
f05             f[i] = f_prime(i), pot[i] = i;
                //expo[i] = 1;
b71         }
3b9         for (int p : primes) {
b9f             if (i*p > lim) break;
569             if (i%p == 0) {
b97                 f[i*p] = f[i / pot[i]] * add_prime(f[pot[i]], p);
                    // se for descomentar, tirar a linha de cima tambem
                    //f[i*p] = f[i / pot[i]] * f_pak(p, expo[i]+1);
                    //expo[i*p] = expo[i]+1;
51f                 pot[i*p] = pot[i] * p;
c2b                 break;
643             } else {
9ef                 f[i*p] = f[i] * f[p];
638                 pot[i*p] = p;
                    //expo[i*p] = 1;
6f7             }
f31         }
1bb     }
350 }
```

1.8 Deteccao de ciclo - Tortoise and Hare

```
// Linear no tanto que tem que andar pra ciclar,
// O(1) de memoria
// Retorna um par com o tanto que tem que andar
// do f0 ate o inicio do ciclo e o tam do ciclo

58d pair<ll, ll> find_cycle() {
273     ll tort = f(f0);
b2b     ll hare = f(f(f0));
b1b     ll t = 0;
683     while (tort != hare) {
b4d         tort = f(tort);
4b2         hare = f(f(hare));
c82         t++;
93d     }
0e8     ll st = 0;
909     tort = f0;
683     while (tort != hare) {
b4d         tort = f(tort);
1a2         hare = f(hare);
397         st++;
c91     }

73d     ll len = 1;
3cd     hare = f(tort);
683     while (tort != hare) {
1a2         hare = f(hare);
040         len++;
f1a     }
ebd     return {st, len};
899 }
```

1.9 Division Trick

```
// Gera o conjunto n/i, pra todo i, em O(sqrt(n))
// copiei do github do tfg50

79c for(int l = 1, r; l <= n; l = r + 1) {
746     r = n / (n / l);
        // n / i has the same value for l <= i <= r
5bf }
```

1.10 Equacao Diofantina Linear

```
// Encontra o numero de solucoes de a*x + b*y = c,
// em que x \in [lx, rx] e y \in [ly, ry]
// Usar o comentario para recuperar as solucoes
// (note que o b ao final eh b/gcd(a, b))
// Cuidado com overflow! Tem que caber o quadrado dos valores
//
// O(log(min(a, b)))

c5e template<typename T> tuple<ll, T, T> ext_gcd(ll a, ll b) {
3bd     if (!a) return {b, 0, 1};
c4b     auto [g, x, y] = ext_gcd<T>(b%a, a);
c59     return {g, y - b/a*x, x};
8a8 }

// numero de solucoes de a*[lx, rx] + b*[ly, ry] = c
14c template<typename T = ll> // usar __int128 se for ate 1e18
2a4 ll diophantine(ll a, ll b, ll c, ll lx, ll rx, ll ly, ll ry) {
c80     if (lx > rx or ly > ry) return 0;
a98     if (a == 0 and b == 0) return c ? 0 : (rx-lx+1)*(ry-ly+1);
8ce     auto [g, x, y] = ext_gcd<T>(abs(a), abs(b));
9c3     if (c % g != 0) return 0;
249     if (a == 0) return (rx-lx+1)*(ly <= c/b and c/b <= ry);
4ce     if (b == 0) return (ry-ly+1)*(lx <= c/a and c/a <= rx);
fb1     x *= a/abs(a) * c/g, y *= b/abs(b) * c/g, a /= g, b /= g;

b20     auto shift = [&](T qt) { x += qt*b, y -= qt*a; };
efa     auto test = [&](T& k, ll mi, ll ma, ll coef, int t) {
866         shift((mi - k)*t / coef);
79d         if (k < mi) shift(coef > 0 ? t : -t);
74d         if (k > ma) return pair<T, T>(rx+2, rx+1);
41f         T x1 = x;
633         shift((ma - k)*t / coef);
c5b         if (k > ma) shift(coef > 0 ? -t : t);
4a9         return pair<T, T>(x1, x);
8e1     };

639     auto [l1, r1] = test(x, lx, rx, b, 1);
38e     auto [l2, r2] = test(y, ly, ry, a, -1);
c43     if (l2 > r2) swap(l2, r2);
50a     T l = max(l1, l2), r = min(r1, r2);
339     if (l > r) return 0;
42f     ll k = (r-l) / abs(b) + 1;
839     return k; // solucoes: x = l + [0, k)*|b|
98e }
```

1.11 Euclides estendido

```
// Acha x e y tal que ax + by = mdc(a, b) (nao eh unico)
// Assume a, b >= 0
//
// O(log(min(a, b)))

2be tuple<ll, ll, ll> ext_gcd(ll a, ll b) {
3bd     if (!a) return {b, 0, 1};
550     auto [g, x, y] = ext_gcd(b%a, a);
c59     return {g, y - b/a*x, x};
354 }
```

1.12 Exponenciacao rapida

```
// (x^y mod m) em O(log(y))

03c ll pow(ll x, ll y, ll m) { // iterativo
c85     ll ret = 1;
1b8     while (y) {
895         if (y & 1) ret = (ret * x) % m;
23b         y >>= 1;
cc5         x = (x * x) % m;
020     }
edf     return ret;
12b }

03c ll pow(ll x, ll y, ll m) { // recursivo
13a     if (!y) return 1;
426     ll ans = pow(x*x%m, y/2, m);
88d     return y%2 ? x*ans%m : ans;
7d4 }
```

1.13 Fast Walsh Hadamard Transform

```
// FWHT<'|'>(f) eh SOS DP
// FWHT<'&'>(f) eh soma de superset DP
// Se chamar com ^, usar tamanho potencia de 2!!
//
// O(n log(n))

382 template<char op, class T> vector<T> FWHT(vector<T> f, bool inv =
    false) {
b75     int n = f.size();
d78     for (int k = 0; (n-1)>>k; k++) for (int i = 0; i < n; i++) if
    (i>>k&1) {
29e         int j = i^(1<<k);
627         if (op == '^') f[j] += f[i], f[i] = f[j] - 2*f[i];
a38         if (op == '|') f[i] += (inv ? -1 : 1) * f[j];
93c         if (op == '&') f[j] += (inv ? -1 : 1) * f[i];
1bb     }
578     if (op == '^' and inv) for (auto& i : f) i /= n;
abe     return f;
50e }

// Generalizacao de FWHT de Xor
//
// Convolucao de soma mod B, usar tamanho potencia de B!!
// Precisa definir o tipo T e a raiz primitiva g
// satisfazendo g^b == g
//
// Se possivel, hardcodar a multiplicacao de matriz
// feita em cada iteracao faz ficar bem mais rapido
//
// O(n b log_b(n))

4fc template<class T>
811 vector<T> FWHT(vector<T> f, int b, T g, bool inv = false) {
b75     int n = f.size();

929     vector<T> w(b);
a7a     w[1] = g;
aab     for (int i = 2; i < b; i++) w[i] = w[i - 1] * g;
dec     w[0] = w[b - 1] * g;

d59     if (inv) reverse(w.begin() + 1, w.end());

c31     for (int pot = 1; pot < n; pot *= b) {
339         for (int i = 0; i < n; i++) if (!(i / pot % b)) {
4e1             vector<T> res(b);
c0c             for (int j = 0; j < b; j++) {
a41                 for (int k = 0; k < b; k++)
a32                     res[j] = res[j] + w[j * k % b] * f[i + k *
    pot];
9c8                 if (inv) res[j] = res[j] / b;
dc3             }
e0e             for (int j = 0; j < b; j++) f[i + j * pot] = res[j];
7f8         }
25e     }

abe     return f;
902 }

// Exemplos da FWHT Generalizada:
//
// mod 7, resposta mod 998244353:
// T = mint, g = 14553391
//
// mod 3, resposta cabe em um long long:
// T = array<ll, 2>, g = {0, 1};
//
// using T = array<ll, 2>;
    // T operator +(const T& a, const T& b) {
    //  return T{a[0] + b[0], a[1] + b[1]};
// }
    // T operator *(const T& a, const T& b) {
    //  return T{a[0] * b[0] - a[1] * b[1],
    //           a[0] * b[1] + a[1] * b[0] - a[1] * b[1]};
// };
    // T operator /(const T& a, const int& b) {
    //  return T{a[0] / b, a[1] / b};
// }
```

1.14 FFT

```
// Chamar convolution com vector<complex<double>> para FFT
// Precisa do mint para NTT
//
// O(n log(n))

// Para FFT
488 void get_roots(bool f, int n, vector<complex<double>>& roots) {
f26     const static double PI = acosl(-1);
71a     for (int i = 0; i < n/2; i++) {
b1e         double alpha = i*((2*PI)/n);
1a1         if (f) alpha = -alpha;
069         roots[i] = {cos(alpha), sin(alpha)};
804     }
de5 }

// Para NTT
9f7 template<int p>
97b void get_roots(bool f, int n, vector<mod_int<p>>& roots) {
1e6     mod_int<p> r;
de9     int ord;
57a     if (p == 998244353) {
9b6         r = 102292;
81b         ord = (1 << 23);
121     } else if (p == 754974721) {
43a         r = 739831874;
f0a         ord = (1 << 24);
d48     } else if (p == 167772161) {
a2a         r = 243;
033         ord = (1 << 25);
5a4     } else assert(false);

547     if (f) r = r^(p - 1 -ord/n);
ee2     else r = r^(ord/n);
be4     roots[0] = 1;
078     for (int i = 1; i < n/2; i++) roots[i] = roots[i-1]*r;
63f }

8a2 template<typename T> void fft(vector<T>& a, bool f, int N,
    vector<int>& rev) {
bc7     for (int i = 0; i < N; i++) if (i < rev[i]) swap(a[i],
    a[rev[i]]);
12b     int l, r, m;
cb4     vector<T> roots(N);
192     for (int n = 2; n <= N; n *= 2) {
0f4         get_roots(f, n, roots);
5dc         for (int pos = 0; pos < N; pos += n) {
432             l = pos + 0, r = pos + n/2, m = 0;
a88             while (m < n/2) {
297                 auto t = roots[m] * a[r];
254                 a[r] = a[l] - t;
b8f                 a[l] = a[l] + t;
2c9                 l++, r++, m++;
d89             }
1fd         }
185     }
235     if (f) {
1c5         auto invN = T(1) / T(N);
557         for (int i = 0; i < N; i++) a[i] = a[i] * invN;
256     }
1b1 }

bf5 template<typename T> vector<T> convolution(vector<T>& a,
    vector<T>& b) {
87a     vector<T> l(a.begin(), a.end()), r(b.begin(), b.end());
e0a     int N = l.size()+r.size()-1;
f03     int n = 1, log_n = 0;
0a4     while (n <= N) n *= 2, log_n++;
808     vector<int> rev(n);
603     for (int i = 0; i < n; i++) {
434         rev[i] = 0;
f44         for (int j = 0; j < log_n; j++) if (i>>j&1)
4ff             rev[i] |= 1 << (log_n-1-j);
256     }
143     assert(N <= n);
fa4     l.resize(n);
7e4     r.resize(n);
56e     fft(l, false, n, rev);
fcf     fft(r, false, n, rev);
917     for (int i = 0; i < n; i++) l[i] *= r[i];
88b     fft(l, true, n, rev);
5e1     l.resize(N);
792     return l;
bd6 }

// NTT
6c8 template<int p, typename T> vector<mod_int<p>> ntt(vector<T>& a,
    vector<T>& b) {
d52     vector<mod_int<p>> A(a.begin(), a.end()), B(b.begin(),
    b.end());
d29     return convolution(A, B);
3bf }

// Convolucao de inteiro
//
// Precisa do CRT
//
// Tabela de valores:
// [0,1]       - <int, 1>
// [-1e5, 1e5] - <ll, 2>
// [-1e9, 1e9] - <__int128, 3>
b3c template<typename T, int mods>
eec vector<T> int_convolution(vector<int>& a, vector<int>& b) {
fe8     static const int M1 = 998244353, M2 = 754974721, M3 =
    167772161;
bf5     auto c1 = ntt<M1>(a, b);
221     auto c2 = (mods >= 2 ? ntt<M2>(a, b) : vector<mod_int<M2>>());
f9b     auto c3 = (mods >= 3 ? ntt<M3>(a, b) : vector<mod_int<M3>>());

2da     vector<T> ans;
5c5     for (int i = 0; i < c1.size(); i++) {
c09         crt<T> at(c1[i].v, M1);
316         if (mods >= 2) at = at * crt<T>(c2[i].v, M2);
987         if (mods >= 3) at = at * crt<T>(c3[i].v, M3);
b2b         ans.push_back(at.a);
26d         if (at.a > at.m/2) ans.back() -= at.m;
b9f     }
ba7     return ans;
5e8 }
```

1.15 Gauss

```
// Resolve sistema linear
// Retornar um par com o numero de solucoes
// e alguma solucao, caso exista
//
// O(n^2 * m)

67a template<typename T>
728 pair<int, vector<T>> gauss(vector<vector<T>> a, vector<T> b) {
6ca     const double eps = 1e-6;
f92     int n = a.size(), m = a[0].size();
2f0     for (int i = 0; i < n; i++) a[i].push_back(b[i]);

3cb     vector<int> where(m, -1);
237     for (int col = 0, row = 0; col < m and row < n; col++) {
f05         int sel = row;
b95         for (int i=row; i<n; ++i)
e55             if (abs(a[i][col]) > abs(a[sel][col])) sel = i;
2c4         if (abs(a[sel][col]) < eps) continue;
1ae         for (int i = col; i <= m; i++)
dd2             swap(a[sel][i], a[row][i]);
2c3         where[col] = row;

0c0         for (int i = 0; i < n; i++) if (i != row) {
96c             T c = a[i][col] / a[row][col];
d5c             for (int j = col; j <= m; j++)
c8f                 a[i][j] -= a[row][j] * c;
490         }
b70         row++;
3d8     }
b1d     vector<T> ans(m, 0);
e1a     for (int i = 0; i < m; i++) if (where[i] != -1)
12a         ans[i] = a[where[i]][m] / a[where[i]][i];
603     for (int i = 0; i < n; i++) {
501         T sum = 0;
a75         for (int j = 0; j < m; j++)
5a9             sum += ans[j] * a[i][j];
b1f         if (abs(sum - a[i][m]) > eps)
6cd             return pair(0, vector<T>());
ec9     }

12e     for (int i = 0; i < m; i++) if (where[i] == -1)
018         return pair(INF, ans);
280     return pair(1, ans);
292 }
```

1.16 Gauss - Z2

```
// D eh dimensao do espaco vetorial
// add(v) - adiciona o vetor v na base (retorna se ele jah pertencia
    ao span da base)
// coord(v) - retorna as coordenadas (c) de v na base atual (basis^T.c
    = v)
// recover(v) - retorna as coordenadas de v nos vetores na ordem em
    que foram inseridos
// coord(v).first e recover(v).first - se v pertence ao span
//
// Complexidade:
// add, coord, recover: O(D^2 / 64)

cd4 template<int D> struct gauss_z2 {
3c1     bitset<D> basis[D], keep[D];
b16     int rk, in;
482     vector<int> id;

cf2     gauss_z2 () : rk(0), in(-1), id(D, -1) {};

04e     bool add(bitset<D> v) {
42c         in++;
fb0         bitset<D> k;
659         for (int i = D - 1; i >= 0; i--) if (v[i]) {
189             if (basis[i][i]) v ^= basis[i], k ^= keep[i];
4e6             else {
ea6                 k[i] = true, id[i] = in, keep[i] = k;
6ce                 basis[i] = v, rk++;
8a6                 return true;
b34             }
09c         }
d1f         return false;
58b     }
0f6     pair<bool, bitset<D>> coord(bitset<D> v) {
944         bitset<D> c;
659         for (int i = D - 1; i >= 0; i--) if (v[i]) {
a39             if (basis[i][i]) v ^= basis[i], c[i] = true;
8af             else return {false, bitset<D>()};
a08         }
5db         return {true, c};
a08     }
330     pair<bool, vector<int>> recover(bitset<D> v) {
22e         auto [span, bc] = coord(v);
af8         if (not span) return {false, {}};
f79         bitset<D> aux;
5a0         for (int i = D - 1; i >= 0; i--) if (bc[i]) aux ^= keep[i];
ea9         vector<int> oc;
ef2         for (int i = D - 1; i >= 0; i--) if (aux[i])
    oc.push_back(id[i]);
001         return {true, oc};
b75     }
688 };
```

1.17 Integracao Numerica

```
// Metodo de Simpson 3/8
// Integra f no intervalo [a, b], erro cresce proporcional a (b - a)^5

676 const int N = 3*100; // multiplo de 3
287 ld integrate(ld a, ld b, function<ld(ld)> f) {
b4d     ld s = 0, h = (b - a)/N;
067     for (int i = 1 ; i < N; i++) s += f(a + i*h)*(i%3 ? 3 : 2);
0da     return (f(a) + s + f(b))*3*h/8;
c7e }
```

1.18 Inverso Modular

```
// Computa o inverso de a modulo b
// Se b eh primo, basta fazer
// a^(b-2)

f0a ll inv(ll a, ll b) {
```

```
ae1     return a > 1 ? b - inv(b%a, a)*b/a : 1;
cf9 }

// computa o inverso modular de 1..MAX-1 modulo um primo
a88 ll inv[MAX]:
0f2 inv[1] = 1;
0fa for (int i = 2; i < MAX; i++) inv[i] = MOD - MOD/i*inv[MOD%i]%MOD;
```

1.19 Karatsuba

```
// Os pragmas podem ajudar
// Para n ~ 2e5, roda em < 1 s
//
// O(n^1.58)

//#pragma GCC optimize("Ofast")
//#pragma GCC target ("avx,avx2")
77a template<typename T> void kar(T* a, T* b, int n, T* r, T* tmp) {
d4c     if (n <= 64) {
510         for (int i = 0; i < n; i++) for (int j = 0; j < n; j++)
212             r[i+j] += a[i] * b[j];
505         return;
bb8     }
194     int mid = n/2;
2d7     T *atmp = tmp, *btmp = tmp+mid, *E = tmp+n;
4f1     memset(E, 0, sizeof(E[0])*n);
c65     for (int i = 0; i < mid; i++) {
c72         atmp[i] = a[i] + a[i+mid];
4b9         btmp[i] = b[i] + b[i+mid];
a3f     }
38a     kar(atmp, btmp, mid, E, tmp+2*n);
b1e     kar(a, b, mid, r, tmp+2*n);
229     kar(a+mid, b+mid, mid, r+n, tmp+2*n);
c65     for (int i = 0; i < mid; i++) {
735         T temp = r[i+mid];
de7         r[i+mid] += E[i] - r[i] - r[i+2*mid];
f1e         r[i+2*mid] += E[i+mid] - temp - r[i+3*mid];
f72     }
28f }

e38 template<typename T> vector<T> karatsuba(vector<T> a, vector<T> b)
    {
ba3     int n = max(a.size(), b.size());
a84     while (n&(n-1)) n++;
ca9     a.resize(n), b.resize(n);
ae0     vector<T> ret(2*n), tmp(4*n);
644     kar(&a[0], &b[0], n, &ret[0], &tmp[0]);
edf     return ret;
f87 }
```

1.20 Logaritmo Discreto

```
// Resolve logaritmo discreto com o algoritmo baby step giant step
// Encontra o menor x tal que a^x = b (mod m)
// Se nao tem, retorna -1
//
// O(sqrt(m) * log(sqrt(m))
d41
da8 int dlog(int b, int a, int m) {
9f8     if (a == 0) return b ? -1 : 1; // caso nao definido
d41
a6e     a %= m, b %= m;
a10     int k = 1, shift = 0;
31e     while (1) {
6e3         int g = gcd(a, m);
d47         if (g == 1) break;
d41
9bc         if (b == k) return shift;
642         if (b % g) return -1;
c36         b /= g, m /= g, shift++;
9ab         k = (ll) k * a / g % m;
515     }
d41
af7     int sq = sqrt(m)+1, giant = 1;
975     for (int i = 0; i < sq; i++) giant = (ll) giant * a % m;
d41
0b5     vector<pair<int, int>> baby;
33f     for (int i = 0, cur = b; i <= sq; i++) {
496         baby.emplace_back(cur, i);
16c         cur = (ll) cur * a % m;
622     }
eb4     sort(baby.begin(), baby.end());
d41
9c9     for (int j = 1, cur = k; j <= sq; j++) {
ace         cur = (ll) cur * giant % m;
78b         auto it = lower_bound(baby.begin(), baby.end(), pair(cur,
    INF));
d26         if (it != baby.begin() and (--it)->first == cur)
ac3             return sq * j - it->second + shift;
b9d     }
d41
daa     return -1;
739 }
```

1.21 Miller-Rabin

```
// Testa se n eh primo, n <= 3 * 10^18
//
// O(log(n)), considerando multiplicacao
// e exponenciacao constantes

d8b ll mul(ll a, ll b, ll m) {
e7a     ll ret = a*b - ll((long double)1/m*a*b+0.5)*m;
074     return ret < 0 ? ret+m : ret;
2f3 }

03c ll pow(ll x, ll y, ll m) {
13a     if (!y) return 1;
dbc     ll ans = pow(mul(x, x, m), y/2, m);
7fa     return y%2 ? mul(x, ans, m) : ans;
539 }

1a2 bool prime(ll n) {
1aa     if (n < 2) return 0;
237     if (n <= 3) return 1;
9de     if (n % 2 == 0) return 0;
f6a     ll r = __builtin_ctzll(n - 1), d = n >> r;

        // com esses primos, o teste funciona garantido para n <= 2^64
        // funciona para n <= 3*10^24 com os primos ate 41
dd1     for (int a : {2, 325, 9375, 28178, 450775, 9780504,
    1795265022}) {
da0         ll x = pow(a, d, n);
709         if (x == 1 or x == n - 1 or a % n == 0) continue;

4a2         for (int j = 0; j < r - 1; j++) {
10f             x = mul(x, x, n);
df0             if (x == n - 1) break;
1ff         }
e1b         if (x != n - 1) return 0;
5b0     }
6a5     return 1;
9a1 }
```

1.22 Multipoint Evaluation And Interpolation

```
// Evaluation:
//  Avalia o polinomio f(x) nos pontos p[0], p[1], ..., p[n-1]
//
// Interpolation:
//  Retorna o polinomio f(x) de grau n que
//  satisfaz f(x) = y pra o conjunto de pontos x, y
//
// Precisa do ntt e
// - do divmod pro evaluate
// - da derivada pro interpolate
//
// O divmod e a derivada estao no arquivo powerSeries.cpp
//
// O(n log^2(n))

55c namespace multipoint {
204     vector<poly> tree;
415     void build(vector<mint>& p) {
2fa         int n = p.size();
ab2         tree.resize(2*n);
f43         for (int i = 0; i < n; i++) tree[n + i] = {-p[i], 1};
917         for (int i = n - 1; i > 0; i--)
e64             tree[i] = convolution(tree[2*i], tree[2*i + 1]);
1f2     }
d21     vector<mint> evaluate(poly& f, vector<mint>& p) {
734         build(p);
2fa         int n = p.size();
305         vector<poly> ans(2 * n);
66a         ans[1] = divmod(f, tree[1]).second;
4cb         for (int i = 2; i < 2 * n; i++)
67d             ans[i] = divmod(ans[i/2], tree[i]).second;
512         vector<mint> results(n);
8f6         for (int i = 0; i < n; i++) results[i] = ans[n + i][0];
238         return results;
084     }
c25     poly prod(vector<mint>& p, int l, int r) {
8f7         if (l == r) return {-p[l], 1};
ee4         int m = (l + r) / 2;
6df         return convolution(prod(p, l, m), prod(p, m + 1, r));
b0b     }
ec4     poly interpolate(vector<mint>& x, vector<mint>& y) {
34a         int n = x.size();
12e         poly p = D(prod(x, 0, n - 1));
beb         auto d = evaluate(p, x);
305         vector<poly> ans(2 * n);
5d0         for (int i = 0; i < n; i++) ans[n + i] = {y[i] / d[i]};
0f2         for (int i = n - 1; i > 0; i--) {
```

```
ccd             poly p1 = convolution(tree[2*i], ans[2*i + 1]);
5c7             poly p2 = convolution(tree[2*i + 1], ans[2*i]);
6ea             ans[i] = p1;
9d8             for (int j = 0; j < p1.size(); j++) ans[i][j] += p2[j];
5df         }
887         return ans[1];
1a1     }
44e }
```

1.23 NTT

```
// Precisa do mint (primitivas de aritmetica modular)
//
// O(n log (n))

4e9 const int MOD = 998244353;
0f4 typedef mod_int<MOD> mint;

c4b void ntt(vector<mint>& a, bool rev) {
6f1     int n = a.size(); auto b = a;
479     assert(!(n&(n-1)));
513     mint g = 1;
459     while ((g^(MOD / 2)) == 1) g += 1;
574     if (rev) g = 1 / g;

e55     for (int step = n / 2; step; step /= 2) {
306         mint w = g^(MOD / (n / step)), wn = 1;
41e         for (int i = 0; i < n/2; i += step) {
c29             for (int j = 0; j < step; j++) {
673                 auto u = a[2 * i + j], v = wn * a[2 * i + j +
    step];
464                 b[i+j] = u + v; b[i + n/2 + j] = u - v;
09e             }
c39             wn = wn * w;
ade         }
257         swap(a, b);
c50     }
1bb     if (rev) {
b18         auto n1 = mint(1) / n;
b28         for (auto& x : a) x *= n1;
eaa     }
574 }

7c4 vector<mint> convolution(const vector<mint>& a, const
    vector<mint>& b) {
03a     vector<mint> l(a.begin(), a.end()), r(b.begin(), b.end());
d71     int N = l.size()+r.size()-1, n = 1;
73b     while (n <= N) n *= 2;
fa4     l.resize(n);
7e4     r.resize(n);
156     ntt(l, false);
557     ntt(r, false);
917     for (int i = 0; i < n; i++) l[i] *= r[i];
de9     ntt(l, true);
5e1     l.resize(N);
792     return l;
4bb }
```

1.24 Operacoes em Polinomios e Series de Potencias

```
// Precisa do NTT
// O exp nao foi bem testado
//
// Fonte:
    github.com/celiopassos/competitive-programming/blob/master/algorithms/
//
// D, I: O(n)
// inv, divmod, log e exp: O(n log(n))

0d8 using poly = vector<mint>;

c8d const int MAGIC = 512;

e47 poly D(poly p) {
cd1     if (p.empty()) return p;
73c     for (int i = 0; i + 1 < p.size(); i++)
9c7         p[i] = (i + 1) * p[i + 1];
087     p.pop_back();
74e     return p;
b66 }

62b poly I(poly p) {
2fa     int n = p.size();
ef6     p.push_back(0);
056     for (int i = n - 1; i >= 0; i--)
5fe         p[i + 1] = p[i] / (i + 1);
481     p[0] = 0;
74e     return p;
809 }

3ef poly inv(poly p) {
640     assert(!p.empty() && p[0] == 1);
253     poly q = {mint(1) / p[0]};
ee3     int n = p.size(), k = 1;
d20     while (k < n) {
539         k *= 2;
d93         q.resize(2 * k);
d01         ntt(q, false);
0ac         poly p0(2 * k);
f39         copy_n(p.begin(), min(k, n), p0.begin());
697         ntt(p0, false);
818         for (int i = 0; i < 2 * k; i++)
eef             q[i] *= 2 - p0[i] * q[i];
ff8         ntt(q, true);
afe         q.resize(k);
cd0     }
ba3     q.resize(n);
bef     return q;
60b }

18f pair<poly, poly> divslow(const poly& a, const poly& b) {
bea     poly q, r = a;
1d1     while (r.size() >= b.size()) {
d01         q.push_back(r.back() / b.back());
e4d         if (q.back() != 0)
c06             for (int i = 0; i < b.size(); i++)
7d9                 r.end()[-i-1] -= q.back() * b.end()[-i-1];
515         r.pop_back();
733     }
539     reverse(q.begin(), q.end());
442     return {q, r};
9b9 }

// retorna (q, r) : a(x) = b(x) * q(x) + r(x)
06b pair<poly, poly> divmod(const poly& a, const poly& b) {
884     if (a.size() < b.size()) return {{}, a};
4ee     if (max(b.size(), a.size() - b.size()) < MAGIC) return
    divslow(a, b);
c83     poly ra = poly(a.rbegin(), a.rend());
b35     poly rb = poly(b.rbegin(), b.rend());
b8b     int k = a.size() - b.size() + 1;
b8b     rb.resize(k);
864     poly irb = inv(move(rb)), q = convolution(ra, irb);
57b     q = poly(q.rend() - k, q.rend());
fe8     poly r = convolution(move(q), b);
f63     for (int i = 0; i < r.size(); i++) r[i] = a[i] - r[i];
46f     while (r.size() > 1 && r.back() == 0) r.pop_back();
442     return {q, r};
8af }

53b poly log(poly p) {
640     assert(!p.empty() && p[0] == 1);
2fa     int n = p.size();
983     auto d = D(p), i = inv(p);
25f     auto r = convolution(d, i);
7c9     r.resize(n - 1);
c7b     return I(move(r));
35c }

84d poly exp(poly p) {
380     assert(p.empty() || p[0] == 0);
5a8     poly q = {1};
ee3     int n = p.size(), k = 1;
d20     while (k < n) {
539         k *= 2;
afe         q.resize(k);
0a7         poly b = log(q);
2d3         for (int i = 0; i < k; i++) b[i] *= -1;
f89         b[0] += 1;
45d         for (int i = 0; i < min(n, k); i++) b[i] += p[i];
b0d         q = convolution(q, b);
afe         q.resize(k);
a3f     }
ba3     q.resize(n);
bef     return q;
f78 }
```

1.25 Pollard's Rho Alg

```
// Usa o algoritmo de deteccao de ciclo de Floyd
// com uma otimizacao na qual o gcd eh acumulado
// A fatoracao nao sai necessariamente ordenada
// O algoritmo rho encontra um fator de n,
// e funciona muito bem quando n possui um fator pequeno
//
// Complexidades (considerando mul constante):
// rho - esperado O(n^(1/4)) no pior caso
// fact - esperado menos que O(n^(1/4) log(n)) no pior caso

d8b ll mul(ll a, ll b, ll m) {
e7a     ll ret = a*b - ll((long double)1/m*a*b+0.5)*m;
074     return ret < 0 ? ret+m : ret;
2f3 }

03c ll pow(ll x, ll y, ll m) {
```

```
13a     if (!y) return 1;
dbc     ll ans = pow(mul(x, x, m), y/2, m);
7fa     return y%2 ? mul(x, ans, m) : ans;
539 }

1a2 bool prime(ll n) {
1aa     if (n < 2) return 0;
237     if (n <= 3) return 1;
9de     if (n % 2 == 0) return 0;

f6a     ll r = __builtin_ctzll(n - 1), d = n >> r;
dd1     for (int a : {2, 325, 9375, 28178, 450775, 9780504,
    1795265022}) {
da0         ll x = pow(a, d, n);
709         if (x == 1 or x == n - 1 or a % n == 0) continue;

4a2         for (int j = 0; j < r - 1; j++) {
10f             x = mul(x, x, n);
df0             if (x == n - 1) break;
1ff         }
e1b         if (x != n - 1) return 0;
5b0     }
6a5     return 1;
9a1 }

9cf ll rho(ll n) {
0f9     if (n == 1 or prime(n)) return n;
f7c     auto f = [n](ll x) {return mul(x, x, n) + 1;};

8a5     ll x = 0, y = 0, t = 30, prd = 2, x0 = 1, q;
533     while (t % 40 != 0 or gcd(prd, n) == 1) {
8a0         if (x==y) x = ++x0, y = f(x);
e13         q = mul(prd, abs(x-y), n);
21f         if (q != 0) prd = q;
450         x = f(x), y = f(f(y)), t++;
379     }
002     return gcd(prd, n);
523 }

5b7 vector<ll> fact(ll n) {
1b9     if (n == 1) return {};
0ec     if (prime(n)) return {n};
0ed     ll d = rho(n);
1de     vector<ll> l = fact(d), r = fact(n / d);
3af     l.insert(l.end(), r.begin(), r.end());
792     return l;
902 }
```

1.26 Produto de dois long long mod m

```
// O(1)

d8b ll mul(ll a, ll b, ll m) { // a*b % m
e7a     ll ret = a*b - ll((long double)1/m*a*b+0.5)*m;
074     return ret < 0 ? ret+m : ret;
2f3 }
```

1.27 Simplex

```
// Maximiza c^T x s.t. Ax <= b, x >= 0
//
// O(2^n), porem executa em O(n^3) no caso medio

395 const double eps = 1e-7;

493 namespace Simplex {
69c     vector<vector<double>> T;
14e     int n, m;
43e     vector<int> X, Y;

c51     void pivot(int x, int y) {
8e6         swap(X[y], Y[x-1]);
d03         for (int i = 0; i <= m; i++) if (i != y) T[x][i] /=
    T[x][y];
33c         T[x][y] = 1/T[x][y];
38b         for (int i = 0; i <= n; i++) if (i != x and abs(T[i][y]) >
    eps) {
774             for (int j = 0; j <= m; j++) if (j != y) T[i][j] -=
    T[i][y] * T[x][j];
3d8             T[i][y] = -T[i][y] * T[x][y];
a7d         }
e05     }

        // Retorna o par (valor maximo, vetor solucao)
6f8     pair<double, vector<double>> simplex(
e9d             vector<vector<double>> A, vector<double> b,
    vector<double> c) {
5bb         n = b.size(), m = c.size();
002         T = vector(n + 1, vector<double>(m + 1));
2d9         X = vector<int>(m);
0c2         Y = vector<int>(n);
115         for (int i = 0; i < m; i++) X[i] = i;
51f         for (int i = 0; i < n; i++) Y[i] = i+m;
5b5         for (int i = 0; i < m; i++) T[0][i] = -c[i];
603         for (int i = 0; i < n; i++) {
ba6             for (int j = 0; j < m; j++) T[i+1][j] = A[i][j];
eca             T[i+1][m] = b[i];
07c         }
667         while (true) {
714             int x = -1, y = -1;
2db             double mn = -eps;
c29             for (int i = 1; i <= n; i++) if (T[i][m] < mn) mn = T[i][m], x = i;
af2             if (x < 0) break;
882             for (int i = 0; i < m; i++) if (T[x][i] < -eps) { y = i; break; }

4a6             if (y < 0) return {-1e18, {}}; // sem solucao para  Ax <= b
7fb             pivot(x, y);
472         }
667         while (true) {
714             int x = -1, y = -1;
2db             double mn = -eps;
562             for (int i = 0; i < m; i++) if (T[0][i] < mn) mn = T[0][i], y = i;
9b0             if (y < 0) break;
034             mn = 1e200;
5af             for (int i = 1; i <= n; i++) if (T[i][y] > eps and T[i][m] / T[i][y] < mn)
48f                 mn = T[i][m] / T[i][y], x = i;

53b             if (x < 0) return {1e18, {}}; // c^T x eh ilimitado
7fb             pivot(x, y);
81e         }
290         vector<double> r(m);
32f         for(int i = 0; i < n; i++) if (Y[i] < m) r[Y[i]] = T[i+1][m];
e59         return {T[0][m], r};
7a4     }
a64 }
```

1.28 Teorema Chines do Resto

```
// Combina equacoes modulares lineares: x = a (mod m)
// O m final eh o lcm dos m's, e a resposta eh unica mod o lcm
// Os m nao precisam ser coprimos
// Se nao tiver solucao, o 'a' vai ser -1
```

```
153 template<typename T> tuple<T, T, T> ext_gcd(T a, T b) {
3bd     if (!a) return {b, 0, 1};
550     auto [g, x, y] = ext_gcd(b%a, a);
c59     return {g, y - b/a*x, x};
537 }

bfe template<typename T = ll> struct crt {
627     T a, m;

5f3     crt() : a(0), m(1) {}
7eb     crt(T a_, T m_) : a(a_), m(m_) {}
911     crt operator * (crt C) {
238         auto [g, x, y] = ext_gcd(m, C.m);
dc0         if ((a - C.a) % g) a = -1;
4f9         if (a == -1 or C.a == -1) return crt(-1, 0);
d09         T lcm = m/g*C.m;
eb2         T ans = a + (x*(C.a-a)/g % (C.m/g))*m;
d8d         return crt((ans % lcm + lcm) % lcm, lcm);
1f2     }
0d9 };
```

1.29 Totiente

```
// O(sqrt(n))

a7e int tot(int n) {
0f6     int ret = n;

505     for (int i = 2; i*i <= n; i++) if (n % i == 0) {
b0c         while (n % i == 0) n /= i;
125         ret -= ret / i;
34a     }
af4     if (n > 1) ret -= ret / n;

edf     return ret;
fae }
```

2 Primitivas

2.1 Aritmetica Modular

```
// O mod tem q ser primo
```

```
429 template<int p> struct mod_int {
c68     ll expo(ll b, ll e) {
c85         ll ret = 1;
c87         while (e) {
cad             if (e % 2) ret = ret * b % p;
9d2             e /= 2, b = b * b % p;
c42         }
edf         return ret;
734     }
1f6     ll inv(ll b) { return expo(b, p-2); }

4d7     using m = mod_int;
d93     int v;
fe0     mod_int() : v(0) {}
e12     mod_int(ll v_) {
019         if (v_ >= p or v_ <= -p) v_ %= p;
bc6         if (v_ < 0) v_ += p;
2e7         v = v_;
7f3     }
74d     m& operator +=(const m& a) {
2fd         v += a.v;
ba5         if (v >= p) v -= p;
357         return *this;
c8b     }
eff     m& operator -=(const m& a) {
8b4         v -= a.v;
cc8         if (v < 0) v += p;
357         return *this;
f8d     }
4c4     m& operator *=(const m& a) {
8a5         v = v * ll(a.v) % p;
357         return *this;
d4c     }
3f9     m& operator /=(const m& a) {
5d6         v = v * inv(a.v) % p;
357         return *this;
62d     }
d65     m operator -(){ return m(-v); }
b3e     m& operator ^=(ll e) {
06d         if (e < 0) {
6e2             v = inv(v);
00c             e = -e;
275         }
284         v = expo(v, e);
            // possivel otimizacao:
            // cuidado com 0^0
            // v = expo(v, e%(p-1));
357         return *this;
6ed     }
423     bool operator ==(const m& a) { return v == a.v; }
69f     bool operator !=(const m& a) { return v != a.v; }

1c6     friend istream& operator >>(istream& in, m& a) {
d1c         ll val; in >> val;
d48         a = m(val);
091         return in;
870     }
44f     friend ostream& operator <<(ostream& out, m a) {
5a0         return out << a.v;
214     }
399     friend m operator +(m a, m b) { return a += b; }
f9e     friend m operator -(m a, m b) { return a -= b; }
9c1     friend m operator *(m a, m b) { return a *= b; }
51b     friend m operator /(m a, m b) { return a /= b; }
08f     friend m operator ^(m a, ll e) { return a ^= e; }
b20 };

055 typedef mod_int<(int)1e9+7> mint;
```

2.2 Big Integer

```
// Complexidades: (para n digitos)
// Soma, subtracao, comparacao - O(n)
// Multiplicacao - O(n log(n))
// Divisao, resto - O(n^2)

864 struct bint {
669     static const int BASE = 1e9;
990     vector<int> v;
3bd     bool neg;

609     bint() : neg(0) {}
d53     bint(int val) : bint() { *this = val; }
e8f     bint(long long val) : bint() { *this = val; }

a0f     void trim() {
f42         while (v.size() and v.back() == 0) v.pop_back();
df8         if (!v.size()) neg = 0;
8e3     }

        // converter de/para string | cin/cout
294     bint(const char* s) : bint() { from_string(string(s)); }
```

```
548     bint(const string& s) : bint() { from_string(s); }
4ab     void from_string(const string& s) {
0a6         v.clear(), neg = 0;
d72         int ini = 0;
8e2         while (ini < s.size() and (s[ini] == '-' or s[ini] == '+' or s[ini] == '0'))
71d             if (s[ini++] == '-') neg = 1;
883         for (int i = s.size()-1; i >= ini; i -= 9) {
05e             int at = 0;
5b1             for (int j = max(ini, i - 8); j <= i; j++) at = 10*at + (s[j]-'0');
1fd             v.push_back(at);
a5a         }
df8         if (!v.size()) neg = 0;
e9a     }
2ff     string to_string() const {
8be         if (!v.size()) return "0";
793         string ret;
73e         if (neg) ret += '-';
3e9         for (int i = v.size()-1; i >= 0; i--) {
582             string at = ::to_string(v[i]);
ced             int add = 9 - at.size();
75e             if (i+1 < v.size()) for (int j = 0; j < add; j++) ret += '0';
f9f             ret += at;
f64         }
edf         return ret;
770     }
d2f     friend istream& operator>>(istream& in, bint& val) {
eb6         string s; in >> s;
966         val = s;
091         return in;
328     }
99d     friend ostream& operator<<(ostream& out, const bint& val) {
8b9         string s = val.to_string();
396         out << s;
fe8         return out;
ce1     }

        // operators
60a     friend bint abs(bint val) {
c5f         val.neg = 0;
d94         return val;
44b     }
bee     friend bint operator-(bint val) {
815         if (val != 0) val.neg ^= 1;
d94         return val;
326     }
41f     bint& operator=(const bint& val) { v = val.v, neg = val.neg; return *this; }
249     bint& operator=(long long val) {
0a6         v.clear(), neg = 0;
3a6         if (val < 0) neg = 1, val *= -1;
fdc         for (; val; val /= BASE) v.push_back(val % BASE);
357         return *this;
220     }
3bd     int cmp(const bint& r) const { // menor: -1 | igual: 0 | maior: 1
b14         if (neg != r.neg) return neg ? -1 : 1;
0bb         if (v.size() != r.v.size()) {
ff7             int ret = v.size() < r.v.size() ? -1 : 1;
91b             return neg ? -ret : ret;
1f6         }
478         for (int i = int(v.size())-1; i >= 0; i--) {
405             if (v[i] != r.v[i]) {
2e5                 int ret = v[i] < r.v[i] ? -1 : 1;
91b                 return neg ? -ret : ret;
9a9             }
c32         }
bb3         return 0;
07d     }
152     friend bool operator<(const bint& l, const bint& r) { return l.cmp(r) == -1; }
c7a     friend bool operator>(const bint& l, const bint& r) { return l.cmp(r) == 1; }
edd     friend bool operator<=(const bint& l, const bint& r) { return l.cmp(r) <= 0; }
954     friend bool operator>=(const bint& l, const bint& r) { return l.cmp(r) >= 0; }
a67     friend bool operator==(const bint& l, const bint& r) { return l.cmp(r) == 0; }
10b     friend bool operator!=(const bint& l, const bint& r) { return l.cmp(r) != 0; }

38e     bint& operator +=(const bint& r) {
6bf         if (!r.v.size()) return *this;
a93         if (neg != r.neg) return *this -= -r;
256         for (int i = 0, c = 0; i < r.v.size() or c; i++) {
e28             if (i == v.size()) v.push_back(0);
08f             v[i] += c + (i < r.v.size() ? r.v[i] : 0);
baa             if ((c = v[i] >= BASE)) v[i] -= BASE;
8bb         }
357         return *this;
ab1     }
```

```
54c     friend bint operator+(bint a, const bint& b) { return a += b; }
9c8     bint& operator -=(const bint& r) {
6bf         if (!r.v.size()) return *this;
524         if (neg != r.neg) return *this += -r;
358         if ((!neg and *this < r) or (neg and r < *this)) {
b10             *this = r - *this;
a10             neg ^= 1;
357             return *this;
807         }
256         for (int i = 0, c = 0; i < r.v.size() or c; i++) {
9ef             v[i] -= c + (i < r.v.size() ? r.v[i] : 0);
c8c             if ((c = v[i] < 0)) v[i] += BASE;
687         }
0eb         trim();
357         return *this;
f72     }
f44     friend bint operator-(bint a, const bint& b) { return a -= b; }

        // operators de * / %
6b0     bint& operator *=(int val) {
bca         if (val < 0) val *= -1, neg ^= 1;
566         for (int i = 0, c = 0; i < v.size() or c; i++) {
e28             if (i == v.size()) v.push_back(0);
352             long long at = (long long) v[i] * val + c;
6a3             v[i] = at % BASE;
b3d             c = at / BASE;
cb1         }
0eb         trim();
357         return *this;
a57     }
480     friend bint operator *(bint a, int b) { return a *= b; }
d5c     friend bint operator *(int a, bint b) { return b *= a; }
13b     using cplx = complex<double>;
bfb     void fft(vector<cplx>& a, bool f, int N, vector<int>& rev)
    const {
bc7         for (int i = 0; i < N; i++) if (i < rev[i]) swap(a[i],
    a[rev[i]]);
bad         vector<cplx> roots(N);
192         for (int n = 2; n <= N; n *= 2) {
4e9             const static double PI = acos(-1);
71a             for (int i = 0; i < n/2; i++) {
40d                 double alpha = (2*PI*i)/n;
1a1                 if (f) alpha = -alpha;
3f6                 roots[i] = cplx(cos(alpha), sin(alpha));
f16             }
3e9             for (int pos = 0; pos < N; pos += n)
898                 for (int l = pos, r = pos+n/2, m = 0; m < n/2;
    l++, r++, m++) {
297                     auto t = roots[m]*a[r];
254                     a[r] = a[l] - t;
b8f                     a[l] = a[l] + t;
b0d                 }
e07         }
3f1         if (!f) return;
08b         auto invN = cplx(1)/cplx(N);
873         for (int i = 0; i < N; i++) a[i] *= invN;
c75     }
0e0     vector<long long> convolution(const vector<int>& a, const
    vector<int>& b) const {
ff9         vector<cplx> l(a.begin(), a.end()), r(b.begin(), b.end());
996         int ln = l.size(), rn = r.size(), N = ln+rn+1, n = 1,
    log_n = 0;
821         while (n <= N) n <<= 1, log_n++;
808         vector<int> rev(n);
603         for (int i = 0; i < n; i++) {
434             rev[i] = 0;
f44             for (int j = 0; j < log_n; j++) if (i>>j&1)
4ff                 rev[i] |= 1 << (log_n-1-j);
256         }
230         l.resize(n), r.resize(n);
a89         fft(l, false, n, rev), fft(r, false, n, rev);
917         for (int i = 0; i < n; i++) l[i] *= r[i];
88b         fft(l, true, n, rev);
7ae         vector<long long> ret;
c14         for (auto& i : l) ret.push_back(round(i.real()));
edf         return ret;
917     }
633     vector<int> convert_base(const vector<int>& a, int from, int
    to) const {
498         static vector<long long> pot(10, 1);
671         if (pot[1] == 1) for (int i = 1; i < 10; i++) pot[i] =
    10*pot[i-1];
4b8         vector<int> ret;
156         long long at = 0;
608         int digits = 0;
941         for (int i : a) {
412             at += i * pot[digits];
035             digits += from;
684             while (digits >= to) {
0c8                 ret.push_back(at % pot[to]);
cf9                 at /= pot[to];
fd4                 digits -= to;
122             }
87b         }
944         ret.push_back(at);
384         while (ret.size() and ret.back() == 0) ret.pop_back();
edf         return ret;
090     }
edb     bint operator*(const bint& r) const { // O(n log(n))
2af         bint ret;
968         ret.neg = neg ^ r.neg;
d5d         auto conv = convolution(convert_base(v, 9, 4),
    convert_base(r.v, 9, 4));
a0e         long long c = 0;
a74         for (auto i : conv) {
f6d             long long at = i+c;
4cb             ret.v.push_back(at % 10000);
a25             c = at / 10000;
773         }
3cb         for (; c; c /= 10000) ret.v.push_back(c%10000);
0e2         ret.v = convert_base(ret.v, 4, 9);
25c         if (!ret.v.size()) ret.neg = 0;
edf         return ret;
c6b     }
359     bint& operator*=(const bint& r) { return *this = *this * r; };
9a3     bint& operator/=(int val) {
d9a         if (val < 0) neg ^= 1, val *= -1;
f18         for (int i = int(v.size())-1, c = 0; i >= 0; i--) {
2a7             long long at = v[i] + c * (long long) BASE;
e02             v[i] = at / val;
fb1             c = at % val;
fdb         }
0eb         trim();
357         return *this;
db6     }
e74     friend bint operator/(bint a, int b) { return a /= b; }
4a9     int operator %=(int val) {
23b         if (val < 0) val *= -1;
156         long long at = 0;
f31         for (int i = int(v.size())-1; i >= 0; i--)
1b3             at = (BASE * at + v[i]) % val;
d22         if (neg) at *= -1;
ce6         return at;
4b4     }
2fb     friend int operator%(bint a, int b) { return a %= b; }
13b     friend pair<bint, bint> divmod(const bint& a_, const bint& b_)
    { // O(n^2)
611         if (a_ == 0) return {0, 0};
d8a         int norm = BASE / (b_.v.back() + 1);
b4e         bint a = abs(a_) * norm;
027         bint b = abs(b_) * norm;
14d         bint q, r;
c91         for (int i = a.v.size() - 1; i >= 0; i--) {
b71             r *= BASE, r += a.v[i];
4ff             long long upper = b.v.size() < r.v.size() ?
    r.v[b.v.size()] : 0;
86d             int lower = b.v.size() - 1 < r.v.size() ?
    r.v[b.v.size() - 1] : 0;
431             int d = (upper * BASE + lower) / b.v.back();
5d4             r -= b*d;
30f             while (r < 0) r += b, d--; // roda O(1) vezes
738             q.v.push_back(d);
c6a         }
a48         reverse(q.v.begin(), q.v.end());
ae2         q.neg = a_.neg ^ b_.neg;
88b         r.neg = a_.neg;
8e5         q.trim(), r.trim();
0ef         return {q, r / norm};
4fd     }
1d8     bint operator/(const bint& val) { return divmod(*this,
    val).first; }
7f9     bint& operator/=(const bint& val) { return *this = *this /
    val; }
1f9     bint operator%(const bint& val) { return divmod(*this,
    val).second; }
df5     bint& operator%=(const bint& val) { return *this = *this %
    val; }
6c3 };
```

2.3 Calendario

```
// Congruencia de Zeller
//
// Os dias da semana correspondem aos restos % 7
// Segunda=0, Terca=1, ..., Domingo=6

74e int get_id(int d, int m, int y) {
c5d     if (m < 3) y--, m += 12;
0ab     return 365 * y + y / 4 - y / 100 + y / 400 + (153 * (m - 3) +
    2) / 5 + d - 307;
ff5 }

ade tuple<int, int, int> date(int id) {
99a     int x = id + 1789995, n = 4 * x / 146097, i, j, d, m, y;
33e     x -= (146097 * n + 3) / 4;
6fc     i = (4000 * (x + 1)) / 1461001;
b1d     x -= 1461 * i / 4 - 31;
dd0     j = 80 * x / 2447, d = x - 2447 * j / 80;
179     x = j / 11;
e85     m = j + 2 - 12 * x, y = 100 * (n - 49) + i + x;
b86     return {d, m, y};
0d9 }
```

2.4 Fracao

```
// Funciona com o Big Int

a4e template<typename T = int> struct frac {
a40     T num, den;
e3f     template<class U, class V>
61d     frac(U num_ = 0, V den_ = 1) : num(num_), den(den_) {
bad         assert(den != 0);
583         if (den < 0) num *= -1, den *= -1;
a51         T g = gcd(abs(num), den);
572         num /= g, den /= g;
fbf     }

51f     friend bool operator<(const frac& l, const frac& r) {
fa0         return l.num * r.den < r.num * l.den;
a4e     }
4b5     friend frac operator+(const frac& l, const frac& r) {
b61         return {l.num*r.den + l.den*r.num, l.den*r.den};
25f     }
74d     friend frac operator-(const frac& l, const frac& r) {
2cd         return {l.num*r.den - l.den*r.num, l.den*r.den};
8a7     }
c80     friend frac operator*(const frac& l, const frac& r) {
510         return {l.num*r.num, l.den*r.den};
14b     }
a1b     friend frac operator/(const frac& l, const frac& r) {
8f3         return {l.num*r.den, l.den*r.num};
b2c     }
012     friend ostream& operator<<(ostream& out, frac f) {
37a         out << f.num << '/' << f.den;
fe8         return out;
b49     }
cdb };
```

2.5 Geometria

```
c83 typedef double ld;
e3b const ld DINF = 1e18;
43a const ld pi = acos(-1.0);
107 const ld eps = 1e-9;

b32 #define sq(x) ((x)*(x))

d97 bool eq(ld a, ld b) {
ba0     return abs(a - b) <= eps;
bfc }

b2a struct pt { // ponto
c1e     ld x, y;
3dd     pt(ld x_ = 0, ld y_ = 0) : x(x_), y(y_) {}
5bc     bool operator < (const pt p) const {
059         if (!eq(x, p.x)) return x < p.x;
f98         if (!eq(y, p.y)) return y < p.y;
bb3         return 0;
f61     }
a83     bool operator == (const pt p) const {
ed0         return eq(x, p.x) and eq(y, p.y);
589     }
cb9     pt operator + (const pt p) const { return pt(x+p.x, y+p.y); }
a24     pt operator - (const pt p) const { return pt(x-p.x, y-p.y); }
4a8     pt operator * (const ld c) const { return pt(x*c  , y*c  ); }
a60     pt operator / (const ld c) const { return pt(x/c  , y/c  ); }
3b6     ld operator * (const pt p) const { return x*p.x + y*p.y; }
6df     ld operator ^ (const pt p) const { return x*p.y - y*p.x; }
5ed     friend istream& operator >> (istream& in, pt& p) {
e37         return in >> p.x >> p.y;
e45     }
a8b };

b3a struct line { // reta
730     pt p, q;
0d6     line() {}
4b8     line(pt p_, pt q_) : p(p_), q(q_) {}
8d7     friend istream& operator >> (istream& in, line& r) {
4cb         return in >> r.p >> r.q;
858     }
7ab };

// PONTO & VETOR

364 ld dist(pt p, pt q) { // distancia
5f3     return hypot(p.y - q.y, p.x - q.x);
c68 }
```

```
9d7 ld dist2(pt p, pt q) { // quadrado da distancia
f24     return sq(p.x - q.x) + sq(p.y - q.y);
80f }

483 ld norm(pt v) { // norma do vetor
490     return dist(pt(0, 0), v);
cf7 }

589 ld angle(pt v) { // angulo do vetor com o eixo x
587     ld ang = atan2(v.y, v.x);
6f8     if (ang < 0) ang += 2*pi;
19c     return ang;
404 }

298 ld sarea(pt p, pt q, pt r) { // area com sinal
606     return ((q-p)^(r-q))/2;
1b1 }

e32 bool col(pt p, pt q, pt r) { // se p, q e r sao colin.
e7d     return eq(sarea(p, q, r), 0);
98c }

0cd bool ccw(pt p, pt q, pt r) { // se p, q, r sao ccw
fa7     return sarea(p, q, r) > eps;
85d }

1ef pt rotate(pt p, ld th) { // rotaciona o ponto th radianos
e5c     return pt(p.x * cos(th) - p.y * sin(th),
ff1             p.x * sin(th) + p.y * cos(th));
41a }

ab1 pt rotate90(pt p) { // rotaciona 90 graus
a0d     return pt(-p.y, p.x);
e4a }

// RETA

edc bool isvert(line r) { // se r eh vertical
87d     return eq(r.p.x, r.q.x);
0fb }

099 bool isinseg(pt p, line r) { // se p pertence ao seg de r
f65     pt a = r.p - p, b = r.q - p;
b04     return eq((a ^ b), 0) and (a * b) < eps;
726 }
```

```
98d ld get_t(pt v, line r) { // retorna t tal que t*v pertence a reta r
6ee     return (r.p^r.q) / ((r.p-r.q)^v);
a0a }

256 pt proj(pt p, line r) { // projecao do ponto p na reta r
bea     if (r.p == r.q) return r.p;
97a     r.q = r.q - r.p; p = p - r.p;
9f8     pt proj = r.q * ((p*r.q) / (r.q*r.q));
2cd     return proj + r.p;
232 }

d5c pt inter(line r, line s) { // r inter s
146     if (eq((r.p - r.q) ^ (s.p - s.q), 0)) return pt(DINF, DINF);
205     r.q = r.q - r.p, s.p = s.p - r.p, s.q = s.q - r.p;
543     return r.q * get_t(r.q, s) + r.p;
111 }

676 bool interseg(line r, line s) { // se o seg de r intersecta o seg
    de s
19b     if (isinseg(r.p, s) or isinseg(r.q, s)
c21         or isinseg(s.p, r) or isinseg(s.q, r)) return 1;

9fa     return ccw(r.p, r.q, s.p) != ccw(r.p, r.q, s.q) and
413             ccw(s.p, s.q, r.p) != ccw(s.p, s.q, r.q);
359 }

fcb ld disttoline(pt p, line r) { // distancia do ponto a reta
89a     return 2 * abs(sarea(p, r.p, r.q)) / dist(r.p, r.q);
1b7 }

bcc ld disttoseg(pt p, line r) { // distancia do ponto ao seg
73d     if ((r.q - r.p)*(p - r.p) < 0) return dist(r.p, p);
951     if ((r.p - r.q)*(p - r.q) < 0) return dist(r.q, p);
a19     return disttoline(p, r);
367 }

11d ld distseg(line a, line b) { // distancia entre seg
4df     if (interseg(a, b)) return 0;

349     ld ret = DINF;
341     ret = min(ret, disttoseg(a.p, b));
ceb     ret = min(ret, disttoseg(a.q, b));
093     ret = min(ret, disttoseg(b.p, a));
448     ret = min(ret, disttoseg(b.q, a));

edf     return ret;
222 }
```

```
// POLIGONO

// corta poligono com a reta r deixando os pontos p tal que
// ccw(r.p, r.q, p)
1a9 vector<pt> cut_polygon(vector<pt> v, line r) { // O(n)
8af     vector<pt> ret;
8a4     for (int j = 0; j < v.size(); j++) {
dac         if (ccw(r.p, r.q, v[j])) ret.push_back(v[j]);
dce         if (v.size() == 1) continue;
030         line s(v[j], v[(j+1)%v.size()]);
ae3         pt p = inter(r, s);
a3d         if (isinseg(p, s)) ret.push_back(p);
d44     }
8a1     ret.erase(unique(ret.begin(), ret.end()), ret.end());
24d     if (ret.size() > 1 and ret.back() == ret[0]) ret.pop_back();
edf     return ret;
253 }

// distancia entre os retangulos a e b (lados paralelos aos eixos)
// assume que ta representado (inferior esquerdo, superior direito)
5f5 ld dist_rect(pair<pt, pt> a, pair<pt, pt> b) {
080     ld hor = 0, vert = 0;
34b     if (a.second.x < b.first.x) hor = b.first.x - a.second.x;
f5f     else if (b.second.x < a.first.x) hor = a.first.x - b.second.x;
4fd     if (a.second.y < b.first.y) vert = b.first.y - a.second.y;
80a     else if (b.second.y < a.first.y) vert = a.first.y - b.second.y;
96f     return dist(pt(0, 0), pt(hor, vert));
630 }

13d ld polarea(vector<pt> v) { // area do poligono
9c5     ld ret = 0;
c6e     for (int i = 0; i < v.size(); i++)
80f         ret += sarea(pt(0, 0), v[i], v[(i + 1) % v.size()]);
d03     return abs(ret);
5df }

// se o ponto ta dentro do poligono: retorna 0 se ta fora,
// 1 se ta no interior e 2 se ta na borda
8e7 int inpol(vector<pt>& v, pt p) { // O(n)
8de     int qt = 0;
f14     for (int i = 0; i < v.size(); i++) {
bda         if (p == v[i]) return 2;
6af         int j = (i+1)%v.size();
e38         if (eq(p.y, v[i].y) and eq(p.y, v[j].y)) {
97f             if ((v[i]-p)*(v[j]-p) < eps) return 2;
5e2             continue;
48b         }
388         bool baixo = v[i].y+eps < p.y;
464         if (baixo == (v[j].y+eps < p.y)) continue;
366         auto t = (p-v[i])^(v[j]-v[i]);
1b4         if (eq(t, 0)) return 2;
839         if (baixo == (t > eps)) qt += baixo ? 1 : -1;
d13     }
b84     return qt != 0;
a64 }

6ff bool interpol(vector<pt> v1, vector<pt> v2) { // se dois poligonos
    se intersectam - O(n*m)
7d1     int n = v1.size(), m = v2.size();
c36     for (int i = 0; i < n; i++) if (inpol(v2, v1[i])) return 1;
ab8     for (int i = 0; i < n; i++) if (inpol(v1, v2[i])) return 1;
523     for (int i = 0; i < n; i++) for (int j = 0; j < m; j++)
0c8         if (interseg(line(v1[i], v1[(i+1)%n]), line(v2[j],
    v2[(j+1)%m]))) return 1;
bb3     return 0;
c58 }

494 ld distpol(vector<pt> v1, vector<pt> v2) { // distancia entre
    poligonos
f6b     if (interpol(v1, v2)) return 0;

349     ld ret = DINF;

1c8     for (int i = 0; i < v1.size(); i++) for (int j = 0; j <
    v2.size(); j++)
6c2         ret = min(ret, distseg(line(v1[i], v1[(i + 1) %
    v1.size()]),
9d9                         line(v2[j], v2[(j + 1) % v2.size()])));
edf     return ret;
125 }

138 vector<pt> convex_hull(vector<pt> v) { // convex hull - O(n log(n))
fca     sort(v.begin(), v.end());
d76     v.erase(unique(v.begin(), v.end()), v.end());
52d     if (v.size() <= 1) return v;
526     vector<pt> l, u;
f14     for (int i = 0; i < v.size(); i++) {
fb2         while (l.size() > 1 and !ccw(l.end()[-2], l.end()[-1],
    v[i]))
364             l.pop_back();
c35         l.push_back(v[i]);
58e     }
3e9     for (int i = v.size() - 1; i >= 0; i--) {
```

```
f19         while (u.size() > 1 and !ccw(u.end()[-2], u.end()[-1],
    v[i]))
7a8             u.pop_back();
a95         u.push_back(v[i]);
0b8     }
cfc     l.pop_back(); u.pop_back();
82b     for (pt i : u) l.push_back(i);
792     return l;
10d }

483 struct convex_pol {
f50     vector<pt> pol;

        // nao pode ter ponto colinear no convex hull
d98     convex_pol() {}
a04     convex_pol(vector<pt> v) : pol(convex_hull(v)) {}

        // se o ponto ta dentro do hull - O(log(n))
8af     bool is_inside(pt p) {
b6e         if (pol.size() == 0) return false;
eae         if (pol.size() == 1) return p == pol[0];
67f         int l = 1, r = pol.size();
40c         while (l < r) {
ee4             int m = (l+r)/2;
48f             if (ccw(p, pol[0], pol[m])) l = m+1;
ef3             else r = m;
91c         }
00a         if (l == 1) return isinseg(p, line(pol[0], pol[1]));
9e7         if (l == pol.size()) return false;
1c0         return !ccw(p, pol[l], pol[l-1]);
6b0     }
        // ponto extremo em relacao a cmp(p, q) = p mais extremo q
        // (copiado de https://github.com/gustavoM32/caderno-zika)
719     int extreme(const function<bool(pt, pt)>& cmp) {
b1c         int n = pol.size();
4a2         auto extr = [&](int i, bool& cur_dir) {
22a             cur_dir = cmp(pol[(i+1)%n], pol[i]);
61a             return !cur_dir and !cmp(pol[(i+n-1)%n], pol[i]);
364         };
63d         bool last_dir, cur_dir;
a0d         if (extr(0, last_dir)) return 0;
993         int l = 0, r = n;
ead         while (l+1 < r) {
ee4             int m = (l+r)/2;
f29             if (extr(m, cur_dir)) return m;
44a             bool rel_dir = cmp(pol[m], pol[l]);
b18             if ((!last_dir and cur_dir) or
261                     (last_dir == cur_dir and rel_dir == cur_dir)) {
8a6                 l = m;
1f1                 last_dir = cur_dir;
94a             } else r = m;
606         }
792         return l;
56c     }
316     int max_dot(pt v) {
ec1         return extreme([&](pt p, pt q) { return p*v > q*v; });
3b7     }
a54     pair<int, int> tangents(pt p) {
ffb         auto L = [&](pt q, pt r) { return ccw(p, r, q); };
8fd         auto R = [&](pt q, pt r) { return ccw(p, q, r); };
fa8         return {extreme(L), extreme(R)};
736     }
3ec };

// CIRCUNFERENCIA

911 pt getcenter(pt a, pt b, pt c) { // centro da circunf dado 3 pontos
174     b = (a + b) / 2;
2ae     c = (a + c) / 2;
98b     return inter(line(b, b + rotate90(a - b)),
3f8             line(c, c + rotate90(a - c)));
a12 }

4b3 vector<pt> circ_line_inter(pt a, pt b, pt c, ld r) { // intersecao
    da circunf (c, r) e reta ab
8af     vector<pt> ret;
f2b     b = b-a, a = a-c;
4b1     ld A = b*b;
20a     ld B = a*b;
2e9     ld C = a*a - r*r;
1fa     ld D = B*B - A*C;
818     if (D < -eps) return ret;
dc5     ret.push_back(c+a+b*(-B+sqrt(D+eps))/A);
20e     if (D > eps) ret.push_back(c+a+b*(-B-sqrt(D))/A);
edf     return ret;
cd8 }

ad2 vector<pt> circ_inter(pt a, pt b, ld r, ld R) { // intersecao da
    circunf (a, r) e (b, R)
8af     vector<pt> ret;
b7e     ld d = dist(a, b);
5ce     if (d > r+R or d+min(r, R) < max(r, R)) return ret;
398     ld x = (d*d-R*R+r*r)/(2*d);
183     ld y = sqrt(r*r-x*x);
```

```
325     pt v = (b-a)/d;
76e     ret.push_back(a+v*x + rotate90(v)*y);
2cb     if (y > 0) ret.push_back(a+v*x - rotate90(v)*y);
edf     return ret;
fb1 }

6e0 bool operator <(const line& a, const line& b) { // comparador pra reta
        // assume que as retas tem p < q
a13     pt v1 = a.q - a.p, v2 = b.q - b.p;
f82     if (!eq(angle(v1), angle(v2))) return angle(v1) < angle(v2);
780     return ccw(a.p, a.q, b.p); // mesmo angulo
27e }
b14 bool operator ==(const line& a, const line& b) {
76c     return !(a < b) and !(b < a);
449 }

// comparador pro set pra fazer sweep line com segmentos
2c4 struct cmp_sweepline {
d80     bool operator () (const line& a, const line& b) const {
            // assume que os segmentos tem p < q
191         if (a.p == b.p) return ccw(a.p, a.q, b.q);
231         if (!eq(a.p.x, a.q.x) and (eq(b.p.x, b.q.x) or a.p.x+eps < b.p.x))
780             return ccw(a.p, a.q, b.p);
dc0         return ccw(a.p, b.q, b.p);
243     }
367 };

// comparador pro set pra fazer sweep angle com segmentos
bef pt dir;
5b0 struct cmp_sweepangle {
d80     bool operator () (const line& a, const line& b) const {
522         return get_t(dir, a) + eps < get_t(dir, b);
653     }
97f };
```

2.6 Geometria - inteiro

```
2de #define sq(x) ((x)*(ll)(x))

b2a struct pt { // ponto
e91     int x, y;
df1     pt(int x_ = 0, int y_ = 0) : x(x_), y(y_) {}
5bc     bool operator < (const pt p) const {
95a         if (x != p.x) return x < p.x;
89c         return y < p.y;
dcd     }
a83     bool operator == (const pt p) const {
d74         return x == p.x and y == p.y;
7b4     }
cb9     pt operator + (const pt p) const { return pt(x+p.x, y+p.y); }
a24     pt operator - (const pt p) const { return pt(x-p.x, y-p.y); }
0ef     pt operator * (const int c) const { return pt(x*c, y*c); }
60d     ll operator * (const pt p) const { return x*(ll)p.x + y*(ll)p.y; }
d86     ll operator ^ (const pt p) const { return x*(ll)p.y - y*(ll)p.x; }
5ed     friend istream& operator >> (istream& in, pt& p) {
e37         return in >> p.x >> p.y;
e45     }
840 };

b3a struct line { // reta
730     pt p, q;
0d6     line() {}
4b8     line(pt p_, pt q_) : p(p_), q(q_) {}
8d7     friend istream& operator >> (istream& in, line& r) {
4cb         return in >> r.p >> r.q;
858     }
7ab };

// PONTO & VETOR

ea8 ll dist2(pt p, pt q) { // quadrado da distancia
f24     return sq(p.x - q.x) + sq(p.y - q.y);
515 }

5a2 ll sarea2(pt p, pt q, pt r) { // 2 * area com sinal
586     return (q-p)^(r-q);
bf4 }

e32 bool col(pt p, pt q, pt r) { // se p, q e r sao colin.
034     return sarea2(p, q, r) == 0;
a08 }

0cd bool ccw(pt p, pt q, pt r) { // se p, q, r sao ccw
276     return sarea2(p, q, r) > 0;
42b }

c31 int quad(pt p) { // quadrante de um ponto
dbb     return (p.x<0)^3*(p.y<0);
fcf }
```

```
2df bool compare_angle(pt p, pt q) { // retorna se ang(p) < ang(q)
9fc     if (quad(p) != quad(q)) return quad(p) < quad(q);
ea1     return ccw(q, pt(0, 0), p);
771 }

ab1 pt rotate90(pt p) { // rotaciona 90 graus
a0d     return pt(-p.y, p.x);
e4a }

// RETA

099 bool isinseg(pt p, line r) { // se p pertence ao seg de r
f65     pt a = r.p - p, b = r.q - p;
2ac     return (a ^ b) == 0 and (a * b) <= 0;
c9f }

676 bool interseg(line r, line s) { // se o seg de r intersecta o seg
    de s
19b     if (isinseg(r.p, s) or isinseg(r.q, s)
c21         or isinseg(s.p, r) or isinseg(s.q, r)) return 1;

9fa     return ccw(r.p, r.q, s.p) != ccw(r.p, r.q, s.q) and
413             ccw(s.p, s.q, r.p) != ccw(s.p, s.q, r.q);
359 }

9e0 int segpoints(line r) { // numero de pontos inteiros no segmento
9ce     return 1 + __gcd(abs(r.p.x - r.q.x), abs(r.p.y - r.q.y));
dd8 }

88a double get_t(pt v, line r) { // retorna t tal que t*v pertence a
    reta r
1ad     return (r.p^r.q) / (double) ((r.p-r.q)^v);
d27 }

// POLIGONO

// quadrado da distancia entre os retangulos a e b (lados paralelos
    aos eixos)
// assume que ta representado (inferior esquerdo, superior direito)
485 ll dist2_rect(pair<pt, pt> a, pair<pt, pt> b) {
c59     int hor = 0, vert = 0;
34b     if (a.second.x < b.first.x) hor = b.first.x - a.second.x;
f5f     else if (b.second.x < a.first.x) hor = a.first.x - b.second.x;
4fd     if (a.second.y < b.first.y) vert = b.first.y - a.second.y;
80a     else if (b.second.y < a.first.y) vert = a.first.y - b.second.y;
869     return sq(hor) + sq(vert);
e13 }

9c3 ll polarea2(vector<pt> v) { // 2 * area do poligono
b73     ll ret = 0;
c6e     for (int i = 0; i < v.size(); i++)
532         ret += sarea2(pt(0, 0), v[i], v[(i + 1) % v.size()]);
d03     return abs(ret);
d5f }

// se o ponto ta dentro do poligono: retorna 0 se ta fora,
// 1 se ta no interior e 2 se ta na borda
8e7 int inpol(vector<pt>& v, pt p) { // O(n)
8de     int qt = 0;
f14     for (int i = 0; i < v.size(); i++) {
bda         if (p == v[i]) return 2;
6af         int j = (i+1)%v.size();
cc6         if (p.y == v[i].y and p.y == v[j].y) {
547             if ((v[i]-p)*(v[j]-p) <= 0) return 2;
5e2             continue;
b47         }
78c         bool baixo = v[i].y < p.y;
057         if (baixo == (v[j].y < p.y)) continue;
366         auto t = (p-v[i])^(v[j]-v[i]);
2ad         if (!t) return 2;
0bb         if (baixo == (t > 0)) qt += baixo ? 1 : -1;
9cf     }
b84     return qt != 0;
afd }

138 vector<pt> convex_hull(vector<pt> v) { // convex hull - O(n log(n))
fca     sort(v.begin(), v.end());
d76     v.erase(unique(v.begin(), v.end()), v.end());
52d     if (v.size() <= 1) return v;
526     vector<pt> l, u;
f14     for (int i = 0; i < v.size(); i++) {
fb2         while (l.size() > 1 and !ccw(l.end()[-2], l.end()[-1],
    v[i]))
364             l.pop_back();
c35         l.push_back(v[i]);
58e     }
3e9     for (int i = v.size() - 1; i >= 0; i--) {
f19         while (u.size() > 1 and !ccw(u.end()[-2], u.end()[-1],
    v[i]))
7a8             u.pop_back();
a95         u.push_back(v[i]);
0b8     }
cfc     l.pop_back(); u.pop_back();
```

```
82b     for (pt i : u) l.push_back(i);
792     return l;
10d }

786 ll interior_points(vector<pt> v) { // pontos inteiros dentro de um
    poligono simples
c4e     ll b = 0;
c6e     for (int i = 0; i < v.size(); i++)
0ce         b += segpoints(line(v[i], v[(i+1)%v.size()])) - 1;
a1c     return (polarea2(v) - b) / 2 + 1;
af2 }

483 struct convex_pol {
f50     vector<pt> pol;

        // nao pode ter ponto colinear no convex hull
d98     convex_pol() {}
a04     convex_pol(vector<pt> v) : pol(convex_hull(v)) {}

        // se o ponto ta dentro do hull - O(log(n))
8af     bool is_inside(pt p) {
b6e         if (pol.size() == 0) return false;
eae         if (pol.size() == 1) return p == pol[0];
67f         int l = 1, r = pol.size();
40c         while (l < r) {
ee4             int m = (l+r)/2;
48f             if (ccw(p, pol[0], pol[m])) l = m+1;
ef3             else r = m;
91c         }
00a         if (l == 1) return isinseg(p, line(pol[0], pol[1]));
9e7         if (l == pol.size()) return false;
1c0         return !ccw(p, pol[l], pol[l-1]);
6b0     }
        // ponto extremo em relacao a cmp(p, q) = p mais extremo q
        // (copiado de https://github.com/gustavoM32/caderno-zika)
719     int extreme(const function<bool(pt, pt)>& cmp) {
b1c         int n = pol.size();
4a2         auto extr = [&](int i, bool& cur_dir) {
22a             cur_dir = cmp(pol[(i+1)%n], pol[i]);
61a             return !cur_dir and !cmp(pol[(i+n-1)%n], pol[i]);
364         };
63d         bool last_dir, cur_dir;
a0d         if (extr(0, last_dir)) return 0;
993         int l = 0, r = n;
ead         while (l+1 < r) {
ee4             int m = (l+r)/2;
f29             if (extr(m, cur_dir)) return m;
44a             bool rel_dir = cmp(pol[m], pol[l]);
b18             if ((!last_dir and cur_dir) or
261                     (last_dir == cur_dir and rel_dir == cur_dir)) {
8a6                 l = m;
1f1                 last_dir = cur_dir;
94a             } else r = m;
606         }
792         return l;
56c     }
316     int max_dot(pt v) {
ec1         return extreme([&](pt p, pt q) { return p*v > q*v; });
3b7     }
a54     pair<int, int> tangents(pt p) {
ffb         auto L = [&](pt q, pt r) { return ccw(p, r, q); };
8fd         auto R = [&](pt q, pt r) { return ccw(p, q, r); };
fa8         return {extreme(L), extreme(R)};
736     }
3ec };

6e0 bool operator <(const line& a, const line& b) { // comparador pra
    reta
        // assume que as retas tem p < q
a13     pt v1 = a.q - a.p, v2 = b.q - b.p;
036     bool b1 = compare_angle(v1, v2), b2 = compare_angle(v2, v1);
73c     if (b1 or b2) return b1;
780     return ccw(a.p, a.q, b.p); // mesmo angulo
b61 }
b14 bool operator ==(const line& a, const line& b) {
76c     return !(a < b) and !(b < a);
449 }

// comparador pro set pra fazer sweep line com segmentos
2c4 struct cmp_sweepline {
d80     bool operator () (const line& a, const line& b) const {
            // assume que os segmentos tem p < q
191         if (a.p == b.p) return ccw(a.p, a.q, b.q);
614         if (a.p.x != a.q.x and (b.p.x == b.q.x or a.p.x < b.p.x))
780             return ccw(a.p, a.q, b.p);
dc0         return ccw(a.p, b.q, b.p);
baf     }
677 };

// comparador pro set pra fazer sweep angle com segmentos
bef pt dir;
5b0 struct cmp_sweepangle {
d80     bool operator () (const line& a, const line& b) const {
261         return get_t(dir, a) < get_t(dir, b);
dc5         }
f6d };
```

2.7 Geometria 3D

```
c83 typedef double ld;
e3b const ld DINF = 1e18;
107 const ld eps = 1e-9;

b32 #define sq(x) ((x)*(x))

d97 bool eq(ld a, ld b) {
ba0         return abs(a - b) <= eps;
bfc }

b2a struct pt { // ponto
2eb         ld x, y, z;
a50         pt(ld x_ = 0, ld y_ = 0, ld z_ = 0) : x(x_), y(y_), z(z_)
    {}
5bc         bool operator < (const pt p) const {
059                 if (!eq(x, p.x)) return x < p.x;
f98                 if (!eq(y, p.y)) return y < p.y;
44c                 if (!eq(z, p.z)) return z < p.z;
bb3                 return 0;
6cd         }
a83         bool operator == (const pt p) const {
41c                 return eq(x, p.x) and eq(y, p.y) and eq(z, p.z);
fb5         }
44b         pt operator + (const pt p) const { return pt(x+p.x, y+p.y,
    z+p.z); }
392         pt operator - (const pt p) const { return pt(x-p.x, y-p.y,
    z-p.z); }
fb7         pt operator * (const ld c) const { return pt(x*c  , y*c  ,
    z*c  ); }
7a1         pt operator / (const ld c) const { return pt(x/c  , y/c  ,
    z/c  ); }
a65         ld operator * (const pt p) const { return x*p.x + y*p.y +
    z*p.z; }
7f6         pt operator ^ (const pt p) const { return pt(y*p.z -
    z*p.y, z*p.x - x*p.z, x*p.y - y*p.x); }
5ed         friend istream& operator >> (istream& in, pt& p) {
9bf                 return in >> p.x >> p.y >> p.z;
5e8         }
3ee };

b3a struct line { // reta
730         pt p, q;
0d6         line() {}
4b8         line(pt p_, pt q_) : p(p_), q(q_) {}
8d7         friend istream& operator >> (istream& in, line& r) {
4cb                 return in >> r.p >> r.q;
858         }
7ab };

79b struct plane { // plano
7e1         array<pt, 3> p;  // pontos que definem o plano
29b         array<ld, 4> eq; // equacao do plano
bb7         plane() {}
fb0         plane(pt p_, pt q_, pt r_) : p({p_, q_, r_}) { build(); }

ca9         friend istream& operator >> (istream& in, plane& P) {
2ab                 return in >> P.p[0] >> P.p[1] >> P.p[2];
70e                 P.build();
544         }
0a8         void build() {
da2                 pt dir = (p[1] - p[0]) ^ (p[2] - p[0]);
7d5                 eq = {dir.x, dir.y, dir.z, dir*p[0]*(-1)};
41a         }
d5d };

// converte de coordenadas polares para cartesianas
// (angulos devem estar em radianos)
// phi eh o angulo com o eixo z (cima) theta eh o angulo de rotacao ao
    redor de z
2fb pt convert(ld rho, ld th, ld phi) {
cf4         return pt(sin(phi) * cos(th), sin(phi) * sin(th),
    cos(phi)) * rho;
a4f }

// projecao do ponto p na reta r
256 pt proj(pt p, line r) {
bea         if (r.p == r.q) return r.p;
97a         r.q = r.q - r.p; p = p - r.p;
9f8         pt proj = r.q * ((p*r.q) / (r.q*r.q));
2cd         return proj + r.p;
232 }

// projecao do ponto p no plano P
b1a pt proj(pt p, plane P) {
7b6         p = p - P.p[0], P.p[1] = P.p[1] - P.p[0], P.p[2] = P.p[2]
    - P.p[0];
b69         pt norm = P.p[1] ^ P.p[2];
6ab         pt proj = p - (norm * (norm * p) / (norm*norm));
```

```
467         return proj + P.p[0];
4a0 }

// distancia
a45 ld dist(pt a, pt b) {
fd9         return sqrt(sq(a.x-b.x) + sq(a.y-b.y) + sq(a.z-b.z));
2d0 }

// distancia ponto reta
137 ld distline(pt p, line r) {
ce1         return dist(p, proj(p, r));
3c4 }

// distancia de ponto para segmento
d43 ld distseg(pt p, line r) {
73d         if ((r.q - r.p)*(p - r.p) < 0) return dist(r.p, p);
951         if ((r.p - r.q)*(p - r.q) < 0) return dist(r.q, p);
200         return distline(p, r);
42c }

// distancia de ponto a plano com sinal
7cc ld sdist(pt p, plane P) {
150         return P.eq[0]*p.x + P.eq[1]*p.y + P.eq[2]*p.z +  P.eq[3];
d49 }

// distancia de ponto a plano
768 ld distplane(pt p, plane P) {
c3e         return abs(sdist(p, P));
33d }

// se ponto pertence a reta
099 bool isinseg(pt p, line r) {
a32         return eq(distseg(p, r), 0);
31a }

// se ponto pertence ao triangulo definido por P.p
cd2 bool isinpol(pt p, vector<pt> v) {
fad         assert(v.size() >= 3);
bf4         pt norm = (v[1]-v[0]) ^ (v[2]-v[1]);
8a4         bool inside = true;
cec         int sign = -1;
f14         for (int i = 0; i < v.size(); i++) {
834                 line r(v[(i+1)%3], v[i]);
2a9                 if (isinseg(p, r)) return true;

4ef                 pt ar = v[(i+1)%3] - v[i];
320                 if (sign == -1) sign = ((ar^(p-v[i]))*norm > 0);
82b                 else if (((ar^(p-v[i]))*norm > 0) != sign) inside
    = false;
15e         }
aca         return inside;
c81 }

// distancia de ponto ate poligono
361 ld distpol(pt p, vector<pt> v) {
3e7         pt p2 = proj(p, plane(v[0], v[1], v[2]));
61a         if (isinpol(p2, v)) return dist(p, p2);
349         ld ret = DINF;
f14         for (int i = 0; i < v.size(); i++) {
6af                 int j = (i+1)%v.size();
5ee                 ret = min(ret, distseg(p, line(v[i], v[j])));
7b2         }
edf         return ret;
a8d }

// intersecao de plano e segmento
// BOTH = o segmento esta no plano
// ONE = um dos pontos do segmento esta no plano
// PARAL = segmento paralelo ao plano
// CONCOR = segmento concorrente ao plano
e51 enum RETCODE {BOTH, ONE, PARAL, CONCOR};
26b pair<RETCODE, pt> intersect(plane P, line r) {
fac     ld d1 = sdist(r.p, P);
f8f     ld d2 = sdist(r.q, P);
53a     if (eq(d1, 0) and eq(d2, 0))
504                 return pair(BOTH, r.p);
72c     if (eq(d1, 0))
847                 return pair(ONE, r.p);
485     if (eq(d2, 0))
168                 return pair(ONE, r.q);
3fb     if ((d1 > 0 and d2 > 0) or (d1 < 0 and d2 < 0)) {
463         if (eq(d1-d2, 0)) return pair(PARAL, pt());
406         return pair(CONCOR, pt());
91c     }
c84     ld frac = d1 / (d1 - d2);
3ff     pt res = r.p + ((r.q - r.p) * frac);
394     return pair(ONE, res);
b92 }

// rotaciona p ao redor do eixo u por um angulo a
787 pt rotate(pt p, pt u, ld a) {
773         u = u / dist(u, pt());
e6f         return u * (u * p) + (u ^ p ^ u) * cos(a) + (u ^ p) *
    sin(a);
```

```
7f0 }
```

2.8 Matriz

```
945 #define MODULAR false
5ed template<typename T> struct matrix : vector<vector<T>> {
14e     int n, m;

30f     void print() {
603         for (int i = 0; i < n; i++) {
70f             for (int j = 0; j < m; j++) cout << (*this)[i][j] << " ";
1fb             cout << endl;
d98         }
101     }

aa3     matrix(int n_, int m_, bool ident = false) :
b14             vector<vector<T>>(n_, vector<T>(m_, 0)), n(n_), m(m_) {
94e         if (ident) {
df7             assert(n == m);
a89             for (int i = 0; i < n; i++) (*this)[i][i] = 1;
359         }
527     }
b83     matrix(const vector<vector<T>>& c) : vector<vector<T>>(c),
a3d         n(c.size()), m(c[0].size()) {}
efc     matrix(const initializer_list<initializer_list<T>>& c) {
f7e         vector<vector<T>> val;
212         for (auto& i : c) val.push_back(i);
303         *this = matrix(val);
c50     }

388     matrix<T> operator*(matrix<T>& r) {
1e2         assert(m == r.n);
82c         matrix<T> M(n, r.m);
d69         for (int i = 0; i < n; i++) for (int k = 0; k < m; k++)
df4             for (int j = 0; j < r.m; j++) {
e34                 T add = (*this)[i][k] * r[k][j];
f98 #if MODULAR
d41 #warning Usar matrix<ll> e soh colocar valores em [0, MOD) na matriz!
8b6                 M[i][j] += add%MOD;
983                 if (M[i][j] >= MOD) M[i][j] -= MOD;
8c1 #else
7bb                 M[i][j] += add;
f2e #endif
620             }
474         return M;
394     }
528     matrix<T> operator^(ll e){
f10         matrix<T> M(n, n, true), at = *this;
c87         while (e) {
2e2             if (e&1) M = M*at;
cc2             e >>= 1;
c80             at = at*at;
eb6         }
474         return M;
ca3     }
582     void apply_transform(matrix M, ll e){
1c3         auto& v = *this;
c87         while (e) {
9ba             if (e&1) v = M*v;
cc2             e >>= 1;
419             M = M*M;
d86         }
4e5     }
70d };
```

2.9 Matroid

```
// Matroids de Grafo e Particao
// De modo geral, toda Matroid contem um build() linear
// e uma funcao constante oracle()
// oracle(i) responde se o conjunto continua independente
// apos adicao do elemento i
// oracle(i, j) responde se o conjunto continua indepente
// apos trocar o elemento i pelo elemento j
//
// Intersecao sem peso O(r^2 n)
// em que n eh o tamanho do conjunto e r eh o tamanho da resposta

// Matroid Grafica
// Matroid das florestas de um grafo
// Um conjunto de arestas eh independente se formam uma floresta
//
// build() : O(n)
// oracle() : O(1)

fda struct graphic_matroid {
5da     int n, m, t;
32c     vector<array<int, 2>> edges;
789     vector<vector<int>> g;
62e     vector<int> comp, in, out;
```

```
513     graphic_matroid(int n_, vector<array<int, 2>> edges_)
a1f         : n(n_), m(edges_.size()), edges(edges_), g(n), comp(n),
    in(n), out(n) {}
315     void dfs(int u) {
ab8         in[u] = t++;
17d         for (auto v : g[u]) if (in[v] == -1)
863             comp[v] = comp[u], dfs(v);
677         out[u] = t;
d83     }
945     void build(vector<int> I) {
a34         t = 0;
741         for (int u = 0; u < n; u++) g[u].clear(), in[u] = -1;
667         for (int e : I) {
d00             auto [u, v] = edges[e];
125             g[u].push_back(v), g[v].push_back(u);
a8a         }
809         for (int u = 0; u < n; u++) if (in[u] == -1)
a7d             comp[u] = u, dfs(u);
207     }
f31     bool is_ancestor(int u, int v) {
a68         return in[u] <= in[v] and in[v] < out[u];
0c2     }
e6b     bool oracle(int e) {
453         return comp[edges[e][0]] != comp[edges[e][1]];
687     }
f75     bool oracle(int e, int f) {
574         if (oracle(f)) return true;
622         int u = edges[e][in[edges[e][0]] < in[edges[e][1]]];
ff2         return is_ancestor(u, edges[f][0]) != is_ancestor(u,
    edges[f][1]);
8a9     }
691 };

// Matroid de particao ou cores
// Um conjunto eh independente se a quantidade de elementos
// de cada cor nao excede a capacidade da cor
// Quando todas as capacidades sao 1, um conjunto eh independente
// se todas as suas cores sao distintas
//
// build() : O(n)
// oracle() : O(1)

994 struct partition_matroid {
501     vector<int> cap, color, d;
608     partition_matroid(vector<int> cap_, vector<int> color_)
04d         : cap(cap_), color(color_), d(cap.size()) {}
945     void build(vector<int> I) {
def         fill(d.begin(), d.end(), 0);
e9d         for (int u : I) d[color[u]]++;
c58     }
514     bool oracle(int u) {
0a1         return d[color[u]] < cap[color[u]];
703     }
f7f     bool oracle(int u, int v) {
2f7         return color[u] == color[v] or oracle(v);
4b4     }
caa };

// Intersecao de matroid sem pesos
// Dadas duas matroids M1 e M2 definidas sobre o mesmo
// conjunto I, retorna o maior subconjunto de I
// que eh independente tanto para M1 quanto para M2
//
// O(r^2*n)

// Matroid "pesada" deve ser a M2
132 template<typename Matroid1, typename Matroid2>
801 vector<int> matroid_intersection(int n, Matroid1 M1, Matroid2 M2) {
f5b     vector<bool> b(n);
a64     vector<int> I[2];
a8b     bool converged = false;
0c1     while (!converged) {
742         I[0].clear(), I[1].clear();
99d         for (int u = 0; u < n; u++) I[b[u]].push_back(u);

09d         M1.build(I[1]), M2.build(I[1]);
289         vector<bool> target(n), pushed(n);
26a         queue<int> q;
5c5         for (int u : I[0]) {
2b2             target[u] = M2.oracle(u);
c1b             if (M1.oracle(u)) pushed[u] = true, q.push(u);
0e6         }
3fe         vector<int> p(n, -1);
07a         converged = true;
402         while (q.size()) {
be1             int u = q.front(); q.pop();
5c6             if (target[u]) {
101                 converged = false;
c32                 for (int v = u; v != -1; v = p[v]) b[v] = !b[v];
c2b                 break;
a80             }
e78             for (int v : I[!b[u]]) if (!pushed[v]) {
34d                 if ((b[u] and M1.oracle(u, v)) or (b[v] and
    M2.oracle(v, u)))
bae                     p[v] = u, pushed[v] = true, q.push(v);
533                 }
1d9             }
5e7         }
b68         return I[1];
381 }

// Intersecao de matroid com pesos
// Dadas duas matroids M1 e M2 e uma funcao de pesos w, todas definidas sobre
// um conjunto I retorna o maior subconjunto de I (desempatado pelo menor peso)
// que eh independente tanto para M1 quanto para M2
// A resposta eh construida incrementando o tamanho conjunto I de 1 em 1
// Se nao tiver custo negativo, nao precisa de SPFA
//
// O(r^3*n) com SPFA
// O(r^2*n*log(n)) com Dijkstra e potencial

42a template<typename T, typename Matroid1, typename Matroid2>
2b5 vector<int> weighted_matroid_intersection(int n, vector<T> w, Matroid1 M1, Matroid2 M2) {
6c9     vector<bool> b(n), target(n), is_inside(n);
563     vector<int> I[2], from(n);
e35     vector<pair<T, int>> d(n);
169     auto check_edge = [&](int u, int v) {
249         return (b[u] and M1.oracle(u, v)) or (b[v] and M2.oracle(v, u));
253     };
667     while (true) {
742         I[0].clear(), I[1].clear();
99d         for (int u = 0; u < n; u++) I[b[u]].push_back(u);
            // I[1] contem o conjunto de tamanho I[1].size() de menor peso
09d         M1.build(I[1]), M2.build(I[1]);
687         for (int u = 0; u < n; u++) {
ea5             target[u] = false, is_inside[u] = false, from[u] = -1;
961             d[u] = {numeric_limits<T>::max(), INF};
392         }
8d3         deque<T> q;
476         sort(I[0].begin(), I[0].end(), [&](int i, int j){ return w[i] < w[j]; });
5c5         for (int u : I[0]) {
2b2             target[u] = M2.oracle(u);
5a7             if (M1.oracle(u)) {
4ef                 if (is_inside[u]) continue;
7cc                 d[u] = {w[u], 0};
427                 if (!q.empty() and d[u] > d[q.front()]) q.push_back(u);
655                 else q.push_front(u);
4ae                 is_inside[u] = true;
764             }
add         }
402         while (q.size()) {
97a             int u = q.front(); q.pop_front();
6f3             is_inside[u] = false;
57a             for (int v : I[!b[u]]) if (check_edge(u, v)) {
9de                 pair<T, int> nd(d[u].first + w[v], d[u].second + 1);
61b                 if (nd < d[v]) {
6ac                     from[v] = u, d[v] = nd;
bd7                     if (is_inside[v]) continue;
eec                     if (q.size() and d[v] > d[q.front()]) q.push_back(v);
275                     else q.push_front(v);
587                     is_inside[v] = true;
b3f                 }
a3b             }
563         }
cc8         pair<T, int> mini = pair(numeric_limits<T>::max(), INF);
489         int targ = -1;
259         for (int u : I[0]) if (target[u] and d[u] < mini)
2b9             mini = d[u], targ = u;
e14         if (targ != -1) for (int u = targ; u != -1; u = from[u])
d89             b[u] = !b[u], w[u] *= -1;
f97         else break;
c7d     }
b68     return I[1];
8e7 }
```

3 Problemas

3.1 Angle Range Intersection

```
// Computa intersecao de angulos
// Os angulos (arcos) precisam ter comprimeiro < pi
// (caso contrario a intersecao eh estranha)
//
// Tudo O(1)
```

```
32a struct angle_range {
75e     static constexpr ld ALL = 1e9, NIL = -1e9;
395     ld l, r;
c77     angle_range() : l(ALL), r(ALL) {}
894     angle_range(ld l_, ld r_) : l(l_), r(r_) { fix(l), fix(r); }

4ee     void fix(ld& theta) {
da7         if (theta == ALL or theta == NIL) return;
323         if (theta > 2*pi) theta -= 2*pi;
868         if (theta < 0) theta += 2*pi;
625     }
2ee     bool empty() { return l == NIL; }
931     bool contains(ld q) {
40f         fix(q);
4d7         if (l == ALL) return true;
fec         if (l == NIL) return false;
6a6         if (l < r) return l < q and q < r;
075         return q > l or q < r;
800     }
9c7     friend angle_range operator &(angle_range p, angle_range q) {
743         if (p.l == ALL or q.l == NIL) return q;
20f         if (q.l == ALL or p.l == NIL) return p;
7d5         if (p.l > p.r and q.l > q.r) return {max(p.l, q.l) ,
    min(p.r, q.r)};
aa6         if (q.l > q.r) swap(p.l, q.l), swap(p.r, q.r);
8d8         if (p.l > p.r) {
249             if (q.r > p.l) return {max(q.l, p.l) , q.r};
6f7             else if (q.l < p.r) return {q.l, min(q.r, p.r)};
270             return {NIL, NIL};
337         }
5a8         if (max(p.l, q.l) > min(p.r, q.r)) return {NIL, NIL};
bcb         return {max(p.l, q.l), min(p.r, q.r)};
142     }
5e1 };
```

3.2 Area da Uniao de Retangulos

```
// O(n log(n))
// 5d8d2f

aa4 namespace seg {
6b3     pair<int, ll> seg[4*MAX];
b1b     ll lazy[4*MAX], *v;
1a8     int n;

e01     pair<int, ll> merge(pair<int, ll> l, pair<int, ll> r){
719         if (l.second == r.second) return {l.first+r.first,
    l.second};
53b         else if (l.second < r.second) return l;
aa0         else return r;
d82     }

6fc     pair<int, ll> build(int p=1, int l=0, int r=n-1) {
3c7         lazy[p] = 0;
bf8         if (l == r) return seg[p] = {1, v[l]};
ee4         int m = (l+r)/2;
432         return seg[p] = merge(build(2*p, l, m), build(2*p+1, m+1,
    r));
f94     }
d9e     void build(int n2, ll* v2) {
680         n = n2, v = v2;
6f2         build();
f8a     }
ceb     void prop(int p, int l, int r) {
208         seg[p].second += lazy[p];
2c9         if (l != r) lazy[2*p] += lazy[p], lazy[2*p+1] += lazy[p];
3c7         lazy[p] = 0;
bf2     }
693     pair<int, ll> query(int a, int b, int p=1, int l=0, int r=n-1)
    {
6b9         prop(p, l, r);
527         if (a <= l and r <= b) return seg[p];
9b7         if (b < l or r < a) return {0, LINF};
ee4         int m = (l+r)/2;
eeb         return merge(query(a, b, 2*p, l, m), query(a, b, 2*p+1,
    m+1, r));
786     }
07c     pair<int, ll> update(int a, int b, int x, int p=1, int l=0,
    int r=n-1) {
6b9         prop(p, l, r);
9a3         if (a <= l and r <= b) {
b94             lazy[p] += x;
6b9             prop(p, l, r);
534             return seg[p];
821         }
e9f         if (b < l or r < a) return seg[p];
ee4         int m = (l+r)/2;
086         return seg[p] = merge(update(a, b, x, 2*p, l, m),
579                 update(a, b, x, 2*p+1, m+1, r));
c65     }
043 };

eb5 ll seg_vec[MAX];
```

```
8be ll area_sq(vector<pair<pair<int, int>, pair<int, int>>> &sq){
28c     vector<pair<pair<int, int>, pair<int, int>>> up;
60a     for (auto it : sq){
619         int x1, y1, x2, y2;
ae0         tie(x1, y1) = it.first;
68e         tie(x2, y2) = it.second;
80f         up.push_back({{x1+1,  1}, {y1, y2}});
aee         up.push_back({{x2+1, -1}, {y1, y2}});
6c3     }
092     sort(up.begin(), up.end());
049     memset(seg_vec, 0, sizeof seg_vec);
6fe     ll H_MAX = MAX;
156     seg::build(H_MAX-1, seg_vec);
7ba     auto it = up.begin();
04b     ll ans = 0;
f14     while (it != up.end()){
07f         ll L = (*it).first.first;
718         while (it != up.end() && (*it).first.first == L){
127             int x, inc, y1, y2;
d35             tie(x, inc) = it->first;
d3d             tie(y1, y2) = it->second;
5d1             seg::update(y1+1, y2, inc);
40d             it++;
9b1         }
852         if (it == up.end()) break;
d8a         ll R = (*it).first.first;

f59         ll W = R-L;
efd         auto jt = seg::query(0, H_MAX-1);
91a         ll H = H_MAX - 1;
e8a         if (jt.second == 0) H -= jt.first;
8df         ans += W*H;
5c8     }
ba7     return ans;
385 }
```

3.3 Area Maxima de Histograma

```
// Assume que todas as barras tem largura 1,
// e altura dada no vetor v
//
// O(n)

15e ll area(vector<int> v) {
b73     ll ret = 0;
4ce     stack<int> s;
        // valores iniciais pra dar tudo certo
447     v.insert(v.begin(), -1);
d56     v.insert(v.end(), -1);
1f8     s.push(0);

0be     for(int i = 0; i < (int) v.size(); i++) {
78e         while (v[s.top()] > v[i]) {
265             ll h = v[s.top()]; s.pop();
de1             ret = max(ret, h * (i - s.top() - 1));
40a         }
18e         s.push(i);
020     }

edf     return ret;
e43 }
```

3.4 Binomial modular

```
// Computa C(n, k) mod m em O(m + log(m) log(n))
// = O(rapido)

97c ll divi[MAX];

398 ll expo(ll a, ll b, ll m) {
1c1     if (!b) return 1;
399     ll ans = expo(a*a%m, b/2, m);
751     if (b%2) ans *= a;
2e9     return ans%m;
754 }

f0a ll inv(ll a, ll b){
bca     return 1<a ? b - inv(b%a,a)*b/a : 1;
041 }

153 template<typename T> tuple<T, T, T> ext_gcd(T a, T b) {
3bd     if (!a) return {b, 0, 1};
550     auto [g, x, y] = ext_gcd(b%a, a);
c59     return {g, y - b/a*x, x};
537 }

bfe template<typename T = ll> struct crt {
627     T a, m;

5f3     crt() : a(0), m(1) {}
7eb     crt(T a_, T m_) : a(a_), m(m_) {}
911     crt operator * (crt C) {
238         auto [g, x, y] = ext_gcd(m, C.m);
dc0         if ((a - C.a) % g) a = -1;
4f9         if (a == -1 or C.a == -1) return crt(-1, 0);
d09         T lcm = m/g*C.m;
eb2         T ans = a + (x*(C.a-a)/g % (C.m/g))*m;
d8d         return crt((ans % lcm + lcm) % lcm, lcm);
1f2     }
0d9 };

6f2 pair<ll, ll> divide_show(ll n, int p, int k, int pak) {
4f7     if (n == 0) return {0, 1};
d02     ll blocos = n/pak, falta = n%pak;
2ce     ll periodo = divi[pak], resto = divi[falta];
616     ll r = expo(periodo, blocos, pak)*resto%pak;

445     auto rec = divide_show(n/p, p, k, pak);
a51     ll y = n/p + rec.first;
bb9     r = r*rec.second % pak;

90f     return {y, r};
533 }

6e6 ll solve_pak(ll n, ll x, int p, int k, int pak) {
d34     divi[0] = 1;
f2b     for (int i = 1; i <= pak; i++) {
901         divi[i] = divi[i-1];
840         if (i%p) divi[i] = divi[i] * i % pak;
51a     }

4ac     auto dn = divide_show(n, p, k, pak), dx = divide_show(x, p, k,
    pak),
162          dnx = divide_show(n-x, p, k, pak);
768     ll y = dn.first-dx.first-dnx.first, r =
b64         (dn.second*inv(dx.second, pak)%pak)*inv(dnx.second,
    pak)%pak;
035     return expo(p, y, pak) * r % pak;
d78 }

9dd ll solve(ll n, ll x, int mod) {
490     vector<pair<int, int>> f;
c3b     int mod2 = mod;
7b4     for (int i = 2; i*i <= mod2; i++) if (mod2%i==0) {
aff         int c = 0;
75b         while (mod2%i==0) mod2 /= i, c++;
2a1         f.push_back({i, c});
fe7     }
0ff     if (mod2 > 1) f.push_back({mod2, 1});
e96     crt ans(0, 1);
a13     for (int i = 0; i < f.size(); i++) {
702         int pak = 1;
7e4         for (int j = 0; j < f[i].second; j++) pak *= f[i].first;
304         ans = ans * crt(solve_pak(n, x, f[i].first, f[i].second,
    pak), pak);
7fd     }
5fb     return ans.a;
689 }
```

3.5 Closest pair of points

```
// O(nlogn)

915 pair<pt, pt> closest_pair_of_points(vector<pt> v) {
3d2     int n = v.size();
fca     sort(v.begin(), v.end());
31c     for (int i = 1; i < n; i++) if (v[i] == v[i-1]) return
    {v[i-1], v[i]};
c20     auto cmp_y = [&](const pt &l, const pt &r) {
b53         if (l.y != r.y) return l.y < r.y;
920         return l.x < r.x;
55a     };
62e     set<pt, decltype(cmp_y)> s(cmp_y);
3d9     int l = 0, r = -1;
6a2     ll d2_min = numeric_limits<ll>::max();
4d5     pt pl, pr;
bd1     const int magic = 5;
a55     while (r+1 < n) {
7f1         auto it = s.insert(v[++r]).first;
c92         int cnt = magic/2;
773         while (cnt-- and it != s.begin()) it--;
a01         cnt = 0;
d68         while (cnt++ < magic and it != s.end()) {
f19             if (!((*it) == v[r])) {
67e                 ll d2 = dist2(*it, v[r]);
74e                 if (d2_min > d2) {
229                     d2_min = d2;
841                     pl = *it;
4f2                     pr = v[r];
7d9                 }
10a             }
40d             it++;
801         }
eb0         while (l < r and sq(v[l].x-v[r].x) > d2_min)
   s.erase(v[l++]);
de6     }
c74     return {pl, pr};
f90 }
```

3.6 Coloracao de Grafo de Intervalo

```
// Colore os intervalos com o numero minimo
// de cores de tal forma que dois intervalos
// que se interceptam tem cores diferentes
// As cores vao de 1 ate n
//
// O(n log(n))

615 vector<int> coloring(vector<pair<int, int>>& v) {
3d2     int n = v.size();
c08     vector<pair<int, pair<int, int>>> ev;
603     for (int i = 0; i < n; i++) {
150         ev.push_back({v[i].first, {1, i}});
cda         ev.push_back({v[i].second, {0, i}});
6a4     }
49e     sort(ev.begin(), ev.end());
360     vector<int> ans(n), avl(n);
265     for (int i = 0; i < n; i++) avl.push_back(n-i);
4bf     for (auto i : ev) {
cbe         if (i.second.first == 1) {
021             ans[i.second.second] = avl.back();
a00             avl.pop_back();
e98         } else avl.push_back(ans[i.second.second]);
3a6     }
ba7     return ans;
83a }
```

3.7 Conectividade Dinamica DC

```
// Offline com Divide and Conquer e
// DSU com rollback
// O(n log^2(n))

8f2 typedef pair<int, int> T;

1cd namespace data {
553     int n, ans;
573     int p[MAX], sz[MAX];
ee6     stack<int> S;

e5b     void build(int n2) {
1e3         n = n2;
8a6         for (int i = 0; i < n; i++) p[i] = i, sz[i] = 1;
0b2         ans = n;
cba     }
1b1     int find(int k) {
006         while (p[k] != k) k = p[k];
839         return k;
c1e     }
072     void add(T x) {
700         int a = x.first, b = x.second;
605         a = find(a), b = find(b);
843         if (a == b) return S.push(-1);
e7d         ans--;
3c6         if (sz[a] > sz[b]) swap(a, b);
4c2         S.push(a);
582         sz[b] += sz[a];
84b         p[a] = b;
e1a     }
5eb     int query() {
ba7         return ans;
35c     }
5cf     void rollback() {
465         int u = S.top(); S.pop();
61c         if (u == -1) return;
270         sz[p[u]] -= sz[u];
546         p[u] = u;
0df         ans++;
456     }
568 };

357 int ponta[MAX]; // outra ponta do intervalo ou -1 se for query
4f0 int ans[MAX], n, q;
487 T qu[MAX];

47b void solve(int l = 0, int r = q-1) {
0b1     if (l >= r) {
8c0         ans[l] = data::query(); // agora a estrutura ta certa
505         return;
f77     }
962     int m = (l+r)/2, qnt = 1;
fc7     for (int i = m+1; i <= r; i++) if (ponta[i]+1 and ponta[i] < l)
37d         data::add(qu[i]), qnt++;
221     solve(l, m);
593     while (--qnt) data::rollback();
a2c     for (int i = l; i <= m; i++) if (ponta[i]+1 and ponta[i] > r)
37d         data::add(qu[i]), qnt++;
37b     solve(m+1, r);
281     while (qnt--) data::rollback();
0d4 }
```

3.8 Conectividade Dinamica LCT

```
// Offline com link-cut trees
// O(n log(n))

1ef namespace lct {
3c9     struct node {
19f         int p, ch[2];
a2a         int val, sub;
aa6         bool rev;
f93         node() {}
54e         node(int v) : p(-1), val(v), sub(v), rev(0) { ch[0] =
    ch[1] = -1; }
cac     };

c53     node t[2*MAX]; // MAXN + MAXQ
99e     map<pair<int, int>, int> aresta;
e4d     int sz;

95a     void prop(int x) {
aa2         if (t[x].rev) {
f95             swap(t[x].ch[0], t[x].ch[1]);
379             if (t[x].ch[0]+1) t[t[x].ch[0]].rev ^= 1;
c3d             if (t[x].ch[1]+1) t[t[x].ch[1]].rev ^= 1;
50e         }
693         t[x].rev = 0;
750     }
564     void update(int x) {
e8d         t[x].sub = t[x].val;
8ca         for (int i = 0; i < 2; i++) if (t[x].ch[i]+1) {
621             prop(t[x].ch[i]);
78d             t[x].sub = min(t[x].sub, t[t[x].ch[i]].sub);
3e4         }
9bf     }
971     bool is_root(int x) {
657         return t[x].p == -1 or (t[t[x].p].ch[0] != x and
    t[t[x].p].ch[1] != x);
cf1     }
ed6     void rotate(int x) {
497         int p = t[x].p, pp = t[p].p;
fc4         if (!is_root(p)) t[pp].ch[t[pp].ch[1] == p] = x;
251         bool d = t[p].ch[0] == x;
461         t[p].ch[!d] = t[x].ch[d], t[x].ch[d] = p;
a76         if (t[p].ch[!d]+1) t[t[p].ch[!d]].p = p;
8fa         t[x].p = pp, t[p].p = x;
444         update(p), update(x);
f31     }
238     int splay(int x) {
18c         while (!is_root(x)) {
497             int p = t[x].p, pp = t[p].p;
77b             if (!is_root(p)) prop(pp);
be5             prop(p), prop(x);
0c5             if (!is_root(p)) rotate((t[pp].ch[0] == p)^(t[p].ch[0]
    == x) ? x : p);
64f             rotate(x);
72c         }
aab         return prop(x), x;
08f     }
f16     int access(int v) {
0eb         int last = -1;
d9f         for (int w = v; w+1; update(last = w), splay(v), w =
    t[v].p)
024             splay(w), t[w].ch[1] = (last == -1 ? -1 : v);
3d3         return last;
294     }
952     void make_tree(int v, int w=INF) { t[v] = node(w); }
82f     bool conn(int v, int w) {
2cf         access(v), access(w);
b9b         return v == w ? true : t[v].p != -1;
ec0     }
277     void rootify(int v) {
5e3         access(v);
a02         t[v].rev ^= 1;
a05     }
a1d     int query(int v, int w) {
b54         rootify(w), access(v);
249         return t[v].sub;
c28     }
204     void link_(int v, int w) {
821         rootify(w);
389         t[w].p = v;
523     }
6b8     void link(int v, int w, int x) { // v--w com peso x
379         int id = MAX + sz++;
110         aresta[make_pair(v, w)] = id;
ab6         make_tree(id, x);
c88         link_(v, id), link_(id, w);
984     }
e63     void cut_(int v, int w) {
b54         rootify(w), access(v);
264         t[v].ch[0] = t[t[v].ch[0]].p = -1;
7cd     }
031     void cut(int v, int w) {
b0f         int id = aresta[make_pair(v, w)];
a4a         cut_(v, id), cut_(id, w);
840     }
0d7 }

893 void dyn_conn() {
c5f     int n, q; cin >> n >> q;
d6e     vector<int> p(2*q, -1); // outra ponta do intervalo
b4f     for (int i = 0; i < n; i++) lct::make_tree(i);
fbf     vector<pair<int, int>> qu(q);
139     map<pair<int, int>, int> m;
abf     for (int i = 0; i < q; i++) {
3c2         char c; cin >> c;
ef6         if (c == '?') continue;
602         int a, b; cin >> a >> b; a--, b--;
d11         if (a > b) swap(a, b);
8a1         qu[i] = {a, b};
8d7         if (c == '+') {
94b             p[i] = i+q, p[i+q] = i;
906             m[make_pair(a, b)] = i;
8a0         } else {
412             int j = m[make_pair(a, b)];
ac2             p[i] = j, p[j] = i;
0da         }
9e5     }
447     int ans = n;
abf     for (int i = 0; i < q; i++) {
87d         if (p[i] == -1) {
886             cout << ans << endl; // numero de comp conexos
5e2             continue;
b35         }
69d         int a = qu[i].first, b = qu[i].second;
c4d         if (p[i] > i) { // +
ac5             if (lct::conn(a, b)) {
18f                 int mi = lct::query(a, b);
993                 if (p[i] < mi) {
dd3                     p[p[i]] = p[i];
5e2                     continue;
474                 }
6f7                 lct::cut(qu[p[mi]].first, qu[p[mi]].second), ans++;
6ea                 p[mi] = mi;
9a9             }
d1d             lct::link(a, b, p[i]), ans--;
9d0         } else if (p[i] != i) lct::cut(a, b), ans++; // -
c03     }
56a }
```

3.9 Conj. Indep. Maximo com Peso em Grafo de Intervalo

```
// Retorna os indices ordenados dos intervalos selecionados
// Se tiver empate, retorna o que minimiza o comprimento total
//
// O(n log(n))

31e vector<int> ind_set(vector<tuple<int, int, int>>& v) {
b27     vector<tuple<int, int, int>> w;
f14     for (int i = 0; i < v.size(); i++) {
e85         w.push_back(tuple(get<0>(v[i]), 0, i));
6f0         w.push_back(tuple(get<1>(v[i]), 1, i));
17f     }
d1d     sort(w.begin(), w.end());

844     vector<int> nxt(v.size());
c22     vector<pair<ll, int>> dp(v.size());
0eb     int last = -1;
723     for (auto [fim, t, i] : w) {
25a         if (t == 0) {
4ca             nxt[i] = last;
5e2             continue;
5fd         }
78b         dp[i] = {0, 0};
cb8         if (last != -1) dp[i] = max(dp[i], dp[last]);
911         pair<ll, int> pega = {get<2>(v[i]), -(get<1>(v[i]) -
    get<0>(v[i]) + 1)};
5d3         if (nxt[i] != -1) pega.first += dp[nxt[i]].first,
    pega.second += dp[nxt[i]].second;
b08         if (pega > dp[i]) dp[i] = pega;
7cb         else nxt[i] = last;
381         last = i;
b7c     }
977     pair<ll, int> ans = {0, 0};
919     int idx = -1;
ceb     for (int i = 0; i < v.size(); i++) if (dp[i] > ans) ans =
    dp[i], idx = i;
4b8     vector<int> ret;
fdd     while (idx != -1) {
```

```
d69         if (get<2>(v[idx]) > 0 and
a05             (nxt[idx] == -1 or get<1>(v[nxt[idx]]) <
    get<0>(v[idx]))) ret.push_back(idx);
e4f         idx = nxt[idx];
042     }
0ea     sort(ret.begin(), ret.end());
edf     return ret;
c4d }
```

3.10 Convex Hull Dinamico

```
// insert - O(log n) amortizado
// is_inside - O(log n)

0b9 struct upper {
af8     set<pt> se;
80b     set<pt>::iterator it;

25c     int is_under(pt p) { // 1 -> inside ; 2 -> border
fe0         it = se.lower_bound(p);
633         if (it == se.end()) return 0;
a94         if (it == se.begin()) return p == *it ? 2 : 0;
ca0         if (ccw(p, *it, *prev(it))) return 1;
402         return ccw(p, *prev(it), *it) ? 0 : 2;
dba     }
eaa     void insert(pt p) {
712         if (is_under(p)) return;

a86         if (it != se.end()) while (next(it) != se.end() and
    !ccw(*next(it), *it, p))
316             it = se.erase(it);
be3         if (it != se.begin()) while (--it != se.begin() and
    !ccw(p, *it, *prev(it)))
316             it = se.erase(it);

0c8         se.insert(p);
5da     }
750 };

06f struct dyn_hull {
d93     upper U, L;

333     int is_inside(pt p) {
632         int u = U.is_under(p), l = L.is_under({-p.x, -p.y});
4cc         if (!u or !l) return 0;
fc0         return max(u, l);
478     }
eaa     void insert(pt p) {
86c         U.insert(p);
925         L.insert({-p.x, -p.y});
64b     }
285     int size() {
7c2         int ans = U.se.size() + L.se.size();
1c9         return ans <= 2 ? ans/2 : ans-2;
ad5     }
65e };
```

3.11 Distancia maxima entre dois pontos

```
// max_dist2(v) - O(n log(n))
// max_dist_manhattan - O(n)

// Quadrado da Distancia Euclidiana (precisa copiar convex_hull, ccw e
    pt)
859 ll max_dist2(vector<pt> v) {
221     v = convex_hull(v);
a14     if (v.size() <= 2) return dist2(v[0], v[1%v.size()]);
04b     ll ans = 0;
323     int n = v.size(), j = 0;
603     for (int i = 0; i < n; i++) {
057         while (!ccw(v[(i+1)%n]-v[i], pt(0, 0), v[(j+1)%n]-v[j])) j
    = (j+1)%n;
e7a         ans = max({ans, dist2(v[i], v[j]), dist2(v[(i+1)%n],
    v[j])});
1f6     }
ba7     return ans;
bda }

// Distancia de Manhattan
c51 template<typename T> T max_dist_manhattan(vector<pair<T, T>> v) {
8eb     T min_sum, max_sum, min_dif, max_dif;
4f5     min_sum = max_sum = v[0].first + v[0].second;
271     min_dif = max_dif = v[0].first - v[0].second;
c25     for (auto [x, y] : v) {
1cb         min_sum = min(min_sum, x+y);
683         max_sum = max(max_sum, x+y);
782         min_dif = min(min_dif, x-y);
af7         max_dif = max(max_dif, x-y);
e3a     }
9f0     return max(max_sum - min_sum, max_dif - min_dif);
4e9 }
```

3.12 Distinct Range Query

```
// build - O(n (log n + log(sigma)))
// query - O(log(sigma))

789 namespace perseg { };

53d int qt[MAX];

edc void build(vector<int>& v) {
3d2     int n = v.size();
16b     perseg::build(n);
663     map<int, int> last;
05e     int at = 0;
603     for (int i = 0; i < n; i++) {
817         if (last.count(v[i])) {
a58             perseg::update(last[v[i]], -1);
69a             at++;
d1f         }
4f2         perseg::update(i, 1);
460         qt[i] = ++at;
efe         last[v[i]] = i;
d6f     }
0f4 }

9e3 int query(int l, int r) {
080     return perseg::query(l, r, qt[r]);
215 }
```

3.13 Distinct Range Query com Update

```
// build - O(n log(n))
// query - O(log^2(n))
// update - O(log^2(n))

774 #include <ext/pb_ds/assoc_container.hpp>
30f #include <ext/pb_ds/tree_policy.hpp>
0d7 using namespace __gnu_pbds;
4fc template <class T>
def     using ord_set = tree<T, null_type, less<T>, rb_tree_tag,
3a1     tree_order_statistics_node_update>;

042 int v[MAX], n, nxt[MAX], prv[MAX];
f60 map<int, set<int> > ocor;
```

```
e04 namespace bit {
686     ord_set<pair<int, int>> bit[MAX];

0a8     void build() {
3e1         for (int i = 1; i <= n; i++) bit[i].insert({nxt[i-1],
    i-1});
78a         for (int i = 1; i <= n; i++) {
edf             int j = i + (i&-i);
d03             if (j <= n) for (auto x : bit[i]) bit[j].insert(x);
5cb         }
af6     }
d3f     int pref(int p, int x) {
7c9         int ret = 0;
bbf         for (; p; p -= p&-p) ret += bit[p].order_of_key({x, -INF});
edf         return ret;
0e1     }
d50     int query(int l, int r, int x) {
e55         return pref(r+1, x) - pref(l, x);
9b4     }
ff3     void update(int p, int x) {
f17         int p2 = p;
5ed         for (p++; p <= n; p += p&-p) {
ca8             bit[p].erase({nxt[p2], p2});
f6b             bit[p].insert({x, p2});
3df         }
151     }
c63 }

0a8 void build() {
383     for (int i = 0; i < n; i++) nxt[i] = INF;
7b3     for (int i = 0; i < n; i++) prv[i] = -INF;
d07     vector<pair<int, int>> t;
348     for (int i = 0; i < n; i++) t.push_back({v[i], i});
3fd     sort(t.begin(), t.end());
603     for (int i = 0; i < n; i++) {
b40         if (i and t[i].first == t[i-1].first)
565             prv[t[i].second] = t[i-1].second;
a8b         if (i+1 < n and t[i].first == t[i+1].first)
12f             nxt[t[i].second] = t[i+1].second;
48d     }

a23     for (int i = 0; i < n; i++) ocor[v[i]].insert(i);

1d7     bit::build();
d44 }

aae void muda(int p, int x) {
```

```
f92     bit::update(p, x);
c3d     nxt[p] = x;
97c }

4ea int query(int a, int b) {
a0a     return b-a+1 - bit::query(a, b, b+1);
511 }

ff3 void update(int p, int x) { // mudar valor na pos. p para x
c0b     if (prv[p] > -INF) muda(prv[p], nxt[p]);
4ae     if (nxt[p] < INF) prv[nxt[p]] = prv[p];

5bf     ocor[v[p]].erase(p);
4b4     if (!ocor[x].size()) {
19d         muda(p, INF);
8d4         prv[p] = -INF;
f6c     } else if (*ocor[x].rbegin() < p) {
5b5         int i = *ocor[x].rbegin();
f64         prv[p] = i;
19d         muda(p, INF);
5f2         muda(i, p);
f36     } else {
d46         int i = *ocor[x].lower_bound(p);
33f         if (prv[i] > -INF) {
f17             muda(prv[i], p);
8f9             prv[p] = prv[i];
bc4         } else prv[p] = -INF;
523         prv[i] = p;
597         muda(p, i);
2ac     }
c96     v[p] = x; ocor[x].insert(p);
38e }
```

3.14 Dominator Points

```
// Se um ponto A tem ambas as coordenadas >= B, dizemos
// que A domina B
// is_dominated(p) fala se existe algum ponto no conjunto
// que domina p
// insert(p) insere p no conjunto
// (se p for dominado por alguem, nao vai inserir)
// o multiset 'quina' guarda informacao sobre os pontos
// nao dominados por um elemento do conjunto que nao dominam
// outro ponto nao dominado por um elemento do conjunto
// No caso, armazena os valores de x+y esses pontos
//
// Complexidades:
// is_dominated - O(log(n))
// insert - O(log(n)) amortizado
// query - O(1)

e2a struct dominator_points {
baf     set<pair<int, int>> se;
4dd     multiset<int> quina;

a85     bool is_dominated(pair<int, int> p) {
80f         auto it = se.lower_bound(p);
633         if (it == se.end()) return 0;
ab4         return it->second >= p.second;
28f     }
99b     void mid(pair<int, int> a, pair<int, int> b, bool rem) {
29a         pair<int, int> m = {a.first+1, b.second+1};
b19         int val = m.first + m.second;
638         if (!rem) quina.insert(val);
731         else quina.erase(quina.find(val));
241     }
7c4     bool insert(pair<int, int> p) {
fb4         if (is_dominated(p)) return 0;
80f         auto it = se.lower_bound(p);
ca9         if (it != se.begin() and it != se.end())
d4a             mid(*prev(it), *it, 1);
1fa         while (it != se.begin()) {
049             it--;
23c             if (it->second > p.second) break;
b86             if (it != se.begin()) mid(*prev(it), *it, 1);
316             it = se.erase(it);
acd         }
433         it = se.insert(p).first;
69e         if (it != se.begin()) mid(*prev(it), *it, 0);
96d         if (next(it) != se.end()) mid(*it, *next(it), 0);
6a5         return 1;
688     }
5eb     int query() {
956         if (!quina.size()) return INF;
add         return *quina.begin();
b8b     }
09f };
```

3.15 DP de Dominacao 3D

```
// Computa para todo ponto i,
// dp[i] = 1 + max_{j dominado por i} dp[j]
// em que ser dominado eh ter as 3 coordenadas menores
// Da pra adaptar facil para outras dps
//
// O(n log^2 n), O(n) de memoria

c53 void lis2d(vector<vector<tuple<int, int, int>>>& v, vector<int>&
    dp, int l, int r) {
893     if (l == r) {
56f         for (int i = 0; i < v[l].size(); i++) {
8b5             int ii = get<2>(v[l][i]);
1ce             dp[ii] = max(dp[ii], 1);
4b0         }
505         return;
3e4     }
ee4     int m = (l+r)/2;
62b     lis2d(v, dp, l, m);

325     vector<tuple<int, int, int>> vv[2];
d44     vector<int> Z;
871     for (int i = l; i <= r; i++) for (auto it : v[i]) {
2ef         vv[i > m].push_back(it);
042         Z.push_back(get<1>(it));
0d1     }
e9f     sort(vv[0].begin(), vv[0].end());
9b5     sort(vv[1].begin(), vv[1].end());
0d1     sort(Z.begin(), Z.end());
573     auto get_z = [&](int z) { return lower_bound(Z.begin(),
    Z.end(), z) - Z.begin(); };
c51     vector<int> bit(Z.size());

181     int i = 0;
e9a     for (auto [y, z, id] : vv[1]) {
6bd         while (i < vv[0].size() and get<0>(vv[0][i]) < y) {
397             auto [y2, z2, id2] = vv[0][i++];
ea0             for (int p = get_z(z2)+1; p <= Z.size(); p += p&-p)
300                 bit[p-1] = max(bit[p-1], dp[id2]);
82c         }
d3b         int q = 0;
fd9         for (int p = get_z(z); p; p -= p&-p) q = max(q, bit[p-1]);
614         dp[id] = max(dp[id], q + 1);
acc     }
c25     lis2d(v, dp, m+1, r);
4d6 }

4de vector<int> solve(vector<tuple<int, int, int>> v) {
3d2     int n = v.size();
cd4     vector<tuple<int, int, int>> vv;
603     for (int i = 0; i < n; i++) {
9be         auto [x, y, z] = v[i];
5bb         vv.emplace_back(x, y, z, i);
64c     }
bd3     sort(vv.begin(), vv.end());

e11     vector<vector<tuple<int, int, int>>> V;
603     for (int i = 0; i < n; i++) {
a5b         int j = i;
808         V.emplace_back();
c01         while (j < n and get<0>(vv[j]) == get<0>(vv[i])) {
ba6             auto [x, y, z, id] = vv[j++];
cbb             V.back().emplace_back(y, z, id);
8bd         }
452         i = j-1;
ac4     }
388     vector<int> dp(n);
839     lis2d(V, dp, 0, V.size()-1);
898     return dp;
b0a }
```

3.16 Gray Code

```
// Gera uma permutacao de 0 a 2^n-1, de forma que
// duas posicoes adjacentes diferem em exatamente 1 bit
//
// O(2^n)

df6 vector<int> gray_code(int n) {
73f     vector<int> ret(1<<n);
f29     for (int i = 0; i < (1<<n); i++) ret[i] = i^(i>>1);
edf     return ret;
840 }
```

3.17 Half-plane intersection

```
// Cada half-plane eh identificado por uma reta e a regiao ccw a ela
//
// O(n log n)

f4f vector<pt> hp_intersection(vector<line> &v) {
9bc     deque<pt> dq = {{INF, INF}, {-INF, INF}, {-INF, -INF}, {INF,
    -INF}};
d41 #warning considerar trocar por compare_angle
de3     sort(v.begin(), v.end(), [&](line r, line s) { return
    angle(r.q-r.p) < angle(s.q-s.p); });

5e9     for(int i = 0; i < v.size() and dq.size() > 1; i++) {
c69         pt p1 = dq.front(), p2 = dq.back();
6c6         while (dq.size() and !ccw(v[i].p, v[i].q, dq.back()))
47b             p1 = dq.back(), dq.pop_back();
0a2         while (dq.size() and !ccw(v[i].p, v[i].q, dq.front()))
7cf             p2 = dq.front(), dq.pop_front();

4d9         if (!dq.size()) break;
606         if (p1 == dq.front() and p2 == dq.back()) continue;
c9b         dq.push_back(inter(v[i], line(dq.back(), p1)));
65c         dq.push_front(inter(v[i], line(dq.front(), p2)));

fdd         if (dq.size() > 1 and dq.back() == dq.front())
    dq.pop_back();
4d8     }
b2b     return vector<pt>(dq.begin(), dq.end());
f56 }
```

3.18 Heap Sort

```
// O(n log n)

f18 void down(vector<int>& v, int n, int i) {
e1f     while ((i = 2*i+1) < n) {
583         if (i+1 < n and v[i] < v[i+1]) i++;
b27         if (v[i] < v[(i-1)/2]) break;
322         swap(v[i], v[(i-1)/2]);
170     }
724 }
eb6 void heap_sort(vector<int>& v) {
3d2     int n = v.size();
61d     for (int i = n/2-1; i >= 0; i--) down(v, n, i);
917     for (int i = n-1; i > 0; i--)
37f         swap(v[0], v[i]), down(v, i, 0);
b33 }
```

3.19 Hungaro

```
// Resolve o problema de assignment (matriz n x n)
// Colocar os valores da matriz em 'a' (pode < 0)
// assignment() retorna um par com o valor do
// assignment minimo, e a coluna escolhida por cada linha
//
// O(n^3)

a6a template<typename T> struct hungarian {
1a8     int n;
a08     vector<vector<T>> a;
f36     vector<T> u, v;
5ff     vector<int> p, way;
f1e     T inf;

c3f     hungarian(int n_) : n(n_), u(n+1), v(n+1), p(n+1), way(n+1) {
b2f         a = vector<vector<T>>(n, vector<T>(n));
1f3         inf = numeric_limits<T>::max();
78f     }
d67     pair<T, vector<int>> assignment() {
78a         for (int i = 1; i <= n; i++) {
8c9             p[0] = i;
625             int j0 = 0;
ce7             vector<T> minv(n+1, inf);
241             vector<int> used(n+1, 0);
016             do {
472                 used[j0] = true;
d24                 int i0 = p[j0], j1 = -1;
7e5                 T delta = inf;
9ac                 for (int j = 1; j <= n; j++) if (!used[j]) {
7bf                     T cur = a[i0-1][j-1] - u[i0] - v[j];
9f2                     if (cur < minv[j]) minv[j] = cur, way[j] = j0;
821                     if (minv[j] < delta) delta = minv[j], j1 = j;
4d1                 }
f63                 for (int j = 0; j <= n; j++)
2c5                     if (used[j]) u[p[j]] += delta, v[j] -= delta;
6ec                     else minv[j] -= delta;
6d4                 j0 = j1;
f4f             } while (p[j0] != 0);
016             do {
4c5                 int j1 = way[j0];
0d7                 p[j0] = p[j1];
6d4                 j0 = j1;
886             } while (j0);
38d         }
306         vector<int> ans(n);
6db         for (int j = 1; j <= n; j++) ans[p[j]-1] = j-1;
da3         return make_pair(-v[0], ans);
979     }
64c };
```

3.20 Inversion Count

```
// Computa o numero de inversoes para transformar
// l em r (se nao tem como, retorna -1)
//
// O(n log(n))

37b template<typename T> ll inv_count(vector<T> l, vector<T> r = {}) {
bb6     if (!r.size()) {
796         r = l;
1bc         sort(r.begin(), r.end());
dfb     }
874     int n = l.size();
8c0     vector<int> v(n), bit(n);
4e9     vector<pair<T, int>> w;
61c     for (int i = 0; i < n; i++) w.push_back({r[i], i+1});
d1d     sort(w.begin(), w.end());
603     for (int i = 0; i < n; i++) {
bf3         auto it = lower_bound(w.begin(), w.end(), make_pair(l[i],
    0));
1bf         if (it == w.end() or it->first != l[i]) return -1; // nao
    da
962         v[i] = it->second;
6c0         it->second = -1;
964     }

04b     ll ans = 0;
45b     for (int i = n-1; i >= 0; i--) {
2d9         for (int j = v[i]-1; j; j -= j&-j) ans += bit[j];
3a1         for (int j = v[i]; j < n; j += j&-j) bit[j]++;
ebe     }
ba7     return ans;
eef }
```

3.21 LIS - recupera

```
// Calcula e retorna uma LIS
//
// O(n.log(n))

121 template<typename T> vector<T> lis(vector<T>& v) {
1fa     int n = v.size(), m = -1;
f0c     vector<T> d(n+1, INF);
aec     vector<int> l(n);
007     d[0] = -INF;

603     for (int i = 0; i < n; i++) {
            // Para non-decreasing use upper_bound()
4fd         int t = lower_bound(d.begin(), d.end(), v[i]) - d.begin();
3ad         d[t] = v[i], l[i] = t, m = max(m, t);
89c     }

4ff     int p = n;
5a9     vector<T> ret;
cdf     while (p--) if (l[p] == m) {
883         ret.push_back(v[p]);
76b         m--;
f83     }
969     reverse(ret.begin(),ret.end());

edf     return ret;
474 }
```

3.22 LIS - tamanho

```
// Calcula o tamanho da LIS
//
// O(n log(n))

84b template<typename T> int lis(vector<T> &v){
2da     vector<T> ans;
5e0     for (T t : v){
            // Para non-decreasing use upper_bound()
fe6         auto it = lower_bound(ans.begin(), ans.end(), t);
d7f         if (it == ans.end()) ans.push_back(t);
b94         else *it = t;
1f5     }
1eb     return ans.size();
402 }
```

3.23 Minimum Enclosing Circle

```
// O(n) com alta probabilidade

22c const double EPS = 1e-12;
878 mt19937 rng((int)
    chrono::steady_clock::now().time_since_epoch().count());

b2a struct pt {
```

```
662     double x, y;
be7     pt(double x_ = 0, double y_ = 0) : x(x_), y(y_) {}
7af     pt operator + (const pt& p) const { return pt(x+p.x, y+p.y); }
b23     pt operator - (const pt& p) const { return pt(x-p.x, y-p.y); }
254     pt operator * (double c) const { return pt(x*c, y*c); }
701     pt operator / (double c) const { return pt(x/c, y/c); }
54d };

2f9 double dot(pt p, pt q) { return p.x*q.x+p.y*q.y; }
dd5 double cross(pt p, pt q) { return p.x*q.y-p.y*q.x; }
e7c double dist(pt p, pt q) { return sqrt(dot(p-q, p-q)); }

3f4 pt center(pt p, pt q, pt r) {
5d9     pt a = p-r, b = q-r;
e84     pt c = pt(dot(a, p+r)/2, dot(b, q+r)/2);
e01     return pt(cross(c, pt(a.y, b.y)), cross(pt(a.x, b.x), c)) /
    cross(a, b);
fc8 }

aa8 struct circle {
f41     pt cen;
c12     double r;
898     circle(pt cen_, double r_) : cen(cen_), r(r_) {}
83c     circle(pt a, pt b, pt c) {
13d         cen = center(a, b, c);
1f1         r = dist(cen, a);
bc1     }
cd5     bool inside(pt p) { return dist(p, cen) < r+EPS; }
2a6 };

806 circle minCirc(vector<pt> v) {
f21     shuffle(v.begin(), v.end(), rng);
ae0     circle ret = circle(pt(0, 0), 0);
618     for (int i = 0; i < v.size(); i++) if (!ret.inside(v[i])) {
16a         ret = circle(v[i], 0);
f11         for (int j = 0; j < i; j++) if (!ret.inside(v[j])) {
881             ret = circle((v[i]+v[j])/2, dist(v[i], v[j])/2);
b8c             for (int k = 0; k < j; k++) if (!ret.inside(v[k]))
43f                 ret = circle(v[i], v[j], v[k]);
5f8         }
6a1     }
edf     return ret;
eba }
```

3.24 Minkowski Sum

```
// Computa A+B = {a+b : a \in A, b \in B}, em que
// A e B sao poligonos convexos
// A+B eh um poligono convexo com no max |A|+|B| pontos
//
// O(|A|+|B|)

539 vector<pt> minkowski(vector<pt> p, vector<pt> q) {
051     auto fix = [](vector<pt>& P) {
515         rotate(P.begin(), min_element(P.begin(), P.end()),
    P.end());
018         P.push_back(P[0]), P.push_back(P[1]);
f24     };
889     fix(p), fix(q);
8af     vector<pt> ret;
692     int i = 0, j = 0;
2ee     while (i < p.size()-2 or j < q.size()-2) {
898         ret.push_back(p[i] + q[j]);
732         auto c = ((p[i+1] - p[i]) ^ (q[j+1] - q[j]));
ebc         if (c >= 0) i = min<int>(i+1, p.size()-2);
81e         if (c <= 0) j = min<int>(j+1, q.size()-2);
9ff     }
edf     return ret;
d7c }

c3e ld dist_convex(vector<pt> p, vector<pt> q) {
dc2     for (pt& i : p) i = i * -1;
44c     auto s = minkowski(p, q);
95d     if (inpol(s, pt(0, 0))) return 0;
921     ld ans = DINF;
073     for (int i = 0; i < s.size(); i++) ans = min(ans,
f04             disttoseg(pt(0, 0), line(s[(i+1)%s.size()], s[i])));
ba7     return ans;
beb }
```

3.25 MO

```
// Para ter o bound abaixo, escolher
// SQ = n / sqrt(q)
//
// O(n * sqrt(q))

0d2 const int MAX = 1e5+10;
6ff const int SQ = sqrt(MAX);
b69 int v[MAX];

b65 int ans, freq[MAX];
```

```
9da inline void insert(int p) {
ae0     int o = v[p];
591     freq[o]++;
992     ans += (freq[o] == 1);
21d }

a25 inline void erase(int p) {
ae0     int o = v[p];
7ee     ans -= (freq[o] == 1);
ba2     freq[o]--;
dc7 }

e51 inline ll hilbert(int x, int y) {
71e     static int N = 1 << (__builtin_clz(0) - __builtin_clz(MAX));
100     int rx, ry, s;
b72     ll d = 0;
43b     for (s = N/2; s > 0; s /= 2) {
c95         rx = (x & s) > 0, ry = (y & s) > 0;
e3e         d += s * ll(s) * ((3 * rx) ^ ry);
d2e         if (ry == 0) {
5aa             if (rx == 1) x = N-1 - x, y = N-1 - y;
9dd             swap(x, y);
e2d         }
888     }
be2     return d;
7fa }

bac #define HILBERT true
617 vector<int> MO(vector<pair<int, int>> &q) {
c3b     ans = 0;
c23     int m = q.size();
3f8     vector<int> ord(m);
be8     iota(ord.begin(), ord.end(), 0);
6a6 #if HILBERT
8c4     vector<ll> h(m);
74c     for (int i = 0; i < m; i++) h[i] = hilbert(q[i].first,
    q[i].second);
075     sort(ord.begin(), ord.end(), [&](int l, int r) { return h[l] <
    h[r]; });
8c1 #else
d01     sort(ord.begin(), ord.end(), [&](int l, int r) {
9c9         if (q[l].first / SQ != q[r].first / SQ) return q[l].first
    < q[r].first;
0db         if ((q[l].first / SQ) % 2) return q[l].second >
    q[r].second;
a66         return q[l].second < q[r].second;
bec     });
f2e #endif
435     vector<int> ret(m);
3d9     int l = 0, r = -1;

8b0     for (int i : ord) {
6c6         int ql, qr;
4f5         tie(ql, qr) = q[i];
026         while (r < qr) insert(++r);
232         while (l > ql) insert(--l);
75e         while (l < ql) erase(l++);
fe8         while (r > qr) erase(r--);
381         ret[i] = ans;
116     }
edf     return ret;
fb7 }
```

3.26 MO - DSU

```
// Dado uma lista de arestas de um grafo, responde
// para cada query(l, r), quantos componentes conexos
// o grafo tem se soh considerar as arestas l, l+1, ..., r
// Da pra adaptar pra usar MO com qualquer estrutura rollbackavel
//
// O(m sqrt(q) log(n))

8d3 struct dsu {
553     int n, ans;
2e3     vector<int> p, sz;
ee6     stack<int> S;

4b8     dsu(int n_) : n(n_), ans(n), p(n), sz(n) {
8a6         for (int i = 0; i < n; i++) p[i] = i, sz[i] = 1;
aae     }
1b1     int find(int k) {
006         while (p[k] != k) k = p[k];
839         return k;
c1e     }
553     void add(pair<int, int> x) {
700         int a = x.first, b = x.second;
605         a = find(a), b = find(b);
843         if (a == b) return S.push(-1);
e7d         ans--;
3c6         if (sz[a] > sz[b]) swap(a, b);
4c2         S.push(a);
582         sz[b] += sz[a];
84b         p[a] = b;
720     }
35c     int query() { return ans; }
5cf     void rollback() {
465         int u = S.top(); S.pop();
61c         if (u == -1) return;
270         sz[p[u]] -= sz[u];
546         p[u] = u;
0df         ans++;
456     }
9c1 };

1a8 int n;
e93 vector<pair<int, int>> ar;

// 9d242b
617 vector<int> MO(vector<pair<int, int>> &q) {
547     int SQ = sqrt(q.size()) + 1;
c23     int m = q.size();
3f8     vector<int> ord(m);
be8     iota(ord.begin(), ord.end(), 0);
d01     sort(ord.begin(), ord.end(), [&](int l, int r) {
9c9             if (q[l].first / SQ != q[r].first / SQ) return
    q[l].first < q[r].first;
a66             return q[l].second < q[r].second;
b90             });
435     vector<int> ret(m);

3bd     dsu small(n);
dd5     for (int i = 0; i < m; i++) {
5ec         auto [l, r] = q[ord[i]];
acc         if (l / SQ == r / SQ) {
00c             for (int k = l; k <= r; k++) small.add(ar[k]);
b99             ret[ord[i]] = small.query();
64e             for (int k = l; k <= r; k++) small.rollback();
259         }
6b0     }

dd5     for (int i = 0; i < m; i++) {
176         dsu D(n);
ae9         int fim = q[ord[i]].first/SQ*SQ + SQ - 1;
e25         int last_r = fim;
ebc         int j = i-1;
00c         while (j+1 < m and q[ord[j+1]].first / SQ ==
    q[ord[i]].first / SQ) {
a0e             auto [l, r] = q[ord[++j]];
f58             if (l / SQ == r / SQ) continue;

59b             while (last_r < r) D.add(ar[++last_r]);
2cf             for (int k = l; k <= fim; k++) D.add(ar[k]);

9b2             ret[ord[j]] = D.query();

572             for (int k = l; k <= fim; k++) D.rollback();
9c8         }
bdf         i = j;
e99     }
edf     return ret;
9d2 }
```

3.27 MO em Arvores

```
// Problema que resolve: https://www.spoj.com/problems/COT2/
//
// Complexidade sendo c = O(update) e SQ = sqrt(n):
// O((n + q) * sqrt(n) * c)

1bc const int MAX = 40010, SQ = 400;

042 vector<int> g[MAX];

c54 namespace LCA { ... }

249 int in[MAX], out[MAX], vtx[2 * MAX];
81b bool on[MAX];

4c3 int dif, freq[MAX];
9e2 vector<int> w;

d9a void dfs(int v, int p, int &t) {
659     vtx[t] = v, in[v] = t++;
18e     for (int u : g[v]) if (u != p) {
c53         dfs(u, v, t);
e0f     }
217     vtx[t] = v, out[v] = t++;
42b }

e5f void update(int p) { // faca alteracoes aqui
bbc     int v = vtx[p];
0ec     if (not on[v]) { // insere vtx v
31c         dif += (freq[w[v]] == 0);
b20         freq[w[v]]++;
cf7     }
4e6     else { // retira o vertice v
0a9         dif -= (freq[w[v]] == 1);
fd3         freq[w[v]]--;
2c8     }
73e     on[v] = not on[v];
ea9 }

a3a vector<tuple<int, int, int>> build_queries(const vector<pair<int,
    int>>& q) {
ea6     LCA::build(0);
f77     vector<tuple<int, int, int>> ret;
aa9     for (auto [l, r] : q){
d24         if (in[r] < in[l]) swap(l, r);
6f9         int p = LCA::lca(l, r);
826         int init = (p == l) ? in[l] : out[l];
07a         ret.emplace_back(init, in[r], in[p]);
b0e     }
edf     return ret;
8e6 }

f31 vector<int> mo_tree(const vector<pair<int, int>>& vq){
6bb     int t = 0;
dab     dfs(0, -1, t);

af1     auto q = build_queries(vq);

f48     vector<int> ord(q.size());
be8     iota(ord.begin(), ord.end(), 0);
d01     sort(ord.begin(), ord.end(), [&] (int l, int r) {
d8d         int bl = get<0>(q[l]) / SQ, br = get<0>(q[r]) / SQ;
596         if (bl != br) return bl < br;
158         else if (bl % 2 == 1) return get<1>(q[l]) < get<1>(q[r]);
f1d         else return get<1>(q[l]) > get<1>(q[r]);
0a8     });

80e     memset(freq, 0, sizeof freq);
bf6     dif = 0;

ff2     vector<int> ret(q.size());
3d9     int l = 0, r = -1;
8b0     for (int i : ord) {
3c7         auto [ql, qr, qp] = q[i];
af7         while (r < qr) update(++r);
d6b         while (l > ql) update(--l);
951         while (l < ql) update(l++);
6a1         while (r > qr) update(r--);
3d8         if (qp < l or qp > r) { // se LCA estah entre as pontas
74b             update(qp);
2e1             ret[i] = dif;
74b             update(qp);
e83         }
0fe         else ret[i] = dif;
0fd     }
edf     return ret;
48d }
```

3.28 Palindromic Factorization

```
// Precisa da eertree
// Computa o numero de formas de particionar cada
// prefixo da string em strings palindromicas
//
// O(n log n), considerando alfabeto O(1)

070 struct eertree { ... };

0e7 ll factorization(string s) {
b19     int n = s.size(), sz = 2;
580     eertree PT(n);
147     vector<int> diff(n+2), slink(n+2), sans(n+2), dp(n+1);
0ec     dp[0] = 1;
78a     for (int i = 1; i <= n; i++) {
c58         PT.add(s[i-1]);
a7c         if (PT.size()+2 > sz) {
6c4             diff[sz] = PT.len[sz] - PT.len[PT.link[sz]];
241             if (diff[sz] == diff[PT.link[sz]])
d6f                 slink[sz] = slink[PT.link[sz]];
f53             else slink[sz] = PT.link[sz];
eb9             sz++;
f6a         }
911         for (int v = PT.last; PT.len[v] > 0; v = slink[v]) {
297             sans[v] = dp[i - (PT.len[slink[v]] + diff[v])];
85d             if (diff[v] == diff[PT.link[v]])
f20                 sans[v] = (sans[v] + sans[PT.link[v]]) % MOD;
071             dp[i] = (dp[i] + sans[v]) % MOD;
e5e         }
fc0     }
5f0     return dp[n];
3a7 }
```

3.29 Parsing de Expressao

```
// Operacoes associativas a esquerda por default
// Para mudar isso, colocar em r_assoc
// Operacoes com maior prioridade sao feitas primeiro

cc1 bool blank(char c) {
f34     return c == ' ';
ec3 }

8e4 bool is_unary(char c) {
f9c     return c == '+' or c == '-';
b6b }

76d bool is_op(char c) {
010     if (is_unary(c)) return true;
31c     return c == '*' or c == '/' or c == '+' or c == '-';
4e4 }

fa3 bool r_assoc(char op) {
        // operator unario - deve ser assoc. a direita
cf0     return op < 0;
c5c }

79d int priority(char op) {
        // operator unario - deve ter precedencia maior
103     if (op < 0) return INF;

727     if (op == '*' or op == '/') return 2;
439     if (op == '+' or op == '-') return 1;
daa     return -1;
966 }

c15 void process_op(stack<int>& st, stack<int>& op) {
88c     char o = op.top(); op.pop();
91c     if (o < 0) {
4e6         o *= -1;
1e2         int l = st.top(); st.pop();
0ff         if (o == '+') st.push(l);
7e9         if (o == '-') st.push(-l);
320     } else {
14c         int r = st.top(); st.pop();
1e2         int l = st.top(); st.pop();
1e4         if (o == '*') st.push(l * r);
f55         if (o == '/') st.push(l / r);
605         if (o == '+') st.push(l + r);
c40         if (o == '-') st.push(l - r);
0aa     }
2b2 }

439 int eval(string& s) {
212     stack<int> st, op;
d0c     bool un = true;
1cf     for (int i = 0; i < s.size(); i++) {
68d         if (blank(s[i])) continue;

139         if (s[i] == '(') {
367             op.push('(');
99d             un = true;
b88         } else if (s[i] == ')') {
709             while (op.top() != '(') process_op(st, op);
75e             op.pop();
ce2             un = false;
003         } else if (is_op(s[i])) {
4d0             char o = s[i];
37c             if (un and is_unary(o)) o *= -1;
ae3             while (op.size() and (
cd6                         (!r_assoc(o) and priority(op.top()) >=
priority(o)) or
c41                         (r_assoc(o) and priority(op.top()) >
priority(o))))
c47                 process_op(st, op);
c00             op.push(o);
99d             un = true;
196         } else {
da8             int val = 0;
c2b             while (i < s.size() and isalnum(s[i]))
8a3                 val = val * 10 + s[i++] - '0';
169             i--;
25d             st.push(val);
ce2             un = false;
442         }
b19     }

7f6     while (op.size()) process_op(st, op);
123     return st.top();
05c }
```

3.30 RMQ com Divide and Conquer

```
// Responde todas as queries em
// O(n log(n))
```

```
f74 typedef pair<pair<int, int>, int> iii;
7c6 #define f first
0ab #define s second

87d int n, q, v[MAX];
e3f iii qu[MAX];
aeb int ans[MAX], pref[MAX], sulf[MAX];

0e3 void solve(int l=0, int r=n-1, int ql=0, int qr=q-1) {
8a3     if (l > r or ql > qr) return;
ee4     int m = (l+r)/2;
1b1     int qL = partition(qu+ql, qu+qr+1, [=](iii x){return x.f.s <
    m;}) - qu;
eb0     int qR = partition(qu+qL, qu+qr+1, [=](iii x){return x.f.f
    <=m;}) - qu;

3cd     pref[m] = sulf[m] = v[m];
9f9     for (int i = m-1; i >= l; i--) pref[i] = min(v[i], pref[i+1]);
ea8     for (int i = m+1; i <= r; i++) sulf[i] = min(v[i], sulf[i-1]);

b2a     for (int i = qL; i < qR; i++)
f3a         ans[qu[i].s] = min(pref[qu[i].f.f], sulf[qu[i].f.s]);

364     solve(l, m-1, ql, qL-1), solve(m+1, r, qR, qr);
13e }
```

3.31 Segment Intersection

```
// Verifica, dado n segmentos, se existe algum par de segmentos
// que se intersecta
//
// O(n log n)

6e0 bool operator < (const line& a, const line& b) { // comparador pro
    sweepline
191     if (a.p == b.p) return ccw(a.p, a.q, b.q);
231     if (!eq(a.p.x, a.q.x) and (eq(b.p.x, b.q.x) or a.p.x+eps <
    b.p.x))
780         return ccw(a.p, a.q, b.p);
dc0     return ccw(a.p, b.q, b.p);
e36 }

8e2 bool has_intersection(vector<line> v) {
576     auto intersects = [&](pair<line, int> a, pair<line, int> b) {
a08         return interseg(a.first, b.first);
3e6     };
e1b     vector<pair<pt, pair<int, int>>> w;
f14     for (int i = 0; i < v.size(); i++) {
876         if (v[i].q < v[i].p) swap(v[i].p, v[i].q);
e1d         w.push_back({v[i].p, {0, i}});
034         w.push_back({v[i].q, {1, i}});
220     }
d1d     sort(w.begin(), w.end());
7f2     set<pair<line, int>> se;
e58     for (auto i : w) {
bfd         line at = v[i.second.second];
292         if (i.second.first == 0) {
145             auto nxt = se.lower_bound({at, i.second.second});
d1e             if (nxt != se.end() and intersects(*nxt, {at,
    i.second.second})) return 1;
257             if (nxt != se.begin() and intersects(*(--nxt), {at,
    i.second.second})) return 1;
78f             se.insert({at, i.second.second});
08b         } else {
884             auto nxt = se.upper_bound({at, i.second.second}), cur
    = nxt, prev = --cur;
b64             if (nxt != se.end() and prev != se.begin()
4fb                 and intersects(*nxt, *(--prev))) return 1;
cca             se.erase(cur);
e27         }
a00     }
bb3     return 0;
196 }
```

3.32 Sequencia de de Brujin

```
// Se passar sem o terceiro parametro, gera um vetor com valores
// em [0, k) de tamanho k^n de forma que todos os subarrays ciclicos
// de tamanho n ocorrem exatamente uma vez
// Se passar com um limite lim, gera o menor vetor com valores
// em [0, k) que possui lim subarrays de tamanho n distintos
// (assume que lim <= k^n)
//
// Linear no tamanho da resposta

860 vector<int> de_brujin(int n, int k, int lim = INF) {
b55     if (k == 1) return vector<int>(lim == INF ? 1 : n, 0);
5f6     vector<int> l = {0}, ret; // l eh lyndon word
667     while (true) {
c86         if (l.size() == 0) {
1b9             if (lim == INF) break;
daf             l.push_back(0);
bae         }
686         if (n % l.size() == 0) for (int i : l) {
728             ret.push_back(i);
c99             if (ret.size() == n+lim-1) return ret;
56e         }
630         int p = l.size();
905         while (l.size() < n) l.push_back(l[l.size()%p]);
e7f         while (l.size() and l.back() == k-1) l.pop_back();
88a         if (l.size()) l.back()++;
2ef     }
edf     return ret;
197 }
```

3.33 Shortest Addition Chain

```
// Computa o menor numero de adicoes para construir
// cada valor, comecando com 1 (e podendo salvar variaveis)
// Retorna um par com a dp e o pai na arvore
// A arvore eh tao que o taminho da raiz (1) ate x
// contem os valores que devem ser criados para gerar x
// A profundidade de x na arvore eh dp[x]
// DP funciona para ateh 300, mas a arvore soh funciona
// para ateh 148

// recuperacao certa soh ateh 148 (erra para 149, 233, 298)
3de pair<vector<int>, vector<int>> addition_chain() {
16f     int MAX = 301;
875     vector<int> dp(MAX), p(MAX);
1ab     for (int n = 2; n < MAX; n++) {
7c0         pair<int, int> val = {INF, -1};
212         for (int i = 1; i < n; i++) for (int j = i; j; j = p[j])
94a             if (j == n-i) val = min(val, pair(dp[i]+1, i));
eb3         tie(dp[n], p[n]) = val;
efe         if (n == 9) p[n] = 8;
ba1         if (n == 149 or n == 233) dp[n]--;
bcd     }
717     return {dp, p};
84f }
```

3.34 Simple Polygon

```
// Verifica se um poligono com n pontos eh simples
//
// O(n log n)

6e0 bool operator < (const line& a, const line& b) { // comparador pro
    sweepline
191     if (a.p == b.p) return ccw(a.p, a.q, b.q);
231     if (!eq(a.p.x, a.q.x) and (eq(b.p.x, b.q.x) or a.p.x+eps <
    b.p.x))
780         return ccw(a.p, a.q, b.p);
dc0     return ccw(a.p, b.q, b.p);
e36 }

6f3 bool simple(vector<pt> v) {
576     auto intersects = [&](pair<line, int> a, pair<line, int> b) {
e72         if ((a.second+1)%v.size() == b.second or
80e             (b.second+1)%v.size() == a.second) return false;
a08         return interseg(a.first, b.first);
1c5     };
41a     vector<line> seg;
e1b     vector<pair<pt, pair<int, int>>> w;
f14     for (int i = 0; i < v.size(); i++) {
0a8         pt at = v[i], nxt = v[(i+1)%v.size()];
828         if (nxt < at) swap(at, nxt);
937         seg.push_back(line(at, nxt));
f7e         w.push_back({at, {0, i}});
69c         w.push_back({nxt, {1, i}});
            // casos degenerados estranhos
ae8         if (isinseg(v[(i+2)%v.size()], line(at, nxt))) return 0;
88d         if (isinseg(v[(i+v.size()-1)%v.size()], line(at, nxt)))
    return 0;
cba     }
d1d     sort(w.begin(), w.end());
7f2     set<pair<line, int>> se;
e58     for (auto i : w) {
ff8         line at = seg[i.second.second];
292         if (i.second.first == 0) {
145             auto nxt = se.lower_bound({at, i.second.second});
7c4             if (nxt != se.end() and intersects(*nxt, {at,
    i.second.second})) return 0;
b34             if (nxt != se.begin() and intersects(*(--nxt), {at,
    i.second.second})) return 0;
78f             se.insert({at, i.second.second});
537         } else {
884             auto nxt = se.upper_bound({at, i.second.second}), cur
    = nxt, prev = --cur;
b64             if (nxt != se.end() and prev != se.begin()
403                 and intersects(*nxt, *(--prev))) return 0;
cca             se.erase(cur);
7be         }
d17     }
6a5     return 1;
af3 }
```

3.35 Steiner Tree

```
// steiner: retorna o peso da menor arvore que cobre os vertices S
// get_steiner: retorna o valor minimo e as arestas de uma solucao
// se nao tiver solucao retorna LINF
//
// grafo nao pode ter pesos negativos
// se so tiver peso nas arestas/vertices pode deletar os vw/w no codigo
//
// k = |S|
// O(3^k * n + 2^k * m log m)

// otimizacao: joga um vertice x do S fora e pegue a resposta em dp[...][x] e reconstrua a arvore a partir dele
// ta comentado no codigo as mudancas necessarias

1a8 int n; // numero de vertices
c0d vector<pair<int, int>> g[MAX]; // {vizinho, peso}
920 ll d[1 << K][MAX]; // dp[mask][v] = arvore minima com o subconjunto mask de S e o vertice v
0b0 ll vw[MAX]; // peso do vertice

c8f ll steiner(const vector<int> &S) {
934     int k = S.size(); // k--;
dea     for (int mask = 0; mask < (1 << k); mask++) for(int v = 0; v < n; v++) d[mask][v] = LINF;
6b8     for (int v = 0; v < n; v++) d[0][v] = vw[v];
254     for (int i = 0; i < k; ++i) d[1 << i][S[i]] = vw[S[i]];
042     for (int mask = 1; mask < (1 << k); mask++) {
b5b         for (int a = (mask - 1) & mask; a; a = (a - 1) & mask) {
638             int b = mask ^ a;
6bf             if (b > a) break;
84d             for (int v = 0; v < n; v++)
2e6                 d[mask][v] = min(d[mask][v], d[a][v] + d[b][v] - vw[v]);
2ab         }
88c         priority_queue<pair<ll, int>> pq;
778         for (int v = 0; v < n; v++) {
6ad             if (d[mask][v] == LINF) continue;
5ca             pq.emplace(-d[mask][v], v);
f2e         }
265         while (pq.size()) {
a25             auto [ndist, u] = pq.top(); pq.pop();
dad             if (-ndist > d[mask][u]) continue;
c38             for (auto [idx, w] : g[u]) if (d[mask][idx] > d[mask][u] + w + vw[idx]) {
679                 d[mask][idx] = d[mask][u] + w + vw[idx];
a2e                 pq.emplace(-d[mask][idx], idx);
07e             }
de5         }
a65     }
478     return d[(1 << k) - 1][S[0]]; // S[k]
704 }

d41 #warning se k=1 a solucao eh a folha isolada e a funcao retorna edg = {}
d41 #warning se k=0 crasha
4d1 pair<ll,vector<pair<int,int>>> get_steiner(const vector<int> &S) {
934     int k = S.size(); // k--;
8ec     ll ans = steiner(S);
c8d     vector<pair<int,int>> edg;
57f     stack<pair<int,int>> stk;
09b     stk.emplace((1 << k) - 1, S[0]); // S[k]
07d     while (!stk.empty()) {
c37         bool cont = 0;
9c6         auto [mask,u] = stk.top();stk.pop();
de2         if ((__builtin_popcount(mask) == 1 and u == S[__bit_width(mask) - 1])) continue;
851         for (auto [idx, w] : g[u]){
bb4             if (d[mask][u] == d[mask][idx] + w + vw[u]) {
fc7                 edg.emplace_back(u, idx);
8ab                 stk.emplace(mask, idx);
a04                 cont = true;
c2b                 break;
342             }
ed9         }
3b5         if (cont) continue;
b5b         for (int a = (mask - 1) & mask; a; a = (a - 1) & mask) {
638             int b = mask ^ a;
1e8             if (d[mask][u] == d[a][u] + d[b][u] - vw[u]) {
be8                 stk.emplace(a, u);
c0a                 stk.emplace(b, u);
a04                 cont = true;
c2b                 break;
f52             }
d29         }
c5c         assert(!mask || cont);
2b1     }
be8     return {ans, edg};
cf6 }
```

3.36 Sweep Direction

```
// Passa por todas as ordenacoes dos pontos definitas por "direcoes"
// Assume que nao existem pontos coincidentes
//
// O(n^2 log n)

4b8 void sweep_direction(vector<pt> v) {
3d2     int n = v.size();
163     sort(v.begin(), v.end(), [](pt a, pt b) {
3a5         if (a.x != b.x) return a.x < b.x;
572         return a.y > b.y;
79a     });
b89     vector<int> at(n);
516     iota(at.begin(), at.end(), 0);
b79     vector<pair<int, int>> swapp;
25e     for (int i = 0; i < n; i++) for (int j = i+1; j < n; j++)
95f         swapp.push_back({i, j}), swapp.push_back({j, i});

269     sort(swapp.begin(), swapp.end(), [&](auto a, auto b) {
134         pt A = rotate90(v[a.first] - v[a.second]);
247         pt B = rotate90(v[b.first] - v[b.second]);
615         if (quad(A) == quad(B) and !sarea2(pt(0, 0), A, B)) return
    a < b;
224         return compare_angle(A, B);
5e7     });
4e6     for (auto par : swapp) {
e24         assert(abs(at[par.first] - at[par.second]) == 1);
a96         int l = min(at[par.first], at[par.second]),
0d3             r = n-1 - max(at[par.first], at[par.second]);
            // l e r sao quantos caras tem de cada lado do par de
                pontos
            // (cada par eh visitado duas vezes)
9cf         swap(v[at[par.first]], v[at[par.second]]);
1c0         swap(at[par.first], at[par.second]);
241     }
6bb }
```

3.37 Triangulacao de Delaunay

```
// Computa a triangulacao de Delaunay, o dual
// do diagrama de Voronoi (a menos de casos degenerados)
// Retorna um grafo indexado pelos indices dos pontos, e as arestas
// sao as arestas da triangulacao
// As arestas partindo de um vertice ja vem ordenadas por angulo,
// ou seja, se o vertice v nao esta no convex hull, (v, v_i, v_{i+1})
// eh um triangulo da triangulacao, em que v_i eh o i-esimo vizinho
// Usa o alg d&c, precisa representar MAX_COOR^4, por isso __int128
// pra aguentar valores ateh 1e9
//
// Propriedades:
// 1 - O grafo tem no max 3n-6 arestas
// 2 - Para todo triangulo, a circunf. que passa pelos 3 pontos
//     nao contem estritamente nenhum ponto
// 3 - A MST euclidiana eh subgrafo desse grafo
// 4 - Cada ponto eh vizinho do ponto mais proximo dele
//
// O(n log n)

2ad typedef struct QuadEdge* Q;
ba5 struct QuadEdge {
53e     int id;
114     pt o;
41e     Q rot, nxt;
3e5     bool used;

3fc     QuadEdge(int id_ = -1, pt o_ = pt(INF, INF)) :
4ba         id(id_), o(o_), rot(nullptr), nxt(nullptr), used(false) {}

00f     Q rev() const { return rot->rot; }
c3c     Q next() const { return nxt; }
188     Q prev() const { return rot->next()->rot; }
0d4     pt dest() const { return rev()->o; }
828 };

91b Q edge(pt from, pt to, int id_from, int id_to) {
c6e     Q e1 = new QuadEdge(id_from, from);
61b     Q e2 = new QuadEdge(id_to, to);
8f6     Q e3 = new QuadEdge;
5ca     Q e4 = new QuadEdge;
e69     tie(e1->rot, e2->rot, e3->rot, e4->rot) = {e3, e4, e2, e1};
f22     tie(e1->nxt, e2->nxt, e3->nxt, e4->nxt) = {e1, e2, e4, e3};
1ad     return e1;
c70 }

d8d void splice(Q a, Q b) {
a6f     swap(a->nxt->rot->nxt, b->nxt->rot->nxt);
da4     swap(a->nxt, b->nxt);
a58 }
```

```
167 void del_edge(Q& e, Q ne) { // delete e and assign e <- ne
cc0     splice(e, e->prev());
eec     splice(e->rev(), e->rev()->prev());
7ea     delete e->rev()->rot, delete e->rev();
524     delete e->rot; delete e;
6b2     e = ne;
18b }

d08 Q conn(Q a, Q b) {
cc5     Q e = edge(a->dest(), b->o, a->rev()->id, b->id);
f2b     splice(e, a->rev()->prev());
d37     splice(e->rev(), b);
6bf     return e;
f78 }

d64 bool in_c(pt a, pt b, pt c, pt p) { // p ta na circunf. (a, b, c) ?
268     __int128 p2 = p*p, A = a*a - p2, B = b*b - p2, C = c*c - p2;
cbe     return sarea2(p, a, b) * C + sarea2(p, b, c) * A + sarea2(p, c, a) * B > 0;
b54 }

540 pair<Q, Q> build_tr(vector<pt>& p, int l, int r) {
09d     if (r-l+1 <= 3) {
2eb         Q a = edge(p[l], p[l+1], l, l+1), b = edge(p[l+1], p[r], l+1, r);
912         if (r-l+1 == 2) return {a, a->rev()};
0ec         splice(a->rev(), b);
c3c         ll ar = sarea2(p[l], p[l+1], p[r]);
1af         Q c = ar ? conn(b, a) : 0;
021         if (ar >= 0) return {a, b->rev()};
9db         return {c->rev(), c};
bce     }
ee4     int m = (l+r)/2;
328     auto [la, ra] = build_tr(p, l, m);
b93     auto [lb, rb] = build_tr(p, m+1, r);
667     while (true) {
b99         if (ccw(lb->o, ra->o, ra->dest())) ra = ra->rev()->prev();
458         else if (ccw(lb->o, ra->o, lb->dest())) lb = lb->rev()->next();
f97         else break;
24a     }
ca5     Q b = conn(lb->rev(), ra);
713     auto valid = [&](Q e) { return ccw(e->dest(), b->dest(), b->o); };
ee1     if (ra->o == la->o) la = b->rev();
63f     if (lb->o == rb->o) rb = b;
667     while (true) {
71e         Q L = b->rev()->next();
d11         if (valid(L)) while (in_c(b->dest(), b->o, L->dest(), L->next()->dest()))
1c0                 del_edge(L, L->next());
c76         Q R = b->prev();
2b0         if (valid(R)) while (in_c(b->dest(), b->o, R->dest(), R->prev()->dest()))
541                 del_edge(R, R->prev());
a3a         if (!valid(L) and !valid(R)) break;
ccd         if (!valid(L) or (valid(R) and in_c(L->dest(), L->o, R->o, R->dest())))
36c                 b = conn(R, b->rev());
666         else b = conn(b->rev(), L->rev());
94d     }
a2b     return {la, rb};
689 }

b58 vector<vector<int>> delaunay(vector<pt> v) {
3d2     int n = v.size();
397     auto tmp = v;
135     vector<int> idx(n);
295     iota(idx.begin(), idx.end(), 0);
fe9     sort(idx.begin(), idx.end(), [&](int l, int r) { return v[l] < v[r]; });
5d8     for (int i = 0; i < n; i++) v[i] = tmp[idx[i]];
780     assert(unique(v.begin(), v.end()) == v.end());
4aa     vector<vector<int>> g(n);
4ec     bool col = true;
a96     for (int i = 2; i < n; i++) if (sarea2(v[i], v[i-1], v[i-2])) col = false;
bf5     if (col) {
aa4         for (int i = 1; i < n; i++)
839             g[idx[i-1]].push_back(idx[i]), g[idx[i]].push_back(idx[i-1]);
96b         return g;
0ae     }
d36     Q e = build_tr(v, 0, n-1).first;
113     vector<Q> edg = {e};
5d1     for (int i = 0; i < edg.size(); e = edg[i++]) {
3ed         for (Q at = e; !at->used; at = at->next()) {
60d             at->used = true;
cf8             g[idx[at->id]].push_back(idx[at->rev()->id]);
15d             edg.push_back(at->rev());
9f2         }
d19     }
96b     return g;
b43 }
```

3.38 Triangulos em Grafos

```
// get_triangles(i) encontra todos os triangulos ijk no grafo
// Custo nas arestas
// retorna {custo do triangulo, {j, k}}
//
// O(m sqrt(m) log(n)) se chamar para todos os vertices

c0d vector<pair<int, int>> g[MAX]; // {para, peso}

d41 #warning o 'g' deve estar ordenado
9a5 vector<pair<int, pair<int, int>>> get_triangles(int i) {
771     vector<pair<int, pair<int, int>>> tri;
b23     for (pair<int, int> j : g[i]) {
2b3         int a = i, b = j.first;
6dd         if (g[a].size() > g[b].size()) swap(a, b);
eb0         for (pair<int, int> c : g[a]) if (c.first != b and c.first
    > j.first) {
525             auto it = lower_bound(g[b].begin(), g[b].end(),
    make_pair(c.first, -INF));
f55             if (it == g[b].end() or it->first != c.first) continue;
0aa             tri.push_back({j.second+c.second+it->second, {a == i ?
    b : a, c.first}});
b5e         }
7e1     }
f5e     return tri;
036 }
```

4 Strings

4.1 Aho-corasick

```
// query retorna o somatorio do numero de matches de
// todas as stringuinhas na stringona
//
// insert - O(|s| log(SIGMA))
// build - O(N), onde N = somatorio dos tamanhos das strings
// query - O(|s|)

ea1 namespace aho {
807     map<char, int> to[MAX];
c87     int link[MAX], idx, term[MAX], exit[MAX], sobe[MAX];

bfc     void insert(string& s) {
05e         int at = 0;
b4f         for (char c : s) {
b68             auto it = to[at].find(c);
1c9             if (it == to[at].end()) at = to[at][c] = ++idx;
361             else at = it->second;
ff4         }
142         term[at]++, sobe[at]++;
6eb     }
d41 #warning nao esquece de chamar build() depois de inserir
0a8     void build() {
26a         queue<int> q;
537         q.push(0);
dff         link[0] = exit[0] = -1;
402         while (q.size()) {
379             int i = q.front(); q.pop();
3c4             for (auto [c, j] : to[i]) {
5da                 int l = link[i];
102                 while (l != -1 and !to[l].count(c)) l = link[l];
7a5                 link[j] = l == -1 ? 0 : to[l][c];
3ab                 exit[j] = term[link[j]] ? link[j] : exit[link[j]];
6f2                 if (exit[j]+1) sobe[j] += sobe[exit[j]];
113                 q.push(j);
f1d             }
367         }
768     }
bc0     int query(string& s) {
86d         int at = 0, ans = 0;
b4f         for (char c : s){
1ca             while (at != -1 and !to[at].count(c)) at = link[at];
5b9             at = at == -1 ? 0 : to[at][c];
2b1             ans += sobe[at];
b85         }
ba7         return ans;
038     }
a30 }
```

4.2 eertree

```
// Constroi a eertree, caractere a caractere
// Inicializar com a quantidade de caracteres maxima
// size() retorna a quantidade de substrings pal. distintas
// depois de chamar propagate(), cada substring palindromica
// ocorre qt[i] vezes. O propagate() retorna o numero de
// substrings pal. com repeticao
//
// O(n) amortizado, considerando alfabeto O(1)

8eb struct eertree {
7cc     vector<vector<int>> t;
42e     int n, last, sz;
745     vector<int> s, len, link, qt;

d36     eertree(int N) {
ec8         t = vector(N+2, vector(26, int()));
cee         s = len = link = qt = vector<int>(N+2);
cd1         s[0] = -1;
288         link[0] = 1, len[0] = 0, link[1] = 1, len[1] = -1;
688         sz = 2, last = 0, n = 1;
8dc     }

244     void add(char c) {
692         s[n++] = c -= 'a';
34f         while (s[n-len[last]-2] != c) last = link[last];
289         if (!t[last][c]) {
dab             int prev = link[last];
553             while (s[n-len[prev]-2] != c) prev = link[prev];
fb2             link[sz] = t[prev][c];
3f5             len[sz] = len[last]+2;
1f8             t[last][c] = sz++;
f8b         }
344         qt[last = t[last][c]]++;
b1d     }
f17     int size() { return sz-2; }
2af     ll propagate() {
b73         ll ret = 0;
ebb         for (int i = n; i > 1; i--) {
fd3             qt[link[i]] += qt[i];
db5             ret += qt[i];
074         }
edf         return ret;
ef6     }
a2e };
```

4.3 KMP

```
// matching(s, t) retorna os indices das ocorrencias
// de s em t
// autKMP constroi o automato do KMP
//
// Complexidades:
// pi - O(n)
// match - O(n + m)
// construir o automato - O(|sigma|*n)
// n = |padrao| e m = |texto|

ea8 template<typename T> vector<int> pi(T s) {
019     vector<int> p(s.size());
725     for (int i = 1, j = 0; i < s.size(); i++) {
a51         while (j and s[j] != s[i]) j = p[j-1];
973         if (s[j] == s[i]) j++;
f8c         p[i] = j;
e0a     }
74e     return p;
f50 }

c10 template<typename T> vector<int> matching(T& s, T& t) {
658     vector<int> p = pi(s), match;
a1b     for (int i = 0, j = 0; i < t.size(); i++) {
6be         while (j and s[j] != t[i]) j = p[j-1];
c4d         if (s[j] == t[i]) j++;
310         if (j == s.size()) match.push_back(i-j+1), j = p[j-1];
028     }
ed8     return match;
c82 }

a2d struct KMPaut : vector<vector<int>> {
47c     KMPaut(){}
6c7     KMPaut (string& s) : vector<vector<int>>(26,
    vector<int>(s.size()+1)) {
503         vector<int> p = pi(s);
04b         auto& aut = *this;
4fa         aut[s[0]-'a'][0] = 1;
19a         for (char c = 0; c < 26; c++)
5d3             for (int i = 1; i <= s.size(); i++)
42b                 aut[c][i] = s[i]-'a' == c ? i+1 : aut[c][p[i-1]];
4bb     }
79b };
```

4.4 Manacher

```
// manacher recebe um vetor de T e retorna o vetor com tamanho dos
    palindromos
// ret[2*i] = tamanho do maior palindromo centrado em i
// ret[2*i+1] = tamanho maior palindromo centrado em i e i+1
```

```
//
// Complexidades:
// manacher - O(n)
// palindrome - <O(n), O(1)>
// pal_end - O(n)

28a template<typename T> vector<int> manacher(const T& s) {
18f     int l = 0, r = -1, n = s.size();
fc9     vector<int> d1(n), d2(n);
603     for (int i = 0; i < n; i++) {
821         int k = i > r ? 1 : min(d1[l+r-i], r-i);
61a         while (i+k < n && i-k >= 0 && s[i+k] == s[i-k]) k++;
61e         d1[i] = k--;
9f6         if (i+k > r) l = i-k, r = i+k;
950     }
e03     l = 0, r = -1;
603     for (int i = 0; i < n; i++) {
a64         int k = i > r ? 0 : min(d2[l+r-i+1], r-i+1); k++;
2c6         while (i+k <= n && i-k >= 0 && s[i+k-1] == s[i-k]) k++;
eaa         d2[i] = --k;
26d         if (i+k-1 > r) l = i-k, r = i+k-1;
4fe     }
c41     vector<int> ret(2*n-1);
e6b     for (int i = 0; i < n; i++) ret[2*i] = 2*d1[i]-1;
e1d     for (int i = 0; i < n-1; i++) ret[2*i+1] = 2*d2[i+1];
edf     return ret;
ebb }

// verifica se a string s[i..j] eh palindromo
cac template<typename T> struct palindrome {
f97     vector<int> man;

b2d     palindrome(const T& s) : man(manacher(s)) {}
9d7     bool query(int i, int j) {
bad         return man[i+j] >= j-i+1;
1e7     }
60c };

// tamanho do maior palindromo que termina em cada posicao
7cb template<typename T> vector<int> pal_end(const T& s) {
e57     vector<int> ret(s.size());
fde     palindrome<T> p(s);
d51     ret[0] = 1;
88e     for (int i = 1; i < s.size(); i++) {
a32         ret[i] = min(ret[i-1]+2, i+1);
6ea         while (!p.query(i-ret[i]+1, i)) ret[i]--;
78e     }
edf     return ret;
8bd }
```

4.5 Min/max suffix/cyclic shift

```
// Computa o indice do menor/maior sufixo/cyclic shift
// da string, lexicograficamente
//
// O(n)

016 template<typename T> int max_suffix(T s, bool mi = false) {
476     s.push_back(*min_element(s.begin(), s.end())-1);
1a4     int ans = 0;
88e     for (int i = 1; i < s.size(); i++) {
eec         int j = 0;
708         while (ans+j < i and s[i+j] == s[ans+j]) j++;
7a2         if (s[i+j] > s[ans+j]) {
b52             if (!mi or i != s.size()-2) ans = i;
e51         } else if (j) i += j-1;
69c     }
ba7     return ans;
f2a }

a1a template<typename T> int min_suffix(T s) {
76b     for (auto& i : s) i *= -1;
09d     s.push_back(*max_element(s.begin(), s.end())+1);
925     return max_suffix(s, true);
ec0 }

97c template<typename T> int max_cyclic_shift(T s) {
163     int n = s.size();
1ad     for (int i = 0; i < n; i++) s.push_back(s[i]);
20a     return max_suffix(s);
d34 }

08a template<typename T> int min_cyclic_shift(T s) {
76b     for (auto& i : s) i *= -1;
7be     return max_cyclic_shift(s);
c7a }
```

4.6 String Hashing

```
// Complexidades:
// construtor - O(|s|)
// operator() - O(1)

878 mt19937 rng((int)
    chrono::steady_clock::now().time_since_epoch().count());

463 int uniform(int l, int r) {
a7f     uniform_int_distribution<int> uid(l, r);
f54     return uid(rng);
d9e }

9e0 template<int MOD> struct str_hash { // 116fcb
c63     static int P;
dcf     vector<ll> h, p;
ea8     str_hash(string s) : h(s.size()), p(s.size()) {
7a2         p[0] = 1, h[0] = s[0];
ad7         for (int i = 1; i < s.size(); i++)
84c             p[i] = p[i - 1]*P%MOD, h[i] = (h[i - 1]*P + s[i])%MOD;
1ef     }
af7     ll operator()(int l, int r) { // retorna hash s[l...r]
749         ll hash = h[r] - (l ? h[l - 1]*p[r - l + 1]%MOD : 0);
dfd         return hash < 0 ? hash + MOD : hash;
3ba     }
116 };
217 template<int MOD> int str_hash<MOD>::P = uniform(256, MOD - 1); //
    l > |sigma|
```

4.7 String Hashing - modulo 2^61 - 1

```
// Quase duas vezes mais lento
//
// Complexidades:
// build - O(|s|)
// operator() - O(1)

9d0 const ll MOD = (1ll<<61) - 1;
e38 ll mulmod(ll a, ll b) {
ff3     const static ll LOWER = (1ll<<30) - 1, GET31 = (1ll<<31) - 1;
410     ll l1 = a&LOWER, h1 = a>>30, l2 = b&LOWER, h2 = b>>30;
d54     ll m = l1*h2 + l2*h1, h = h1*h2;
784     ll ans = l1*l2 + (h>>1) + ((h&1)<<60) + (m>>31) +
    ((m&GET31)<<30) + 1;
1dd     ans = (ans&MOD) + (ans>>61), ans = (ans&MOD) + (ans>>61);
c0f     return ans - 1;
f98 }

798 mt19937_64
    rng(chrono::steady_clock::now().time_since_epoch().count());

f89 ll uniform(ll l, ll r) {
969     uniform_int_distribution<ll> uid(l, r);
f54     return uid(rng);
cac }

d7d struct str_hash {
c20     static ll P;
dcf     vector<ll> h, p;
ea8     str_hash(string s) : h(s.size()), p(s.size()) {
7a2         p[0] = 1, h[0] = s[0];
ad7         for (int i = 1; i < s.size(); i++)
632             p[i] = mulmod(p[i - 1], P), h[i] = (mulmod(h[i - 1],
    P) + s[i])%MOD;
507     }
af7     ll operator()(int l, int r) { // retorna hash s[l...r]
538         ll hash = h[r] - (l ? mulmod(h[l - 1], p[r - l + 1]) : 0);
dfd         return hash < 0 ? hash + MOD : hash;
544     }
148 };
6c5 ll str_hash::P = uniform(256, MOD - 1); // l > |sigma|
```

4.8 Suffix Array - O(n log n)

```
// kasai recebe o suffix array e calcula lcp[i],
// o lcp entre s[sa[i],...,n-1] e s[sa[i+1],..,n-1]
//
// Complexidades:
// suffix_array - O(n log(n))
// kasai - O(n)

733 vector<int> suffix_array(string s) {
b38     s += "$";
043     int n = s.size(), N = max(n, 260);
2f3     vector<int> sa(n), ra(n);
29b     for(int i = 0; i < n; i++) sa[i] = i, ra[i] = s[i];

0a2     for(int k = 0; k < n; k ? k *= 2 : k++) {
5ce         vector<int> nsa(sa), nra(n), cnt(N);

fae         for(int i = 0; i < n; i++) nsa[i] = (nsa[i]-k+n)%n,
    cnt[ra[i]]++;
4c4         for(int i = 1; i < N; i++) cnt[i] += cnt[i-1];
368         for(int i = n-1; i+1; i--) sa[--cnt[ra[nsa[i]]]] = nsa[i];
28f         for(int i = 1, r = 0; i < n; i++) nra[sa[i]] = r +=
   ra[sa[i]] !=
f86             ra[sa[i-1]] or ra[(sa[i]+k)%n] != ra[(sa[i-1]+k)%n];
26b         ra = nra;
d5e         if (ra[sa[n-1]] == n-1) break;
11e     }
057     return vector<int>(sa.begin()+1, sa.end());
ff3 }

481 vector<int> kasai(string s, vector<int> sa) {
232     int n = s.size(), k = 0;
408     vector<int> ra(n), lcp(n);
676     for (int i = 0; i < n; i++) ra[sa[i]] = i;

740     for (int i = 0; i < n; i++, k -= !!k) {
199         if (ra[i] == n-1) { k = 0; continue; }
1de         int j = sa[ra[i]+1];
891         while (i+k < n and j+k < n and s[i+k] == s[j+k]) k++;
d98         lcp[ra[i]] = k;
a07     }
5ed     return lcp;
fbe }
```

4.9 Suffix Array - O(n)

```
// Rapidao
// Computa o suffix array em 'sa', o rank em 'rnk'
// e o lcp em 'lcp'
// query(i, j) retorna o LCP entre s[i..n-1] e s[j..n-1]
//
// Complexidades
// O(n) para construir
// query - O(1)

1a5 template<typename T> struct rmq {
517     vector<T> v;
fcc     int n; static const int b = 30;
70e     vector<int> mask, t;

183     int op(int x, int y) { return v[x] <= v[y] ? x : y; }
ee1     int msb(int x) { return __builtin_clz(1)-__builtin_clz(x); }
c92     int small(int r, int sz = b) { return
   r-msb(mask[r]&((1<<sz)-1)); }
6ad     rmq() {}
43c     rmq(const vector<T>& v_) : v(v_), n(v.size()), mask(n), t(n) {
2e5         for (int i = 0, at = 0; i < n; mask[i++] = at |= 1) {
a61             at = (at<<1)&((1<<b)-1);
c00             while (at and op(i-msb(at&-at), i) == i) at ^= at&-at;
c2f         }
ea4         for (int i = 0; i < n/b; i++) t[i] = small(b*i+b-1);
39d         for (int j = 1; (1<<j) <= n/b; j++) for (int i = 0;
   i+(1<<j) <= n/b; i++)
ba5             t[n/b*j+i] = op(t[n/b*(j-1)+i],
   t[n/b*(j-1)+i+(1<<(j-1))]);
41a     }
e34     int index_query(int l, int r) {
27b         if (r-l+1 <= b) return small(r, r-l+1);
e80         int x = l/b+1, y = r/b-1;
fd3         if (x > y) return op(small(l+b-1), small(r));
a4e         int j = msb(y-x+1);
ea3         int ans = op(small(l+b-1), op(t[n/b*j+x],
   t[n/b*j+y-(1<<j)+1]));
be6         return op(ans, small(r));
62a     }
093     T query(int l, int r) { return v[index_query(l, r)]; }
bab };

9d7 struct suffix_array {
ac0     string s;
1a8     int n;
5b4     vector<int> sa, cnt, rnk, lcp;
2de     rmq<int> RMQ;

d6e     bool cmp(int a1, int b1, int a2, int b2, int a3=0, int b3=0) {
91d         return a1 != b1 ? a1 < b1 : (a2 != b2 ? a2 < b2 : a3 < b3);
82d     }
4a4     template<typename T> void radix(int* fr, int* to, T* r, int N,
   int k) {
c17         cnt = vector<int>(k+1, 0);
bac         for (int i = 0; i < N; i++) cnt[r[fr[i]]]++;
703         for (int i = 1; i <= k; i++) cnt[i] += cnt[i-1];
000         for (int i = N-1; i+1; i--) to[--cnt[r[fr[i]]]] = fr[i];
6f3     }
d66     void rec(vector<int>& v, int k) {
a76         auto &tmp = rnk, &m0 = lcp;
3a9         int N = v.size()-3, sz = (N+2)/3, sz2 = sz+N/3;
7f8         vector<int> R(sz2+3);
74f         for (int i = 1, j = 0; j < sz2; i += i%3) R[j++] = i;

b30         radix(&R[0], &tmp[0], &v[0]+2, sz2, k);
207         radix(&tmp[0], &R[0], &v[0]+1, sz2, k);
5f1         radix(&R[0], &tmp[0], &v[0]+0, sz2, k);
```

```
af5         int dif = 0;
ed9         int l0 = -1, l1 = -1, l2 = -1;
d81         for (int i = 0; i < sz2; i++) {
8de             if (v[tmp[i]] != l0 or v[tmp[i]+1] != l1 or
   v[tmp[i]+2] != l2)
b43                 l0 = v[tmp[i]], l1 = v[tmp[i]+1], l2 =
   v[tmp[i]+2], dif++;
199             if (tmp[i]%3 == 1) R[tmp[i]/3] = dif;
1f5             else R[tmp[i]/3+sz] = dif;
d18         }

47f         if (dif < sz2) {
146             rec(R, dif);
746             for (int i = 0; i < sz2; i++) R[sa[i]] = i+1;
105         } else for (int i = 0; i < sz2; i++) sa[R[i]-1] = i;

6f4         for (int i = 0, j = 0; j < sz2; i++) if (sa[i] < sz)
   tmp[j++] = 3*sa[i];
7ce         radix(&tmp[0], &m0[0], &v[0], sz, k);
74d         for (int i = 0; i < sz2; i++)
c9e             sa[i] = sa[i] < sz ? 3*sa[i]+1 : 3*(sa[i]-sz)+2;

332         int at = sz2+sz-1, p = sz-1, p2 = sz2-1;
1c9         while (p >= 0 and p2 >= 0) {
3b3             if ((sa[p2]%3==1 and cmp(v[m0[p]], v[sa[p2]],
   R[m0[p]/3],
0ce                 R[sa[p2]/3+sz])) or (sa[p2]%3==2 and cmp(v[m0[p]],
   v[sa[p2]],
af6                 v[m0[p]+1], v[sa[p2]+1], R[m0[p]/3+sz],
   R[sa[p2]/3+1])))
300                 sa[at--] = sa[p2--];
cb0             else sa[at--] = m0[p--];
214         }
f2b         while (p >= 0) sa[at--] = m0[p--];
eb6         if (N%3==1) for (int i = 0; i < N; i++) sa[i] = sa[i+1];
ee6     }

938     suffix_array(const string& s_) : s(s_), n(s.size()), sa(n+3),
e62             cnt(n+1), rnk(n), lcp(n-1) {
9fe         vector<int> v(n+3);
f9b         for (int i = 0; i < n; i++) v[i] = i;
eba         radix(&v[0], &rnk[0], &s[0], n, 256);
e6d         int dif = 1;
830         for (int i = 0; i < n; i++)
419             v[rnk[i]] = dif += (i and s[rnk[i]] != s[rnk[i-1]]);
7cf         if (n >= 2) rec(v, dif);
fb9         sa.resize(n);

76f         for (int i = 0; i < n; i++) rnk[sa[i]] = i;
892         for (int i = 0, k = 0; i < n; i++, k -= !!k) {
668             if (rnk[i] == n-1) {
5a4                 k = 0;
5e2                 continue;
9df             }
39a             int j = sa[rnk[i]+1];
891             while (i+k < n and j+k < n and s[i+k] == s[j+k]) k++;
825             lcp[rnk[i]] = k;
a3e         }
9ff         RMQ = rmq<int>(lcp);
9a8     }

588     int query(int i, int j) {
d97         if (i == j) return n-i;
223         i = rnk[i], j = rnk[j];
c3a         return RMQ.query(min(i, j), max(i, j)-1);
940     }
71c     pair<int, int> next(int L, int R, int i, char c) {
024         int l = L, r = R+1;
40c         while (l < r) {
ee4             int m = (l+r)/2;
e7e             if (i+sa[m] >= n or s[i+sa[m]] < c) l = m+1;
ef3             else r = m;
ebe         }
575         if (l == R+1 or s[i+sa[l]] > c) return {-1, -1};
eb7         L = l;

9e2         l = L, r = R+1;
40c         while (l < r) {
ee4             int m = (l+r)/2;
1a1             if (i+sa[m] >= n or s[i+sa[m]] <= c) l = m+1;
ef3             else r = m;
b5b         }
56a         R = l-1;
e13         return {L, R};
71b     }
        // quantas vezes 't' ocorre em 's' - O(|t| log n)
66d     int count_substr(string& t) {
b2b         int L = 0, R = n-1;
c9d         for (int i = 0; i < t.size(); i++) {
de0             tie(L, R) = next(L, R, i, t[i]);
4fc             if (L == -1) return 0;
cff         }
fbf         return R-L+1;
```

```
aaa     }

        // exemplo de f que resolve o problema
        //
            https://codeforces.com/edu/course/2/lesson/2/5/practice/contest/269656/problem/D
57e     ll f(ll k) { return k*(k+1)/2; }

e68     ll dfs(int L, int R, int p) { // dfs na suffix tree chamado em
    pre ordem
c54         int ext = L != R ? RMQ.query(L, R-1) : n - sa[L];

            // Tem 'ext - p' substrings diferentes que ocorrem 'R-L+1'
                vezes
            // O LCP de todas elas eh 'ext'
f80         ll ans = (ext-p)*f(R-L+1);

            // L eh terminal, e folha sse L == R
63c         if (sa[L]+ext == n) L++;

            // se for um SA de varias strings separadas como s#t$u&,
                usar no lugar do if de cima
            //  (separadores < 'a', diferentes e inclusive no final)
            // while (L <= R && (sa[L]+ext == n || s[sa[L]+ext] <
                'a')) {
            //   L++;
            // }

add         while (L <= R) {
5a8             int idx = L != R ? RMQ.index_query(L, R-1) : -1;
5ef             if (idx == -1 or lcp[idx] != ext) idx = R;

478             ans += dfs(L, idx, ext);
28d             L = idx+1;
590         }
ba7         return ans;
e21     }

        // sum over substrings: computa, para toda substring t
            distinta de s,
        // \sum f(# ocorrencias de t em s) - O (n)
ca8     ll sos() { return dfs(0, n-1, 0); }
6fa };
```

4.10 Suffix Array Dinamico

```
// Mantem o suffix array, lcp e rank de uma string,
// premitindo push_front e pop_front
// O operador [i] return um par com sa[i] e lcp[i]
// lcp[i] tem o lcp entre sa[i] e sa[i-1] (lcp[0] = 0)
//
// Construir sobre uma string de tamanho n: O(n log n)
// push_front e pop_front: O(log n) amortizado

2fe struct dyn_sa {
3c9     struct node {
1d4         int sa, lcp;
ed1         node *l, *r, *p;
f0d         int sz, mi;
17b         node(int sa_, int lcp_, node* p_) : sa(sa_), lcp(lcp_),
543             l(NULL), r(NULL), p(p_), sz(1), mi(lcp) {}
01e         void update() {
58f             sz = 1, mi = lcp;
bd7             if (l) sz += l->sz, mi = min(mi, l->mi);
a54             if (r) sz += r->sz, mi = min(mi, r->mi);
27c         }
574     };

bb7     node* root;
295     vector<ll> tag; // tag of a suffix (reversed id)
ac0     string s; // reversed

cf4     dyn_sa() : root(NULL) {}
e45     dyn_sa(string s_) : dyn_sa() {
ae4         reverse(s_.begin(), s_.end());
519         for (char c : s_) push_front(c);
2a7     }
a86     ~dyn_sa() {
609         vector<node*> q = {root};
402         while (q.size()) {
e5d             node* x = q.back(); q.pop_back();
ee9             if (!x) continue;
1c7             q.push_back(x->l), q.push_back(x->r);
bf0             delete x;
653         }
8c1     }

73c     int size(node* x) { return x ? x->sz : 0; }
08e     int mirror(int i) { return s.size()-1 - i; }
580     bool cmp(int i, int j) {
a29         if (s[i] != s[j]) return s[i] < s[j];
5b4         if (i == 0 or j == 0) return i < j;
988         return tag[i-1] < tag[j-1];
```

```
9fd     }
919     void fix_path(node* x) { while (x) x->update(), x = x->p; }
245     void flatten(vector<node*>& v, node* x) {
8c8         if (!x) return;
e96         flatten(v, x->l);
2a2         v.push_back(x);
42d         flatten(v, x->r);
01f     }
964     void build(vector<node*>& v, node*& x, node* p, int L, int R, ll l, ll r) {
04c         if (L > R) return void(x = NULL);
331         int M = (L+R)/2;
3e3         ll m = (l+r)/2;
7e5         x = v[M];
63e         x->p = p;
bb3         tag[x->sa] = m;
ae0         build(v, x->l, x, L, M-1, l, m-1), build(v, x->r, x, M+1, R, m+1, r);
ca8         x->update();
a3a     }
82f     void fix(node*& x, node* p, ll l, ll r) {
7f0         if (3*max(size(x->l), size(x->r)) <= 2*size(x)) return x->update();
3d1         vector<node*> v;
0cc         flatten(v, x);
ea9         build(v, x, p, 0, v.size()-1, l, r);
b86     }
b19     node* next(node* x) {
728         if (x->r) {
a91             x = x->r;
347             while (x->l) x = x->l;
ea5             return x;
e7d         }
402         while (x->p and x->p->r == x) x = x->p;
137         return x->p;
48b     }
b68     node* prev(node* x) {
e41         if (x->l) {
a26             x = x->l;
93c             while (x->r) x = x->r;
ea5             return x;
9be         }
6a1         while (x->p and x->p->l == x) x = x->p;
137         return x->p;
73e     }

4f7     int get_lcp(node* x, node* y) {
75a         if (!x or !y) return 0; // change defaut value here
e51         if (s[x->sa] != s[y->sa]) return 0;
843         if (x->sa == 0 or y->sa == 0) return 1;
4d0         return 1 + query(mirror(x->sa-1), mirror(y->sa-1));
8d6     }
ad6     void add_suf(node*& x, node* p, int id, ll l, ll r) {
91e         if (!x) {
8e3             x = new node(id, 0, p);
8e2             node *prv = prev(x), *nxt = next(x);
65d             int lcp_cur = get_lcp(prv, x), lcp_nxt = get_lcp(x, nxt);
ca3             if (nxt) nxt->lcp = lcp_nxt, fix_path(nxt);
71f             x->lcp = lcp_cur;
7b4             tag[id] = (l+r)/2;
ca8             x->update();
505             return;
d0e         }
4a3         if (cmp(id, x->sa)) add_suf(x->l, x, id, l, tag[x->sa]-1);
c3a         else add_suf(x->r, x, id, tag[x->sa]+1, r);
3db         fix(x, p, l, r);
c98     }
ec2     void push_front(char c) {
cc7         s += c;
493         tag.push_back(-1);
05e         add_suf(root, NULL, s.size() - 1, 0, 1e18);
1f2     }

7f3     void rem_suf(node*& x, int id) {
6cf         if (x->sa != id) {
864             if (tag[id] < tag[x->sa]) return rem_suf(x->l, id);
e6f             return rem_suf(x->r, id);
2ae         }
2cf         node* nxt = next(x);
09b         if (nxt) nxt->lcp = min(nxt->lcp, x->lcp), fix_path(nxt);

b20         node *p = x->p, *tmp = x;
f3f         if (!x->l or !x->r) {
2fd             x = x->l ? x->l : x->r;
753             if (x) x->p = p;
696         } else {
7f7             for (tmp = x->l, p = x; tmp->r; tmp = tmp->r) p = tmp;
f2a             x->sa = tmp->sa, x->lcp = tmp->lcp;
482             if (tmp->l) tmp->l->p = p;
14c             if (p->l == tmp) p->l = tmp->l;
a94             else p->r = tmp->l;
97c         }
b5e         fix_path(p);
```

```
7c3            delete tmp;
510        }
15b        void pop_front() {
abe            if (!s.size()) return;
342            s.pop_back();
436            rem_suf(root, s.size());
c6e            tag.pop_back();
987        }

530        int query(node* x, ll l, ll r, ll a, ll b) {
e51            if (!x or tag[x->sa] == -1 or r < a or b < l) return
    s.size();
ef5            if (a <= l and r <= b) return x->mi;
8eb            int ans = s.size();
e1f            if (a <= tag[x->sa] and tag[x->sa] <= b) ans = min(ans,
    x->lcp);
d99            ans = min(ans, query(x->l, l, tag[x->sa]-1, a, b));
261            ans = min(ans, query(x->r, tag[x->sa]+1, r, a, b));
ba7            return ans;
4c8        }
588        int query(int i, int j) { // lcp(s[i..], s[j..])
209            if (i == j) return s.size() - i;
29e            ll a = tag[mirror(i)], b = tag[mirror(j)];
710            int ret = query(root, 0, 1e18, min(a, b)+1, max(a, b));
edf            return ret;
84e        }
           // optional: get rank[i], sa[i] and lcp[i]
044        int rank(int i) {
396            i = mirror(i);
52f            node* x = root;
7c9            int ret = 0;
f4c            while (x) {
33e                if (tag[x->sa] < tag[i]) {
f9d                    ret += size(x->l)+1;
a91                    x = x->r;
6dc                } else x = x->l;
a19            }
edf            return ret;
153        }
649        pair<int, int> operator[](int i) {
52f            node* x = root;
31e            while (1) {
d4d                if (i < size(x->l)) x = x->l;
4e6                else {
85f                    i -= size(x->l);
e03                    if (!i) return {mirror(x->sa), x->lcp};
040                    i--, x = x->r;
b9b                }
7a2            }
90c        }
4c2 };
```

4.11 Suffix Automaton

```
// Automato que aceita os sufixos de uma string
// Todas as funcoes sao lineares

16e namespace sam {
c1a     int cur, sz, len[2*MAX], link[2*MAX], acc[2*MAX];
0b8     int nxt[2*MAX][26];

e6a     void add(int c) {
17a         int at = cur;
9a6         len[sz] = len[cur]+1, cur = sz++;
500         while (at != -1 and !nxt[at][c]) nxt[at][c] = cur, at =
    link[at];
7ea         if (at == -1) { link[cur] = 0; return; }
654         int q = nxt[at][c];
fd9         if (len[q] == len[at]+1) { link[cur] = q; return; }
31f         int qq = sz++;
2c3         len[qq] = len[at]+1, link[qq] = link[q];
9a9         for (int i = 0; i < 26; i++) nxt[qq][i] = nxt[q][i];
e76         while (at != -1 and nxt[at][c] == q) nxt[at][c] = qq, at =
    link[at];
8b8         link[cur] = link[q] = qq;
61a     }
94e     void build(string& s) {
889         cur = 0, sz = 0, len[0] = 0, link[0] = -1, sz++;
9fe         for (auto i : s) add(i-'a');
17a         int at = cur;
121         while (at) acc[at] = 1, at = link[at];
0e7     }

        // coisas que da pra fazer:
28c     ll distinct_substrings() {
04b         ll ans = 0;
a1e         for (int i = 1; i < sz; i++) ans += len[i] - len[link[i]];
ba7         return ans;
0d7     }
a6c     string longest_common_substring(string& S, string& T) {
419         build(S);
111         int at = 0, l = 0, ans = 0, pos = -1;
d59         for (int i = 0; i < T.size(); i++) {
```

```
f2c             while (at and !nxt[at][T[i]-'a']) at = link[at], l =
    len[at];
efa             if (nxt[at][T[i]-'a']) at = nxt[at][T[i]-'a'], l++;
749             else at = 0, l = 0;
a1a             if (l > ans) ans = l, pos = i;
2b3         }
20f         return T.substr(pos-ans+1, ans);
930     }
46e     ll dp[2*MAX];
455     ll paths(int i) {
2a8         auto& x = dp[i];
dee         if (x) return x;
483         x = 1;
71c         for (int j = 0; j < 26; j++) if (nxt[i][j]) x +=
    paths(nxt[i][j]);
ea5         return x;
d88     }
105     void kth_substring(int k, int at=0) { // k=1 : menor substring
    lexicog.
9d2         for (int i = 0; i < 26; i++) if (k and nxt[at][i]) {
d58             if (paths(nxt[at][i]) >= k) {
d02                 cout << char('a'+i);
c43                 kth_substring(k-1, nxt[at][i]);
505                 return;
69a             }
5f4             k -= paths(nxt[at][i]);
ef6         }
a13     }
c37 };
```

4.12 Trie

```
// trie T() constroi uma trie para o alfabeto das letras minusculas
// trie T(tamanho do alfabeto, menor caracter) tambem pode ser usado
//
// T.insert(s) - O(|s|*sigma)
// T.erase(s) - O(|s|)
// T.find(s) retorna a posicao, -1 se nao achar - O(|s|)
// T.count_pref(s) numero de strings que possuem s como prefixo -
    O(|s|)

ab5 struct trie {
e1a     vector<vector<int>> to;
450     vector<int> end, pref;
af0     int sigma; char norm;
bb1     trie(int sigma_=26, char norm_='a') : sigma(sigma_),
    norm(norm_) {
58a         to = {vector<int>(sigma)};
86e         end = {0}, pref = {0};
fe1     }
64e     void insert(string s) {
c67         int x = 0;
7e7         for (auto c : s) {
008             int &nxt = to[x][c-norm];
dd7             if (!nxt) {
0aa                 nxt = to.size();
526                 to.push_back(vector<int>(sigma));
770                 end.push_back(0), pref.push_back(0);
933             }
827             x = nxt, pref[x]++;
34c         }
421         end[x]++, pref[0]++;
e66     }
6b2     void erase(string s) {
c67         int x = 0;
b4f         for (char c : s) {
008             int &nxt = to[x][c-norm];
10c             x = nxt, pref[x]--;
d8e             if (!pref[x]) nxt = 0;
885         }
104         end[x]--, pref[0]--;
8bf     }
aee     int find(string s) {
c67         int x = 0;
7e7         for (auto c : s) {
2ec             x = to[x][c-norm];
59b             if (!x) return -1;
42d         }
ea5         return x;
ecc     }
839     int count_pref(string s) {
b25         int id = find(s);
fc1         return id >= 0 ? pref[id] : 0;
560     }
8ca };
```

4.13 Z

```
// z[i] = lcp(s, s[i..n))
//
// Complexidades:
```

```
// z - O(|s|)
// match - O(|s| + |p|)

a19 vector<int> get_z(string s) {
163     int n = s.size();
2b1     vector<int> z(n, 0);

fae     int l = 0, r = 0;
6f5     for (int i = 1; i < n; i++) {
0af         if (i <= r) z[i] = min(r - i + 1, z[i - l]);
457         while (i + z[i] < n and s[z[i]] == s[i + z[i]]) z[i]++;
65e         if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
5cd     }

070     return z;
74a }
```

5 Estruturas

5.1 BIT

```
// BIT de soma 0-based
//
// upper_bound(x) retorna o menor p tal que pref(p) > x
//
// Complexidades:
// build - O(n)
// update - O(log(n))
// query - O(log(n))
// upper_bound - O(log(n))

8eb struct Bit {
1a8     int n;
406     vector<ll> bit;
e86     Bit(int _n=0) : n(_n), bit(n + 1) {}
70f     Bit(vector<int>& v) : n(v.size()), bit(n + 1) {
78a         for (int i = 1; i <= n; i++) {
671             bit[i] += v[i - 1];
edf             int j = i + (i & -i);
b8a             if (j <= n) bit[j] += bit[i];
806         }
e89     }
625     void update(int i, ll x) { // soma x na posicao i
b64         for (i++; i <= n; i += i & -i) bit[i] += x;
d67     }
462     ll pref(int i) { // soma [0, i]
b73         ll ret = 0;
4d3         for (i++; i; i -= i & -i) ret += bit[i];
edf         return ret;
0ef     }
02a     ll query(int l, int r) {  // soma [l, r]
89b         return pref(r) - pref(l - 1);
ca8     }
014     int upper_bound(ll x) {
1ba         int p = 0;
0af         for (int i = __lg(n); i+1; i--)
6f5             if (p + (1<<i) <= n and bit[p + (1<<i)] <= x)
68e                 x -= bit[p += (1 << i)];
74e         return p;
fdd     }
502 };
```

5.2 BIT 2D

```
// BIT de soma, update incrementa posicao
// Tem que construir com um vetor com todos os pontos
// que vc quer um dia atualizar (os pontos q vc vai chamar update)
//
// Complexidades:
// construir - O(n log(n))
// update e query - O(log^2(n))

a6b template<class T = int> struct bit2d {
acf     vector<T> X;
a84     vector<vector<T>> Y, t;

709     int ub(vector<T>& v, T x) {
dde         return upper_bound(v.begin(), v.end(), x) - v.begin();
9cc     }
5cb     bit2d(vector<pair<T, T>> v) {
2e1         for (auto [x, y] : v) X.push_back(x);
fd4         sort(X.begin(), X.end());
1ee         X.erase(unique(X.begin(), X.end()), X.end());

d56         t.resize(X.size() + 1);
d12         Y.resize(t.size());
3d0         sort(v.begin(), v.end(), [](auto a, auto b) {
e8f             return a.second < b.second; });
961         for (auto [x, y] : v) for (int i = ub(X, x); i < t.size();
   i += i&-i)
b75             if (!Y[i].size() or Y[i].back() != y)
    Y[i].push_back(y);

7c7         for (int i = 0; i < t.size(); i++) t[i].resize(Y[i].size()
    + 1);
8cc     }

e78     void update(T x, T y, T v) {
2a9         for (int i = ub(X, x); i < t.size(); i += i&-i)
cd2             for (int j = ub(Y[i], y); j < t[i].size(); j += j&-j)
    t[i][j] += v;
533     }

5d2     T query(T x, T y) {
966         T ans = 0;
c54         for (int i = ub(X, x); i; i -= i&-i)
4fb             for (int j = ub(Y[i], y); j; j -= j&-j) ans += t[i][j];
ba7         return ans;
62d     }
46d     T query(T x1, T y1, T x2, T y2) {
fcf         return query(x2, y2)-query(x2, y1-1)-query(x1-1,
    y2)+query(x1-1, y1-1);
232     }
6a7 };
```

5.3 BIT com update em range

```
// Operacoes 0-based
// query(l, r) retorna a soma de v[l..r]
// update(l, r, x) soma x em v[l..r]
//
// Complexidades:
// build - O(n)
// query - O(log(n))
// update - O(log(n))

e04 namespace bit {
3ba     ll bit[2][MAX+2];
1a8     int n;

61c     void build(int n2, int* v) {
1e3         n = n2;
535         for (int i = 1; i <= n; i++)
edd             bit[1][min(n+1, i+(i&-i))] += bit[1][i] += v[i-1];
db0     }
637     ll get(int x, int i) {
b73         ll ret = 0;
360         for (; i; i -= i&-i) ret += bit[x][i];
edf         return ret;
99c     }
20c     void add(int x, int i, ll val) {
503         for (; i <= n; i += i&-i) bit[x][i] += val;
bf6     }
162     ll get2(int p) {
c7c         return get(0, p) * p + get(1, p);
153     }
02a     ll query(int l, int r) {
ff5         return get2(r+1) - get2(l);
633     }
089     void update(int l, int r, ll x) {
e5f         add(0, l+1, x), add(0, r+2, -x);
f58         add(1, l+1, -x*l), add(1, r+2, x*(r+1));
e5f     }
f91 };
```

5.4 BIT-Sort Tree

```
// Tipo uma MergeSort Tree usando Bit
// Apesar da complexidade ser pior, fica melhor na pratica.
//
// query(l, r, k) retorna o numero de elementos menores que k
// no intervalo [l, r]
//
// Usa O(n log(n)) de memoria
//
// Complexidades:
// construir - O(n log^2(n))
// query - O(log^2(n))

6fa template<typename T> struct ms_bit {
1a8     int n;
b2f     vector<vector<T>> bit;

899     ms_bit(vector<T>& v) : n(v.size()), bit(n+1) {
830         for (int i = 0; i < n; i++)
d51             for (int j = i+1; j <= n; j += j&-j)
dad                 bit[j].push_back(v[i]);
535         for (int i = 1; i <= n; i++)
eec             sort(bit[i].begin(), bit[i].end());
b4d     }

257     int p_query(int i, T k) {
```

```
7c9         int ret = 0;
be8         for (i++; i; i -= i&-i)
1bd             ret += lower_bound(bit[i].begin(), bit[i].end(), k) -
    bit[i].begin();
edf         return ret;
6f9     }
690     int query(int l, int r, T k) {
83d         return p_query(r, k) - p_query(l-1, k);
bcc     }
8d0 };
```

5.5 Convex Hull Trick Dinamico

```
// para double, use LINF = 1/.0, div(a, b) = a/b
// update(x) atualiza o ponto de intersecao da reta x
// overlap(x) verifica se a reta x sobrepoe a proxima
// add(a, b) adiciona reta da forma ax + b
// query(x) computa maximo de ax + b para entre as retas
//
// O(log(n)) amortizado por insercao
// O(log(n)) por query

72c struct Line {
073     mutable ll a, b, p;
8e3     bool operator<(const Line& o) const { return a < o.a; }
abf     bool operator<(ll x) const { return p < x; }
469 };

326 struct dynamic_hull : multiset<Line, less<>> {
33a     ll div(ll a, ll b) {
a20         return a / b - ((a ^ b) < 0 and a % b);
a8a     }

bbb     void update(iterator x) {
b2a         if (next(x) == end()) x->p = LINF;
772         else if (x->a == next(x)->a) x->p = x->b >= next(x)->b ?
    LINF : -LINF;
424         else x->p = div(next(x)->b - x->b, x->a - next(x)->a);
0c4     }

71c     bool overlap(iterator x) {
f18         update(x);
cfa         if (next(x) == end()) return 0;
a4a         if (x->a == next(x)->a) return x->b >= next(x)->b;
d40         return x->p >= next(x)->p;
901     }

176     void add(ll a, ll b) {
1c7         auto x = insert({a, b, 0});
4ab         while (overlap(x)) erase(next(x)), update(x);
dbc         if (x != begin() and !overlap(prev(x))) x = prev(x),
    update(x);
0fc         while (x != begin() and overlap(prev(x)))
4d2             x = prev(x), erase(next(x)), update(x);
48f     }

4ad     ll query(ll x) {
229         assert(!empty());
7d1         auto l = *lower_bound(x);
d41 #warning cuidado com overflow!
aba         return l.a * x + l.b;
3f5     }
8f2 };
```

5.6 Convex Hull Trick Estatico

```
// adds tem que serem feitos em ordem de slope
// queries tem que ser feitas em ordem de x
//
// add O(1) amortizado, get O(1) amortizado

4b5 struct CHT {
942     int it;
ac1     vector<ll> a, b;
45e     CHT():it(0){}
0bb     ll eval(int i, ll x){
93d         return a[i]*x + b[i];
b2a     }
63a     bool useless(){
a20         int sz = a.size();
35f         int r = sz-1, m = sz-2, l = sz-3;
d41 #warning cuidado com overflow!
d71         return (b[l] - b[r])*(a[m] - a[l]) <
413             (b[l] - b[m])*(a[r] - a[l]);
a0c     }
bf4     void add(ll A, ll B){
7f5         a.push_back(A); b.push_back(B);
565         while (!a.empty()){
233             if ((a.size() < 3) || !useless()) break;
ecb             a.erase(a.end() - 2);
568             b.erase(b.end() - 2);
b21         }
d27         it = min(it, int(a.size()) - 1);
6df     }
81b     ll get(ll x){
46a         while (it+1 < a.size()){
3c4             if (eval(it+1, x) > eval(it, x)) it++;
f97             else break;
fe9         }
420         return eval(it, x);
b44     }
450 };
```

5.7 DSU

```
// Une dois conjuntos e acha a qual conjunto um elemento pertence por
    seu id
//
// find e unite: O(a(n)) ~= O(1) amortizado

8d3 struct dsu {
825     vector<int> id, sz;

b33     dsu(int n) : id(n), sz(n, 1) { iota(id.begin(), id.end(), 0); }

0cf     int find(int a) { return a == id[a] ? a : id[a] = find(id[a]);
    }

440     void unite(int a, int b) {
605         a = find(a), b = find(b);
d54         if (a == b) return;
956         if (sz[a] < sz[b]) swap(a, b);
6d0         sz[a] += sz[b], id[b] = a;
ea7     }
8e1 };

// DSU de bipartido
//
// Une dois vertices e acha a qual componente um vertice pertence
// Informa se a componente de um vertice e bipartida
//
// find e unite: O(log(n))

8d3 struct dsu {
6f7     vector<int> id, sz, bip, c;

5b4     dsu(int n) : id(n), sz(n, 1), bip(n, 1), c(n) {
db8         iota(id.begin(), id.end(), 0);
f25     }

ef0     int find(int a) { return a == id[a] ? a : find(id[a]); }
f30     int color(int a) { return a == id[a] ? c[a] : c[a] ^
    color(id[a]); }

440     void unite(int a, int b) {
263         bool change = color(a) == color(b);
605         a = find(a), b = find(b);
a89         if (a == b) {
4ed             if (change) bip[a] = 0;
505             return;
32d         }

956         if (sz[a] < sz[b]) swap(a, b);
efe         if (change) c[b] = 1;
2cd         sz[a] += sz[b], id[b] = a, bip[a] &= bip[b];
22b     }
118 };


// DSU Persistente
//
// Persistencia parcial, ou seja, tem que ir
// incrementando o 't' no une
//
// find e unite: O(log(n))

8d3 struct dsu {
33c     vector<int> id, sz, ti;

733     dsu(int n) : id(n), sz(n, 1), ti(n, -INF) {
db8         iota(id.begin(), id.end(), 0);
aad     }

5e6     int find(int a, int t) {
6ba         if (id[a] == a or ti[a] > t) return a;
ea5         return find(id[a], t);
6cb     }

fa0     void unite(int a, int b, int t) {
84f         a = find(a, t), b = find(b, t);
d54         if (a == b) return;
956         if (sz[a] < sz[b]) swap(a, b);
35d         sz[a] += sz[b], id[b] = a, ti[b] = t;
513     }
6c6 };
```

```
// DSU com rollback
//
// checkpoint(): salva o estado atual de todas as variaveis
// rollback(): retorna para o valor das variaveis para
// o ultimo checkpoint
//
// Sempre que uma variavel muda de valor, adiciona na stack
//
// find e unite: O(log(n))
// checkpoint: O(1)
// rollback: O(m) em que m e o numero de vezes que alguma
// variavel mudou de valor desde o ultimo checkpoint

8d3 struct dsu {
825     vector<int> id, sz;
27c     stack<stack<pair<int&, int>>> st;

98d     dsu(int n) : id(n), sz(n, 1) {
1cc         iota(id.begin(), id.end(), 0), st.emplace();
8cd     }

bdf     void save(int &x) { st.top().emplace(x, x); }

30d     void checkpoint() { st.emplace(); }

5cf     void rollback() {
ba9         while(st.top().size()) {
6bf             auto [end, val] = st.top().top(); st.top().pop();
149             end = val;
f9a         }
25a         st.pop();
3c6     }

ef0     int find(int a) { return a == id[a] ? a : find(id[a]); }

440     void unite(int a, int b) {
605         a = find(a), b = find(b);
d54         if (a == b) return;
956         if (sz[a] < sz[b]) swap(a, b);
803         save(sz[a]), save(id[b]);
6d0         sz[a] += sz[b], id[b] = a;
1b9     }
c6e };
```

5.8 Li-Chao Tree

```
// Adiciona retas (ax+b), e computa o minimo entre as retas
// em um dado 'x'
// Cuidado com overflow!
// Se tiver overflow, tenta comprimir o 'x' ou usar
// convex hull trick
//
// O(log(MA-MI)), O(n) de memoria

5b0 template<ll MI = ll(-1e9), ll MA = ll(1e9)> struct lichao {
b3a     struct line {
12d         ll a, b;
cef         array<int, 2> ch;
fdf         line(ll a_ = 0, ll b_ = LINF) :
423             a(a_), b(b_), ch({-1, -1}) {}
888         ll operator ()(ll x) { return a*x + b; }
d1d     };
17b     vector<line> ln;

df8     int ch(int p, int d) {
e85         if (ln[p].ch[d] == -1) {
9af             ln[p].ch[d] = ln.size();
cdc             ln.emplace_back();
bc1         }
ef2         return ln[p].ch[d];
86a     }
021     lichao() { ln.emplace_back(); }

c33     void add(line s, ll l=MI, ll r=MA, int p=0) {
3e3         ll m = (l+r)/2;
911         bool L = s(l) < ln[p](l);
d37         bool M = s(m) < ln[p](m);
03b         bool R = s(r) < ln[p](r);
825         if (M) swap(ln[p], s), swap(ln[p].ch, s.ch);
cac         if (s.b == LINF) return;
f6d         if (L != M) add(s, l, m-1, ch(p, 0));
898         else if (R != M) add(s, m+1, r, ch(p, 1));
76e     }
092     ll query(int x, ll l=MI, ll r=MA, int p=0) {
11b         ll m = (l+r)/2, ret = ln[p](x);
9db         if (ret == LINF) return ret;
529         if (x < m) return min(ret, query(x, l, m-1, ch(p, 0)));
81a         return min(ret, query(x, m+1, r, ch(p, 1)));
fba     }
59b };
```

5.9 Li-Chao Tree - Lazy

```
// Sendo N = MA-MI:
// insert({a, b}) minimiza tudo com ax+b - O(log N)
// insert({a, b}, l, r) minimiza com ax+b no range [l, r] - O(log^2 N)
// shift({a, b}) soma ax+b em tudo - O(1)
// shift({a, b}, l, r) soma ax+b no range [l, r] - O(log^2 N)
// query(x) retorna o valor da posicao x - O(log N)
//
// No inicio eh tudo LINF, se inserir {0, 0} fica tudo 0
//
// O(n log N) de memoria ; O(n) de memoria se nao usar as operacoes de range

41c template<int MI = int(-1e9), int MA = int(1e9)> struct lichao {
b3a     struct line {
12d         ll a, b;
158         ll la, lb; // lazy
cef         array<int, 2> ch;
fdf         line(ll a_ = 0, ll b_ = LINF) :
b09             a(a_), b(b_), la(0), lb(0), ch({-1, -1}) {}
888         ll operator ()(ll x) { return a*x + b; }
92e     };
17b     vector<line> ln;

df8     int ch(int p, int d) {
e85         if (ln[p].ch[d] == -1) {
9af             ln[p].ch[d] = ln.size();
cdc             ln.emplace_back();
bc1         }
ef2         return ln[p].ch[d];
86a     }
021     lichao() { ln.emplace_back(); }

ceb     void prop(int p, int l, int r) {
ff8         if (ln[p].la == 0 and ln[p].lb == 0) return;
1d3         ln[p].a += ln[p].la, ln[p].b += ln[p].lb;
579         if (l != r) {
b9e             int pl = ch(p, 0), pr = ch(p, 1);
0d7             ln[pl].la += ln[p].la, ln[pl].lb += ln[p].lb;
fa8             ln[pr].la += ln[p].la, ln[pr].lb += ln[p].lb;
77f         }
01e         ln[p].la = ln[p].lb = 0;
89b     }

c06     ll query(int x, int p=0, int l=MI, int r=MA) {
6b9         prop(p, l, r);
6f3         ll ret = ln[p](x);
33b         if (ln[p].ch[0] == -1 and ln[p].ch[1] == -1) return ret;
90d         int m = l + (r-l)/2;
da9         if (x <= m) return min(ret, query(x, ch(p, 0), l, m));
c55         return min(ret, query(x, ch(p, 1), m+1, r));
953     }

5df     void push(line s, int p, int l, int r) {
6b9         prop(p, l, r);
90d         int m = l + (r-l)/2;
911         bool L = s(l) < ln[p](l);
d37         bool M = s(m) < ln[p](m);
03b         bool R = s(r) < ln[p](r);
c3f         if (M) swap(ln[p].a, s.a), swap(ln[p].b, s.b);
cac         if (s.b == LINF) return;
c49         if (L != M) push(s, ch(p, 0), l, m);
29e         else if (R != M) push(s, ch(p, 1), m+1, r);
ceb     }
a8e     void insert(line s, int a=MI, int b=MA, int p=0, int l=MI, int r=MA) {
6b9         prop(p, l, r);
2d3         if (a <= l and r <= b) return push(s, p, l, r);
1dd         if (b < l or r < a) return;
90d         int m = l + (r-l)/2;
f1e         insert(s, a, b, ch(p, 0), l, m);
952         insert(s, a, b, ch(p, 1), m+1, r);
375     }

97a     void shift(line s, int a=MI, int b=MA, int p=0, int l=MI, int r=MA) {
6b9         prop(p, l, r);
90d         int m = l + (r-l)/2;
9a3         if (a <= l and r <= b) {
ada             ln[p].la += s.a, ln[p].lb += s.b;
505             return;
570         }
1dd         if (b < l or r < a) return;
fdd         if (ln[p].b != LINF) {
751             push(ln[p], ch(p, 0), l, m);
ade             push(ln[p], ch(p, 1), m+1, r);
c2f             ln[p].a = 0, ln[p].b = LINF;
199         }
a04         shift(s, a, b, ch(p, 0), l, m);
e7d         shift(s, a, b, ch(p, 1), m+1, r);
d43     }
285 };
```

5.10 MergeSort Tree

```
// Se for construida sobre um array:
//      count(i, j, a, b) retorna quantos
//      elementos de v[i..j] pertencem a [a, b]
//      report(i, j, a, b) retorna os indices dos
//      elementos de v[i..j] que pertencem a [a, b]
//      retorna o vetor ordenado
// Se for construida sobre pontos (x, y):
//      count(x1, x2, y1, y2) retorna quantos pontos
//      pertencem ao retangulo (x1, y1), (x2, y2)
//      report(x1, x2, y1, y2) retorna os indices dos pontos que
//      pertencem ao retangulo (x1, y1), (x2, y2)
//      retorna os pontos ordenados lexicograficamente
//      (assume x1 <= x2, y1 <= y2)
//
// kth(y1, y2, k) retorna o indice do ponto com k-esimo menor
// x dentre os pontos que possuem y em [y1, y2] (0 based)
// Se quiser usar para achar k-esimo valor em range, construir
// com ms_tree t(v, true), e chamar kth(l, r, k)
//
// Usa O(n log(n)) de memoria
//
// Complexidades:
// construir - O(n log(n))
// count - O(log(n))
// report - O(log(n) + k) para k indices retornados
// kth - O(log(n))

c6c template <typename T = int> struct ms_tree {
6f7     vector<tuple<T, T, int>> v;
1a8     int n;
5ee     vector<vector<tuple<T, T, int>>> t; // {y, idx, left}
6ae     vector<T> vy;

78c     ms_tree(vector<pair<T, T>>& vv) : n(vv.size()), t(4*n), vy(n) {
e80         for (int i = 0; i < n; i++) v.push_back({vv[i].first,
    vv[i].second, i});
fca         sort(v.begin(), v.end());
224         build(1, 0, n-1);
01a         for (int i = 0; i < n; i++) vy[i] = get<0>(t[1][i+1]);
45e     }
dac     ms_tree(vector<T>& vv, bool inv = false) { // inv: inverte
    indice e valor
8e8         vector<pair<T, T>> v2;
e1e         for (int i = 0; i < vv.size(); i++)
196             inv ? v2.push_back({vv[i], i}) : v2.push_back({i,
    vv[i]});
cca         *this = ms_tree(v2);
f23     }
2c6     void build(int p, int l, int r) {
1d2         t[p].push_back({get<0>(v[l]), get<0>(v[r]), 0}); //
    {min_x, max_x, 0}
5c8         if (l == r) return t[p].push_back({get<1>(v[l]),
    get<2>(v[l]), 0});
ee4         int m = (l+r)/2;
bd9         build(2*p, l, m), build(2*p+1, m+1, r);

32d         int L = 0, R = 0;
a03         while (t[p].size() <= r-l+1) {
68e             int left = get<2>(t[p].back());
4aa             if (L > m-l or (R+m+1 <= r and t[2*p+1][1+R] <
    t[2*p][1+L])) {
8cf                 t[p].push_back(t[2*p+1][1 + R++]);
da0                 get<2>(t[p].back()) = left;
5e2                 continue;
ce0             }
249             t[p].push_back(t[2*p][1 + L++]);
339             get<2>(t[p].back()) = left+1;
208         }
2eb     }

dd3     int get_l(T y) { return lower_bound(vy.begin(), vy.end(), y) -
    vy.begin(); }
ebb     int get_r(T y) { return upper_bound(vy.begin(), vy.end(), y) -
    vy.begin(); }

f62     int count(T x1, T x2, T y1, T y2) {
902         function<int(int, int, int)> dfs = [&](int p, int l, int
    r) {
7c6             if (l == r or x2 < get<0>(t[p][0]) or get<1>(t[p][0])
    < x1) return 0;
2bb             if (x1 <= get<0>(t[p][0]) and get<1>(t[p][0]) <= x2)
    return r-l;
784             int nl = get<2>(t[p][l]), nr = get<2>(t[p][r]);
eb6             return dfs(2*p, nl, nr) + dfs(2*p+1, l-nl, r-nr);
122         };
7cb         return dfs(1, get_l(y1), get_r(y2));
f65     }
002     vector<int> report(T x1, T x2, T y1, T y2) {
4b8         vector<int> ret;
85e         function<void(int, int, int)> dfs = [&](int p, int l, int
    r) {
882             if (l == r or x2 < get<0>(t[p][0]) or get<1>(t[p][0])
    < x1) return;
8da                 if (x1 <= get<0>(t[p][0]) and get<1>(t[p][0]) <= x2) {
e00                     for (int i = l; i < r; i++)
    ret.push_back(get<1>(t[p][i+1]));
505                     return;
067                 }
784                 int nl = get<2>(t[p][l]), nr = get<2>(t[p][r]);
194                 dfs(2*p, nl, nr), dfs(2*p+1, l-nl, r-nr);
12b             };
8ad             dfs(1, get_l(y1), get_r(y2));
edf             return ret;
668         }
985         int kth(T y1, T y2, int k) {
902             function<int(int, int, int)> dfs = [&](int p, int l, int
    r) {
150                 if (k >= r-l) {
941                     k -= r-l;
daa                     return -1;
b8d                 }
8da                 if (r-l == 1) return get<1>(t[p][l+1]);
784                 int nl = get<2>(t[p][l]), nr = get<2>(t[p][r]);
072                 int left = dfs(2*p, nl, nr);
3b6                 if (left != -1) return left;
04d                 return dfs(2*p+1, l-nl, r-nr);
a1b             };
7cb             return dfs(1, get_l(y1), get_r(y2));
635         }
1ce };
```

5.11 Min queue - deque

```
// Tudo O(1) amortizado

1dc template<class T> struct minqueue {
2d8     deque<pair<T, int>> q;

3fc     void push(T x) {
56e         int ct = 1;
953         while (q.size() and x < q.front().first)
75f             ct += q.front().second, q.pop_front();
987         q.emplace_front(x, ct);
e8d     }
42d     void pop() {
aa2         if (q.back().second > 1) q.back().second--;
c51         else q.pop_back();
5fd     }
ea6     T min() { return q.back().first; }
c13 };
```

5.12 Min queue - stack

```
// Tudo O(1) amortizado

557 template<class T> struct minstack {
81f     stack<pair<T, T>> s;

3fc     void push(T x) {
12b         if (!s.size()) s.push({x, x});
9d9         else s.emplace(x, std::min(s.top().second, x));
f8d     }
4f0     T top() { return s.top().first; }
94a     T pop() {
1f2         T ans = s.top().first;
2eb         s.pop();
ba7         return ans;
013     }
614     int size() { return s.size(); }
13b     T min() { return s.top().second; }
4c0 };

1dc template<class T> struct minqueue {
cdc     minstack<T> s1, s2;

7cd     void push(T x) { s1.push(x); }
c96     void move() {
d4d         if (s2.size()) return;
d92         while (s1.size()) {
7ae             T x = s1.pop();
489             s2.push(x);
656         }
ef1     }
787     T front() { return move(), s2.top(); }
23a     T pop() { return move(), s2.pop(); }
7f3     int size() { return s1.size()+s2.size(); }
19c     T min() {
cd6         if (!s1.size()) return s2.min();
58e         else if (!s2.size()) return s1.min();
31d         return std::min(s1.min(), s2.min());
9c7     }
6d3 };
```

5.13 Order Statistic Set

```
// Funciona do C++11 pra cima

774 #include <ext/pb_ds/assoc_container.hpp>
30f #include <ext/pb_ds/tree_policy.hpp>
0d7 using namespace __gnu_pbds;
4fc template <class T>
def     using ord_set = tree<T, null_type, less<T>, rb_tree_tag,
3a1     tree_order_statistics_node_update>;

// para declarar:
// ord_set<int> s;
// coisas do set normal funcionam:
// for (auto i : s) cout << i << endl;
// cout << s.size() << endl;
// k-esimo maior elemento O(log|s|):
// k=0: menor elemento
// cout << *s.find_by_order(k) << endl;
// quantos sao menores do que k O(log|s|):
// cout << s.order_of_key(k) << endl;

// Para fazer um multiset, tem que
// usar ord_set<pair<int, int>> com o
// segundo parametro sendo algo para diferenciar
// os ementos iguais.
// s.order_of_key({k, -INF}) vai retornar o
// numero de elementos < k
```

5.14 Priority Queue DS

```
// Mantem updates aplicados em uma estrutura de dados
// que permita rollback e nao seja amortizada.
// Cada update possui uma prioridade,
// sendo possivel remover o update com maior prioridade.
// Os updates devem ser comutativos, ou seja, o estado
// da estrutura deve ser o mesmo independente da ordem
// que eles sejam aplicados.
//
// Complexidades:
// update - O(log(n) + T(n))
// query - T(n)
// pop - O(log(n) * T(n)) amortizado
//
// onde T(n) eh a complexidade do update

// assumes all priorities are distinct
945 template<typename DS, typename UPD> struct priority_queue_ds {
df4     DS D;
a7e     vector<tuple<UPD, int, int>> upd; // {u, p, idx_in_pos}
866     set<pair<int, int>> st;
927     vector<int> pos;

cf0     priority_queue_ds(int n) : D(n) {}

6af     void update(UPD u, int p) {
9ab         D.update(u);
d07         st.emplace(p, pos.size());
6ca         upd.emplace_back(u, p, pos.size());
e3d         pos.push_back(upd.size() - 1);
6af     }

427     int query(int a) {
aa3         return D.find(a);
2d3     }

42d     void pop() {
25f         int k = 1, min_p; // k = number of pops we will do
43e         vector<tuple<UPD, int, int>> small, big;
639         auto it = st.end();
231         for (int qt = 0; qt++ < (k+1)/2;) {
049             it--;
3ab             min_p = it->first;
80f             int i = pos[it->second];
e82             if (qt > 1) big.push_back(upd[i]);
84b             k = max<int>(k, upd.size() - i);
b9a         }

b3d         for (int i = 0; i < k; i++) {
a62             D.rollback();
6d8             auto [u, p, idx] = upd.rbegin()[i];
86d             if (p < min_p) small.emplace_back(u, p, idx);
588         }

23e         st.erase(prev(st.end()));
623         upd.erase(upd.end() - k, upd.end());

a25         small.insert(small.end(), big.rbegin(), big.rend());
06f         for (auto [u, p, idx] : small) {
9ab             D.update(u);
c8e             upd.emplace_back(u, p, idx);
a7d             pos[idx] = upd.size() - 1;
ec7         }
bd1     }
54a };
```

5.15 Range color

```
// update(l, r, c) colore o range [l, r] com a cor c,
// e retorna os ranges que foram coloridos {l, r, cor}
// query(i) returna a cor da posicao i
//
// Complexidades (para q operacoes):
// update - O(log(q)) amortizado
// query - O(log(q))

df6 template<typename T> struct color {
f0c     set<tuple<int, int, T>> se;

071     vector<tuple<int, int, T>> update(int l, int r, T val) {
9c4         auto it = se.upper_bound({r, INF, val});
753         if (it != se.begin() and get<1>(*prev(it)) > r) {
e91             auto [L, R, V] = *--it;
3f0             se.erase(it);
bfd             se.emplace(L, r, V), se.emplace(r+1, R, V);
683         }
d9e         it = se.lower_bound({l, -INF, val});
516         if (it != se.begin() and get<1>(*prev(it)) >= l) {
e91             auto [L, R, V] = *--it;
3f0             se.erase(it);
75a             se.emplace(L, l-1, V), it = se.emplace(l, R, V).first;
b65         }
d7b         vector<tuple<int, int, T>> ret;
7a1         for (; it != se.end() and get<0>(*it) <= r; it = se.erase(it))
8c0             ret.push_back(*it);
b4a         se.emplace(l, r, val);
edf         return ret;
b6c     }
ff9     T query(int i) {
c31         auto it = se.upper_bound({i, INF, T()});
8e7         if (it == se.begin() or get<1>(*--it) < i) return -1; // nao tem
53d         return get<2>(*it);
daf     }
9e9 };
```

5.16 RMQ <O(n), O(1)> - min queue

```
// O(n) pra buildar, query O(1)
// Se tiver varios minimos, retorna
// o de menor indice

1a5 template<typename T> struct rmq {
517     vector<T> v;
fcc     int n; static const int b = 30;
70e     vector<int> mask, t;

183     int op(int x, int y) { return v[x] <= v[y] ? x : y; }
ee1     int msb(int x) { return __builtin_clz(1)-__builtin_clz(x); }
c92     int small(int r, int sz = b) { return r-msb(mask[r]&((1<<sz)-1)); }
6ad     rmq() {}
43c     rmq(const vector<T>& v_) : v(v_), n(v.size()), mask(n), t(n) {
2e5         for (int i = 0, at = 0; i < n; mask[i++] = at |= 1) {
a61             at = (at<<1)&((1<<b)-1);
c00             while (at and op(i-msb(at&-at), i) == i) at ^= at&-at;
c2f         }
ea4         for (int i = 0; i < n/b; i++) t[i] = small(b*i+b-1);
39d         for (int j = 1; (1<<j) <= n/b; j++) for (int i = 0; i+(1<<j) <= n/b; i++)
ba5             t[n/b*j+i] = op(t[n/b*(j-1)+i], t[n/b*(j-1)+i+(1<<(j-1))]);
41a     }
e34     int index_query(int l, int r) {
27b         if (r-l+1 <= b) return small(r, r-l+1);
e80         int x = l/b+1, y = r/b-1;
fd3         if (x > y) return op(small(l+b-1), small(r));
a4e         int j = msb(y-x+1);
ea3         int ans = op(small(l+b-1), op(t[n/b*j+x], t[n/b*j+y-(1<<j)+1]));
be6         return op(ans, small(r));
62a     }
093     T query(int l, int r) { return v[index_query(l, r)]; }
bab };
```

5.17 SegTreap

```
// Muda uma posicao do plano, e faz query de operacao
// associativa e comutativa em retangulo
// Mudar ZERO e op
// Esparso nas duas coordenadas, inicialmente eh tudo ZERO
```

```
//
// Para query com distancia de manhattan <= d, faca
// nx = x+y, ny = x-y
// Update em (nx, ny), query em ((nx-d, ny-d), (nx+d, ny+d))
//
// Valores no X tem que ser de 0 ateh NX
// Para q operacoes, usa O(q log(NX)) de memoria, e as
// operacoes custa O(log(q) log(NX))

55b const int ZERO = INF;
560 const int op(int l, int r) { return min(l, r); }

878 mt19937 rng((int)
    chrono::steady_clock::now().time_since_epoch().count());

aa1 template<typename T> struct treap {
3c9     struct node {
b19         node *l, *r;
ee1         int p;
850         pair<ll, ll> idx; // {y, x}
36d         T val, mi;
bc2         node(ll x, ll y, T val_) : l(NULL), r(NULL), p(rng()),
1b5             idx(pair(y, x)), val(val_), mi(val) {}
01e         void update() {
d6e             mi = val;
182             if (l) mi = op(mi, l->mi);
b68             if (r) mi = op(mi, r->mi);
282         }
6e1     };

bb7     node* root;

84b     treap() { root = NULL; }
cec     ~treap() {
609         vector<node*> q = {root};
402         while (q.size()) {
e5d             node* x = q.back(); q.pop_back();
ee9             if (!x) continue;
1c7             q.push_back(x->l), q.push_back(x->r);
bf0             delete x;
653         }
50e     }
225     treap(treap&& t) : treap() { swap(root, t.root); }

bcf     void join(node* l, node* r, node*& i) { // assume que l < r
986         if (!l or !r) return void(i = l ? l : r);
80e         if (l->p > r->p) join(l->r, r, l->r), i = l;
fa0         else join(l, r->l, r->l), i = r;
bda         i->update();
671     }
c82     void split(node* i, node*& l, node*& r, pair<ll, ll> idx) {
26a         if (!i) return void(r = l = NULL);
13c         if (i->idx < idx) split(i->r, i->r, r, idx), l = i;
d26         else split(i->l, l, i->l, idx), r = i;
bda         i->update();
143     }
d3b     void update(ll x, ll y, T v) {
df9         node *L, *M, *R;
8b2         split(root, M, R, pair(y, x+1)), split(M, L, M, pair(y,
    x));
1e4         if (M) M->val = M->mi = v;
9e5         else M = new node(x, y, v);
69d         join(L, M, M), join(M, R, root);
58e     }
91b     T query(ll ly, ll ry) {
df9         node *L, *M, *R;
1c0         split(root, M, R, pair(ry, LINF)), split(M, L, M, pair(ly,
    0));
0f7         T ret = M ? M->mi : ZERO;
69d         join(L, M, M), join(M, R, root);
edf         return ret;
1ae     }
bdf };

46a template<typename T> struct segtreap {
c4f     vector<treap<T>> seg;
6e7     vector<int> ch[2];
e4e     ll NX;

253     segtreap(ll NX_) : seg(1), NX(NX_) { ch[0].push_back(-1),
    ch[1].push_back(-1); }

a71     int get_ch(int i, int d){
e51         if (ch[d][i] == -1) {
2d6             ch[d][i] = seg.size();
23e             seg.emplace_back();
842             ch[0].push_back(-1), ch[1].push_back(-1);
3e1         }
968         return ch[d][i];
bb6     }

10c     T query(ll lx, ll rx, ll ly, ll ry, int p, ll l, ll r) {
003         if (rx < l or r < lx) return ZERO;
f0f         if (lx <= l and r <= rx) return seg[p].query(ly, ry);
e6a         ll m = l + (r-l)/2;
354         return op(query(lx, rx, ly, ry, get_ch(p, 0), l, m),
060                 query(lx, rx, ly, ry, get_ch(p, 1), m+1, r));
a5e     }
f48     T query(ll lx, ll rx, ll ly, ll ry) { return query(lx, rx, ly,
    ry, 0, 0, NX); }

249     void update(ll x, ll y, T val, int p, ll l, ll r) {
73c         if (l == r) return seg[p].update(x, y, val);
e6a         ll m = l + (r-l)/2;
cc5         if (x <= m) update(x, y, val, get_ch(p, 0), l, m);
5a2         else update(x, y, val, get_ch(p, 1), m+1, r);
980         seg[p].update(x, y, val);
cc2     }
517     void update(ll x, ll y, T val) { update(x, y, val, 0, 0, NX); }
40a };
```

5.18 SegTree

```
// Recursiva com Lazy Propagation
// Query: soma do range [a, b]
// Update: soma x em cada elemento do range [a, b]
// Pode usar a seguinte funcao para indexar os nohs:
// f(l, r) = (l+r)|(l!=r), usando 2N de memoria
//
// Complexidades:
// build - O(n)
// query - O(log(n))
// update - O(log(n))

aa4 namespace seg {
005     ll seg[4*MAX], lazy[4*MAX];
052     int n, *v;

d22     ll build(int p=1, int l=0, int r=n-1) {
3c7         lazy[p] = 0;
6cd         if (l == r) return seg[p] = v[l];
ee4         int m = (l+r)/2;
193         return seg[p] = build(2*p, l, m) + build(2*p+1, m+1, r);
c71     }
0d8     void build(int n2, int* v2) {
680         n = n2, v = v2;
6f2         build();
acb     }
ceb     void prop(int p, int l, int r) {
cdf         seg[p] += lazy[p]*(r-l+1);
2c9         if (l != r) lazy[2*p] += lazy[p], lazy[2*p+1] += lazy[p];
3c7         lazy[p] = 0;
c10     }
2c3     ll query(int a, int b, int p=1, int l=0, int r=n-1) {
6b9         prop(p, l, r);
527         if (a <= l and r <= b) return seg[p];
786         if (b < l or r < a) return 0;
ee4         int m = (l+r)/2;
b1f         return query(a, b, 2*p, l, m) + query(a, b, 2*p+1, m+1, r);
4c5     }
cfb     ll update(int a, int b, int x, int p=1, int l=0, int r=n-1) {
6b9         prop(p, l, r);
9a3         if (a <= l and r <= b) {
b94             lazy[p] += x;
6b9             prop(p, l, r);
534             return seg[p];
821         }
e9f         if (b < l or r < a) return seg[p];
ee4         int m = (l+r)/2;
fdb         return seg[p] = update(a, b, x, 2*p, l, m) +
7fd             update(a, b, x, 2*p+1, m+1, r);
75c     }
0af };


// Se tiver uma seg de max, da pra descobrir em O(log(n))
// o primeiro e ultimo elemento >= val numa range:

// primeira posicao >= val em [a, b] (ou -1 se nao tem)
119 int get_left(int a, int b, int val, int p=1, int l=0, int r=n-1) {
6b9     prop(p, l, r);
f38     if (b < l or r < a or seg[p] < val) return -1;
205     if (r == l) return l;
ee4     int m = (l+r)/2;
753     int x = get_left(a, b, val, 2*p, l, m);
50e     if (x != -1) return x;
c3c     return get_left(a, b, val, 2*p+1, m+1, r);
68c }

// ultima posicao >= val em [a, b] (ou -1 se nao tem)
992 int get_right(int a, int b, int val, int p=1, int l=0, int r=n-1) {
6b9     prop(p, l, r);
f38     if (b < l or r < a or seg[p] < val) return -1;
205     if (r == l) return l;
ee4     int m = (l+r)/2;
1b1     int x = get_right(a, b, val, 2*p+1, m+1, r);
50e     if (x != -1) return x;
6a7     return get_right(a, b, val, 2*p, l, m);
1b7 }

// Se tiver uma seg de soma sobre um array nao negativo v, da pra
// descobrir em O(log(n)) o maior j tal que v[i]+v[i+1]+...+v[j-1] <
    val
6a9 int lower_bound(int i, ll& val, int p, int l, int r) {
6b9     prop(p, l, r);
6e8     if (r < i) return n;
b5d     if (i <= l and seg[p] < val) {
bff         val -= seg[p];
041         return n;
634     }
3ce     if (l == r) return l;
ee4     int m = (l+r)/2;
514     int x = lower_bound(i, val, 2*p, l, m);
ee0     if (x != n) return x;
8b9     return lower_bound(i, val, 2*p+1, m+1, r);
2b8 }
```

5.19 SegTree 2D Iterativa

```
// Consultas 0-based
// Um valor inicial em (x, y) deve ser colocado em seg[x+n][y+n]
// Query: soma do retangulo ((x1, y1), (x2, y2))
// Update: muda o valor da posicao (x, y) para val
// Nao pergunte como que essa coisa funciona
//
// Para query com distancia de manhattan <= d, faca
// nx = x+y, ny = x-y
// Update em (nx, ny), query em ((nx-d, ny-d), (nx+d, ny+d))
//
// Se for de min/max, pode tirar os if's da 'query', e fazer
// sempre as 4 operacoes. Fica mais rapido
//
// Complexidades:
// build - O(n^2)
// query - O(log^2(n))
// update - O(log^2(n))

731 int seg[2*MAX][2*MAX], n;

0a8 void build() {
919     for (int x = 2*n; x; x--) for (int y = 2*n; y; y--) {
c81         if (x < n) seg[x][y] = seg[2*x][y] + seg[2*x+1][y];
fe9         if (y < n) seg[x][y] = seg[x][2*y] + seg[x][2*y+1];
d51     }
499 }

251 int query(int x1, int y1, int x2, int y2) {
827     int ret = 0, y3 = y1 + n, y4 = y2 + n;
83e     for (x1 += n, x2 += n; x1 <= x2; ++x1 /= 2, --x2 /= 2)
0f2         for (y1 = y3, y2 = y4; y1 <= y2; ++y1 /= 2, --y2 /= 2) {
554             if (x1%2 == 1 and y1%2 == 1) ret += seg[x1][y1];
6b0             if (x1%2 == 1 and y2%2 == 0) ret += seg[x1][y2];
c01             if (x2%2 == 0 and y1%2 == 1) ret += seg[x2][y1];
5d4             if (x2%2 == 0 and y2%2 == 0) ret += seg[x2][y2];
2d0         }

edf     return ret;
ff1 }

767 void update(int x, int y, int val) {
66a     int y2 = y += n;
192     for (x += n; x; x /= 2, y = y2) {
970         if (x >= n) seg[x][y] = val;
ba9         else seg[x][y] = seg[2*x][y] + seg[2*x+1][y];

3b1         while (y /= 2) seg[x][y] = seg[x][2*y] + seg[x][2*y+1];
d8d     }
62e }
```

5.20 SegTree Beats

```
// query(a, b) - {{min(v[a..b]), max(v[a..b])}, sum(v[a..b])}
// updatemin(a, b, x) faz com que v[i] <- min(v[i], x),
// para i em [a, b]
// updatemax faz o mesmo com max, e updatesum soma x
// em todo mundo do intervalo [a, b]
//
// Complexidades:
// build - O(n)
// query - O(log(n))
// update - O(log^2 (n)) amortizado
// (se nao usar updatesum, fica log(n) amortizado)

7c6 #define f first
0ab #define s second

f39 namespace beats {
3c9     struct node {
```

```
526         int tam;
125         ll sum, lazy; // lazy pra soma
4f3         ll mi1, mi2, mi; // mi = #mi1
c61         ll ma1, ma2, ma; // ma = #ma1

426         node(ll x = 0) {
ba6             sum = mi1 = ma1 = x;
b29             mi2 = LINF, ma2 = -LINF;
62c             mi = ma = tam = 1;
c60             lazy = 0;
b00         }
770         node(const node& l, const node& r) {
a95             sum = l.sum + r.sum, tam = l.tam + r.tam;
c60             lazy = 0;
797             if (l.mi1 > r.mi1) {
230                 mi1 = r.mi1, mi = r.mi;
ea2                 mi2 = min(l.mi1, r.mi2);
f1e             } else if (l.mi1 < r.mi1) {
e34                 mi1 = l.mi1, mi = l.mi;
4b3                 mi2 = min(r.mi1, l.mi2);
ef2             } else {
a39                 mi1 = l.mi1, mi = l.mi+r.mi;
83d                 mi2 = min(l.mi2, r.mi2);
a92             }
cd0             if (l.ma1 < r.ma1) {
6a0                 ma1 = r.ma1, ma = r.ma;
96d                 ma2 = max(l.ma1, r.ma2);
3c0             } else if (l.ma1 > r.ma1) {
ae0                 ma1 = l.ma1, ma = l.ma;
2ca                 ma2 = max(r.ma1, l.ma2);
da8             } else {
db2                 ma1 = l.ma1, ma = l.ma+r.ma;
c05                 ma2 = max(l.ma2, r.ma2);
11c             }
1ba         }
4b4         void setmin(ll x) {
55e             if (x >= ma1) return;
463             sum += (x - ma1)*ma;
be5             if (mi1 == ma1) mi1 = x;
0a0             if (mi2 == ma1) mi2 = x;
b81             ma1 = x;
0c3         }
6cb         void setmax(ll x) {
e25             if (x <= mi1) return;
7e8             sum += (x - mi1)*mi;
0bb             if (ma1 == mi1) ma1 = x;
c32             if (ma2 == mi1) ma2 = x;
1ff                 mi1 = x;
a86             }
4cf             void setsum(ll x) {
fe8                 mi1 += x, mi2 += x, ma1 += x, ma2 += x;
620                 sum += x*tam;
c46                 lazy += x;
b53             }
47f         };

62b         node seg[4*MAX];
052         int n, *v;

93b         node build(int p=1, int l=0, int r=n-1) {
d84             if (l == r) return seg[p] = {v[l]};
ee4             int m = (l+r)/2;
3d6             return seg[p] = {build(2*p, l, m), build(2*p+1, m+1, r)};
444         }
0d8         void build(int n2, int* v2) {
680             n = n2, v = v2;
6f2             build();
acb         }
ceb         void prop(int p, int l, int r) {
8ce             if (l == r) return;
abd             for (int k = 0; k < 2; k++) {
d07                 if (seg[p].lazy) seg[2*p+k].setsum(seg[p].lazy);
843                 seg[2*p+k].setmin(seg[p].ma1);
f79                 seg[2*p+k].setmax(seg[p].mi1);
585             }
431             seg[p].lazy = 0;
7ee         }
055         pair<pair<ll, ll>, ll> query(int a, int b, int p=1, int l=0,
    int r=n-1) {
e07             if (b < l or r < a) return {{LINF, -LINF}, 0};
9be             if (a <= l and r <= b) return {{seg[p].mi1, seg[p].ma1},
    seg[p].sum};
6b9             prop(p, l, r);
ee4             int m = (l+r)/2;
e6f             auto L = query(a, b, 2*p, l, m), R = query(a, b, 2*p+1,
    m+1, r);
96d             return {{min(L.f.f, R.f.f), max(L.f.s, R.f.s)}, L.s+R.s};
e9d         }
2c8         node updatemin(int a, int b, ll x, int p=1, int l=0, int
    r=n-1) {
744             if (b < l or r < a or seg[p].ma1 <= x) return seg[p];
309             if (a <= l and r <= b and seg[p].ma2 < x) {
ccd                 seg[p].setmin(x);
534                 return seg[p];
bbf         }
6b9         prop(p, l, r);
ee4         int m = (l+r)/2;
96a         return seg[p] = {updatemin(a, b, x, 2*p, l, m),
4db                         updatemin(a, b, x, 2*p+1, m+1, r)};
aad     }
044     node updatemax(int a, int b, ll x, int p=1, int l=0, int
    r=n-1) {
b59         if (b < l or r < a or seg[p].mi1 >= x) return seg[p];
a9e         if (a <= l and r <= b and seg[p].mi2 > x) {
e8a             seg[p].setmax(x);
534             return seg[p];
e9b         }
6b9         prop(p, l, r);
ee4         int m = (l+r)/2;
ee3         return seg[p] = {updatemax(a, b, x, 2*p, l, m),
98b                         updatemax(a, b, x, 2*p+1, m+1, r)};
323     }
aee     node updatesum(int a, int b, ll x, int p=1, int l=0, int
    r=n-1) {
e9f         if (b < l or r < a) return seg[p];
9a3         if (a <= l and r <= b) {
8f4             seg[p].setsum(x);
534             return seg[p];
596         }
6b9         prop(p, l, r);
ee4         int m = (l+r)/2;
7b6         return seg[p] = {updatesum(a, b, x, 2*p, l, m),
483                         updatesum(a, b, x, 2*p+1, m+1, r)};
111     }
0d2 };
```

5.21 SegTree Colorida

```
// Cada posicao tem um valor e uma cor
// O construtor receve um vector de {valor, cor}
// e o numero de cores (as cores devem estar em [0, c-1])
// query(c, a, b) retorna a soma dos valores
// de todo mundo em [a, b] que tem cor c
// update(c, a, b, x) soma x em todo mundo em
// [a, b] que tem cor c
// paint(c1, c2, a, b) faz com que todo mundo
// em [a, b] que tem cor c1 passe a ter cor c2
//
// Complexidades:
// construir - O(n log(n)) espaco e tempo
// query - O(log(n))
// update - O(log(n))
// paint - O(log(n)) amortizado

04f struct seg_color {
3c9     struct node {
b19         node *l, *r;
0f9         int cnt;
9ca         ll val, lazy;
277         node() : l(NULL), r(NULL), cnt(0), val(0), lazy(0) {}
01e         void update() {
d0a             cnt = 0, val = 0;
bc4             for (auto i : {l, r}) if (i) {
c89                 i->prop();
281                 cnt += i->cnt, val += i->val;
68d             }
554         }
a9c         void prop() {
2dd             if (!lazy) return;
3f7             val += lazy*(ll)cnt;
b64             for (auto i : {l, r}) if (i) i->lazy += lazy;
c60             lazy = 0;
e24         }
514     };

1a8     int n;
9b0     vector<node*> seg;

6e0     seg_color(vector<pair<int, int>>& v, int c) : n(v.size()),
    seg(c, NULL) {
830         for (int i = 0; i < n; i++)
9b7             seg[v[i].second] = insert(seg[v[i].second], i,
    v[i].first, 0, n-1);
94a     }
3c7     ~seg_color() {
dde         queue<node*> q;
3a6         for (auto i : seg) q.push(i);
402         while (q.size()) {
20b             auto i = q.front(); q.pop();
dab             if (!i) continue;
7c7             q.push(i->l), q.push(i->r);
5ce             delete i;
c60         }
139     }

40b     node* insert(node* at, int idx, int val, int l, int r) {
1a4         if (!at) at = new node();
232         if (l == r) return at->cnt = 1, at->val = val, at;
ee4         int m = (l+r)/2;
137         if (idx <= m) at->l = insert(at->l, idx, val, l, m);
3e6         else at->r = insert(at->r, idx, val, m+1, r);
cff         return at->update(), at;
d6e     }
870     ll query(node* at, int a, int b, int l, int r) {
61b         if (!at or b < l or r < a) return 0;
d9f         at->prop();
cb2         if (a <= l and r <= b) return at->val;
ee4         int m = (l+r)/2;
4c4         return query(at->l, a, b, l, m) + query(at->r, a, b, m+1, r);
8c3     }
e54     ll query(int c, int a, int b) { return query(seg[c], a, b, 0, n-1); }
91c     void update(node* at, int a, int b, int x, int l, int r) {
fba         if (!at or b < l or r < a) return;
d9f         at->prop();
9a3         if (a <= l and r <= b) {
e9a             at->lazy += x;
cb2             return void(at->prop());
051         }
ee4         int m = (l+r)/2;
0b0         update(at->l, a, b, x, l, m), update(at->r, a, b, x, m+1, r);
7b4         at->update();
9fd     }
a40     void update(int c, int a, int b, int x) { update(seg[c], a, b, x, 0, n-1); }
70c     void paint(node*& from, node*& to, int a, int b, int l, int r) {
10f         if (to == from or !from or b < l or r < a) return;
e85         from->prop();
889         if (to) to->prop();
9a3         if (a <= l and r <= b) {
24d             if (!to) {
38f                 to = from;
140                 from = NULL;
505                 return;
e5f             }
ee4             int m = (l+r)/2;
1cb             paint(from->l, to->l, a, b, l, m), paint(from->r, to->r, a, b, m+1, r);
72d             to->update();
270             delete from;
140             from = NULL;
505             return;
a0e         }
019         if (!to) to = new node();
ee4         int m = (l+r)/2;
1cb         paint(from->l, to->l, a, b, l, m), paint(from->r, to->r, a, b, m+1, r);
45a         from->update(), to->update();
4aa     }
471     void paint(int c1, int c2, int a, int b) { paint(seg[c1], seg[c2], a, b, 0, n-1); }
293 };
```

5.22 SegTree Esparsa - Lazy

```
// Query: soma do range [a, b]
// Update: flipa os valores de [a, b]
// O MAX tem q ser Q log N para Q updates
//
// Complexidades:
// build - O(1)
// query - O(log(n))
// update - O(log(n))

aa4 namespace seg {
6de     int seg[MAX], lazy[MAX], R[MAX], L[MAX], ptr;
e9a     int get_l(int i){
3db         if (L[i] == 0) L[i] = ptr++;
a96         return L[i];
b6e     }
943     int get_r(int i){
71b         if (R[i] == 0) R[i] = ptr++;
283         return R[i];
43a     }

e71     void build() { ptr = 2; }

ceb     void prop(int p, int l, int r) {
b77         if (!lazy[p]) return;
76c         seg[p] = r-l+1 - seg[p];
213         if (l != r) lazy[get_l(p)]^=lazy[p], lazy[get_r(p)]^=lazy[p];
3c7         lazy[p] = 0;
20b     }

158     int query(int a, int b, int p=1, int l=0, int r=N-1) {
6b9         prop(p, l, r);
786         if (b < l or r < a) return 0;
527         if (a <= l and r <= b) return seg[p];

ee4         int m = (l+r)/2;
818         return query(a, b, get_l(p), l, m)+query(a, b, get_r(p),
    m+1, r);
0d9     }

51f     int update(int a, int b, int p=1, int l=0, int r=N-1) {
6b9         prop(p, l, r);
e9f         if (b < l or r < a) return seg[p];
9a3         if (a <= l and r <= b) {
ab6             lazy[p] ^= 1;
6b9             prop(p, l, r);
534             return seg[p];
8e4         }
ee4         int m = (l+r)/2;
43a         return seg[p] = update(a, b, get_l(p), l, m)+update(a, b,
    get_r(p), m+1, r);
1dc     }
dc3 };
```

5.23 SegTree Esparsa - O(q) memoria

```
// Query: min do range [a, b]
// Update: troca o valor de uma posicao
// Usa O(q) de memoria para q updates
//
// Complexidades:
// query - O(log(n))
// update - O(log(n))

13d template<typename T> struct seg {
3c9     struct node {
d53         node* ch[2];
970         char d;
ca0         T v;

c4e         T mi;

d4e         node(int d_, T v_, T val) : d(d_), v(v_) {
e71             ch[0] = ch[1] = NULL;
d6e             mi = val;
065         }
b32         node(node* x) : d(x->d), v(x->v), mi(x->mi) {
c99             ch[0] = x->ch[0], ch[1] = x->ch[1];
cb7         }
01e         void update() {
909             mi = numeric_limits<T>::max();
151             for (int i = 0; i < 2; i++) if (ch[i])
b5a                 mi = min(mi, ch[i]->mi);
fe3         }
530     };

bb7     node* root;
9c5     char n;

ba7     seg() : root(NULL), n(0) {}
512     ~seg() {
4c0         std::vector<node*> q = {root};
402         while (q.size()) {
e5d             node* x = q.back(); q.pop_back();
ee9             if (!x) continue;
73f             q.push_back(x->ch[0]), q.push_back(x->ch[1]);
bf0             delete x;
d3e         }
d8c     }

1a6     char msb(T v, char l, char r) { // msb in range (l, r]
8e4         for (char i = r; i > l; i--) if (v>>i&1) return i;
daa         return -1;
688     }
430     void cut(node* at, T v, char i) {
677         char d = msb(v ^ at->v, at->d, i);
23b         if (d == -1) return; // no need to split
ebf         node* nxt = new node(at);
d43         at->ch[v>>d&1] = NULL;
34f         at->ch[!(v>>d&1)] = nxt;
150         at->d = d;
0b3     }

6e5     node* update(node* at, T idx, T val, char i) {
c8c         if (!at) return new node(-1, idx, val);
d67         cut(at, idx, i);
1a2         if (at->d == -1) { // leaf
792             at->mi = val;
ce6             return at;
a6f         }
b29         bool dir = idx>>at->d&1;
c8f         at->ch[dir] = update(at->ch[dir], idx, val, at->d-1);
7b4         at->update();
ce6         return at;
76d     }
85c     void update(T idx, T val) {
8f4         while (idx>>n) n++;
61e         root = update(root, idx, val, n-1);
79d     }

9d8     T query(node* at, T a, T b, T l, T r, char i) {
df0         if (!at or b < l or r < a) return numeric_limits<T>::max();
fd3         if (a <= l and r <= b) return at->mi;
841         T m = l + (r-l)/2;
c85         if (at->d < i) {
c59             if ((at->v>>i&1) == 0) return query(at, a, b, l, m,
    i-1);
ca4             else return query(at, a, b, m+1, r, i-1);
934         }
373         return min(query(at->ch[0], a, b, l, m, i-1),
    query(at->ch[1], a, b, m+1, r, i-1));
2db     }
034     T query(T l, T r) { return query(root, l, r, 0, (T(1)<<n)-1,
    n-1); }
d7f };
```

5.24 SegTree Iterativa

```
// Consultas 0-based
// Valores iniciais devem estar em (seg[n], ... , seg[2*n-1])
// Query: soma do range [a, b]
// Update: muda o valor da posicao p para x
//
// Complexidades:
// build - O(n)
// query - O(log(n))
// update - O(log(n))

6a4 int seg[2 * MAX];
1a8 int n;

0a8 void build() {
d15     for (int i = n - 1; i; i--) seg[i] = seg[2*i] + seg[2*i+1];
9a8 }

4ea int query(int a, int b) {
7c9     int ret = 0;
728     for(a += n, b += n; a <= b; ++a /= 2, --b /= 2) {
4ea         if (a % 2 == 1) ret += seg[a];
244         if (b % 2 == 0) ret += seg[b];
ac0     }
edf     return ret;
24a }

ff3 void update(int p, int x) {
37d     seg[p += n] = x;
c8c     while (p /= 2) seg[p] = seg[2*p] + seg[2*p+1];
02d }
```

5.25 SegTree Iterativa com Lazy Propagation

```
// Query: soma do range [a, b]
// Update: soma x em cada elemento do range [a, b]
// Para mudar, mudar as funcoes junta, poe e query
// LOG = ceil(log2(MAX))
//
// Complexidades:
// build - O(n)
// query - O(log(n))
// update - O(log(n))

aa4 namespace seg {
6db     ll seg[2*MAX], lazy[2*MAX];
1a8     int n;

9b3     ll junta(ll a, ll b) {
534         return a+b;
e26     }

        // soma x na posicao p de tamanho tam
1b4     void poe(int p, ll x, int tam, bool prop=1) {
517         seg[p] += x*tam;
6ae         if (prop and p < n) lazy[p] += x;
8bc     }

        // atualiza todos os pais da folha p
b1e     void sobe(int p) {
d5a         for (int tam = 2; p /= 2; tam *= 2) {
4ca             seg[p] = junta(seg[2*p], seg[2*p+1]);
388             poe(p, lazy[p], tam, 0);
acd         }
b76     }

        // propaga o caminho da raiz ate a folha p
a0a     void prop(int p) {
076         int tam = 1 << (LOG-1);
0a8         for (int s = LOG; s; s--, tam /= 2) {
```

```
4b1             int i = p >> s;
27c             if (lazy[i]) {
860                 poe(2*i, lazy[i], tam);
e38                 poe(2*i+1, lazy[i], tam);
b97                 lazy[i] = 0;
de8             }
3ed         }
e29     }

61c     void build(int n2, int* v) {
1e3         n = n2;
95f         for (int i = 0; i < n; i++) seg[n+i] = v[i];
c41         for (int i = n-1; i; i--) seg[i] = junta(seg[2*i], seg[2*i+1]);
f4c         for (int i = 0; i < 2*n; i++) lazy[i] = 0;
8bb     }

4f3     ll query(int a, int b) {
b73         ll ret = 0;
b48         for (prop(a+=n), prop(b+=n); a <= b; ++a/=2, --b/=2) {
a8e             if (a%2 == 1) ret = junta(ret, seg[a]);
c58             if (b%2 == 0) ret = junta(ret, seg[b]);
510         }
edf         return ret;
38b     }

a28     void update(int a, int b, int x) {
c2d         int a2 = a += n, b2 = b += n, tam = 1;
0ff         for (; a <= b; ++a/=2, --b/=2, tam *= 2) {
32a             if (a%2 == 1) poe(a, x, tam);
9da             if (b%2 == 0) poe(b, x, tam);
9bc         }
0f7         sobe(a2), sobe(b2);
adc     }
6dc };
```

5.26 SegTree PA

```
// Segtree de PA
// update_set(l, r, A, R) seta [l, r] para PA(A, R),
// update_add soma PA(A, R) em [l, r]
// query(l, r) retorna a soma de [l, r]
//
// PA(A, R) eh a PA: [A+R, A+2R, A+3R, ... ]
//
// Complexidades:
// construir - O(n)
// update_set, update_add, query - O(log(n))

dc7 struct seg_pa {
350     struct Data {
8f5         ll sum;
662         ll set_a, set_r, add_a, add_r;
9b7         Data() : sum(0), set_a(LINF), set_r(0), add_a(0), add_r(0) {}
eb6     };
16a     vector<Data> seg;
1a8     int n;

d45     seg_pa(int n_) {
e95         n = n_;
fc3         seg = vector<Data>(4*n);
ce0     }

ceb     void prop(int p, int l, int r) {
d5a         int tam = r-l+1;
c3f         ll &sum = seg[p].sum, &set_a = seg[p].set_a, &set_r = seg[p].set_r,
a1b             &add_a = seg[p].add_a, &add_r = seg[p].add_r;

c02         if (set_a != LINF) {
660             set_a += add_a, set_r += add_r;
06e             sum = set_a*tam + set_r*tam*(tam+1)/2;
579             if (l != r) {
ee4                 int m = (l+r)/2;

886                 seg[2*p].set_a = set_a;
358                 seg[2*p].set_r = set_r;
ed6                 seg[2*p].add_a = seg[2*p].add_r = 0;

f0c                 seg[2*p+1].set_a = set_a + set_r * (m-l+1);
471                 seg[2*p+1].set_r = set_r;
d48                 seg[2*p+1].add_a = seg[2*p+1].add_r = 0;
a97             }
823             set_a = LINF, set_r = 0;
953             add_a = add_r = 0;
da7         } else if (add_a or add_r) {
18b             sum += add_a*tam + add_r*tam*(tam+1)/2;
579             if (l != r) {
ee4                 int m = (l+r)/2;

ff0                 seg[2*p].add_a += add_a;
ec0                 seg[2*p].add_r += add_r;
06c                 seg[2*p+1].add_a += add_a + add_r * (m-l+1);
a6d                 seg[2*p+1].add_r += add_r;
8af             }
953             add_a = add_r = 0;
ab7         }
07f     }

0b7     int inter(pair<int, int> a, pair<int, int> b) {
98c         if (a.first > b.first) swap(a, b);
eef         return max(0, min(a.second, b.second) - b.first + 1);
628     }
be1     ll set(int a, int b, ll aa, ll rr, int p, int l, int r) {
6b9         prop(p, l, r);
457         if (b < l or r < a) return seg[p].sum;
9a3         if (a <= l and r <= b) {
91c             seg[p].set_a = aa;
774             seg[p].set_r = rr;
6b9             prop(p, l, r);
254             return seg[p].sum;
8ee         }
ee4         int m = (l+r)/2;
963         int tam_l = inter({l, m}, {a, b});
c34         return seg[p].sum = set(a, b, aa, rr, 2*p, l, m) +
365             set(a, b, aa + rr * tam_l, rr, 2*p+1, m+1, r);
8e2     }
f55     void update_set(int l, int r, ll aa, ll rr) {
6f7         set(l, r, aa, rr, 1, 0, n-1);
913     }
5f6     ll add(int a, int b, ll aa, ll rr, int p, int l, int r) {
6b9         prop(p, l, r);
457         if (b < l or r < a) return seg[p].sum;
9a3         if (a <= l and r <= b) {
359             seg[p].add_a += aa;
1ee             seg[p].add_r += rr;
6b9             prop(p, l, r);
254             return seg[p].sum;
d19         }
ee4         int m = (l+r)/2;
963         int tam_l = inter({l, m}, {a, b});
586         return seg[p].sum = add(a, b, aa, rr, 2*p, l, m) +
695             add(a, b, aa + rr * tam_l, rr, 2*p+1, m+1, r);
904     }
848     void update_add(int l, int r, ll aa, ll rr) {
afa         add(l, r, aa, rr, 1, 0, n-1);
81e     }
f45     ll query(int a, int b, int p, int l, int r) {
6b9         prop(p, l, r);
786         if (b < l or r < a) return 0;
e9a         if (a <= l and r <= b) return seg[p].sum;
ee4         int m = (l+r)/2;
b1f         return query(a, b, 2*p, l, m) + query(a, b, 2*p+1, m+1, r);
f6e     }
bfc     ll query(int l, int r) { return query(l, r, 1, 0, n-1); }
bc4 };
```

5.27 SegTree Persistente

```
// SegTree de soma, update de somar numa posicao
//
// query(a, b, t) retorna a query de [a, b] na versao t
// update(a, x, t) faz um update v[a]+=x a partir da
// versao de t, criando uma nova versao e retornando seu id
// Por default, faz o update a partir da ultima versao
//
// build - O(n)
// query - O(log(n))
// update - O(log(n))

54a const int MAX = 1e5+10, UPD = 1e5+10, LOG = 18;
6de const int MAXS = 2*MAX+UPD*LOG;

f6e namespace perseg {
bd6     ll seg[MAXS];
f4e     int rt[UPD], L[MAXS], R[MAXS], cnt, t;
052     int n, *v;

3c4     ll build(int p, int l, int r) {
6cd         if (l == r) return seg[p] = v[l];
855         L[p] = cnt++, R[p] = cnt++;
ee4         int m = (l+r)/2;
275         return seg[p] = build(L[p], l, m) + build(R[p], m+1, r);
39d     }
0d8     void build(int n2, int* v2) {
680         n = n2, v = v2;
856         rt[0] = cnt++;
c50         build(0, 0, n-1);
a2e     }
f45     ll query(int a, int b, int p, int l, int r) {
786         if (b < l or r < a) return 0;
527         if (a <= l and r <= b) return seg[p];
ee4         int m = (l+r)/2;
1ed         return query(a, b, L[p], l, m) + query(a, b, R[p], m+1, r);
4d2     }
182     ll query(int a, int b, int tt) {
c13         return query(a, b, rt[tt], 0, n-1);
726     }
bb3     ll update(int a, int x, int lp, int p, int l, int r) {
747         if (l == r) return seg[p] = seg[lp]+x;
ee4         int m = (l+r)/2;
ab8         if (a <= m)
b48             return seg[p] = update(a, x, L[lp], L[p]=cnt++, l, m)
    + seg[R[p]=R[lp]];
8a9         return seg[p] = seg[L[p]=L[lp]] + update(a, x, R[lp],
    R[p]=cnt++, m+1, r);
788     }
6f6     int update(int a, int x, int tt=t) {
ab3         update(a, x, rt[tt], rt[++t]=cnt++, 0, n-1);
e0d         return t;
d63     }
26f };
```

5.28 SegTree Persistente com Lazy

```
// Nao propaga, meio estranho de mexer, mas da
//
// query(a, b, t) retorna a query de [a, b] na versao t
// update(a, b, x, t) faz um update v[a..b]+=x a partir da
// versao de t, criando uma nova versao e retornando seu id
// Por default, faz o update a partir da ultima versao
//
// build - O(n)
// query - O(log(n))
// update - O(log(n))

54a const int MAX = 1e5+10, UPD = 1e5+10, LOG = 18;
ab3 const int MAXS = 2*MAX + 4*UPD*LOG;

f6e namespace perseg {
9eb     int seg[MAXS];
f4e     int rt[UPD], L[MAXS], R[MAXS], cnt, t;
052     int n, *v;

adf     int build(int p, int l, int r) {
6cd         if (l == r) return seg[p] = v[l];
855         L[p] = cnt++, R[p] = cnt++;
ee4         int m = (l+r)/2;
01d         return seg[p] = max(build(L[p], l, m), build(R[p], m+1,
    r));
ffd     }
0d8     void build(int n2, int *v2) {
680         n = n2, v = v2;
856         rt[0] = cnt++;
c50         build(0, 0, n-1);
a2e     }
976     int query(int a, int b, int p, int l, int r) {
27b         if (b < l or r < a) return -INF;
793         if (a <= l and r <= b) return lazy[p] + seg[p];
ee4         int m = (l+r)/2;
7a2         int ret = lazy[p] + max(query(a, b, L[p], l, m), query(a,
    b, R[p], m+1, r));
edf         return ret;
9a7     }
442     int query(int a, int b, int tt) {
c13         return query(a, b, rt[tt], 0, n-1);
a05     }
bc1     int update(int a, int b, int x, int lp, int p, int l, int r) {
3f6         tie(seg[p], lazy[p], L[p], R[p]) = {seg[lp], lazy[lp],
    L[lp], R[lp]};
847         if (b < l or r < a) return seg[p] + lazy[p];
32a         if (a <= l and r <= b) return seg[p] + (lazy[p] += x);

ee4         int m = (l+r)/2;
24a         seg[p] = max(update(a, b, x, L[lp], L[p] = cnt++, l, m),
bdb                      update(a, b, x, R[lp], R[p] = cnt++, m+1, r));
1ed         lazy[p] = lazy[lp];
1b7         return seg[p] + lazy[p];
877     }
cbf     int update(int a, int b, int x, int tt=t) {
aa8         assert(tt <= t);
661         update(a, b, x, rt[tt], rt[++t]=cnt++, 0, n-1);
e0d         return t;
aad     }
f27 };
```

5.29 SlopeTrick

```
// Armazena uma estrutura convexa piecewise linear
// Permite adicionar slopes sem peso e realizar query de minimo
// Comentarios acima das funcoes para explicar o que cada uma faz

2f8 template<typename T> struct SlopeTrick {
64e     T inf = numeric_limits<T>::max() / 3;
acc     T min_f;
f32     priority_queue<T, vector<T>, less<>> L;
```

```
6ef     priority_queue<T, vector<T>, greater<>> R;
a20     T add_l, add_r;

055     T top_R() {
a34         if (R.empty()) return inf;
ffe         else return R.top() + add_r;
074     }

70c     T pop_R() {
66f         T val = top_R();
8e0         if (R.size()) R.pop();
d94         return val;
21d     }

d9d     T top_L() {
b7b         if (L.empty()) return -inf;
470         else return L.top() + add_l;
31d     }

821     T pop_L() {
66a         T val = top_L();
1e0         if (L.size()) L.pop();
d94         return val;
dfd     }

86d     size_t size() {
7ff         return L.size() + R.size();
c4b     }

0e8     SlopeTrick() : min_f(0), add_l(0), add_r(0) {};

        // return {min f(x), lx, rx}
        // Em que [lx, rx] eh o intervalo que atinge o minimo
5ee     array<T, 3> query() {
e8a         return {min_f, top_L(), top_R()};
14f     }

        // f(x) += a
ad4     void add_all(T a) {
f8c         min_f += a;
78a     }

        // add \_
        // f(x) += max(a - x, 0)
60a     void add_a_minus_x(T a) {
8c6         min_f += max(T(0), a - top_R());
cdb         R.push(a - add_r);
416         L.push(pop_R() - add_l);
44c     }

        // add _/
        // f(x) += max(x - a, 0)
7a9     void add_x_minus_a(T a) {
b36         min_f += max(T(0), top_L() - a);
988         L.push(a - add_l);
e5a         R.push(pop_L() - add_r);
f3a     }

        // add \/
        // f(x) += abs(x - a)
825     void add_abs(T a) {
9cc         add_a_minus_x(a);
e55         add_x_minus_a(a);
639     }

        // \/ -> \_
        // f_{new} (x) = min f(y) (y <= x)
73b     void clear_right() {
b8e         while (R.size()) R.pop();
2b3     }

        // \/ -> _/
        // f_{new} (x) = min f(y) (y >= x)
fd5     void clear_left() {
e21         while (L.size()) L.pop();
bc4     }

        // \/ -> \_/
        // f_{new} (x) = min f(y) (x-b <= y <= x-a)
564     void shift(T a, T b) {
25b         assert(a <= b);
b95         add_l += a;
165         add_r += b;
29a     }

        // \/. -> .\/
        // f_{new} (x) = f(x - a)
5d6     void shift(T a) {
a77         shift(a, a);
af1     }

        // Retorna f(x)
        // O(size)
c2a     T get(T x) {
```

```
7ce         auto L2 = L;
202         auto R2 = R;
bf4         T ret = min_f;
6a9         while (L.size()) {
efd             ret += max(T(0), pop_L() - x);
e50         }
886         while (R.size()) {
97b             ret += max(T(0), x - pop_R());
8ef         }
98a         L = L2, R = R2;
edf         return ret;
093     }

        // O(min(size, st.size))
9e9     void merge(SlopeTrick &st) {
f68         if (st.size() > size()) {
079             swap(*this, st);
788         }
1a3         while (st.R.size()) {
85b             add_x_minus_a(st.pop_R());
2c5         }
8c6         while (st.L.size()) {
897             add_a_minus_x(st.pop_L());
b31         }
eaf         min_f += st.min_f;
3df     }
495 };
```

5.30 Sparse Table

```
// Resolve RMQ
// MAX2 = log(MAX)
//
// Complexidades:
// build - O(n log(n))
// query - O(1)

cca namespace sparse {
710     int m[MAX2][MAX], n;
61c     void build(int n2, int* v) {
1e3         n = n2;
78e         for (int i = 0; i < n; i++) m[0][i] = v[i];
a1c         for (int j = 1; (1<<j) <= n; j++) for (int i = 0; i+(1<<j)
    <= n; i++)
5d5             m[j][i] = min(m[j-1][i], m[j-1][i+(1<<(j-1))]);
cae     }
4ea     int query(int a, int b) {
ee5         int j = __builtin_clz(1) - __builtin_clz(b-a+1);
dc3         return min(m[j][a], m[j][b-(1<<j)+1]);
fba     }
7aa }
```

5.31 Sparse Table Disjunta

```
// Resolve qualquer operacao associativa
// MAX2 = log(MAX)
//
// Complexidades:
// build - O(n log(n))
// query - O(1)

cca namespace sparse {
9bf     int m[MAX2][2*MAX], n, v[2*MAX];
5f7     int op(int a, int b) { return min(a, b); }
0d8     void build(int n2, int* v2) {
1e3         n = n2;
df4         for (int i = 0; i < n; i++) v[i] = v2[i];
a84         while (n&(n-1)) n++;
3d2         for (int j = 0; (1<<j) < n; j++) {
1c0             int len = 1<<j;
d9b             for (int c = len; c < n; c += 2*len) {
332                 m[j][c] = v[c], m[j][c-1] = v[c-1];
668                 for (int i = c+1; i <  c+len; i++) m[j][i] =
    op(m[j][i-1], v[i]);
432                 for (int i = c-2; i >= c-len; i--) m[j][i] =
    op(v[i], m[j][i+1]);
eda             }
f4d         }
ce3     }
9e3     int query(int l, int r) {
f13         if (l == r) return v[l];
e6d         int j = __builtin_clz(1) - __builtin_clz(l^r);
d67         return op(m[j][l], m[j][r]);
a7b     }
fd8 }
```

5.32 Splay Tree

```
// SEMPRE QUE DESCER NA ARVORE, DAR SPLAY NO
// NODE MAIS PROFUNDO VISITADO
// Todas as operacoes sao O(log(n)) amortizado
// Se quiser colocar mais informacao no node,
// mudar em 'update'

538 template<typename T> struct splaytree {
3c9     struct node {
183         node *ch[2], *p;
e4d         int sz;
f48         T val;
da0         node(T v) {
696             ch[0] = ch[1] = p = NULL;
a26             sz = 1;
250             val = v;
2d0         }
01e         void update() {
a26             sz = 1;
c7c             for (int i = 0; i < 2; i++) if (ch[i]) {
d5f                 sz += ch[i]->sz;
486             }
f45         }
aa3     };

bb7     node* root;

fbc     splaytree() { root = NULL; }
214     splaytree(const splaytree& t) {
cbf         throw logic_error("Nao copiar a splaytree!");
1f1     }
891     ~splaytree() {
609         vector<node*> q = {root};
402         while (q.size()) {
e5d             node* x = q.back(); q.pop_back();
ee9             if (!x) continue;
73f             q.push_back(x->ch[0]), q.push_back(x->ch[1]);
bf0             delete x;
d3e         }
837     }

94f     void rotate(node* x) { // x vai ficar em cima
d9b         node *p = x->p, *pp = p->p;
ecf         if (pp) pp->ch[pp->ch[1] == p] = x;
286         bool d = p->ch[0] == x;
d63         p->ch[!d] = x->ch[d], x->ch[d] = p;
bad         if (p->ch[!d]) p->ch[!d]->p = p;
fc2         x->p = pp, p->p = x;
1ea         p->update(), x->update();
007     }
3fa     node* splay(node* x) {
a39         if (!x) return x;
4ea         root = x;
3cf         while (x->p) {
d9b             node *p = x->p, *pp = p->p;
359             if (!pp) return rotate(x), x; // zig
e3c             if ((pp->ch[0] == p)^(p->ch[0] == x))
a2b                 rotate(x), rotate(x); // zigzag
4b2             else rotate(p), rotate(x); // zigzig
028         }
ea5         return x;
379     }
313     node* insert(T v, bool lb=0) {
b64         if (!root) return lb ? NULL : root = new node(v);
002         node *x = root, *last = NULL;;
31e         while (1) {
5d7             bool d = x->val < v;
0fd             if (!d) last = x;
c2e             if (x->val == v) break;
c16             if (x->ch[d]) x = x->ch[d];
4e6             else {
dea                 if (lb) break;
055                 x->ch[d] = new node(v);
99c                 x->ch[d]->p = x;
30e                 x = x->ch[d];
c2b                 break;
68a             }
1ab         }
0b6         splay(x);
61c         return lb ? splay(last) : x;
622     }
c0c     int size() { return root ? root->sz : 0; }
2ca     int count(T v) { return insert(v, 1) and root->val == v; }
111     node* lower_bound(T v) { return insert(v, 1); }
26b     void erase(T v) {
446         if (!count(v)) return;
bce         node *x = root, *l = x->ch[0];
268         if (!l) {
8b1             root = x->ch[1];
32e             if (root) root->p = NULL;
8f3             return delete x;
a86         }
5e7         root = l, l->p = NULL;
902         while (l->ch[1]) l = l->ch[1];
bab         splay(l);
f0e         l->ch[1] = x->ch[1];
7d9         if (l->ch[1]) l->ch[1]->p = l;
bf0         delete x;
62a         l->update();
007     }
24a     int order_of_key(T v) {
62b         if (!lower_bound(v)) return root ? root->sz : 0;
1cc         return root->ch[0] ? root->ch[0]->sz : 0;
b00     }
db6     node* find_by_order(int k) {
084         if (k >= size()) return NULL;
52f         node* x = root;
31e         while (1) {
20f             if (x->ch[0] and x->ch[0]->sz >= k+1) x = x->ch[0];
4e6             else {
a1c                 if (x->ch[0]) k -= x->ch[0]->sz;
1dc                 if (!k) return splay(x);
eb8                 k--, x = x->ch[1];
aca             }
e01         }
0de     }
19c     T min() {
52f         node* x = root;
6f6         while (x->ch[0]) x = x->ch[0]; // max -> ch[1]
3e9         return splay(x)->val;
70e     }
4ff };
```

5.33 Splay Tree Implicita

```
// vector da NASA
// Um pouco mais rapido q a treap
// O construtor a partir do vector
// eh linear, todas as outras operacoes
// custam O(log(n)) amortizado

081 template<typename T> struct splay {
3c9     struct node {
183         node *ch[2], *p;
e4d         int sz;
875         T val, sub, lazy;
aa6         bool rev;
da0         node(T v) {
696             ch[0] = ch[1] = p = NULL;
a26             sz = 1;
1e4             sub = val = v;
c60             lazy = 0;
b67             rev = false;
48f         }
a9c         void prop() {
0ec             if (lazy) {
924                 val += lazy, sub += lazy*sz;
091                 if (ch[0]) ch[0]->lazy += lazy;
1a8                 if (ch[1]) ch[1]->lazy += lazy;
a98             }
1bb             if (rev) {
80a                 swap(ch[0], ch[1]);
628                 if (ch[0]) ch[0]->rev ^= 1;
adc                 if (ch[1]) ch[1]->rev ^= 1;
30a             }
a32             lazy = 0, rev = 0;
6bf         }
01e         void update() {
0c3             sz = 1, sub = val;
c7c             for (int i = 0; i < 2; i++) if (ch[i]) {
05f                 ch[i]->prop();
d5f                 sz += ch[i]->sz;
4a1                 sub += ch[i]->sub;
6c1             }
e98         }
b4a     };

bb7     node* root;

5d9     splay() { root = NULL; }
9b1     splay(node* x) {
4ea         root = x;
32e         if (root) root->p = NULL;
371     }
1b7     splay(vector<T> v) { // O(n)
950         root = NULL;
806         for (T i : v) {
2a0             node* x = new node(i);
bd1             x->ch[0] = root;
37a             if (root) root->p = x;
4ea             root = x;
a0a             root->update();
17c         }
c6b     }
a9e     splay(const splay& t) {
e62         throw logic_error("Nao copiar a splay!");
d4d     }
5ab     ~splay() {
609         vector<node*> q = {root};
402         while (q.size()) {
```

```
e5d                 node* x = q.back(); q.pop_back();
ee9                 if (!x) continue;
73f                 q.push_back(x->ch[0]), q.push_back(x->ch[1]);
bf0                 delete x;
d3e             }
a1c         }

73c         int size(node* x) { return x ? x->sz : 0; }
94f         void rotate(node* x) { // x vai ficar em cima
d9b             node *p = x->p, *pp = p->p;
ecf             if (pp) pp->ch[pp->ch[1] == p] = x;
286             bool d = p->ch[0] == x;
d63             p->ch[!d] = x->ch[d], x->ch[d] = p;
bad             if (p->ch[!d]) p->ch[!d]->p = p;
fc2             x->p = pp, p->p = x;
1ea             p->update(), x->update();
007         }
6a0         node* splaya(node* x) {
a39             if (!x) return x;
be6             root = x, x->update();
3cf             while (x->p) {
d9b                 node *p = x->p, *pp = p->p;
359                 if (!pp) return rotate(x), x; // zig
e3c                 if ((pp->ch[0] == p)^(p->ch[0] == x))
a2b                     rotate(x), rotate(x); // zigzag
4b2                 else rotate(p), rotate(x); // zigzig
028             }
ea5             return x;
21a         }
a7f         node* find(int v) {
a2e             if (!root) return NULL;
52f             node *x = root;
6cd             int key = 0;
31e             while (1) {
857                 x->prop();
ba1                 bool d = key + size(x->ch[0]) < v;
877                 if (key + size(x->ch[0]) != v and x->ch[d]) {
15e                     if (d) key += size(x->ch[0])+1;
30e                     x = x->ch[d];
a30                 } else break;
3c3             }
152             return splaya(x);
f19         }
c0c         int size() { return root ? root->sz : 0; }
c26         void join(splay<T>& l) { // assume que l < *this
690             if (!size()) swap(root, l.root);
579             if (!size() or !l.size()) return;
```

```
bee             node* x = l.root;
31e             while (1) {
857                 x->prop();
34d                 if (!x->ch[1]) break;
bd8                 x = x->ch[1];
fa3             }
147             l.splaya(x), root->prop(), root->update();
42b             x->ch[1] = root, x->ch[1]->p = x;
0aa             root = l.root, l.root = NULL;
a0a             root->update();
7e6         }
5ed         node* split(int v) { // retorna os elementos < v
398             if (v <= 0) return NULL;
060             if (v >= size()) {
f87                 node* ret = root;
950                 root = NULL;
8c9                 ret->update();
edf                 return ret;
d0f             }
adc             find(v);
a59             node* l = root->ch[0];
4df             root->ch[0] = NULL;
5a3             if (l) l->p = NULL;
a0a             root->update();
792             return l;
826         }
511         T& operator [](int i) {
9d4             find(i);
ae0             return root->val;
829         }
231         void push_back(T v) { // O(1)
a01             node* r = new node(v);
0de             r->ch[0] = root;
b11             if (root) root->p = r;
b13             root = r, root->update();
315         }
b7a         T query(int l, int r) {
95f             splay<T> M(split(r+1));
5ff             splay<T> L(M.split(l));
d1c             T ans = M.root->sub;
49c             M.join(L), join(M);
ba7             return ans;
ca3         }
41f         void update(int l, int r, T s) {
95f             splay<T> M(split(r+1));
5ff             splay<T> L(M.split(l));
996             M.root->lazy += s;
```

```
49c         M.join(L), join(M);
9e9     }
8c1     void reverse(int l, int r) {
95f         splay<T> M(split(r+1));
5ff         splay<T> L(M.split(l));
945         M.root->rev ^= 1;
49c         M.join(L), join(M);
c1a     }
2fb     void erase(int l, int r) {
95f         splay<T> M(split(r+1));
5ff         splay<T> L(M.split(l));
dcc         join(L);
68e     }
a35 };
```

5.34 Split-Merge Set

```
// Representa um conjunto de inteiros nao negativos
// Todas as operacoes custam O(log(N)),
// em que N = maior elemento do set,
// exceto o merge, que custa O(log(N)) amortizado
// Usa O(min(N, n log(N))) de memoria, sendo 'n' o
// numero de elementos distintos no set

2dc template<typename T, bool MULTI=false, typename SIZE_T=int> struct
    sms {
3c9     struct node {
b19         node *l, *r;
15f         SIZE_T cnt;
658         node() : l(NULL), r(NULL), cnt(0) {}
01e         void update() {
a01             cnt = 0;
d8a             if (l) cnt += l->cnt;
e49             if (r) cnt += r->cnt;
74d         }
84f     };

bb7     node* root;
fd0     T N;

f34     sms() : root(NULL), N(0) {}
83b     sms(T v) : sms() { while (v >= N) N = 2*N+1; }
5e1     sms(const sms& t) : root(NULL), N(t.N) {
3af         for (SIZE_T i = 0; i < t.size(); i++) {
a0f             T at = t[i];
e6d             SIZE_T qt = t.count(at);
a43             insert(at, qt);
f42             i += qt-1;
1e9         }
ea8     }
a96     sms(initializer_list<T> v) : sms() { for (T i : v) insert(i); }
2dd     ~sms() {
609         vector<node*> q = {root};
402         while (q.size()) {
e5d             node* x = q.back(); q.pop_back();
ee9             if (!x) continue;
1c7             q.push_back(x->l), q.push_back(x->r);
bf0             delete x;
653         }
f0d     }

fdc     friend void swap(sms& a, sms& b) {
49e         swap(a.root, b.root), swap(a.N, b.N);
984     }
83e     sms& operator =(const sms& v) {
768         sms tmp = v;
420         swap(tmp, *this);
357         return *this;
e9b     }
d06     SIZE_T size() const { return root ? root->cnt : 0; }
17f     SIZE_T count(node* x) const { return x ? x->cnt : 0; }
75a     void clear() {
0a0         sms tmp;
4ac         swap(*this, tmp);
fcb     }
a06     void expand(T v) {
bc3         for (; N < v; N = 2*N+1) if (root) {
63c             node* nroot = new node();
956             nroot->l = root;
897             root = nroot;
a0a             root->update();
dd9         }
9f0     }

b14     node* insert(node* at, T idx, SIZE_T qt, T l, T r) {
1a4         if (!at) at = new node();
893         if (l == r) {
435             at->cnt += qt;
beb             if (!MULTI) at->cnt = 1;
ce6             return at;
a53         }
841         T m = l + (r-l)/2;
a02         if (idx <= m) at->l = insert(at->l, idx, qt, l, m);
```

```
8d9         else at->r = insert(at->r, idx, qt, m+1, r);
cff         return at->update(), at;
83b     }
cf7     void insert(T v, SIZE_T qt=1) { // insere 'qt' ocorrencias de 'v'
882         if (qt <= 0) return erase(v, -qt);
72b         assert(v >= 0);
f52         expand(v);
5e9         root = insert(root, v, qt, 0, N);
f62     }

f06     node* erase(node* at, T idx, SIZE_T qt, T l, T r) {
28c         if (!at) return at;
54b         if (l == r) at->cnt = at->cnt < qt ? 0 : at->cnt - qt;
4e6         else {
841             T m = l + (r-l)/2;
281             if (idx <= m) at->l = erase(at->l, idx, qt, l, m);
ba1             else at->r = erase(at->r, idx, qt, m+1, r);
7b4             at->update();
d3d         }
135         if (!at->cnt) delete at, at = NULL;
ce6         return at;
e1f     }
43d     void erase(T v, SIZE_T qt=1) { // remove 'qt' ocorrencias de 'v'
9c3         if (v < 0 or v > N or !qt) return;
9dc         if (qt < 0) insert(v, -qt);
b1d         root = erase(root, v, qt, 0, N);
b32     }
8d6     void erase_all(T v) { // remove todos os 'v'
347         if (v < 0 or v > N) return;
9f2         root = erase(root, v, numeric_limits<SIZE_T>::max(), 0, N);
569     }

0fe     SIZE_T count(node* at, T a, T b, T l, T r) const {
61b         if (!at or b < l or r < a) return 0;
0fe         if (a <= l and r <= b) return at->cnt;
841         T m = l + (r-l)/2;
84a         return count(at->l, a, b, l, m) + count(at->r, a, b, m+1, r);
4e6     }
0a9     SIZE_T count(T v) const { return count(root, v, v, 0, N); }
ffc     SIZE_T order_of_key(T v) { return count(root, 0, v-1, 0, N); }
df2     SIZE_T lower_bound(T v) { return order_of_key(v); }

e68     const T operator [](SIZE_T i) const { // i-esimo menor elemento
809         assert(i >= 0 and i < size());
c43         node* at = root;
4a5         T l = 0, r = N;
40c         while (l < r) {
841             T m = l + (r-l)/2;
5c2             if (count(at->l) > i) at = at->l, r = m;
4e6             else {
b4a                 i -= count(at->l);
ded                 at = at->r; l = m+1;
fa6             }
41a         }
792         return l;
67f     }

78c     node* merge(node* l, node* r) {
347         if (!l or !r) return l ? l : r;
504         if (!l->l and !l->r) { // folha
599             if (MULTI) l->cnt += r->cnt;
55d             delete r;
792             return l;
92c         }
f58         l->l = merge(l->l, r->l), l->r = merge(l->r, r->r);
f4f         l->update(), delete r;
792         return l;
06a     }
f59     void merge(sms& s) { // mergeia dois sets
068         if (N > s.N) swap(*this, s);
785         expand(s.N);
938         root = merge(root, s.root);
ee2         s.root = NULL;
2f6     }

dc6     node* split(node*& x, SIZE_T k) {
7ca         if (k <= 0 or !x) return NULL;
6d0         node* ret = new node();
386         if (!x->l and !x->r) x->cnt -= k, ret->cnt += k;
4e6         else {
85e             if (k <= count(x->l)) ret->l = split(x->l, k);
4e6             else {
06f                 ret->r = split(x->r, k - count(x->l));
cfd                 swap(x->l, ret->l);
63b             }
674             ret->update(), x->update();
379         }
d5b         if (!x->cnt) delete x, x = NULL;
edf         return ret;
f18     }
02b     void split(SIZE_T k, sms& s) { // pega os 'k' menores
```

```
e63         s.clear();
6e5         s.root = split(root, min(k, size()));
e3c         s.N = N;
9a6     }
        // pega os menores que 'k'
131     void split_val(T k, sms& s) { split(order_of_key(k), s); }
2d2 };
```

5.35 SQRT Tree

```
// RMQ em O(log log n) com O(n log log n) pra buildar
// Funciona com qualquer operacao associativa
// Tao rapido quanto a sparse table, mas usa menos memoria
// (log log (1e9) < 5, entao a query eh praticamente O(1))
//
// build - O(n log log n)
// query - O(log log n)

97a namespace sqrtTree {
052     int n, *v;
ec7     int pref[4][MAX], sulf[4][MAX], getl[4][MAX], entre[4][MAX],
    sz[4];

5f7     int op(int a, int b) { return min(a, b); }
c72     inline int getblk(int p, int i) { return (i-getl[p][i])/sz[p];
    }
2c6     void build(int p, int l, int r) {
bc8         if (l+1 >= r) return;
368         for (int i = l; i <= r; i++) getl[p][i] = l;
f16         for (int L = l; L <= r; L += sz[p]) {
191             int R = min(L+sz[p]-1, r);
89c             pref[p][L] = v[L], sulf[p][R] = v[R];
59f             for (int i = L+1; i <= R; i++) pref[p][i] =
    op(pref[p][i-1], v[i]);
d9a             for (int i = R-1; i >= L; i--) sulf[p][i] = op(v[i],
    sulf[p][i+1]);
221             build(p+1, L, R);
c7b         }
695         for (int i = 0; i <= sz[p]; i++) {
ca5             int at = entre[p][l+i*sz[p]+i] = sulf[p][l+i*sz[p]];
759             for (int j = i+1; j <= sz[p]; j++)
    entre[p][l+i*sz[p]+j] = at =
23a                     op(at, sulf[p][l+j*sz[p]]);
c51         }
861     }
0d8     void build(int n2, int* v2) {
680         n = n2, v = v2;
44c         for (int p = 0; p < 4; p++) sz[p] = n2 = sqrt(n2);
c50         build(0, 0, n-1);
940     }
9e3     int query(int l, int r) {
792         if (l+1 >= r) return l == r ? v[l] : op(v[l], v[r]);
1ba         int p = 0;
4ba         while (getblk(p, l) == getblk(p, r)) p++;
9e4         int ans = sulf[p][l], a = getblk(p, l)+1, b = getblk(p,
    r)-1;
8bf         if (a <= b) ans = op(ans, entre[p][getl[p][l]+a*sz[p]+b]);
dea         return op(ans, pref[p][r]);
589     }
8ff }
```

5.36 Treap

```
// Todas as operacoes custam
// O(log(n)) com alta probabilidade, exceto meld
// meld custa O(log^2 n) amortizado com alta prob.,
// e permite unir duas treaps sem restricao adicional
// Na pratica, esse meld tem constante muito boa e
// o pior caso eh meio estranho de acontecer

878 mt19937 rng((int)
    chrono::steady_clock::now().time_since_epoch().count());

aa1 template<typename T> struct treap {
3c9     struct node {
b19         node *l, *r;
284         int p, sz;
36d         T val, mi;
4c7         node(T v) : l(NULL), r(NULL), p(rng()), sz(1), val(v),
    mi(v) {}
01e         void update() {
a26             sz = 1;
d6e             mi = val;
bd7             if (l) sz += l->sz, mi = min(mi, l->mi);
a54             if (r) sz += r->sz, mi = min(mi, r->mi);
660         }
c1b     };

bb7     node* root;

84b     treap() { root = NULL; }
2d8     treap(const treap& t) {
```

```
465         throw logic_error("Nao copiar a treap!");
1e9     }
cec     ~treap() {
609         vector<node*> q = {root};
402         while (q.size()) {
e5d             node* x = q.back(); q.pop_back();
ee9             if (!x) continue;
1c7             q.push_back(x->l), q.push_back(x->r);
bf0             delete x;
653         }
50e     }

73c     int size(node* x) { return x ? x->sz : 0; }
b2b     int size() { return size(root); }
bcf     void join(node* l, node* r, node*& i) { // assume que l < r
986         if (!l or !r) return void(i = l ? l : r);
80e         if (l->p > r->p) join(l->r, r, l->r), i = l;
fa0         else join(l, r->l, r->l), i = r;
bda         i->update();
671     }
ece     void split(node* i, node*& l, node*& r, T v) {
26a         if (!i) return void(r = l = NULL);
f05         if (i->val < v) split(i->r, i->r, r, v), l = i;
807         else split(i->l, l, i->l, v), r = i;
bda         i->update();
2cd     }
3fc     void split_leq(node* i, node*& l, node*& r, T v) {
26a         if (!i) return void(r = l = NULL);
181         if (i->val <= v) split_leq(i->r, i->r, r, v), l = i;
58f         else split_leq(i->l, l, i->l, v), r = i;
bda         i->update();
70f     }
e13     int count(node* i, T v) {
6b4         if (!i) return 0;
352         if (i->val == v) return 1;
8d0         if (v < i->val) return count(i->l, v);
4d0         return count(i->r, v);
5e6     }
26d     void index_split(node* i, node*& l, node*& r, int v, int key =
    0) {
26a         if (!i) return void(r = l = NULL);
c10         if (key + size(i->l) < v) index_split(i->r, i->r, r, v,
    key+size(i->l)+1), l = i;
e5a         else index_split(i->l, l, i->l, v, key), r = i;
bda         i->update();
ccf     }
a1f     int count(T v) {
e06         return count(root, v);
980     }
c27     void insert(T v) {
980         if (count(v)) return;
031         node *L, *R;
d42         split(root, L, R, v);
585         node* at = new node(v);
59f         join(L, at, L);
a28         join(L, R, root);
37c     }
26b     void erase(T v) {
df9         node *L, *M, *R;
b6b         split_leq(root, M, R, v), split(M, L, M, v);
f17         if (M) delete M;
f38         M = NULL;
a28         join(L, R, root);
b92     }
e77     void meld(treap& t) { // segmented merge
4a6         node *L = root, *R = t.root;
950         root = NULL;
6b1         while (L or R) {
fe2             if (!L or (L and R and L->mi > R->mi)) std::swap(L, R);
5e1             if (!R) join(root, L, root), L = NULL;
3c9             else if (L->mi == R->mi) {
a76                 node* LL;
439                 split(L, LL, L, R->mi+1);
359                 delete LL;
2a3             } else {
a76                 node* LL;
537                 split(L, LL, L, R->mi);
dbb                 join(root, LL, root);
f4f             }
576         }
689         t.root = NULL;
8e7     }
651 };
```

5.37 Treap Implicita

```
// Todas as operacoes custam
// O(log(n)) com alta probabilidade

878 mt19937 rng((int)
    chrono::steady_clock::now().time_since_epoch().count());

aa1 template<typename T> struct treap {
```

```
3c9     struct node {
b19         node *l, *r;
284         int p, sz;
875         T val, sub, lazy;
aa6         bool rev;
8dc         node(T v) : l(NULL), r(NULL), p(rng()), sz(1), val(v),
    sub(v), lazy(0), rev(0) {}
a9c         void prop() {
0ec             if (lazy) {
924                 val += lazy, sub += lazy*sz;
b87                 if (l) l->lazy += lazy;
d3b                 if (r) r->lazy += lazy;
cea             }
1bb             if (rev) {
e4f                 swap(l, r);
dc8                 if (l) l->rev ^= 1;
f2f                 if (r) r->rev ^= 1;
3e5             }
a32             lazy = 0, rev = 0;
ca6         }
01e         void update() {
0c3             sz = 1, sub = val;
a09             if (l) l->prop(), sz += l->sz, sub += l->sub;
095             if (r) r->prop(), sz += r->sz, sub += r->sub;
360         }
d37     };

bb7     node* root;

84b     treap() { root = NULL; }
2d8     treap(const treap& t) {
465         throw logic_error("Nao copiar a treap!");
1e9     }
cec     ~treap() {
609         vector<node*> q = {root};
402         while (q.size()) {
e5d             node* x = q.back(); q.pop_back();
ee9             if (!x) continue;
1c7             q.push_back(x->l), q.push_back(x->r);
bf0             delete x;
653         }
50e     }

73c     int size(node* x) { return x ? x->sz : 0; }
b2b     int size() { return size(root); }
bcf     void join(node* l, node* r, node*& i) { // assume que l < r
986         if (!l or !r) return void(i = l ? l : r);
161         l->prop(), r->prop();
80e         if (l->p > r->p) join(l->r, r, l->r), i = l;
fa0         else join(l, r->l, r->l), i = r;
bda         i->update();
b57     }
a20     void split(node* i, node*& l, node*& r, int v, int key = 0) {
26a         if (!i) return void(r = l = NULL);
c89         i->prop();
5bd         if (key + size(i->l) < v) split(i->r, i->r, r, v,
    key+size(i->l)+1), l = i;
219         else split(i->l, l, i->l, v, key), r = i;
bda         i->update();
d37     }
231     void push_back(T v) {
2e0         node* i = new node(v);
7ab         join(root, i, root);
46b     }
b7a     T query(int l, int r) {
df9         node *L, *M, *R;
dca         split(root, M, R, r+1), split(M, L, M, l);
d43         T ans = M->sub;
69d         join(L, M, M), join(M, R, root);
ba7         return ans;
1f7     }
41f     void update(int l, int r, T s) {
df9         node *L, *M, *R;
dca         split(root, M, R, r+1), split(M, L, M, l);
8f6         M->lazy += s;
69d         join(L, M, M), join(M, R, root);
29f     }
8c1     void reverse(int l, int r) {
df9         node *L, *M, *R;
dca         split(root, M, R, r+1), split(M, L, M, l);
66a         M->rev ^= 1;
69d         join(L, M, M), join(M, R, root);
ea8     }
139 };
```

5.38 Treap Persistent Implicita

```
// Todas as operacoes custam
// O(log(n)) com alta probabilidade

6cf mt19937_64 rng((int)
    chrono::steady_clock::now().time_since_epoch().count());
```

```
3c9 struct node {
b19     node *l, *r;
f14     ll sz, val, sub;
304     node(ll v) : l(NULL), r(NULL), sz(1), val(v), sub(v) {}
c12     node(node* x) : l(x->l), r(x->r), sz(x->sz), val(x->val),
    sub(x->sub) {}
01e     void update() {
0c3         sz = 1, sub = val;
77e         if (l) sz += l->sz, sub += l->sub;
d6e         if (r) sz += r->sz, sub += r->sub;
124         sub %= MOD;
472     }
95f };

bc9 ll size(node* x) { return x ? x->sz : 0; }
761 void update(node* x) { if (x) x->update(); }
828 node* copy(node* x) { return x ? new node(x) : NULL; }

b02 node* join(node* l, node* r) {
e1f     if (!l or !r) return l ? copy(l) : copy(r);
48b     node* ret;
49f     if (rng() % (size(l) + size(r)) < size(l)) {
7eb         ret = copy(l);
cc1         ret->r = join(ret->r, r);
784     } else {
4c5         ret = copy(r);
551         ret->l = join(l, ret->l);
7a0     }
74f     return update(ret), ret;
2cc }

723 void split(node* x, node*& l, node*& r, ll v, ll key = 0) {
421     if (!x) return void(l = r = NULL);
b4b     if (key + size(x->l) < v) {
72f         l = copy(x);
d70         split(l->r, l->r, r, v, key+size(l->l)+1);
710     } else {
303         r = copy(x);
417         split(r->l, l, r->l, v, key);
3d1     }
da2     update(l), update(r);
666 }

f9e vector<node*> treap;

139 void init(const vector<ll>& v) {
bbd     treap = {NULL};
969     for (auto i : v) treap[0] = join(treap[0], new node(i));
286 }
```

5.39 Wavelet Tree

```
// Usa O(sigma + n log(sigma)) de memoria,
// onde sigma = MAXN - MINN
// Depois do build, o v fica ordenado
// count(i, j, x, y) retorna o numero de elementos de
// v[i, j) que pertencem a [x, y]
// kth(i, j, k) retorna o elemento que estaria
// na poscicao k-1 de v[i, j), se ele fosse ordenado
// sum(i, j, x, y) retorna a soma dos elementos de
// v[i, j) que pertencem a [x, y]
// sumk(i, j, k) retorna a soma dos k-esimos menores
// elementos de v[i, j) (sum(i, j, 1) retorna o menor)
//
// Complexidades:
// build - O(n log(sigma))
// count - O(log(sigma))
// kth   - O(log(sigma))
// sum   - O(log(sigma))
// sumk  - O(log(sigma))

597 int n, v[MAX];
578 vector<int> esq[4*(MAXN-MINN)], pref[4*(MAXN-MINN)];

f8d void build(int b = 0, int e = n, int p = 1, int l = MINN, int r =
    MAXN) {
58f     int m = (l+r)/2; esq[p].push_back(0); pref[p].push_back(0);
f2f     for (int i = b; i < e; i++) {
6b9         esq[p].push_back(esq[p].back()+(v[i]<=m));
26f         pref[p].push_back(pref[p].back()+v[i]);
206     }
8ce     if (l == r) return;
3a7     int m2 = stable_partition(v+b, v+e, [=](int i){return i <=
    m;}) - v;
347     build(b, m2, 2*p, l, m), build(m2, e, 2*p+1, m+1, r);
0fb }

540 int count(int i, int j, int x, int y, int p = 1, int l = MINN, int
    r = MAXN) {
2ad     if (y < l or r < x) return 0;
4db     if (x <= l and r <= y) return j-i;
ddc     int m = (l+r)/2, ei = esq[p][i], ej = esq[p][j];
0a5     return count(ei, ej, x, y, 2*p, l, m)+count(i-ei, j-ej, x, y,
    2*p+1, m+1, r);
3cf }

f62 int kth(int i, int j, int k, int p=1, int l = MINN, int r = MAXN) {
3ce     if (l == r) return l;
ddc     int m = (l+r)/2, ei = esq[p][i], ej = esq[p][j];
585     if (k <= ej-ei) return kth(ei, ej, k, 2*p, l, m);
28b     return kth(i-ei, j-ej, k-(ej-ei), 2*p+1, m+1, r);
8b6 }

f2c int sum(int i, int j, int x, int y, int p = 1, int l = MINN, int r
    = MAXN) {
2ad     if (y < l or r < x) return 0;
2a9     if (x <= l and r <= y) return pref[p][j]-pref[p][i];
ddc     int m = (l+r)/2, ei = esq[p][i], ej = esq[p][j];
43b     return sum(ei, ej, x, y, 2*p, l, m) + sum(i-ei, j-ej, x, y,
    2*p+1, m+1, r);
b6d }

b84 int sumk(int i, int j, int k, int p = 1, int l = MINN, int r =
    MAXN) {
8a1     if (l == r) return l*k;
ddc     int m = (l+r)/2, ei = esq[p][i], ej = esq[p][j];
50c     if (k <= ej-ei) return sumk(ei, ej, k, 2*p, l, m);
4c9     return pref[2*p][ej]-pref[2*p][ei]+sumk(i-ei, j-ej, k-(ej-ei),
    2*p+1, m+1, r);
940 }
```

6 Grafos

6.1 AGM Direcionada

```
// Fala o menor custo para selecionar arestas tal que
// o vertice 'r' alcance todos
// Se nao tem como, retorna LINF
//
// O(m log(n))

3c9 struct node {
f31     pair<ll, int> val;
4e4     ll lazy;
b19     node *l, *r;
f93     node() {}
c53     node(pair<int, int> v) : val(v), lazy(0), l(NULL), r(NULL) {}

a9c     void prop() {
768         val.first += lazy;
b87         if (l) l->lazy += lazy;
d3b         if (r) r->lazy += lazy;
c60         lazy = 0;
05b     }
296 };
de5 void merge(node*& a, node* b) {
c11     if (!a) swap(a, b);
802     if (!b) return;
626     a->prop(), b->prop();
d04     if (a->val > b->val) swap(a, b);
4b0     merge(rand()%2 ? a->l : a->r, b);
b82 }
d01 pair<ll, int> pop(node*& R) {
e8f     R->prop();
22e     auto ret = R->val;
af0     node* tmp = R;
3f3     merge(R->l, R->r);
6c9     R = R->l;
3e4     if (R) R->lazy -= ret.first;
7c3     delete tmp;
edf     return ret;
c4e }
6f6 void apaga(node* R) { if (R) apaga(R->l), apaga(R->r), delete R; }

f13 ll dmst(int n, int r, vector<pair<pair<int, int>, int>>& ar) {
94e     vector<int> p(n); iota(p.begin(), p.end(), 0);
a23     function<int(int)> find = [&](int k) { return
    p[k]==k?k:p[k]=find(p[k]); };
2d7     vector<node*> h(n);
56f     for (auto e : ar) merge(h[e.first.second], new node({e.second,
    e.first.first}));
fd1     vector<int> pai(n, -1), path(n);
66e     pai[r] = r;
04b     ll ans = 0;

603     for (int i = 0; i < n; i++) { // vai conectando todo mundo
2a3         int u = i, at = 0;
cae         while (pai[u] == -1) {
daa             if (!h[u]) { // nao tem
947                 for (auto i : h) apaga(i);
77c                 return LINF;
dd1             }
167             path[at++] = u, pai[u] = i;
55e             auto [mi, v] = pop(h[u]);
64c             ans += mi;

5e2             if (pai[u = find(v)] == i) { // ciclo
86f                 while (find(v = path[--at]) != u)
621                     merge(h[u], h[v]), h[v] = NULL, p[find(v)] = u;
57a                 pai[u] = -1;
0d8             }
ce8         }
5df     }
947     for (auto i : h) apaga(i);
ba7     return ans;
e02 }
```

6.2 Articulation Points

```
// Computa os pontos de articulacao (vertices criticos) de um grafo
//
// art[i] armazena o numero de novas componentes criadas ao deletar
    vertice i
// se art[i] >= 1, entao vertice i eh ponto de articulacao
//
// O(n+m)

1a8 int n;
789 vector<vector<int>> g;
4ce stack<int> s;
b66 vector<int> id, art;

3e1 int dfs_art(int i, int& t, int p = -1) {
cf0     int lo = id[i] = t++;
18e     s.push(i);
cac     for (int j : g[i]) if (j != p) {
9a3         if (id[j] == -1) {
206             int val = dfs_art(j, t, i);
0c3             lo = min(lo, val);

588             if (val >= id[i]) {
66a                 art[i]++;
bd9                 while (s.top() != j) s.pop();
2eb                 s.pop();
1f3             }
                // if (val > id[i]) aresta i-j eh ponte
238         }
328         else lo = min(lo, id[j]);
762     }
3bd     if (p == -1 and art[i]) art[i]--;
253     return lo;
8e1 }

d79 void compute_art_points() {
597     id = vector<int>(n, -1);
a62     art = vector<int>(n, 0);
6bb     int t = 0;
d41     for (int i = 0; i < n; i++) if (id[i] == -1)
625         dfs_art(i, t, -1);
379 }
```

6.3 Bellman-Ford

```
// Calcula a menor distancia
// entre a e todos os vertices e
// detecta ciclo negativo
// Retorna 1 se ha ciclo negativo
// Nao precisa representar o grafo,
// soh armazenar as arestas
//
// O(nm)

14e int n, m;
248 int d[MAX];
e93 vector<pair<int, int>> ar; // vetor de arestas
9e2 vector<int> w;              // peso das arestas

6be bool bellman_ford(int a) {
8ec     for (int i = 0; i < n; i++) d[i] = INF;
8a8     d[a] = 0;

4e3     for (int i = 0; i <= n; i++)
891         for (int j = 0; j < m; j++) {
6e4             if (d[ar[j].second] > d[ar[j].first] + w[j]) {
705                 if (i == n) return 1;

e93                 d[ar[j].second] = d[ar[j].first] + w[j];
84b             }
a82         }

bb3     return 0;
6eb }
```

6.4 Block-Cut Tree

```
// Cria a block-cut tree, uma arvore com os blocos
// e os pontos de articulacao
// Blocos sao componentes 2-vertice-conexos maximais
// Uma 2-coloracao da arvore eh tal que uma cor sao
// os blocos, e a outra cor sao os pontos de art.
// Funciona para grafo nao conexo
//
// art[i] responde o numero de novas componentes conexas
// criadas apos a remocao de i do grafo g
// Se art[i] >= 1, i eh ponto de articulacao
//
// Para todo i < blocks.size()
// blocks[i] eh uma componente 2-vertice-conexa maximal
// edgblocks[i] sao as arestas do bloco i
// tree[i] eh um vertice da arvore que corresponde ao bloco i
//
// pos[i] responde a qual vertice da arvore vertice i pertence
// Arvore tem no maximo 2n vertices
//
// O(n+m)

d10 struct block_cut_tree {
d8e     vector<vector<int>> g, blocks, tree;
43b     vector<vector<pair<int, int>>> edgblocks;
4ce     stack<int> s;
6c0     stack<pair<int, int>> s2;
2bb     vector<int> id, art, pos;

763     block_cut_tree(vector<vector<int>> g_) : g(g_) {
af1         int n = g.size();
37a         id.resize(n, -1), art.resize(n), pos.resize(n);
6f2         build();
6bd     }

df6     int dfs(int i, int& t, int p = -1) {
cf0         int lo = id[i] = t++;
18e         s.push(i);

827         if (p != -1) s2.emplace(i, p);
53f         for (int j : g[i]) if (j != p and id[j] != -1)
    s2.emplace(i, j);

cac         for (int j : g[i]) if (j != p) {
9a3             if (id[j] == -1) {
121                 int val = dfs(j, t, i);
0c3                 lo = min(lo, val);

588                 if (val >= id[i]) {
66a                     art[i]++;
483                     blocks.emplace_back(1, i);
110                     while (blocks.back().back() != j)
138                         blocks.back().push_back(s.top()), s.pop();

128                     edgblocks.emplace_back(1, s2.top()), s2.pop();
47e                     while (edgblocks.back().back() != pair(j, i))
bce                         edgblocks.back().push_back(s2.top()),
    s2.pop();
870                 }
                    // if (val > id[i]) aresta i-j eh ponte
85c             }
328             else lo = min(lo, id[j]);
344         }

3bd         if (p == -1 and art[i]) art[i]--;
253         return lo;
726     }

0a8     void build() {
6bb         int t = 0;
abf         for (int i = 0; i < g.size(); i++) if (id[i] == -1) dfs(i,
    t, -1);

56c         tree.resize(blocks.size());
f7d         for (int i = 0; i < g.size(); i++) if (art[i])
965             pos[i] = tree.size(), tree.emplace_back();

973         for (int i = 0; i < blocks.size(); i++) for (int j :
    blocks[i]) {
403             if (!art[j]) pos[j] = i;
101             else tree[i].push_back(pos[j]),
    tree[pos[j]].push_back(i);
3df         }
c03     }
056 };
```

6.5 Blossom

```
// Matching maximo em grafo geral
//
// O(n^3)
// Se for bipartido, nao precisa da funcao
// 'contract', e roda em O(nm)

042 vector<int> g[MAX];
128 int match[MAX]; // match[i] = com quem i esta matchzado ou -1
1f1 int n, pai[MAX], base[MAX], vis[MAX];
26a queue<int> q;

107 void contract(int u, int v, bool first = 1) {
165     static vector<bool> bloss;
fbe     static int l;
418     if (first) {
a47         bloss = vector<bool>(n, 0);
042         vector<bool> teve(n, 0);
ddf         int k = u; l = v;
31e         while (1) {
297             teve[k = base[k]] = 1;
116             if (match[k] == -1) break;
dfa             k = pai[match[k]];
68b         }
d31         while (!teve[l = base[l]]) l = pai[match[l]];
5d6     }
2e9     while (base[u] != l) {
e29         bloss[base[u]] = bloss[base[match[u]]] = 1;
8fa         pai[u] = v;
0b0         v = match[u];
a51         u = pai[match[u]];
58e     }
71c     if (!first) return;
95e     contract(v, u, 0);
6ee     for (int i = 0; i < n; i++) if (bloss[base[i]]) {
594         base[i] = l;
ca7         if (!vis[i]) q.push(i);
29a         vis[i] = 1;
857     }
e35 }

f10 int getpath(int s) {
88f     for (int i = 0; i < n; i++) base[i] = i, pai[i] = -1, vis[i] =
    0;
ded     vis[s] = 1; q = queue<int>(); q.push(s);
402     while (q.size()) {
be1         int u = q.front(); q.pop();
bdc         for (int i : g[u]) {
7a2             if (base[i] == base[u] or match[u] == i) continue;
e35             if (i == s or (match[i] != -1 and pai[match[i]] != -1))
4f2                 contract(u, i);
e2e             else if (pai[i] == -1) {
545                 pai[i] = u;
f6a                 if (match[i] == -1) return i;
818                 i = match[i];
29d                 vis[i] = 1; q.push(i);
90e             }
0b5         }
634     }
daa     return -1;
a16 }

83f int blossom() {
1a4     int ans = 0;
315     memset(match, -1, sizeof(match));
2e3     for (int i = 0; i < n; i++) if (match[i] == -1)
f76         for (int j : g[i]) if (match[j] == -1) {
1bc             match[i] = j;
f1d             match[j] = i;
0df             ans++;
c2b             break;
723         }
da8     for (int i = 0; i < n; i++) if (match[i] == -1) {
7e3         int j = getpath(i);
5f2         if (j == -1) continue;
0df         ans++;
3a0         while (j != -1) {
ef0             int p = pai[j], pp = match[p];
348             match[p] = j;
fe9             match[j] = p;
55d             j = pp;
797         }
f70     }
ba7     return ans;
fcd }
```

6.6 Centro de arvore

```
// Retorna o diametro e o(s) centro(s) da arvore
// Uma arvore tem sempre um ou dois centros e estes estao no meio do
   diametro
//
// O(n)

042 vector<int> g[MAX];
df1 int d[MAX], par[MAX];

544 pair<int, vector<int>> center() {
```

```
a95     int f, df;
36d     function<void(int)> dfs = [&] (int v) {
d47         if (d[v] > df) f = v, df = d[v];
e68         for (int u : g[v]) if (u != par[v])
1a5             d[u] = d[v] + 1, par[u] = v, dfs(u);
90d     };

1b0     f = df = par[0] = -1, d[0] = 0;
41e     dfs(0);
c2d     int root = f;
0f6     f = df = par[root] = -1, d[root] = 0;
14e     dfs(root);

761     vector<int> c;
87e     while (f != -1) {
999         if (d[f] == df/2 or d[f] == (df+1)/2) c.push_back(f);
19c         f = par[f];
3bf     }

00f     return {df, c};
9c7 }
```

6.7 Centroid

```
// Computa os 2 centroids da arvore
//
// O(n)

97a int n, subsize[MAX];
042 vector<int> g[MAX];

98f void dfs(int k, int p=-1) {
bd2     subsize[k] = 1;
6e5     for (int i : g[k]) if (i != p) {
801         dfs(i, k);
2e3         subsize[k] += subsize[i];
1b2     }
5a5 }

2e8 int centroid(int k, int p=-1, int size=-1) {
e73     if (size == -1) size = subsize[k];
8df     for (int i : g[k]) if (i != p) if (subsize[i] > size/2)
bab         return centroid(i, k, size);
839     return k;
b6a }

f20 pair<int, int> centroids(int k=0) {
051     dfs(k);
909     int i = centroid(k), i2 = i;
8dd     for (int j : g[i]) if (2*subsize[j] == subsize[k]) i2 = j;
0cb     return {i, i2};
cf4 }
```

6.8 Centroid decomposition

```
// decomp(0, k) computa numero de caminhos com 'k' arestas
// Mudar depois do comentario
//
// O(n log(n))

042 vector<int> g[MAX];
ba8 int sz[MAX], rem[MAX];

747 void dfs(vector<int>& path, int i, int l=-1, int d=0) {
547     path.push_back(d);
75f     for (int j : g[i]) if (j != l and !rem[j]) dfs(path, j, i,
    d+1);
3e9 }

071 int dfs_sz(int i, int l=-1) {
02c     sz[i] = 1;
e5c     for (int j : g[i]) if (j != l and !rem[j]) sz[i] += dfs_sz(j,
    i);
191     return sz[i];
86b }

85a int centroid(int i, int l, int size) {
994     for (int j : g[i]) if (j != l and !rem[j] and sz[j] > size / 2)
735         return centroid(j, i, size);
d9a     return i;
96e }

d79 ll decomp(int i, int k) {
106     int c = centroid(i, i, dfs_sz(i));
a67     rem[c] = 1;

        // gasta O(n) aqui - dfs sem ir pros caras removidos
04b     ll ans = 0;
020     vector<int> cnt(sz[i]);
878     cnt[0] = 1;
0a8     for (int j : g[c]) if (!rem[j]) {
5b4         vector<int> path;
baf         dfs(path, j);
1a1         for (int d : path) if (0 <= k-d-1 and k-d-1 < sz[i])
285             ans += cnt[k-d-1];
e8b         for (int d : path) cnt[d+1]++;
fa2     }

1c1     for (int j : g[c]) if (!rem[j]) ans += decomp(j, k);
3f1     rem[c] = 0;
ba7     return ans;
193 }
```

6.9 Centroid Tree

```
// Constroi a centroid tree
// p[i] eh o pai de i na centroid-tree
// dist[i][k] = distancia na arvore original entre i
// e o k-esimo ancestral na arvore da centroid
//
// O(n log(n)) de tempo e memoria

845 vector<int> g[MAX], dist[MAX];
c1e int sz[MAX], rem[MAX], p[MAX];

071 int dfs_sz(int i, int l=-1) {
02c     sz[i] = 1;
e5c     for (int j : g[i]) if (j != l and !rem[j]) sz[i] += dfs_sz(j,
    i);
191     return sz[i];
86b }

85a int centroid(int i, int l, int size) {
994     for (int j : g[i]) if (j != l and !rem[j] and sz[j] > size / 2)
735         return centroid(j, i, size);
d9a     return i;
96e }

324 void dfs_dist(int i, int l, int d=0) {
541     dist[i].push_back(d);
5a1     for (int j : g[i]) if (j != l and !rem[j])
82a         dfs_dist(j, i, d+1);
645 }

27e void decomp(int i, int l = -1) {
106     int c = centroid(i, i, dfs_sz(i));
1b9     rem[c] = 1, p[c] = l;
534     dfs_dist(c, c);
a2a     for (int j : g[c]) if (!rem[j]) decomp(j, c);
ebd }

76c void build(int n) {
235     for (int i = 0; i < n; i++) rem[i] = 0, dist[i].clear();
867     decomp(0);
96b     for (int i = 0; i < n; i++) reverse(dist[i].begin(),
    dist[i].end());
a78 }
```

6.10 Dijkstra

```
// encontra menor distancia de x
// para todos os vertices
// se ao final do algoritmo d[i] = LINF,
// entao x nao alcanca i
//
// O(m log(n))

eff ll d[MAX];
c0d vector<pair<int, int>> g[MAX]; // {vizinho, peso}

1a8 int n;

abc void dijkstra(int v) {
22c     for (int i = 0; i < n; i++) d[i] = LINF;
a7f     d[v] = 0;
88c     priority_queue<pair<ll, int>> pq;
b32     pq.emplace(0, v);

265     while (pq.size()) {
a25         auto [ndist, u] = pq.top(); pq.pop();
953         if (-ndist > d[u]) continue;

cda         for (auto [idx, w] : g[u]) if (d[idx] > d[u] + w) {
331             d[idx] = d[u] + w;
a84             pq.emplace(-d[idx], idx);
c56         }
e5c     }
fec }
```

6.11 Dinitz

```
// O(min(m * max_flow, n^2 m))
// Grafo com capacidades 1: O(min(m sqrt(m), m * n^(2/3)))
// Todo vertice tem grau de entrada ou saida 1: O(m sqrt(n))

472 struct dinitz {
61f     const bool scaling = false; // com scaling -> O(nm
    log(MAXCAP)),
206     int lim;                     // com constante alta
670     struct edge {
358         int to, cap, rev, flow;
7f9         bool res;
d36         edge(int to_, int cap_, int rev_, bool res_)
a94             : to(to_), cap(cap_), rev(rev_), flow(0), res(res_) {}
f70     };

002     vector<vector<edge>> g;
216     vector<int> lev, beg;
a71     ll F;
190     dinitz(int n) : g(n), F(0) {}

087     void add(int a, int b, int c) {
bae         g[a].emplace_back(b, c, g[b].size(), false);
4c6         g[b].emplace_back(a, 0, g[a].size()-1, true);
5c2     }
123     bool bfs(int s, int t) {
90f         lev = vector<int>(g.size(), -1); lev[s] = 0;
64c         beg = vector<int>(g.size(), 0);
8b2         queue<int> q; q.push(s);
402         while (q.size()) {
be1             int u = q.front(); q.pop();
bd9             for (auto& i : g[u]) {
dbc                 if (lev[i.to] != -1 or (i.flow == i.cap)) continue;
b4f                 if (scaling and i.cap - i.flow < lim) continue;
185                 lev[i.to] = lev[u] + 1;
8ca                 q.push(i.to);
f97             }
e87         }
0de         return lev[t] != -1;
742     }
dfb     int dfs(int v, int s, int f = INF) {
50b         if (!f or v == s) return f;
88f         for (int& i = beg[v]; i < g[v].size(); i++) {
027             auto& e = g[v][i];
206             if (lev[e.to] != lev[v] + 1) continue;
ee0             int foi = dfs(e.to, s, min(f, e.cap - e.flow));
749             if (!foi) continue;
3c5             e.flow += foi, g[e.to][e.rev].flow -= foi;
45c                 return foi;
618         }
bb3         return 0;
4b1     }
ff6     ll max_flow(int s, int t) {
a86         for (lim = scaling ? (1<<30) : 1; lim; lim /= 2)
9d1             while (bfs(s, t)) while (int ff = dfs(s, t)) F += ff;
4ff         return F;
8b9     }
86f };

// Recupera as arestas do corte s-t
dbd vector<pair<int, int>> get_cut(dinitz& g, int s, int t) {
f07     g.max_flow(s, t);
68c     vector<pair<int, int>> cut;
1b0     vector<int> vis(g.g.size(), 0), st = {s};
321     vis[s] = 1;
3c6     while (st.size()) {
b17         int u = st.back(); st.pop_back();
322         for (auto e : g.g[u]) if (!vis[e.to] and e.flow < e.cap)
c17             vis[e.to] = 1, st.push_back(e.to);
d14     }
481     for (int i = 0; i < g.g.size(); i++) for (auto e : g.g[i])
9d2         if (vis[i] and !vis[e.to] and !e.res) cut.emplace_back(i,
    e.to);
d1b     return cut;
1e8 }
```

6.12 Dominator Tree

```
// Codigo do Kawakami. Se vira pra usar ai
//
// build - O(m log(n))
// dominates - O(1)

1a8 int n;

bbf namespace d_tree {
042     vector<int> g[MAX];

        // The dominator tree
b39     vector<int> tree[MAX];
5af     int dfs_l[MAX], dfs_r[MAX];

        // Auxiliary data
a2e     vector<int> rg[MAX], bucket[MAX];
3ef     int idom[MAX], sdom[MAX], prv[MAX], pre[MAX];
44b     int ancestor[MAX], label[MAX];
563     vector<int> preorder;

76a     void dfs(int v) {
6a1         static int t = 0;
db6         pre[v] = ++t;
767         sdom[v] = label[v] = v;
a3d         preorder.push_back(v);
d08         for (int nxt: g[v]) {
56c             if (sdom[nxt] == -1) {
eed                 prv[nxt] = v;
900                 dfs(nxt);
f48             }
2b5             rg[nxt].push_back(v);
5a1         }
d6a     }
62e     int eval(int v) {
c93         if (ancestor[v] == -1) return v;
a75         if (ancestor[ancestor[v]] == -1) return label[v];
f33         int u = eval(ancestor[v]);
b49         if (pre[sdom[u]] < pre[sdom[label[v]]]) label[v] = u;
66e         ancestor[v] = ancestor[u];
c24         return label[v];
0b9     }
4b2     void dfs2(int v) {
6a1         static int t = 0;
330         dfs_l[v] = t++;
5e0         for (int nxt: tree[v]) dfs2(nxt);
8e2         dfs_r[v] = t++;
cfa     }
c2c     void build(int s) {
603         for (int i = 0; i < n; i++) {
e6f             sdom[i] = pre[i] = ancestor[i] = -1;
2e1             rg[i].clear();
50a             tree[i].clear();
666             bucket[i].clear();
3ba         }
772         preorder.clear();
c6c         dfs(s);
12b         if (preorder.size() == 1) return;
3c7         for (int i = int(preorder.size()) - 1; i >= 1; i--) {
6c6             int w = preorder[i];
a52             for (int v: rg[w]) {
5c1                 int u = eval(v);
a17                 if (pre[sdom[u]] < pre[sdom[w]]) sdom[w] = sdom[u];
018             }
680             bucket[sdom[w]].push_back(w);
ea7             ancestor[w] = prv[w];
b99             for (int v: bucket[prv[w]]) {
5c1                 int u = eval(v);
977                 idom[v] = (u == v) ? sdom[v] : u;
aff             }
2cc             bucket[prv[w]].clear();
0a3         }
d0c         for (int i = 1; i < preorder.size(); i++) {
6c6             int w = preorder[i];
14b             if (idom[w] != sdom[w]) idom[w] = idom[idom[w]];
32f             tree[idom[w]].push_back(w);
c58         }
8ac         idom[s] = sdom[s] = -1;
1b6         dfs2(s);
d09     }

        // Whether every path from s to v passes through u
490     bool dominates(int u, int v) {
c75         if (pre[v] == -1) return 1; // vacuously true
2ea         return dfs_l[u] <= dfs_l[v] && dfs_r[v] <= dfs_r[u];
332     }
ce9 };
```

6.13 Euler Path / Euler Cycle

```
// Para declarar: 'euler<true> E(n);' se quiser
// direcionado e com 'n' vertices
// As funcoes retornam um par com um booleano
// indicando se possui o cycle/path que voce pediu,
// e um vector de {vertice, id da aresta para chegar no vertice}
// Se for get_path, na primeira posicao o id vai ser -1
// get_path(src) tenta achar um caminho ou ciclo euleriano
// comecando no vertice 'src'.
// Se achar um ciclo, o primeiro e ultimo vertice serao 'src'.
// Se for um P3, um possiveo retorno seria [0, 1, 2, 0]
// get_cycle() acha um ciclo euleriano se o grafo for euleriano.
// Se for um P3, um possivel retorno seria [0, 1, 2]
// (vertie inicial nao repete)
//
// O(n+m)

63f template<bool directed=false> struct euler {
1a8     int n;
4c0     vector<vector<pair<int, int>>> g;
d63     vector<int> used;
30f     euler(int n_) : n(n_), g(n) {}
50f     void add(int a, int b) {
4cd         int at = used.size();
c51         used.push_back(0);
74e         g[a].emplace_back(b, at);
fab         if (!directed) g[b].emplace_back(a, at);
411     }
d41 #warning chamar para o src certo!
eed     pair<bool, vector<pair<int, int>>> get_path(int src) {
baf         if (!used.size()) return {true, {}};
b25         vector<int> beg(n, 0);
4ec         for (int& i : used) i = 0;
            // {{vertice, anterior}, label}
363         vector<pair<pair<int, int>, int>> ret, st = {{{src, -1},
    -1}};
3c6         while (st.size()) {
8ff             int at = st.back().first.first;
002             int& it = beg[at];
8a1             while (it < g[at].size() and used[g[at][it].second])
    it++;
8e4             if (it == g[at].size()) {
9dd                 if (ret.size() and ret.back().first.second != at)
b82                     return {false, {}};
420                 ret.push_back(st.back()), st.pop_back();
2c0             } else {
daa                 st.push_back({{g[at][it].first, at},
    g[at][it].second});
eb8                 used[g[at][it].second] = 1;
396             }
b3a         }
a19         if (ret.size() != used.size()+1) return {false, {}};
f77         vector<pair<int, int>> ans;
fdf         for (auto i : ret) ans.emplace_back(i.first.first,
    i.second);
459         reverse(ans.begin(), ans.end());
997         return {true, ans};
844     }
9b6     pair<bool, vector<pair<int, int>>> get_cycle() {
baf         if (!used.size()) return {true, {}};
ad1         int src = 0;
34b         while (!g[src].size()) src++;
687         auto ans = get_path(src);
33c         if (!ans.first or ans.second[0].first !=
    ans.second.back().first)
b82             return {false, {}};
350         ans.second[0].second = ans.second.back().second;
8b8         ans.second.pop_back();
ba7         return ans;
48f     }
711 };
```

6.14 Floyd-Warshall

```
// encontra o menor caminho entre todo
// par de vertices e detecta ciclo negativo
// returna 1 sse ha ciclo negativo
// d[i][i] deve ser 0
// para i != j, d[i][j] deve ser w se ha uma aresta
// (i, j) de peso w, INF caso contrario
//
// O(n^3)

1a8 int n;
ae5 int d[MAX][MAX];

73c bool floyd_warshall() {
e22     for (int k = 0; k < n; k++)
830     for (int i = 0; i < n; i++)
f90     for (int j = 0; j < n; j++)
0ab         d[i][j] = min(d[i][j], d[i][k] + d[k][j]);

830     for (int i = 0; i < n; i++)
753         if (d[i][i] < 0) return 1;

bb3     return 0;
192 }
```

6.15 Functional Graph

```
// rt[i] fala o ID da raiz associada ao vertice i
// d[i] fala a profundidade (0 sse ta no ciclo)
// pos[i] fala a posicao de i no array que eh a concat. dos ciclos
// build(f, val) recebe a funcao f e o custo de ir de
// i para f[i] (por default, val = f)
// f_k(i, k) fala onde i vai parar se seguir k arestas
// path(i, k) fala o custo (soma) seguir k arestas a partir de i
// Se quiser outra operacao, da pra alterar facil o codigo
// Codigo um pouco louco, tenho que admitir
//
// build - O(n)
// f_k   - O(log(min(n, k)))
// path  - O(log(min(n, k)))

6ef namespace func_graph {
1a8     int n;
ce2     int f[MAX], vis[MAX], d[MAX];
f82     int p[MAX], pp[MAX], rt[MAX], pos[MAX];
ebd     int sz[MAX], comp;
6a9     vector<vector<int>> ciclo;
405     ll val[MAX], jmp[MAX], seg[2*MAX];

97c     ll op(ll a, ll b) { return a+b; }; // mudar a operacao aqui
27b     void dfs(int i, int t = 2) {
9c9         vis[i] = t;
f09         if (vis[f[i]] >= 2) { // comeca ciclo - f[i] eh o rep.
e0a             d[i] = 0, rt[i] = comp;
74c             sz[comp] = t - vis[f[i]] + 1;
97b             p[i] = pp[i] = i, jmp[i] = val[i];
15c             ciclo.emplace_back();
bfb             ciclo.back().push_back(i);
a22         } else {
c16             if (!vis[f[i]]) dfs(f[i], t+1);
8c0             rt[i] = rt[f[i]];
195             if (sz[comp]+1) { // to no ciclo
d0f                 d[i] = 0;
97b                 p[i] = pp[i] = i, jmp[i] = val[i];
bfb                 ciclo.back().push_back(i);
c20             } else { // nao to no ciclo
00d                 d[i] = d[f[i]]+1, p[i] = f[i];
511                 pp[i] = 2*d[pp[f[i]]] == d[pp[pp[f[i]]]]+d[f[i]] ?
    pp[pp[f[i]]] : f[i];
114                 jmp[i] = pp[i] == f[i] ? val[i] : op(val[i],
    op(jmp[f[i]], jmp[pp[f[i]]]));
db8             }
003         }
e4a         if (f[ciclo[rt[i]][0]] == i) comp++; // fim do ciclo
29a         vis[i] = 1;
0ba     }
1da     void build(vector<int> f_, vector<int> val_ = {}) {
bcb         n = f_.size(), comp = 0;
527         if (!val_.size()) val_ = f_;
830         for (int i = 0; i < n; i++)
998             f[i] = f_[i], val[i] = val_[i], vis[i] = 0, sz[i] = -1;

e74         ciclo.clear();
158         for (int i = 0; i < n; i++) if (!vis[i]) dfs(i);
6bb         int t = 0;
daa         for (auto& c : ciclo) {
336             reverse(c.begin(), c.end());
ea5             for (int j : c) {
85b                 pos[j] = t;
948                 seg[n+t] = val[j];
c82                 t++;
25e             }
cbc         }
dc1         for (int i = n-1; i; i--) seg[i] = op(seg[2*i],
    seg[2*i+1]);
90b     }

283     int f_k(int i, ll k) {
1b1         while (d[i] and k) {
77b             int big = d[i] - d[pp[i]];
ded             if (big <= k) k -= big, i = pp[i];
584             else k--, i = p[i];
09c         }
77e         if (!k) return i;
a19         return ciclo[rt[i]][(pos[i] - pos[ciclo[rt[i]][0]] + k) %
    sz[rt[i]]];
f34     }
047     ll path(int i, ll k) {
3cf         auto query = [&](int l, int r) {
3e4             ll q = 0;
47a             for (l += n, r += n; l <= r; ++l/=2, --r/=2) {
27e                 if (l%2 == 1) q = op(q, seg[l]);
1f2                 if (r%2 == 0) q = op(q, seg[r]);
598             }
bef             return q;
6e1         };
b73         ll ret = 0;
1b1         while (d[i] and k) {
77b             int big = d[i] - d[pp[i]];
327             if (big <= k) k -= big, ret = op(ret, jmp[i]), i =
    pp[i];
f9e             else k--, ret = op(ret, val[i]), i = p[i];
7e3         }
e3c         if (!k) return ret;
a9e         int first = pos[ciclo[rt[i]][0]], last =
    pos[ciclo[rt[i]].back()];

            // k/sz[rt[i]] voltas completas
430         if (k/sz[rt[i]]) ret = op(ret, k/sz[rt[i]] * query(first,
    last));

9af         k %= sz[rt[i]];
e3c         if (!k) return ret;
8ea         int l = pos[i], r = first + (pos[i] - first + k - 1) %
    sz[rt[i]];
982         if (l <= r) return op(ret, query(l, r));
687         return op(ret, op(query(l, last), query(first, r)));
380     }
51f }
```

6.16 HLD - aresta

```
// SegTree de soma
// query / update de soma das arestas
//
// Complexidades:
// build - O(n)
// query_path - O(log^2 (n))
// update_path - O(log^2 (n))
// query_subtree - O(log(n))
// update_subtree - O(log(n))

// namespace seg { ... }

826 namespace hld {
c0d     vector<pair<int, int> > g[MAX];
e65     int pos[MAX], sz[MAX];
7c0     int sobe[MAX], pai[MAX];
096     int h[MAX], v[MAX], t;

0ce     void build_hld(int k, int p = -1, int f = 1) {
180         v[pos[k] = t++] = sobe[k]; sz[k] = 1;
418         for (auto& i : g[k]) if (i.first != p) {
dd2             auto [u, w] = i;
a76             sobe[u] = w; pai[u] = k;
0c1             h[u] = (i == g[k][0] ? h[k] : u);
da7             build_hld(u, k, f); sz[k] += sz[u];

865             if (sz[u] > sz[g[k][0].first] or g[k][0].first == p)
9a3                 swap(i, g[k][0]);
804         }
667         if (p*f == -1) build_hld(h[k] = k, -1, t = 0);
4dd     }
1f8     void build(int root = 0) {
a34         t = 0;
295         build_hld(root);
c83         seg::build(t, v);
ea2     }
3fc     ll query_path(int a, int b) {
2d5         if (a == b) return 0;
aa1         if (pos[a] < pos[b]) swap(a, b);

29b         if (h[a] == h[b]) return seg::query(pos[b]+1, pos[a]);
fca         return seg::query(pos[h[a]], pos[a]) +
    query_path(pai[h[a]], b);
87f     }
920     void update_path(int a, int b, int x) {
d54         if (a == b) return;
aa1         if (pos[a] < pos[b]) swap(a, b);

881         if (h[a] == h[b]) return (void)seg::update(pos[b]+1,
    pos[a], x);
701         seg::update(pos[h[a]], pos[a], x); update_path(pai[h[a]],
    b, x);
dbf     }
d0a     ll query_subtree(int a) {
b9f         if (sz[a] == 1) return 0;
2f6         return seg::query(pos[a]+1, pos[a]+sz[a]-1);
77f     }
acc     void update_subtree(int a, int x) {
a5a         if (sz[a] == 1) return;
9cd         seg::update(pos[a]+1, pos[a]+sz[a]-1, x);
a46     }
7be     int lca(int a, int b) {
aa1         if (pos[a] < pos[b]) swap(a, b);
ca5         return h[a] == h[b] ? b : lca(pai[h[a]], b);
219     }
599 }
```

6.17 HLD - vertice

```
// SegTree de soma
// query / update de soma dos vertices
//
// Complexidades:
// build - O(n)
// query_path - O(log^2 (n))
// update_path - O(log^2 (n))
// query_subtree - O(log(n))
// update_subtree - O(log(n))

// namespace seg { ... }

826 namespace hld {
```

```
042     vector<int> g[MAX];
e65     int pos[MAX], sz[MAX];
bd4     int peso[MAX], pai[MAX];
096     int h[MAX], v[MAX], t;

0ce     void build_hld(int k, int p = -1, int f = 1) {
b18         v[pos[k] = t++] = peso[k]; sz[k] = 1;
b94         for (auto& i : g[k]) if (i != p) {
78d             pai[i] = k;
26e             h[i] = (i == g[k][0] ? h[k] : i);
193             build_hld(i, k, f); sz[k] += sz[i];

cd1             if (sz[i] > sz[g[k][0]] or g[k][0] == p) swap(i,
    g[k][0]);
d94         }
667         if (p*f == -1) build_hld(h[k] = k, -1, t = 0);
a83     }
1f8     void build(int root = 0) {
a34         t = 0;
295         build_hld(root);
c83         seg::build(t, v);
ea2     }
3fc     ll query_path(int a, int b) {
aa1         if (pos[a] < pos[b]) swap(a, b);

4bf         if (h[a] == h[b]) return seg::query(pos[b], pos[a]);
fca         return seg::query(pos[h[a]], pos[a]) +
    query_path(pai[h[a]], b);
c17     }
920     void update_path(int a, int b, int x) {
aa1         if (pos[a] < pos[b]) swap(a, b);

198         if (h[a] == h[b]) return (void)seg::update(pos[b], pos[a],
    x);
701         seg::update(pos[h[a]], pos[a], x); update_path(pai[h[a]],
    b, x);
421     }
d0a     ll query_subtree(int a) {
b3e         return seg::query(pos[a], pos[a]+sz[a]-1);
ba2     }
acc     void update_subtree(int a, int x) {
a22         seg::update(pos[a], pos[a]+sz[a]-1, x);
480     }
7be     int lca(int a, int b) {
aa1         if (pos[a] < pos[b]) swap(a, b);
ca5         return h[a] == h[b] ? b : lca(pai[h[a]], b);
219     }
de3 }
```

6.18 HLD sem Update

```
// query de min do caminho
//
// Complexidades:
// build - O(n)
// query_path - O(log(n))

826 namespace hld {
c0d     vector<pair<int, int> > g[MAX];
e65     int pos[MAX], sz[MAX];
7c0     int sobe[MAX], pai[MAX];
096     int h[MAX], v[MAX], t;
ea2     int men[MAX], seg[2*MAX];

0ce     void build_hld(int k, int p = -1, int f = 1) {
180         v[pos[k] = t++] = sobe[k]; sz[k] = 1;
418         for (auto& i : g[k]) if (i.first != p) {
1f5             sobe[i.first] = i.second; pai[i.first] = k;
6fa             h[i.first] = (i == g[k][0] ? h[k] : i.first);
87b             men[i.first] = (i == g[k][0] ? min(men[k], i.second) :
    i.second);
4b2             build_hld(i.first, k, f); sz[k] += sz[i.first];

bc3             if (sz[i.first] > sz[g[k][0].first] or g[k][0].first
    == p)
9a3                 swap(i, g[k][0]);
ea4         }
667         if (p*f == -1) build_hld(h[k] = k, -1, t = 0);
8ec     }
1f8     void build(int root = 0) {
a34         t = 0;
295         build_hld(root);
3ae         for (int i = 0; i < t; i++) seg[i+t] = v[i];
8db         for (int i = t-1; i; i--) seg[i] = min(seg[2*i],
    seg[2*i+1]);
ea5     }
f04     int query_path(int a, int b) {
490         if (a == b) return INF;
aa1         if (pos[a] < pos[b]) swap(a, b);

98f         if (h[a] != h[b]) return min(men[a], query_path(pai[h[a]],
    b));
46b         int ans = INF, x = pos[b]+1+t, y = pos[a]+t;
646         for (; x <= y; ++x/=2, --y/=2) ans = min({ans, seg[x], seg[y]});
ba7         return ans;
3a9     }
ee6 };
```

6.19 Hopcroft Karp

```
// Computa matching maximo em grafo bipartido
// 'n' e 'm' sao quantos vertices tem em cada particao
// chamar add(i, j) para add aresta entre o cara i
// da particao A, e o cara j da particao B
// (entao i < n, j < m)
//
// O(|E| * sqrt(|V|)) com constante baixa
// Para grafos esparsos gerados aleatoriamente, roda em O(|E| * log(|V|))
// com alta probabilidade

878 mt19937 rng((int) chrono::steady_clock::now().time_since_epoch().count());

958 struct hopcroft_karp {
14e     int n, m;
789     vector<vector<int>> g;
5ea     vector<int> dist, nxt, ma, mb;

605     hopcroft_karp(int n_, int m_) : n(n_), m(m_), g(n),
ac5         dist(n), nxt(n), ma(n, -1), mb(m, -1) {}

ba6     void add(int a, int b) { g[a].push_back(b); }

caf     bool dfs(int i) {
32b         for (int &id = nxt[i]; id < g[i].size(); id++) {
d9b             int j = g[i][id];
dd2             if (mb[j] == -1 or (dist[mb[j]] == dist[i] + 1 and dfs(mb[j]))) {
bfe                 ma[i] = j, mb[j] = i;
8a6                 return true;
c96             }
cf0         }
d1f         return false;
0de     }
838     bool bfs() {
85d         for (int i = 0; i < n; i++) dist[i] = n;
26a         queue<int> q;
ad2         for (int i = 0; i < n; i++) if (ma[i] == -1) {
d6b             dist[i] = 0;
3f2             q.push(i);
030         }
43f         bool rep = 0;
402         while (q.size()) {
379             int i = q.front(); q.pop();
48e             for (int j : g[i]) {
096                 if (mb[j] == -1) rep = 1;
395                 else if (dist[mb[j]] > dist[i] + 1) {
998                     dist[mb[j]] = dist[i] + 1;
a21                     q.push(mb[j]);
040                 }
36e             }
fc5         }
d14         return rep;
ad7     }
bf7     int matching() {
7c9         int ret = 0;
5a8         for (auto& i : g) shuffle(i.begin(), i.end(), rng);
6d4         while (bfs()) {
c79             for (int i = 0; i < n; i++) nxt[i] = 0;
830             for (int i = 0; i < n; i++)
475                 if (ma[i] == -1 and dfs(i)) ret++;
939         }
edf         return ret;
b77     }
cd2 };
```

6.20 Isomorfismo de arvores

```
// thash() retorna o hash da arvore (usando centroids como vertices especiais).
// Duas arvores sao isomorfas sse seu hash eh o mesmo
//
// O(|V|.log(|V|))

91f map<vector<int>, int> mphash;

df6 struct tree {
1a8     int n;
789     vector<vector<int>> g;
347     vector<int> sz, cs;

1b5     tree(int n_) : n(n_), g(n_), sz(n_) {}
```

```
76b     void dfs_centroid(int v, int p) {
588         sz[v] = 1;
fa7         bool cent = true;
18e         for (int u : g[v]) if (u != p) {
365             dfs_centroid(u, v), sz[v] += sz[u];
e90             if(sz[u] > n/2) cent = false;
ece         }
1f6         if (cent and n - sz[v] <= n/2) cs.push_back(v);
368     }
784     int fhash(int v, int p) {
544         vector<int> h;
332         for (int u : g[v]) if (u != p) h.push_back(fhash(u, v));
1c9         sort(h.begin(), h.end());
3ac         if (!mphash.count(h)) mphash[h] = mphash.size();
bbc         return mphash[h];
748     }
38f     ll thash() {
23a         cs.clear();
3a5         dfs_centroid(0, -1);
16d         if (cs.size() == 1) return fhash(cs[0], -1);
772         ll h1 = fhash(cs[0], cs[1]), h2 = fhash(cs[1], cs[0]);
fae         return (min(h1, h2) << 30) + max(h1, h2);
138     }
4dd };


// Versao mais rapida com hash, ideal para hash de floresta.
// subtree_hash(v, p) retorna o hash da subarvore enraizada em v com
    pai p.
// tree_hash() retorna o hash da arvore.
// forest_hash() retorna o hash da floresta.
// use o vetor forb[] para marcar vertices que nao podem ser visitados.
//
// O(|V|.log(|V|))

c8a mt19937
    rng(chrono::steady_clock::now().time_since_epoch().count());

426 int uniform(ll l, ll r) {
969     uniform_int_distribution<ll> uid(l, r);
f54     return uid(rng);
5fc }

3e4 const int MOD = 1e9 + 7;
58c const int H = 13;
db1 const int P = uniform(1, MOD-1);
325 const int P2 = uniform(1, MOD-1);

df6 struct tree {
2e2     int fn;
789     vector<vector<int>> g;
347     vector<int> sz, cs;
268     vector<bool> forb;

f73     tree(int n_) : fn(n_), g(n_), sz(n_), forb(n_) {}

762     void dfs_size(int v, int p) {
588         sz[v] = 1;
d8a         for (int u : g[v]) if (u != p and !forb[u]) {
db5             dfs_size(u, v), sz[v] += sz[u];
896         }
156     }
301     void dfs_centroid(int v, int p, int n) {
fa7         bool cent = true;
d8a         for (int u : g[v]) if (u != p and !forb[u]) {
e1f             dfs_centroid(u, v, n);
e90             if(sz[u] > n/2) cent = false;
235         }
1f6         if (cent and n - sz[v] <= n/2) cs.push_back(v);
188     }
1fc     int subtree_hash(int v, int p) {
3a7         int h = H;
d8a         for (int u : g[v]) if (u != p and !forb[u]) {
d36             h = ll(h) * (P + subtree_hash(u, v)) %  MOD;
cf2         }
81c         return h;
d83     }
126     int tree_hash(int v=0) {
23a         cs.clear();
575         dfs_size(v, -1);
8a5         dfs_centroid(v, -1, sz[v]);
d8d         if (cs.size() == 1) return subtree_hash(cs[0], -1);
098         assert (cs.size() == 2);
403         int h1 = subtree_hash(cs[0], cs[1]);
c49         int h2 = subtree_hash(cs[1], cs[0]);
1ae         return ll(P + h1) * (P + h2) % MOD;
ad7     }
c50     int forest_hash() {
5d7         fill(sz.begin(), sz.end(), 0);
eb4         int hash = 1;
778         for (int v = 0; v < fn; v++) if (!sz[v] and !forb[v]) {
f21             hash = hash * ll(P2 + tree_hash(v)) % MOD;
cc2         }
34e         return hash;
2ce     }
2fe };
```

6.21 Johnson

```
// funciona igual ao Floyd-Warshall
// encontra o menor caminho entre todo
// par de vertices e retorna 1 sse tem
// ciclo negativo no grafo
//
// O(nm log(m))

1b8 vector<pair<int, ll>> g[MAX]; // {vizinho, peso}
1a5 ll d[MAX][MAX];

e33 bool johnson(int n) {
61e     vector<ll> h(n, 0);
4e3     for (int i = 0; i <= n; i++)
84d         for (int v = 0; v < n; v++)
aa2             for (auto [u, w] : g[v]) if (h[u] > h[v] + w) {
705                 if (i == n) return 1;
e47                 h[u] = h[v] + w;
c36             }

603     for (int i = 0; i < n; i++) {
bb0         for (int j = 0; j < n; j++) d[i][j] = LINF;
682         d[i][i] = 0;
88c         priority_queue<pair<ll, int>> pq;
99c         pq.emplace(0, i);
265         while (pq.size()) {
d82             auto [ndist, v] = pq.top(); pq.pop();
a3f             if (-ndist > d[i][v]) continue;

209             for (auto [u, w] : g[v]) {
5a5                 w += h[v] - h[u];
f18                 if (d[i][u] > d[i][v] + w) {
db9                     d[i][u] = d[i][v] + w;
558                     pq.emplace(-d[i][u], u);
ef1                 }
f7f             }
a90         }
f90         for (int j = 0; j < n; j++)
fa1             d[i][j] += h[j] - h[i];
8f9     }

bb3     return 0;
d3d }
```

6.22 Kosaraju

```
// O(n + m)

1a8 int n;
042 vector<int> g[MAX];
58d vector<int> gi[MAX]; // grafo invertido
c5a int vis[MAX];
ee6 stack<int> S;
a52 int comp[MAX]; // componente conexo de cada vertice

1ca void dfs(int k) {
59a     vis[k] = 1;
54f     for (int i = 0; i < (int) g[k].size(); i++)
8d5         if (!vis[g[k][i]]) dfs(g[k][i]);

58f     S.push(k);
89c }

436 void scc(int k, int c) {
59a     vis[k] = 1;
52c     comp[k] = c;
ff0     for (int i = 0; i < (int) gi[k].size(); i++)
bf6         if (!vis[gi[k][i]]) scc(gi[k][i], c);
088 }

db8 void kosaraju() {
991     for (int i = 0; i < n; i++) vis[i] = 0;
158     for (int i = 0; i < n; i++) if (!vis[i]) dfs(i);

991     for (int i = 0; i < n; i++) vis[i] = 0;
d32     while (S.size()) {
70b         int u = S.top();
7de         S.pop();
f43         if (!vis[u]) scc(u, u);
207     }
e21 }
```

6.23 Kruskal

```
// Gera e retorna uma AGM e seu custo total a partir do vetor de
   arestas (edg)
```

```
// do grafo
//
// O(m log(m) + m a(m))
// 864875

1b9 vector<tuple<int, int, int>> edg; // {peso,[x,y]}

// DSU em O(a(n))
4a6 void dsu_build();
d78 int find(int a);
369 void unite(int a, int b);

c67 pair<ll, vector<tuple<int, int, int>>> kruskal(int n) {
8d2     dsu_build(n);
e31     sort(edg.begin(), edg.end());

854     ll cost = 0;
979     vector<tuple<int, int, int>> mst;
fea     for (auto [w,x,y] : edg) if (find(x) != find(y)) {
9de         mst.emplace_back(w, x, y);
45f         cost += w;
05a         unite(x,y);
ca2     }
5df     return {cost, mst};
b6a }
```

6.24 Kuhn

```
// Computa matching maximo em grafo bipartido
// 'n' e 'm' sao quantos vertices tem em cada particao
// chamar add(i, j) para add aresta entre o cara i
// da particao A, e o cara j da particao B
// (entao i < n, j < m)
// Para recuperar o matching, basta olhar 'ma' e 'mb'
// 'recover' recupera o min vertex cover como um par de
// {caras da particao A, caras da particao B}
//
// O(|V| * |E|)
// Na pratica, parece rodar tao rapido quanto o Dinitz

878 mt19937 rng((int)
    chrono::steady_clock::now().time_since_epoch().count());

6c6 struct kuhn {
14e     int n, m;
789     vector<vector<int>> g;
d3f     vector<int> vis, ma, mb;

40e     kuhn(int n_, int m_) : n(n_), m(m_), g(n),
8af         vis(n+m), ma(n, -1), mb(m, -1) {}

ba6     void add(int a, int b) { g[a].push_back(b); }

caf     bool dfs(int i) {
29a         vis[i] = 1;
29b         for (int j : g[i]) if (!vis[n+j]) {
8c9             vis[n+j] = 1;
2cf             if (mb[j] == -1 or dfs(mb[j])) {
bfe                 ma[i] = j, mb[j] = i;
8a6                 return true;
b17             }
82a         }
d1f         return false;
4ef     }
bf7     int matching() {
1ae         int ret = 0, aum = 1;
5a8         for (auto& i : g) shuffle(i.begin(), i.end(), rng);
392         while (aum) {
618             for (int j = 0; j < m; j++) vis[n+j] = 0;
c5d             aum = 0;
830             for (int i = 0; i < n; i++)
01f                 if (ma[i] == -1 and dfs(i)) ret++, aum = 1;
085         }
edf         return ret;
2ee     }
b0d };

ebf pair<vector<int>, vector<int>> recover(kuhn& K) {
e80     K.matching();
50c     int n = K.n, m = K.m;
9d0     for (int i = 0; i < n+m; i++) K.vis[i] = 0;
bde     for (int i = 0; i < n; i++) if (K.ma[i] == -1) K.dfs(i);
8ad     vector<int> ca, cb;
576     for (int i = 0; i < n; i++) if (!K.vis[i]) ca.push_back(i);
f24     for (int i = 0; i < m; i++) if (K.vis[n+i]) cb.push_back(i);
aad     return {ca, cb};
55f }
```

6.25 LCA com binary lifting

```
// Assume que um vertice eh ancestral dele mesmo, ou seja,
// se a eh ancestral de b, lca(a, b) = a
// MAX2 = ceil(log(MAX))
//
// Complexidades:
// build - O(n log(n))
// lca - O(log(n))

677 vector<vector<int> > g(MAX);
41c int n, p;
e75 int pai[MAX2][MAX];
999 int in[MAX], out[MAX];

1ca void dfs(int k) {
fdf     in[k] = p++;
54f     for (int i = 0; i < (int) g[k].size(); i++)
9b7         if (in[g[k][i]] == -1) {
ba6             pai[0][g[k][i]] = k;
c38             dfs(g[k][i]);
e2d         }
26f     out[k] = p++;
691 }

c11 void build(int raiz) {
a67     for (int i = 0; i < n; i++) pai[0][i] = i;
c63     p = 0, memset(in, -1, sizeof in);
ecb     dfs(raiz);

        // pd dos pais
511     for (int k = 1; k < MAX2; k++) for (int i = 0; i < n; i++)
d38         pai[k][i] = pai[k - 1][pai[k - 1][i]];
530 }

00f bool anc(int a, int b) { // se a eh ancestral de b
bfe     return in[a] <= in[b] and out[a] >= out[b];
2d6 }

7be int lca(int a, int b) {
86d     if (anc(a, b)) return a;
e52     if (anc(b, a)) return b;

        // sobe a
f70     for (int k = MAX2 - 1; k >= 0; k--)
acf         if (!anc(pai[k][a], b)) a = pai[k][a];

847     return pai[0][a];
5c4 }

// Alternativamente:
// 'binary lifting' gastando O(n) de memoria
// Da pra add folhas e fazer queries online
// 3 vezes o tempo do binary lifting normal
//
// build - O(n)
// kth, lca, dist - O(log(n))

9c6 int d[MAX], p[MAX], pp[MAX];

d40 void set_root(int i) { p[i] = pp[i] = i, d[i] = 0; }

e9d void add_leaf(int i, int u) {
e0b     p[i] = u, d[i] = d[u]+1;
b15     pp[i] = 2*d[pp[u]] == d[pp[pp[u]]]+d[u] ? pp[pp[u]] : u;
33f }

c37 int kth(int i, int k) {
4e3     int dd = max(0, d[i]-k);
935     while (d[i] > dd) i = d[pp[i]] >= dd ? pp[i] : p[i];
d9a     return i;
f3c }

7be int lca(int a, int b) {
a69     if (d[a] < d[b]) swap(a, b);
6cd     while (d[a] > d[b]) a = d[pp[a]] >= d[b] ? pp[a] : p[a];
984     while (a != b) {
932         if (pp[a] != pp[b]) a = pp[a], b = pp[b];
e7c         else a = p[a], b = p[b];
4ea     }
3f5     return a;
21d }

4fe int dist(int a, int b) { return d[a]+d[b]-2*d[lca(a,b)]; }

042 vector<int> g[MAX];

3ab void build(int i, int pai=-1) {
5cf     if (pai == -1) set_root(i);
15f     for (int j : g[i]) if (j != pai) {
d31         add_leaf(j, i);
b21         build(j, i);
43b     }
74a }
```

6.26 LCA com HLD

```
// Assume que um vertice eh ancestral dele mesmo, ou seja,
// se a eh ancestral de b, lca(a, b) = a
// Para buildar pasta chamar build(root)
// anc(a, b) responde se 'a' eh ancestral de 'b'
//
// Complexidades:
// build - O(n)
// lca - O(log(n))
// anc - O(1)

042 vector<int> g[MAX];
713 int pos[MAX], h[MAX], sz[MAX];
ff1 int pai[MAX], t;

8bf void build(int k, int p = -1, int f = 1) {
bce     pos[k] = t++; sz[k] = 1;
e26     for (int& i : g[k]) if (i != p) {
78d         pai[i] = k;
26e         h[i] = (i == g[k][0] ? h[k] : i);
cb8         build(i, k, f); sz[k] += sz[i];

cd1         if (sz[i] > sz[g[k][0]] or g[k][0] == p) swap(i, g[k][0]);
917     }
3da     if (p*f == -1) t = 0, h[k] = k, build(k, -1, 0);
1b9 }

7be int lca(int a, int b) {
aa1     if (pos[a] < pos[b]) swap(a, b);
ca5     return h[a] == h[b] ? b : lca(pai[h[a]], b);
219 }

00f bool anc(int a, int b) {
db5     return pos[a] <= pos[b] and pos[b] <= pos[a]+sz[a]-1;
272 }
```

6.27 LCA com RMQ

```
// Assume que um vertice eh ancestral dele mesmo, ou seja,
// se a eh ancestral de b, lca(a, b) = a
// dist(a, b) retorna a distancia entre a e b
//
// Complexidades:
// build - O(n)
// lca - O(1)
// dist - O(1)

1a5 template<typename T> struct rmq {
517     vector<T> v;
fcc     int n; static const int b = 30;
70e     vector<int> mask, t;

18e     int op(int x, int y) { return v[x] < v[y] ? x : y; }
ee1     int msb(int x) { return __builtin_clz(1)-__builtin_clz(x); }
6ad     rmq() {}
43c     rmq(const vector<T>& v_) : v(v_), n(v.size()), mask(n), t(n) {
2e5         for (int i = 0, at = 0; i < n; mask[i++] = at |= 1) {
a61             at = (at<<1)&((1<<b)-1);
76a             while (at and op(i, i-msb(at&-at)) == i) at ^= at&-at;
53c         }
243         for (int i = 0; i < n/b; i++) t[i] =
   b*i+b-1-msb(mask[b*i+b-1]);
39d         for (int j = 1; (1<<j) <= n/b; j++) for (int i = 0;
   i+(1<<j) <= n/b; i++)
ba5             t[n/b*j+i] = op(t[n/b*(j-1)+i],
   t[n/b*(j-1)+i+(1<<(j-1))]);
2d3     }
c92     int small(int r, int sz = b) { return
   r-msb(mask[r]&((1<<sz)-1)); }
b7a     T query(int l, int r) {
27b         if (r-l+1 <= b) return small(r, r-l+1);
7bf         int ans = op(small(l+b-1), small(r));
e80         int x = l/b+1, y = r/b-1;
e25         if (x <= y) {
a4e             int j = msb(y-x+1);
002             ans = op(ans, op(t[n/b*j+x], t[n/b*j+y-(1<<j)+1]));
4b6         }
ba7         return ans;
6bf     }
021 };

065 namespace lca {
042     vector<int> g[MAX];
8ec     int v[2*MAX], pos[MAX], dep[2*MAX];
8bd     int t;
2de     rmq<int> RMQ;

4cf     void dfs(int i, int d = 0, int p = -1) {
c97         v[t] = i, pos[i] = t, dep[t++] = d;
cac         for (int j : g[i]) if (j != p) {
8ec             dfs(j, d+1, i);
cf2             v[t] = i, dep[t++] = d;
843         }
d6a     }
789     void build(int n, int root) {
a34         t = 0;
14e         dfs(root);
3f4         RMQ = rmq<int>(vector<int>(dep, dep+2*n-1));
657     }
7be     int lca(int a, int b) {
ab7         a = pos[a], b = pos[b];
9c0         return v[RMQ.query(min(a, b), max(a, b))];
5db     }
b5d     int dist(int a, int b) {
670         return dep[pos[a]] + dep[pos[b]] - 2*dep[pos[lca(a, b)]];
5b7     }
645 }
```

6.28 Line Tree

```
// Reduz min-query em arvore para RMQ
// Se o grafo nao for uma arvore, as queries
// sao sobre a arvore geradora maxima
// Queries de minimo
//
// build - O(n log(n))
// query - O(log(n))

1a8 int n;

3ae namespace linetree {
f37     int id[MAX], seg[2*MAX], pos[MAX];
43f     vector<int> v[MAX], val[MAX];
430     vector<pair<int, pair<int, int> > > ar;

dc6     void add(int a, int b, int p) { ar.push_back({p, {a, b}}); }
0a8     void build() {
b09         sort(ar.rbegin(), ar.rend());
0e3         for (int i = 0; i < n; i++) id[i] = i, v[i] = {i},
    val[i].clear();
8bb         for (auto i : ar) {
c91             int a = id[i.second.first], b = id[i.second.second];
f6f             if (a == b) continue;
c58             if (v[a].size() < v[b].size()) swap(a, b);
fb8             for (auto j : v[b]) id[j] = a, v[a].push_back(j);
482             val[a].push_back(i.first);
78b             for (auto j : val[b]) val[a].push_back(j);
e39             v[b].clear(), val[b].clear();
012         }
8e8         vector<int> vv;
2ce         for (int i = 0; i < n; i++) for (int j = 0; j <
    v[i].size(); j++) {
e52             pos[v[i][j]] = vv.size();
941             if (j + 1 < v[i].size()) vv.push_back(val[i][j]);
1cb             else vv.push_back(0);
475         }
bb4         for (int i = n; i < 2*n; i++) seg[i] = vv[i-n];
69e         for (int i = n-1; i; i--) seg[i] = min(seg[2*i],
    seg[2*i+1]);
9fe     }
4ea     int query(int a, int b) {
596         if (id[a] != id[b]) return 0; // nao estao conectados
ab7         a = pos[a], b = pos[b];
d11         if (a > b) swap(a, b);
199         b--;
38a         int ans = INF;
513         for (a += n, b += n; a <= b; ++a/=2, --b/=2) ans =
    min({ans, seg[a], seg[b]});
ba7         return ans;
952     }
00f };
```

6.29 Link-cut Tree

```
// Link-cut tree padrao
//
// Todas as operacoes sao O(log(n)) amortizado

1ef namespace lct {
3c9     struct node {
19f         int p, ch[2];
062         node() { p = ch[0] = ch[1] = -1; }
f43     };

5f3     node t[MAX];

971     bool is_root(int x) {
657         return t[x].p == -1 or (t[t[x].p].ch[0] != x and
    t[t[x].p].ch[1] != x);
cf1     }
ed6     void rotate(int x) {
497         int p = t[x].p, pp = t[p].p;
fc4         if (!is_root(p)) t[pp].ch[t[pp].ch[1] == p] = x;
251         bool d = t[p].ch[0] == x;
461         t[p].ch[!d] = t[x].ch[d], t[x].ch[d] = p;
a76         if (t[p].ch[!d]+1) t[t[p].ch[!d]].p = p;
8fa             t[x].p = pp, t[p].p = x;
49b         }
07c         void splay(int x) {
18c             while (!is_root(x)) {
497                 int p = t[x].p, pp = t[p].p;
0c5                 if (!is_root(p)) rotate((t[pp].ch[0] == p)^(t[p].ch[0]
    == x) ? x : p);
64f                 rotate(x);
d8d             }
4fa         }
f16         int access(int v) {
0eb             int last = -1;
01a             for (int w = v; w+1; last = w, splay(v), w = t[v].p)
024                 splay(w), t[w].ch[1] = (last == -1 ? -1 : v);
3d3             return last;
0a4         }
e89         int find_root(int v) {
5e3             access(v);
3de             while (t[v].ch[0]+1) v = t[v].ch[0];
f05             return splay(v), v;
ee7         }
142         void link(int v, int w) { // v deve ser raiz
5e3             access(v);
10d             t[v].p = w;
c56         }
4e6         void cut(int v) { // remove aresta de v pro pai
5e3             access(v);
264             t[v].ch[0] = t[t[v].ch[0]].p = -1;
5f5         }
bbb         int lca(int v, int w) {
948             return access(v), access(w);
b6d         }
e4e }
```

6.30 Link-cut Tree - aresta

```
// Valores nas arestas
// rootify(v) torna v a raiz de sua arvore
// query(v, w) retorna a soma do caminho v--w
// update(v, w, x) soma x nas arestas do caminho v--w
//
// Todas as operacoes sao O(log(n)) amortizado

1ef namespace lct {
3c9     struct node {
19f         int p, ch[2];
810         ll val, sub;
aa6         bool rev;
04a         int sz, ar;
4e4         ll lazy;
f93         node() {}
7a8         node(int v, int ar_) :
546         p(-1), val(v), sub(v), rev(0), sz(ar_), ar(ar_), lazy(0) {
b07             ch[0] = ch[1] = -1;
53b         }
6e0     };

c53     node t[2*MAX]; // MAXN + MAXQ
99e     map<pair<int, int>, int> aresta;
e4d     int sz;

95a     void prop(int x) {
dc1         if (t[x].lazy) {
25e             if (t[x].ar) t[x].val += t[x].lazy;
2ab             t[x].sub += t[x].lazy*t[x].sz;
edc             if (t[x].ch[0]+1) t[t[x].ch[0]].lazy += t[x].lazy;
942             if (t[x].ch[1]+1) t[t[x].ch[1]].lazy += t[x].lazy;
1ba         }
aa2         if (t[x].rev) {
f95             swap(t[x].ch[0], t[x].ch[1]);
379             if (t[x].ch[0]+1) t[t[x].ch[0]].rev ^= 1;
c3d             if (t[x].ch[1]+1) t[t[x].ch[1]].rev ^= 1;
50e         }
230         t[x].lazy = 0, t[x].rev = 0;
f9d     }
564     void update(int x) {
1a3         t[x].sz = t[x].ar, t[x].sub = t[x].val;
8ca         for (int i = 0; i < 2; i++) if (t[x].ch[i]+1) {
621             prop(t[x].ch[i]);
c4f             t[x].sz += t[t[x].ch[i]].sz;
269             t[x].sub += t[t[x].ch[i]].sub;
400         }
28b     }
971     bool is_root(int x) {
657         return t[x].p == -1 or (t[t[x].p].ch[0] != x and
    t[t[x].p].ch[1] != x);
cf1     }
ed6     void rotate(int x) {
497         int p = t[x].p, pp = t[p].p;
fc4         if (!is_root(p)) t[pp].ch[t[pp].ch[1] == p] = x;
251         bool d = t[p].ch[0] == x;
461         t[p].ch[!d] = t[x].ch[d], t[x].ch[d] = p;
a76         if (t[p].ch[!d]+1) t[t[p].ch[!d]].p = p;
8fa         t[x].p = pp, t[p].p = x;
444         update(p), update(x);
f31     }
238     int splay(int x) {
18c         while (!is_root(x)) {
497             int p = t[x].p, pp = t[p].p;
77b             if (!is_root(p)) prop(pp);
be5             prop(p), prop(x);
0c5             if (!is_root(p)) rotate((t[pp].ch[0] == p)^(t[p].ch[0] == x) ? x : p);
64f             rotate(x);
72c         }
aab         return prop(x), x;
08f     }
f16     int access(int v) {
0eb         int last = -1;
d9f         for (int w = v; w+1; update(last = w), splay(v), w = t[v].p)
024             splay(w), t[w].ch[1] = (last == -1 ? -1 : v);
3d3         return last;
294     }
9f1     void make_tree(int v, int w=0, int ar=0) { t[v] = node(w, ar); }
e89     int find_root(int v) {
13f         access(v), prop(v);
9f0         while (t[v].ch[0]+1) v = t[v].ch[0], prop(v);
637         return splay(v);
16a     }
82f     bool conn(int v, int w) {
2cf         access(v), access(w);
b9b         return v == w ? true : t[v].p != -1;
ec0     }
277     void rootify(int v) {
5e3         access(v);
a02         t[v].rev ^= 1;
a05     }
971     ll query(int v, int w) {
b54         rootify(w), access(v);
249         return t[v].sub;
652     }
3fa     void update(int v, int w, int x) {
b54         rootify(w), access(v);
12c         t[v].lazy += x;
74f     }
204     void link_(int v, int w) {
821         rootify(w);
389         t[w].p = v;
523     }
6b8     void link(int v, int w, int x) { // v--w com peso x
379         int id = MAX + sz++;
110         aresta[make_pair(v, w)] = id;
a88         make_tree(id, x, 1);
c88         link_(v, id), link_(id, w);
58c     }
e63     void cut_(int v, int w) {
b54         rootify(w), access(v);
264         t[v].ch[0] = t[t[v].ch[0]].p = -1;
7cd     }
031     void cut(int v, int w) {
b0f         int id = aresta[make_pair(v, w)];
a4a         cut_(v, id), cut_(id, w);
840     }
bbb     int lca(int v, int w) {
5e3         access(v);
a8b         return access(w);
524     }
9ce }
```

6.31 Link-cut Tree - vertice

```
// Valores nos vertices
// make_tree(v, w) cria uma nova arvore com um
// vertice soh com valor 'w'
// rootify(v) torna v a raiz de sua arvore
// query(v, w) retorna a soma do caminho v--w
// update(v, w, x) soma x nos vertices do caminho v--w
//
// Todas as operacoes sao O(log(n)) amortizado

1ef namespace lct {
3c9     struct node {
19f         int p, ch[2];
810         ll val, sub;
aa6         bool rev;
e4d         int sz;
4e4         ll lazy;
f93         node() {}
aa0         node(int v) : p(-1), val(v), sub(v), rev(0), sz(1), lazy(0) {
b07             ch[0] = ch[1] = -1;
c4e         }
2b7     };
5f3     node t[MAX];

95a     void prop(int x) {
dc1         if (t[x].lazy) {
9f7             t[x].val += t[x].lazy, t[x].sub += t[x].lazy*t[x].sz;
edc             if (t[x].ch[0]+1) t[t[x].ch[0]].lazy += t[x].lazy;
942             if (t[x].ch[1]+1) t[t[x].ch[1]].lazy += t[x].lazy;
e26         }
aa2         if (t[x].rev) {
f95             swap(t[x].ch[0], t[x].ch[1]);
379             if (t[x].ch[0]+1) t[t[x].ch[0]].rev ^= 1;
c3d             if (t[x].ch[1]+1) t[t[x].ch[1]].rev ^= 1;
50e         }
230         t[x].lazy = 0, t[x].rev = 0;
c62     }
564     void update(int x) {
ec2         t[x].sz = 1, t[x].sub = t[x].val;
8ca         for (int i = 0; i < 2; i++) if (t[x].ch[i]+1) {
621             prop(t[x].ch[i]);
c4f             t[x].sz += t[t[x].ch[i]].sz;
269             t[x].sub += t[t[x].ch[i]].sub;
400         }
da7     }
971     bool is_root(int x) {
657         return t[x].p == -1 or (t[t[x].p].ch[0] != x and
    t[t[x].p].ch[1] != x);
cf1     }
ed6     void rotate(int x) {
497         int p = t[x].p, pp = t[p].p;
fc4         if (!is_root(p)) t[pp].ch[t[pp].ch[1] == p] = x;
251         bool d = t[p].ch[0] == x;
461         t[p].ch[!d] = t[x].ch[d], t[x].ch[d] = p;
a76         if (t[p].ch[!d]+1) t[t[p].ch[!d]].p = p;
8fa         t[x].p = pp, t[p].p = x;
444         update(p), update(x);
f31     }
238     int splay(int x) {
18c         while (!is_root(x)) {
497             int p = t[x].p, pp = t[p].p;
77b             if (!is_root(p)) prop(pp);
be5             prop(p), prop(x);
0c5             if (!is_root(p)) rotate((t[pp].ch[0] == p)^(t[p].ch[0]
    == x) ? x : p);
64f             rotate(x);
72c         }
aab         return prop(x), x;
08f     }
f16     int access(int v) {
0eb         int last = -1;
d9f         for (int w = v; w+1; update(last = w), splay(v), w =
    t[v].p)
024             splay(w), t[w].ch[1] = (last == -1 ? -1 : v);
3d3         return last;
294     }
f17     void make_tree(int v, int w) { t[v] = node(w); }
e89     int find_root(int v) {
13f         access(v), prop(v);
9f0         while (t[v].ch[0]+1) v = t[v].ch[0], prop(v);
637         return splay(v);
16a     }
f94     bool connected(int v, int w) {
2cf         access(v), access(w);
b9b         return v == w ? true : t[v].p != -1;
ec6     }
277     void rootify(int v) {
5e3         access(v);
a02         t[v].rev ^= 1;
a05     }
971     ll query(int v, int w) {
b54         rootify(w), access(v);
249         return t[v].sub;
652     }
3fa     void update(int v, int w, int x) {
b54         rootify(w), access(v);
12c         t[v].lazy += x;
74f     }
142     void link(int v, int w) {
821         rootify(w);
389         t[w].p = v;
8a8     }
031     void cut(int v, int w) {
b54         rootify(w), access(v);
264         t[v].ch[0] = t[t[v].ch[0]].p = -1;
d9a     }
bbb     int lca(int v, int w) {
5e3         access(v);
a8b         return access(w);
524     }
f9f }
```

6.32 Max flow com lower bound

```
// add(a, b, l, r):
```

```
//  adiciona aresta de a pra b, onde precisa passar f de fluxo, l <= f
    <= r
// add(a, b, c):
//  adiciona aresta de a pra b com capacidade c
//
// Mesma complexidade do Dinitz

cd5 struct lb_max_flow : dinitz {
5ce     vector<int> d;
d8c     lb_max_flow(int n) : dinitz(n + 2), d(n, 0) {}
b12     void add(int a, int b, int l, int r) {
c97         d[a] -= l;
f1b         d[b] += l;
4c0         dinitz::add(a, b, r - l);
ed4     }
087     void add(int a, int b, int c) {
0f3         dinitz::add(a, b, c);
039     }
7a1     bool has_circulation() {
50c         int n = d.size();

854         ll cost = 0;
603         for (int i = 0; i < n; i++) {
c69             if (d[i] > 0) {
f56                 cost += d[i];
57a                 dinitz::add(n, i, d[i]);
c72             } else if (d[i] < 0) {
b76                 dinitz::add(i, n+1, -d[i]);
b42             }
676         }

067         return (dinitz::max_flow(n, n+1) == cost);
110     }
7bd     bool has_flow(int src, int snk) {
387         dinitz::add(snk, src, INF);
e40         return has_circulation();
cc1     }
4eb     ll max_flow(int src, int snk) {
ee8         if (!has_flow(src, snk)) return -1;
4ad         dinitz::F = 0;
fe5         return dinitz::max_flow(src, snk);
619     }
e8f };
```

6.33 MinCostMaxFlow

```
// min_cost_flow(s, t, f) computa o par (fluxo, custo)
// com max(fluxo) <= f que tenha min(custo)
// min_cost_flow(s, t) -> Fluxo maximo de custo minimo de s pra t
// Se for um dag, da pra substituir o SPFA por uma DP pra nao
// pagar O(nm) no comeco
// Se nao tiver aresta com custo negativo, nao precisa do SPFA
//
// O(nm + f * m log n)

123 template<typename T> struct mcmf {
670     struct edge {
b75         int to, rev, flow, cap; // para, id da reversa, fluxo,
    capacidade
7f9         bool res; // se eh reversa
635         T cost; // custo da unidade de fluxo
892         edge() : to(0), rev(0), flow(0), cap(0), cost(0),
    res(false) {}
1d7         edge(int to_, int rev_, int flow_, int cap_, T cost_, bool
    res_)
f8d             : to(to_), rev(rev_), flow(flow_), cap(cap_),
    res(res_), cost(cost_) {}
723     };

002     vector<vector<edge>> g;
168     vector<int> par_idx, par;
f1e     T inf;
a03     vector<T> dist;

b22     mcmf(int n) : g(n), par_idx(n), par(n),
    inf(numeric_limits<T>::max()/3) {}

91c     void add(int u, int v, int w, T cost) { // de u pra v com cap
    w e custo cost
2fc         edge a = edge(v, g[v].size(), 0, w, cost, false);
234         edge b = edge(u, g[u].size(), 0, 0, -cost, true);

b24         g[u].push_back(a);
c12         g[v].push_back(b);
0ed     }

8bc     vector<T> spfa(int s) { // nao precisa se nao tiver custo
    negativo
871         deque<int> q;
3d1         vector<bool> is_inside(g.size(), 0);
577         dist = vector<T>(g.size(), inf);

a93         dist[s] = 0;
a30         q.push_back(s);
ecb         is_inside[s] = true;

14d         while (!q.empty()) {
b1e             int v = q.front();
ced             q.pop_front();
48d             is_inside[v] = false;

76e             for (int i = 0; i < g[v].size(); i++) {
9d4                 auto [to, rev, flow, cap, res, cost] = g[v][i];
e61                 if (flow < cap and dist[v] + cost < dist[to]) {
943                     dist[to] = dist[v] + cost;

ed6                     if (is_inside[to]) continue;
020                     if (!q.empty() and dist[to] > dist[q.front()])
    q.push_back(to);
b33                     else q.push_front(to);
b52                     is_inside[to] = true;
2d1                 }
8cd             }
f2c         }
8d7         return dist;
96c     }
2a2     bool dijkstra(int s, int t, vector<T>& pot) {
489         priority_queue<pair<T, int>, vector<pair<T, int>>,
    greater<>> q;
577         dist = vector<T>(g.size(), inf);
a93         dist[s] = 0;
115         q.emplace(0, s);
402         while (q.size()) {
91b             auto [d, v] = q.top();
833             q.pop();
68b             if (dist[v] < d) continue;
76e             for (int i = 0; i < g[v].size(); i++) {
9d4                 auto [to, rev, flow, cap, res, cost] = g[v][i];
e8c                 cost += pot[v] - pot[to];
e61                 if (flow < cap and dist[v] + cost < dist[to]) {
943                     dist[to] = dist[v] + cost;
441                     q.emplace(dist[to], to);
88b                     par_idx[to] = i, par[to] = v;
873                 }
de3             }
9d4         }
1d4         return dist[t] < inf;
c68     }

3d2     pair<int, T> min_cost_flow(int s, int t, int flow = INF) {
3dd         vector<T> pot(g.size(), 0);
9e4         pot = spfa(s); // mudar algoritmo de caminho minimo aqui

d22         int f = 0;
ce8         T ret = 0;
4a0         while (f < flow and dijkstra(s, t, pot)) {
bda             for (int i = 0; i < g.size(); i++)
d2a                 if (dist[i] < inf) pot[i] += dist[i];

71b             int mn_flow = flow - f, u = t;
045             while (u != s){
90f                 mn_flow = min(mn_flow,
07d                     g[par[u]][par_idx[u]].cap -
    g[par[u]][par_idx[u]].flow);
3d1                 u = par[u];
935             }

1f2             ret += pot[t] * mn_flow;

476             u = t;
045             while (u != s) {
e09                 g[par[u]][par_idx[u]].flow += mn_flow;
d98                 g[u][g[par[u]][par_idx[u]].rev].flow -= mn_flow;
3d1                 u = par[u];
bcc             }

04d             f += mn_flow;
36d         }

15b         return make_pair(f, ret);
cc3     }

        // Opcional: retorna as arestas originais por onde passa flow
            = cap
182     vector<pair<int,int>> recover() {
24a         vector<pair<int,int>> used;
2a4         for (int i = 0; i < g.size(); i++) for (edge e : g[i])
587             if(e.flow == e.cap && !e.res) used.push_back({i,
    e.to});
f6b         return used;
390     }
697 };
```

6.34 Prufer code

```
// Traduz de lista de arestas para prufer code
// e vice-versa
// Os vertices tem label de 0 a n-1
// Todo array com n-2 posicoes e valores de
// 0 a n-1 sao prufer codes validos
//
// O(n)

47d vector<int> to_prufer(vector<pair<int, int>> tree) {
1fa     int n = tree.size()+1;
2cf     vector<int> d(n, 0);
4aa     vector<vector<int>> g(n);
f87     for (auto [a, b] : tree) d[a]++, d[b]++,
f60         g[a].push_back(b), g[b].push_back(a);
c5a     vector<int> pai(n, -1);
260     queue<int> q; q.push(n-1);
402     while (q.size()) {
be1         int u = q.front(); q.pop();
34c         for (int v : g[u]) if (v != pai[u])
9c9             pai[v] = u, q.push(v);
70d     }
399     int idx, x;
897     idx = x = find(d.begin(), d.end(), 1) - d.begin();
4b8     vector<int> ret;
b28     for (int i = 0; i < n-2; i++) {
d4b         int y = pai[x];
e81         ret.push_back(y);
666         if (--d[y] == 1 and y < idx) x = y;
367         else idx = x = find(d.begin()+idx+1, d.end(), 1) -
    d.begin();
5f9     }
edf     return ret;
d3b }

4d8 vector<pair<int, int>> from_prufer(vector<int> p) {
455     int n = p.size()+2;
126     vector<int> d(n, 1);
650     for (int i : p) d[i]++;
85b     p.push_back(n-1);
399     int idx, x;
897     idx = x = find(d.begin(), d.end(), 1) - d.begin();
1df     vector<pair<int, int>> ret;
b06     for (int y : p) {
dab         ret.push_back({x, y});
666         if (--d[y] == 1 and y < idx) x = y;
367         else idx = x = find(d.begin()+idx+1, d.end(), 1) -
    d.begin();
c3b     }
edf     return ret;
765 }
```

6.35 Rooted Euler Tour Tree

```
// Mantem uma floresta enraizada dinamicamente
// e permite queries/updates em sub-arvore
//
// Chamar ETT E(n, w), passando n = numero de vertices
// e w = vector com os valores de cada vertice
//
// link(v, u) cria uma aresta de v pra u, de forma que u se torna
// o pai de v (eh preciso que v seja raiz anteriormente)
// cut(v) corta a resta de v para o pai
// get_root(v) retorna a raiz de v
// condense(v) remove v e faz seus filhos apontarem para pai(v)
// erase(v) remove v e corta suas arestas
// - Se usar condense ou erase, nao pode usar get_root!
// vtx_query e vtx_update operam sobre valor de um vertice
// subtree_query e subtree_update operam sobre sub-arvore
// is_ancestor(v, u) retorna se v eh ancestral de u
//
// Tudo O(log(n)) com alta probabilidade

878 mt19937 rng((int)
    chrono::steady_clock::now().time_since_epoch().count());

0f9 template<typename T> struct rooted_ett {
83c     const static T ZERO = T();
3c9     struct node {
ed1         node *l, *r, *p;
fa4         int pr, sz;
875         T val, sub, lazy;
53e         int id; // id, ou -1 se nao eh first
5ef         int qt_f; // numero de 'first' na subarvore
3f6         node(T v, int id_) : l(NULL), r(NULL), p(NULL), pr(rng()),
3ee             sz(1), val(v), sub(v), lazy(), id(id_), qt_f(id >= 0)
    {}
a9c         void prop() {
9fc             if (lazy != ZERO) {
1e2                 if (id >= 0) val += lazy;
2cf                 sub += lazy * qt_f;
b87                 if (l) l->lazy += lazy;
d3b                 if (r) r->lazy += lazy;
f74             }
520             lazy = ZERO;
4e7         }
01e         void update() {
c21             sz = 1, sub = val, qt_f = id >= 0;
171             if (l) l->prop(), sz += l->sz, sub += l->sub, qt_f += l->qt_f;
117             if (r) r->prop(), sz += r->sz, sub += r->sub, qt_f += r->qt_f;
93e         }
4f5     };

73c     int size(node* x) { return x ? x->sz : 0; }
bcf     void join(node* l, node* r, node*& i) {
986         if (!l or !r) return void(i = l ? l : r);
161         l->prop(), r->prop();
ff5         if (l->pr > r->pr) join(l->r, r, l->r), l->r->p = i = l;
982         else join(l, r->l, r->l), r->l->p = i = r;
bda         i->update();
84d     }
a20     void split(node* i, node*& l, node*& r, int v, int key = 0) {
26a         if (!i) return void(r = l = NULL);
c89         i->prop();
d9e         if (key + size(i->l) < v) {
448             split(i->r, i->r, r, v, key+size(i->l)+1), l = i;
a21             if (r) r->p = NULL;
6e8             if (i->r) i->r->p = i;
396         } else {
98d             split(i->l, l, i->l, v, key), r = i;
5a3             if (l) l->p = NULL;
899             if (i->l) i->l->p = i;
18b         }
bda         i->update();
134     }
3fa     pair<node*, int> get_idx(node* i) { // {root, idx}
6cf         int ret = size(i->l);
65d         for (; i->p; i = i->p) if (i != i->p->l)
fc8             ret += size(i->p->l) + 1;
c89         i->prop();
e51         return {i, ret};
fa4     }

4fb     vector<node*> first, last;

96c     rooted_ett(int n, vector<T> w) : first(n), last(n) {
aab         node* tmp;
603         for (int i = 0; i < n; i++) {
761             first[i] = new node(w[i], i);
45f             join(first[i], last[i] = new node(ZERO, -1), tmp);
d89         }
b6c     }
562     rooted_ett(const rooted_ett& t) { throw logic_error("Nao copiar a ETT!"); }
27b     ~rooted_ett() {
d79         for (int i = 0; i < first.size(); i++) delete first[i], delete last[i];
94f     }

d69     tuple<node*, int, int> get_range(int i) {
8ea         auto [root, l_idx] = get_idx(first[i]);
1bc         return {root, l_idx, get_idx(last[i]).second};
86e     }
d72     tuple<node*, node*, node*> split3(node* root, int l, int r) {
df9         node *L, *M, *R;
cf1         split(root, M, R, r+1);
f1a         split(M, L, M, l);
b2d         return {L, M, R};
681     }
97c     node* extract_range(node*& root, int l, int r) {
ace         auto [L, M, R] = split3(root, l, r);
a28         join(L, R, root);
474         return M;
b38     }
ead     void join3(node* L, node* M, node* R) {
1f7         join(L, M, M), join(M, R, R);
275     }

7af     void link(int v, int u) { // parent(v) <-- u
c9f         auto [root_v, l, r] = get_range(v);
b1f         root_v = extract_range(root_v, l, r);

61c         if (u == -1) return;

1aa         auto [root, idx] = get_idx(first[u]);
031         node *L, *R;
f07         split(root, L, R, idx+1);
c59         join3(L, root_v, R);
e39     }
4e6     void cut(int v) { // corta de v pro pai
d05         link(v, -1);
d06     }
fcc     int get_root(int v) {
20c         auto [root, idx] = get_idx(first[v]);
f6e         while (root->l) root = root->l;
43c         return root->id;
00b     }
```

```
        // se usar condense ou erase, nao pode usar get_root!
1ab     void condense(int v) { // remove v, pai dos filhos vira pai(v)
b73         auto [root, l, r] = get_range(v);
a56         auto E2 = extract_range(root, r, r);
00d         auto E1 = extract_range(root, l, l);
468         join(E1, E2, E2);
eb6     }
930     void erase(int v) { // remove v e arestas adjacentes
124         cut(v);
f75         condense(v);
a9c     }

c74     T vtx_query(int v) {
20c         auto [root, idx] = get_idx(first[v]);
5a1         auto [L, M, R] = split3(root, idx, idx);
d43         T ans = M->sub;
20c         join3(L, M, R);
ba7         return ans;
010     }
8e6     void vtx_update(int v, T val) {
20c         auto [root, idx] = get_idx(first[v]);
5a1         auto [L, M, R] = split3(root, idx, idx);
409         M->lazy += val;
6aa         M->prop();
20c         join3(L, M, R);
385     }
c99     T subtree_query(int v) {
b73         auto [root, l, r] = get_range(v);
ace         auto [L, M, R] = split3(root, l, r);
d43         T ans = M->sub;
20c         join3(L, M, R);
ba7         return ans;
40f     }
80d     void subtree_update(int v, T val) {
b73         auto [root, l, r] = get_range(v);
ace         auto [L, M, R] = split3(root, l, r);
409         M->lazy += val;
20c         join3(L, M, R);
afb     }

a8e     bool is_ancestor(int v, int u) { // se v eh ancestral de u
497         auto [root_v, l_v, r_v] = get_range(v);
3c7         auto [root_u, l_u, r_u] = get_range(u);
b9c         return root_v == root_u and l_v <= l_u and r_u <= r_v;
786     }
7b9 };
```

6.36 Sack (DSU em arvores)

```
// Responde queries de todas as sub-arvores
// offline
//
// O(n log(n))

6bf int sz[MAX], cor[MAX], cnt[MAX];
042 vector<int> g[MAX];

6df void build(int k, int d=0) {
e8f     sz[k] = 1;
01a     for (auto& i : g[k]) {
30f         build(i, d+1); sz[k] += sz[i];
925         if (sz[i] > sz[g[k][0]]) swap(i, g[k][0]);
011     }
189 }

74f void compute(int k, int x, bool dont=1) {
de9     cnt[cor[k]] += x;
828     for (int i = dont; i < g[k].size(); i++)
b5c         compute(g[k][i], x, 0);
896 }

dc4 void solve(int k, bool keep=0) {
32a     for (int i = int(g[k].size())-1; i >= 0; i--)
b4c         solve(g[k][i], !i);
4a0     compute(k, 1);

        // agora cnt[i] tem quantas vezes a cor
        // i aparece na sub-arvore do k

830     if (!keep) compute(k, -1, 0);
8bc }
```

6.37 Stable Marriage

```
// Emparelha todos os elementos de A com elementos de B
// de forma que nao exista um par x \in A, y \in B
// e x nao pareado com y tal que x prefira parear com y
// e y prefira parear com x.
//
// a[i] contem os elementos de B ordenados por preferencia de i
// b[j] contem os elementos de A ordenados por preferencia de j
// |A| <= |B|
//
// Retorna um vetor v de tamanho |A| onde v[i] guarda o match de i.
//
// O(|A| * |B|)

380 vector<int> stable_marriage(vector<vector<int>> &a,
    vector<vector<int>> &b) {
652     int n = a.size(), m = b.size();
83e     assert(a[0].size() == m and b[0].size() == n and n <= m);
017     vector<int> match(m, -1), it(n, 0);
e6f     vector inv_b(m, vector<int>(n));
a34     for (int i = 0; i < m; i++) for (int j = 0; j < n; j++)
9f2         inv_b[i][b[i][j]] = j;

26a     queue<int> q;
5af     for (int i = 0; i < n; i++) q.push(i);
402     while (q.size()) {
379         int i = q.front(); q.pop();
4b8         int j = a[i][it[i]];

57c         if (match[j] == -1) match[j] = i;
02d         else if (inv_b[j][i] < inv_b[j][match[j]]) {
5d1             q.emplace(match[j]);
e7d             it[match[j]]++;
f1d             match[j] = i;
bc4         } else q.emplace(i), it[i]++;
258     }

825     vector<int> ret(n);
d72     for (int i = 0; i < m; i++) if (match[i] != -1) ret[match[i]]
    = i;
edf     return ret;
0ff }
```

6.38 Tarjan para SCC

```
// O(n + m)

042 vector<int> g[MAX];
4ce stack<int> s;
a42 int vis[MAX], comp[MAX];
3fd int id[MAX];

// se quiser comprimir ciclo ou achar ponte em grafo nao direcionado,
// colocar um if na dfs para nao voltar pro pai da DFS tree
f32 int dfs(int i, int& t) {
cf0     int lo = id[i] = t++;
18e     s.push(i);
0c2     vis[i] = 2;

48e     for (int j : g[i]) {
740         if (!vis[j]) lo = min(lo, dfs(j, t));
994         else if (vis[j] == 2) lo = min(lo, id[j]);
d64     }

        // aresta de i pro pai eh uma ponte (no caso nao direcionado)
3de     if (lo == id[i]) while (1) {
3c3         int u = s.top(); s.pop();
9c5         vis[u] = 1, comp[u] = i;
2ef         if (u == i) break;
266     }

253     return lo;
38a }

f93 void tarjan(int n) {
6bb     int t = 0;
991     for (int i = 0; i < n; i++) vis[i] = 0;

3be     for (int i = 0; i < n; i++) if (!vis[i]) dfs(i, t);
ea1 }
```

6.39 Top Tree

```
// lct<T> t(n, w) armazena uma floresta enraizada dinamica com 'n'
// vertices indexados de 0 a n-1 com peso w[i] do tipo T.
// Varias operacoes podem ser realizadas.
//
// Complexidades:
// Construcao - O(n) espaco e tempo
// Todas as outras operacoes - O(log(n)) amortizado, \Theta(n) pior
    caso
// O(n) memoria em qualquer momento
//
// 1. Consultas estruturais
// - find_root(v): raiz do vertice v (seguindo arestas pai)
// - lca(v, u): ancestral comum mais baixo entre v e u, ou -1 se nao
    existir
// - connected(v, u): se v e u estao no mesmo componente conexo
// - parent(v, k): k-esimo pai de v (seguindo k arestas), ou -1 se nao
    existir
// - depth(v): numero de arestas entre v e sua raiz
// - dist(v, u): distancia entre v e u na arvore nao direcionada
// - subtree_size(v): numero de vertices na sub-arvore de v
// - adjacent(v, u): se existe uma aresta {v, u}
//
// 2. Atualizacoes estruturais
// - evert(v): rotaciona para que v seja a raiz de seu componente
// - link(v, u): adiciona aresta v -> u (pai de v eh u)
//   . Pre-condicao: nao deve criar ciclo!
//   . Se v nao eh raiz, eh feito evert(v)
// - cut(v): remove aresta de v para seu pai
// - cut(v, u): remove aresta {v, u}
//   . Pre-condicao: aresta {v, u} deve existir!
//
// 3. Operacoes em vertices
// - t[v]: retorna w[v]
//   . Pode ser usado para atualizar: t[v]++
//
// Para as seguintes operacoes que agregam w:
//
// Defina uma tag lazy associativa e comutativa para combinar
   atualizacoes em 'struct lazy'
// - Defina o que eh a tag lazy (int)
// - Defina o elemento neutro (0)
// - Defina como combinar tags lazy em 'operator+=' (+=)
// - Defina como aplicar a tag lazy em um valor em 'apply_lazy(val,
   lz)' (+=)
//
// Defina um agregado associativo em 'struct data':
// - Defina o que serah mantido (soma/minimo/tamanho/...)
//   . Manter tamanho eh obrigatorio!
// - Defina o elemento neutro (0/INF/...)
// - Defina como agregar em 'operator+=' (+=/min=/...)
// - Defina como aplicar tag lazy no agregado em 'apply_lazy(lz)'
// - Se a operacao nao for comutativa:
//   . Defina como reverter um intervalo em reverse()
//
// 4. Operacoes em caminhos
// - path_query(v): retorna o agregado de w de v ate sua raiz
//   . Nessa ordem, para operacoes nao comutativas!
// - path_query(v, u): retorna o agregado de w de v ateh u (nao
   direcionado)
//   . Nessa ordem, para operacoes nao comutativas!
// - path_update(v, u, val): aplica lazy(val) em w de v ate u (nao
   direcionado)
//
// 5. Operacoes em sub-arvores
// - subtree_query(v, exclude_v): retorna o agregado de w na
   sub-arvore de v
//   . Se exclude_v, exclui v do agregado
//   . Ordem nao eh definida para operacoes nao comutativas!
// - subtree_update(v, val, exclude_v): aplica lazy(val) em w na
   sub-arvore de v
//   . Se exclude_v, exclui v da atualizacao
//
// Para a proxima operacao que combina agregados de filhos:
//
// Defina uma combinacao associativa e comutativa de filhos em
   'data_child'
// - Defina o elemento neutro
// - Defina o valor de um filho, dado seu agregado
   (tamanho/tamanho^2/soma/...)
// - Defina como combinar valores de filhos (+=/min=/...)
//
// 6. Combinacao de dados dos filhos
// - combine_children(v): retorna a combinacao dos filhos de v
//   . Se voce definir data_child() = 1 e combinar com +=,
//     retorna numero de filhos

b5d template<typename T> struct lct {
07c     struct lazy {
f48         T val;

d26         lazy(T val_ = 0) : val(val_) {}
be2         lazy& operator +=(const lazy& lz) {
17c             val += lz.val;
357             return *this;
fed         }
2e2     };
64b     void apply_lazy(T& val, const lazy& lz) {
17c         val += lz.val;
073     };
994     struct data {
026         T sum;
e4d         int sz;

62f         data() : sum(0), sz(0) {}
ce5         data(T val) : sum(val), sz(1) {}

382         data& operator +=(const data& d) {
4b6             sum += d.sum;
73c             sz += d.sz;
357             return *this;
cbb         }
66a         friend data operator +(data a, const data& b) { return a
```

```
    += b; }

c50             void apply_lazy(const lazy& lz) {
977                 if (sz == 0) return;
2a5                 sum += lz.val * sz;
b76             }
ae9             void reverse() {}
5a9         };
97c         struct data_child {
0f9             int cnt;

afd             data_child() : cnt(0) {}
9d4             data_child(data sub) { // 'sub' representa dado da
    sub-arvore
88d                 cnt = 1; // para numero de filhos
7ad             }
b76             data_child& operator +=(const data_child& d) {
eb8                 cnt += d.cnt;
357                 return *this;
64a             }
ef4             friend data_child operator +(data_child a, const
    data_child& b) { return a += b; }
52c         };

3c9         struct node {
f48             T val;
2fc             int p, ch[4];
7e4             data path, sub, all;
d9d             data_child data_ch;
43e             lazy lz_path, lz_sub;
533             bool rev, fake;

373             node() : p(-1), path(), sub(), all(), data_ch(),
    lz_path(), lz_sub(), rev(0), fake(1) {
dc5                 memset(ch, -1, sizeof ch);
cf9             }
a94             node(T val_) : node() {
d9b                 val = val_;
446                 path = all = data(val);
e12                 data_ch = data_child(all);
e7c                 fake = false;
168             }
44d         };

bbb         vector<node> t;
08f         vector<int> fakes;
ce0         int free_fake;

61e         lct(int n, vector<T> w = {}) : t(2*n), fakes(n), free_fake(n) {
9a9             if (!w.size()) w = vector<T>(n, T());
461             for (int i = 0; i < n; i++) t[i] = node(w[i]);
2b9             iota(fakes.begin(), fakes.end(), n);
1b0         }

ec8         void prop_rev(int v) {
501             if (v == -1) return;

953             t[v].path.reverse();
b5a             swap(t[v].ch[0], t[v].ch[1]);
a02             t[v].rev ^= 1;
a20         }
f9b         void prop_path(int v, const lazy& lz) {
53a             if (v == -1 or t[v].fake) return;

906             apply_lazy(t[v].val, lz);

6ff             t[v].path.apply_lazy(lz);
1e8             t[v].lz_path += lz;
361             t[v].all = t[v].path + t[v].sub;
4df             t[v].data_ch = data_child(t[v].all);
3df         }
cbb         void prop_sub(int v, const lazy& lz, bool virt) {
501             if (v == -1) return;

db2             t[v].sub.apply_lazy(lz);
8a0             t[v].lz_sub += lz;
91b             if (!t[v].fake and virt) prop_path(v, lz);
46e             else t[v].all = t[v].path + t[v].sub;
58d         }
098         void prop(int v) { // lazy nao inclui o vertice
501             if (v == -1) return;
2d0             if (t[v].rev) {
8c3                 for (int i = 0; i < 2; i++) prop_rev(t[v].ch[i]);
1e3                 t[v].rev = false;
ae5             }
dd1             if (t[v].lz_path.val != lazy().val) {
492                 for (int i = 0; i < 2; i++) prop_path(t[v].ch[i],
    t[v].lz_path);
806                 t[v].lz_path = lazy();
048             }
b96             if (t[v].lz_sub.val != lazy().val) {
6a5                 for (int i = 0; i < 4; i++) prop_sub(t[v].ch[i],
    t[v].lz_sub, i > 1);
65e                 t[v].lz_sub = lazy();
```

```
aa2         }
db4     }
437     node get_ch(int v, int c) {
17f         return t[v].ch[c] == -1 ? node() : t[t[v].ch[c]];
ee8     }
31f     void update(int v) {
ef6         if (!t[v].fake) t[v].path = get_ch(v, 1).path + data(t[v].val) + get_ch(v, 0).path;
119         t[v].sub = get_ch(v, 0).sub + get_ch(v, 1).sub + get_ch(v, 2).all + get_ch(v, 3).all;
361         t[v].all = t[v].path + t[v].sub;

f06         if (t[v].fake) t[v].data_ch = get_ch(v, 2).data_ch + get_ch(v, 3).data_ch;
bc2         else t[v].data_ch = data_child(t[v].all);
e3b     }

8b9     int dir(int v) {
4cb         if (t[v].p == -1) return -1;
c04         for (int i = 0; i < 4; i++) if (t[t[v].p].ch[i] == v) return i;
daa         return -1;
66b     }
679     bool is_root(int v) {
252         if (t[v].p == -1) return true;
caf         return t[v].fake ? !t[t[v].p].fake : (dir(v)&2);
287     }
a5e     void rotate(int v) {
078         int p = t[v].p, pp = t[p].p, dir_v = dir(v), dir_p = dir(p);

8b6         attach(p, dir_v, t[v].ch[dir_v^1]);
a67         attach(v, dir_v^1, p);
6c7         if (dir_p+1) attach(pp, dir_p, v);
72e         else t[v].p = pp;
8c5     }
ec1     void splay(int v) {
5e3         prop(v);
7d3         while (!is_root(v)) {
931             int p = t[v].p, pp = t[p].p;
bf6             if (pp+1) prop(pp);
a51             prop(p), prop(v);
c31             if (!is_root(p)) rotate(dir(v) == dir(p) ? p : v);
e61             rotate(v);
5d2         }
bed     }
29f     void attach(int v, int c, int u) {
908         t[v].ch[c] = u;
c99         if (u+1) t[u].p = v;
35b         update(v);
93d     }
e07     void add_virt_ch(int v, int u) {
61c         if (u == -1) return;
b51         for (int i = 2; i < 4; i++) if (t[v].ch[i] == -1) {
dab             attach(v, i, u);
505             return;
019         }
680         int w = fakes[--free_fake];
782         attach(w, 2, t[v].ch[2]);
6ef         attach(w, 3, u), attach(v, 2, w);
999     }
e26     void prop_path(int v) {
9b1         if (t[v].fake) prop_path(t[v].p);
5e3         prop(v);
a98     }
4f3     void rem_virt_ch(int v) {
672         int p = t[v].p;
038         prop_path(p);
656         if (t[p].fake) {
676             int pp = t[p].p;
f23             attach(pp, dir(p), t[p].ch[dir(v)^1]);
7e2             if (t[pp].fake) splay(pp);
ffe             fakes[free_fake++] = p;
df2         } else attach(p, dir(v), -1);
07f         t[v].p = -1;
a88     }
7f6     int real_par(int v) {
31a         v = t[v].p;
ae6         if (!t[v].fake) return v;
bf5         splay(v);
fe7         return t[v].p;
973     }
f16     int access(int v) { // retorna ultimo vertice acessado
35b         update(v);
bf5         splay(v);
c4b         add_virt_ch(v, t[v].ch[1]);
b49         attach(v, 1, -1);
aee         int w = v;
7c5         while (t[v].p+1) {
425             w = real_par(v);
50a             splay(w);

323             rem_virt_ch(v);
547             add_virt_ch(w, t[w].ch[1]);
```

```
b4e                 attach(w, 1, v);

bf5                 splay(v);
da0             }
df1             return w;
e96         }

            // consultas estruturais
e89         int find_root(int v) {
5e3             access(v);
9f0             while (t[v].ch[0]+1) v = t[v].ch[0], prop(v);
f05             return splay(v), v;
a26         }
c5d         int lca(int v, int u) {
b77             int ret = (access(v), access(u));
a2a             if (v != u and t[v].p == -1) return -1;
edf             return ret;
872         }
05f         bool connected(int v, int u) {
d16             return lca(v, u) != -1;
1a1         }
ba2         int parent(int v, int k = 1) {
5e3             access(v);
217             if (k >= t[v].path.sz) return -1;

667             while (true) {
c0f                 int sz_r = t[v].ch[1] == -1 ? 0 :
    t[t[v].ch[1]].path.sz;
c4a                 if (k < sz_r) v = t[v].ch[1], prop(v);
4e6                 else {
733                     k -= sz_r;
3fd                     if (k == 0) return splay(v), v;
01b                     v = t[v].ch[0], prop(v);
9bc                     k--;
acf                 }
077             }
67d         }
baa         int depth(int v) {
5e3             access(v);
192             return t[v].path.sz - 1;
ec2         }
419         int dist(int v, int u) {
de8             return depth(v) + depth(u) - 2*depth(lca(v, u));
1f5         }
363         int subtree_size(int v) {
5e3             access(v);
f37             return get_ch(v, 2).all.sz + get_ch(v, 3).all.sz + 1;
eba         }
417         bool adjacent(int v, int u) {
54a             return connected(v, u) and dist(v, u) == 1;
eb0         }

            // atualizacoes estruturais
adb         void evert(int v) { // rotaciona para que v seja a raiz
5e3             access(v);
533             prop_rev(v);
bd9         }
7af         void link(int v, int u) { // adiciona aresta v -> u (pai de v
    eh u) - nao pode criar ciclo!
5c2             evert(v), access(u);
7b7             add_virt_ch(u, v);
d4a         }
4e6         void cut(int v) { // corta aresta de v para o pai
5e3             access(v);
b1d             if (t[v].ch[0]+1) t[t[v].ch[0]].p = -1;
5ff             t[v].ch[0] = -1;
35b             update(v);
8ad         }
af7         void cut(int v, int u) { // aresta {v, u} deve existir!
8c6             cut(depth(v) > depth(u) ? v : u);
416         }

            // operacao de vertice
5f1         T& operator[](int v) { // w[v] (pode ser alterado)
5e3             access(v);
337             return t[v].val;
492         }

            // operacoes de caminho - vertices devem estar no mesmo
                componente!
7f4         data path_query(int v) { // de v para a raiz
5e3             access(v);
1bb             return t[v].path;
2c5         }
76d         data path_query(int v, int u) { // de v para u
db2             int rt = find_root(v);
708             evert(u), access(v);
b4e             auto ret = t[v].path;
e70             evert(rt);
edf             return ret;
267         }
ee6         void path_update(int v, int u, T val) {
db2             int rt = find_root(v);
708             evert(u), access(v);
```

```
715         prop_path(v, lazy(val));
e70         evert(rt);
cfe     }

        // operacoes de sub-arvore
acc     data subtree_query(int v, bool exclude_v = false) {
5e3         access(v);
ffe         data ret = get_ch(v, 2).all + get_ch(v, 3).all;
827         if (!exclude_v) ret += data(t[v].val);
edf         return ret;
696     }
56f     void subtree_update(int v, T val, bool exclude_v = false) {
5e3         access(v);
207         prop_sub(t[v].ch[2], lazy(val), true);
bb9         prop_sub(t[v].ch[3], lazy(val), true);
bf9         if (!exclude_v) apply_lazy(t[v].val, lazy(val));
7b9     }

        // combinacao de dados das sub-arvores dos filhos
26e     data_child combine_children(int v) {
5e3         access(v);
db4         return get_ch(v, 2).data_ch + get_ch(v, 3).data_ch;
303     }
a22 };
```

6.40 Topological Sort

```
// Retorna uma ordenacaoo topologica de g
// Se g nao for DAG retorna um vetor vazio
//
// O(n + m)

042 vector<int> g[MAX];

b6a vector<int> topo_sort(int n) {
46e     vector<int> ret(n,-1), vis(n,0);

f51     int pos = n-1, dag = 1;
36d     function<void(int)> dfs = [&](int v) {
cca         vis[v] = 1;
440         for (auto u : g[v]) {
152             if (vis[u] == 1) dag = 0;
532             else if (!vis[u]) dfs(u);
e37         }
d44         ret[pos--] = v, vis[v] = 2;
57e     };

158     for (int i = 0; i < n; i++) if (!vis[i]) dfs(i);

d8f     if (!dag) ret.clear();
edf     return ret;
d6b }
```

6.41 Unrooted Euler Tour Tree

```
// Mantem uma floresta dinamicamente
// e permite queries/updates em componentes
//
// Chamar ETT E(n, w), passando n = numero de vertices
// e w = vector com os valores de cada vertice
//
// link(v, u) cria uma aresta entre v e u
// cut(v, u)  corta aresta entre v e u
// get_root(v) retorna a raiz de v
// branc_query(v, u) e branch_update(v, u) operam sobre o componente
// conexo de v se arestas {v, u} fosse removida do grafo
// comp_query(v) e comp_update(v, val) operam sobre o compoente conexo
    de v
// point_update(v, val) atualiza o valor do vertice v
// point_update(v, val, u) atualiza o valor da aresta {v, u}
//
// Tudo O(log(n)) com alta probabilidade

878 mt19937 rng((int)
    chrono::steady_clock::now().time_since_epoch().count());

9c9 template<typename T> struct unrooted_ett {
83c     const static T ZERO = T();
3c9     struct node {
ed1         node *l, *r, *p;
1c1         bool is_vtx;
eba         int pr, sz, vtx_cnt;
875         T val, sub, lazy;
b5f         node(T v, bool is_vtx_ = false) : l(NULL), r(NULL),
    p(NULL), is_vtx(is_vtx_),
4ef             pr(rng()), sz(1), vtx_cnt(is_vtx), val(v), sub(v),
    lazy(ZERO) {}
a9c         void prop() {
9fc             if (lazy != ZERO) {
094                 if (is_vtx) val += lazy;
489                 sub += lazy * vtx_cnt;
b87                 if (l) l->lazy += lazy;
d3b                 if (r) r->lazy += lazy;
f2b             }
520             lazy = ZERO;
d34         }
01e         void update() {
1eb             sz = 1, vtx_cnt = is_vtx, sub = val;
ac2             for (auto& i : {l, r}) if (i) {
c89                 i->prop();
fe4                 sz += i->sz, vtx_cnt += i->vtx_cnt;
ea7                 sub += i->sub;
ac7             }
3e7         }
eaf     };

73c     int size(node* x) { return x ? x->sz : 0; }
bcf     void join(node* l, node* r, node*& i) {
986         if (!l or !r) return void(i = l ? l : r);
161         l->prop(), r->prop();
ff5         if (l->pr > r->pr) join(l->r, r, l->r), l->r->p = i = l;
982         else join(l, r->l, r->l), r->l->p = i = r;
bda         i->update();
84d     }
a20     void split(node* i, node*& l, node*& r, int v, int key = 0) {
26a         if (!i) return void(r = l = NULL);
c89         i->prop();
d9e         if (key + size(i->l) < v) {
448             split(i->r, i->r, r, v, key+size(i->l)+1), l = i;
a21             if (r) r->p = NULL;
6e8             if (i->r) i->r->p = i;
396         } else {
98d             split(i->l, l, i->l, v, key), r = i;
5a3             if (l) l->p = NULL;
899             if (i->l) i->l->p = i;
18b         }
bda         i->update();
134     }
3fa     pair<node*, int> get_idx(node* i) { // {root, idx}
6cf         int ret = size(i->l);
65d         for (; i->p; i = i->p) if (i != i->p->l)
fc8             ret += size(i->p->l) + 1;
c89         i->prop();
e51         return {i, ret};
fa4     }

        // as treaps sao disjuntas!
3cc     map<pair<int, int>, node*> mp;
```

```
72a     unrooted_ett(int n, const vector<T>& w) { // w = valor de cada
    vertice
98f         for (int i = 0; i < n; i++) mp[pair(i, i)] = new
    node(w[i], true);
045     }
8d2     unrooted_ett(const unrooted_ett& t) { throw logic_error("Nao
    copiar a ETT!"); }
057     ~unrooted_ett() { for (auto [__, ptr] : mp) delete ptr; }

57a     void link(int v, int u, T val = ZERO) {
0e6         auto [root_v, idx_v] = get_idx(mp[pair(v, v)]);
a60         node *L_v, *R_v;
f8c         split(root_v, L_v, R_v, idx_v);
f7a         join(R_v, L_v, root_v);

a2b         auto [root, idx] = get_idx(mp[pair(u, u)]);
031         node *L, *R;
680         split(root, L, R, idx);
1d6         join(L, mp[pair(u, v)] = new node(val), L); // w[(u, v)]
7ce         join(L, root_v, L);
37b         join(L, mp[pair(v, u)] = new node(ZERO), L); // w[(v, u)]
0c7         join(L, R, R);
1b4     }
85c     node* cut(int v, int u) { // retorna a raiz de v
4c2         auto [root, l] = get_idx(mp[pair(u, v)]);
4a4         int r = get_idx(mp[pair(v, u)]).second;
541         bool rev = false;
19f         if (l > r) swap(l, r), rev = true;

bae         node *L, *E1, *M, *E2, *R;
c6e         split(root, E2, R, r+1);
a8c         split(E2, M, E2, r);
b7a         split(M, E1, M, l+1);
1aa         split(E1, L, E1, l);

739         delete E1;
ef6         delete E2;
ac8         mp.erase(pair(v, u));
717         mp.erase(pair(u, v));

0c7         join(L, R, R);

b31         return rev ? R : M;
ca8     }

        // opera sobre component(v) em F \ {v, u}
13f     T branch_query(int v, int u) {
```

```
4da         auto root = cut(v, u);
e38         T ret = root->sub;
32e         link(v, u);
edf         return ret;
0e3     }
2bf     void branch_update(int v, int u, T val) {
4da         auto root = cut(v, u);
fc6         root->lazy += val;
32e         link(v, u);
827     }

        // opera sobre component(v)
b3d     T comp_query(int v) {
da0         auto root = get_idx(mp[pair(v, v)]).first;
6f0         return root->sub;
4fd     }
623     void comp_update(int v, T val) {
da0         auto root = get_idx(mp[pair(v, v)]).first;
fc6         root->lazy += val;
0e4     }

        // w[v] += val, ou w[(v, u)] += val
e21     void point_update(int v, T val, int u = -1) {
e4a         if (u == -1) u = v;
ca9         auto [root, idx] = get_idx(mp[pair(v, u)]);

df9         node *L, *M, *R;
d26         split(root, M, R, idx+1);
70b         split(M, L, M, idx);

409         M->lazy += val;

675         join(L, M, M);
087         join(M, R, R);
3f0     }
9b3 };
```

6.42 Vertex cover

```
// Encontra o tamanho do vertex cover minimo
// Da pra alterar facil pra achar os vertices
// Parece rodar com < 2 s pra N = 90
//
// O(n * 1.38^n)

76a namespace cover {
5a4     const int MAX = 96;
042     vector<int> g[MAX];
823     bitset<MAX> bs[MAX];
1a8     int n;

697     void add(int i, int j) {
bd0         if (i == j) return;
78c         n = max({n, i+1, j+1});
200         bs[i][j] = bs[j][i] = 1;
203     }

6c0     int rec(bitset<MAX> m) {
1a4         int ans = 0;
25b         for (int x = 0; x < n; x++) if (m[x]) {
002             bitset<MAX> comp;
4bf             function<void(int)> dfs = [&](int i) {
b96                 comp[i] = 1, m[i] = 0;
0c3                 for (int j : g[i]) if (m[j]) dfs(j);
815             };
963             dfs(x);

d34             int ma, deg = -1, cyc = 1;
417             for (int i = 0; i < n; i++) if (comp[i]) {
d0b                 int d = (bs[i]&comp).count();
18a                 if (d <= 1) cyc = 0;
c1f                 if (d > deg) deg = d, ma = i;
d8e             }
269             if (deg <= 2) { // caminho ou ciclo
340                 ans += (comp.count() + cyc) / 2;
5e2                 continue;
702             }
3f9             comp[ma] = 0;

                // ou ta no cover, ou nao ta no cover
1dd             ans += min(1 + rec(comp), deg + rec(comp & ~bs[ma]));
6e6         }
ba7         return ans;
2ec     }
f5c     int solve() {
3c5         bitset<MAX> m;
603         for (int i = 0; i < n; i++) {
939             m[i] = 1;
f90             for (int j = 0; j < n; j++)
741                 if (bs[i][j]) g[i].push_back(j);
13e         }
4f9         return rec(m);
708     }
9c5 }
```

6.43 Virtual Tree

```
// Comprime uma arvore dado um conjunto S de vertices, de forma que
// o conjunto de vertices da arvore comprimida contenha S e seja
// minimal e fechado sobre a operacao de LCA
// Se |S| = k, a arvore comprimida tem menos que 2k vertices
// As arestas de virt possuem a distancia do vertice ate o vizinho
// Retorna a raiz da virtual tree
//
// lca::pos deve ser a ordem de visitacao no dfs
// voce pode usar o LCAcomHLD, por exemplo
//
// O(k log(k))

b36 vector<pair<int, int>> virt[MAX];

d41 #warning lembrar de buildar o LCA antes
c14 int build_virt(vector<int> v) {
b46     auto cmp = [&](int i, int j) { return lca::pos[i] <
    lca::pos[j]; };
074     sort(v.begin(), v.end(), cmp);
e85     for (int i = v.size()-1; i; i--) v.push_back(lca::lca(v[i],
    v[i-1]));
074     sort(v.begin(), v.end(), cmp);
d76     v.erase(unique(v.begin(), v.end()), v.end());
37c     for (int i = 0; i < v.size(); i++) virt[v[i]].clear();
197     for (int i = 1; i < v.size(); i++) virt[lca::lca(v[i-1],
    v[i])].clear();
ad7     for (int i = 1; i < v.size(); i++) {
51b         int parent = lca::lca(v[i-1], v[i]);
290         int d = lca::dist(parent, v[i]);
d41 #warning soh to colocando aresta descendo
4d0         virt[parent].emplace_back(v[i], d);
fe5     }
832     return v[0];
142 }
```

7 DP

7.1 Divide and Conquer DP

```
// Particiona o array em k subarrays
// minimizando o somatorio das queries
//
// O(k n log n), assumindo quer query(l, r) eh O(1)

547 ll dp[MAX][2];

94b void solve(int k, int l, int r, int lk, int rk) {
de6     if (l > r) return;
109     int m = (l+r)/2, p = -1;
d2b     auto& ans = dp[m][k&1] = LINF;
6e2     for (int i = max(m, lk); i <= rk; i++) {
07b         ll at = dp[i+1][~k&1] + query(m, i);
57d         if (at < ans) ans = at, p = i;
8f5     }
1ee     solve(k, l, m-1, lk, p), solve(k, m+1, r, p, rk);
d3e }

cf1 ll DC(int n, int k) {
321     dp[n][0] = dp[n][1] = 0;
f27     for (int i = 0; i < n; i++) dp[i][0] = LINF;
b76     for (int i = 1; i <= k; i++) solve(i, 0, n-i, 0, n-i);
8e7     return dp[0][k&1];
5e9 }
```

7.2 Longest Common Subsequence

```
// Computa a LCS entre dois arrays usando
// o algoritmo de Hirschberg para recuperar
//
// O(n*m), O(n+m) de memoria

eaf int lcs_s[MAX], lcs_t[MAX];
a6d int dp[2][MAX];

// dp[0][j] = max lcs(s[li...ri], t[lj, lj+j])
d12 void dp_top(int li, int ri, int lj, int rj) {
d13     memset(dp[0], 0, (rj-lj+1)*sizeof(dp[0][0]));
753     for (int i = li; i <= ri; i++) {
9aa         for (int j = rj; j >= lj; j--)
83b             dp[0][j - lj] = max(dp[0][j - lj],
741             (lcs_s[i] == lcs_t[j]) + (j > lj ? dp[0][j-1 - lj] :
    0));
04c         for (int j = lj+1; j <= rj; j++)
939             dp[0][j - lj] = max(dp[0][j - lj], dp[0][j-1 -lj]);
09f     }
58f }

// dp[1][j] = max lcs(s[li...ri], t[lj+j, rj])
ca0 void dp_bottom(int li, int ri, int lj, int rj) {
0dd     memset(dp[1], 0, (rj-lj+1)*sizeof(dp[1][0]));
3a2     for (int i = ri; i >= li; i--) {
49c         for (int j = lj; j <= rj; j++)
dbb             dp[1][j - lj] = max(dp[1][j - lj],
4da             (lcs_s[i] == lcs_t[j]) +  (j < rj ? dp[1][j+1 - lj] :
    0));
6ca         for (int j = rj-1; j >= lj; j--)
769             dp[1][j - lj] = max(dp[1][j - lj], dp[1][j+1 - lj]);
19b     }
e8a }

93c void solve(vector<int>& ans, int li, int ri, int lj, int rj) {
2ad     if (li == ri){
49c         for (int j = lj; j <= rj; j++)
f5b             if (lcs_s[li] == lcs_t[j]){
a66                 ans.push_back(lcs_t[j]);
c2b                 break;
840             }
505         return;
126     }
534     if (lj == rj){
753         for (int i = li; i <= ri; i++){
88f             if (lcs_s[i] == lcs_t[lj]){
531                 ans.push_back(lcs_s[i]);
c2b                 break;
68a             }
a03         }
505         return;
76d     }
a57     int mi = (li+ri)/2;
ade     dp_top(li, mi, lj, rj), dp_bottom(mi+1, ri, lj, rj);

d7a     int j_ = 0, mx = -1;

aee     for (int j = lj-1; j <= rj; j++) {
da8         int val = 0;
2bb         if (j >= lj) val += dp[0][j - lj];
b9e         if (j < rj) val += dp[1][j+1 - lj];

ba8         if (val >= mx) mx = val, j_ = j;
14e     }
6f1     if (mx == -1) return;
c2a     solve(ans, li, mi, lj, j_), solve(ans, mi+1, ri, j_+1, rj);
dd5 }

058 vector<int> lcs(const vector<int>& s, const vector<int>& t) {
953     for (int i = 0; i < s.size(); i++) lcs_s[i] = s[i];
577     for (int i = 0; i < t.size(); i++) lcs_t[i] = t[i];
dab     vector<int> ans;
599     solve(ans, 0, s.size()-1, 0, t.size()-1);
ba7     return ans;
17c }
```

7.3 Mochila

```
// Resolve mochila, recuperando a resposta
//
// O(n * cap), O(n + cap) de memoria

add int v[MAX], w[MAX]; // valor e peso
582 int dp[2][MAX_CAP];

// DP usando os itens [l, r], com capacidade = cap
0d6 void get_dp(int x, int l, int r, int cap) {
f8f     memset(dp[x], 0, (cap+1)*sizeof(dp[x][0]));
574     for (int i = l; i <= r; i++) for (int j = cap; j >= 0; j--)
3a9         if (j - w[i] >= 0) dp[x][j] = max(dp[x][j], v[i] + dp[x][j
    - w[i]]);
b95 }

5ab void solve(vector<int>& ans, int l, int r, int cap) {
893     if (l == r) {
9ff         if (w[l] <= cap) ans.push_back(l);
505         return;
13a     }
ee4     int m = (l+r)/2;
283     get_dp(0, l, m, cap), get_dp(1, m+1, r, cap);
056     int left_cap = -1, opt = -INF;
c94     for (int j = 0; j <= cap; j++)
2f2         if (int at = dp[0][j] + dp[1][cap - j]; at > opt)
91d             opt = at, left_cap = j;
da3     solve(ans, l, m, left_cap), solve(ans, m+1, r, cap - left_cap);
d75 }

0d7 vector<int> knapsack(int n, int cap) {
dab     vector<int> ans;
1e0     solve(ans, 0, n-1, cap);
ba7     return ans;
e4d }
```

7.4 SOS DP

```
// O(n 2^n)

// soma de sub-conjunto
e03 vector<ll> sos_dp(vector<ll> f) {
6c0     int N = __builtin_ctz(f.size());
e59     assert((1<<N) == f.size());

5a5     for (int i = 0; i < N; i++) for (int mask = 0; mask < (1<<N);
    mask++)
796         if (mask>>i&1) f[mask] += f[mask^(1<<i)];
abe     return f;
bec }

// soma de super-conjunto
e03 vector<ll> sos_dp(vector<ll> f) {
6c0     int N = __builtin_ctz(f.size());
e59     assert((1<<N) == f.size());

5a5     for (int i = 0; i < N; i++) for (int mask = 0; mask < (1<<N);
    mask++)
a3c         if (~mask>>i&1) f[mask] += f[mask^(1<<i)];
abe     return f;
dbd }
```

7.5 Subset sum

```
// Retorna max(x <= t tal que existe subset de w que soma x)
//
// O(n * max(w))
// O(max(w)) de memoria

efd int subset_sum(vector<int> w, int t) {
bb5     int pref = 0, k = 0;
417     while (k < w.size() and pref + w[k] <= t) pref += w[k++];
1e7     if (k == w.size()) return pref;
444     int W = *max_element(w.begin(), w.end());
44d     vector<int> last, dp(2*W, -1);
d7b     dp[W - (t-pref)] = k;
54d     for (int i = k; i < w.size(); i++) {
288         last = dp;
15f         for (int x = 0; x < W; x++) dp[x+w[i]] = max(dp[x+w[i]],
    last[x]);
17b         for (int x = 2*W - 1; x > W; x--)
303             for (int j = max(0, last[x]); j < dp[x]; j++)
595                 dp[x-w[j]] = max(dp[x-w[j]], j);
867     }
2fb     int ans = t;
1c1     while (dp[W - (t-ans)] < 0) ans--;
ba7     return ans;
d88 }
```

8 Extra

8.1 pragma.cpp

```
// Otimizacoes agressivas, pode deixar mais rapido ou mais devagar
#pragma GCC optimize("Ofast")
// Auto explicativo
#pragma GCC optimize("unroll-loops")
// Vetorizacao
#pragma GCC target("avx2")
// Para operacoes com bits
#pragma GCC target("bmi,bmi2,popcnt,lzcnt")
```

8.2 stress.sh

```
P=a
make ${P} ${P}2 gen || exit 1
for ((i = 1; ; i++)) do
    ./gen $i > in
    ./${P} < in > out
    ./${P}2 < in > out2
    if (! cmp -s out out2) then
        echo "--> entrada:"
        cat in
        echo "--> saida1:"
        cat out
        echo "--> saida2:"
        cat out2
        break;
    fi
    echo $i
done
```

8.3 vimrc

```
189 "" {
d79 set ts=4 sw=4 mouse=a nu ai si undofile
7c9 function H(l)
496     return system("sed '/^#/d' | cpp -dD -P -fpreprocessed | tr -d
    '[:space:]' | md5sum", a:l)
0be endfunction
329 function P() range
dd8     for i in range(a:firstline, a:lastline)
ccc         let l = getline(i)
139         call cursor(i, len(l))
7c9         echo H(getline(search('{}'[1], 'bc', i) ? searchpair('{',
    '', '}', 'bn') : i, i))[0:2] l
bf9     endfor
0be endfunction
90e vmap <C-H> :call P()<CR>
de2 "" }
```

8.4 debug.cpp

```
void debug_out(string s, int line) { cerr << endl; }
template<typename H, typename... T>
void debug_out(string s, int line, H h, T... t) {
    if (s[0] != ',') cerr << "Line(" << line << ") ";
    do { cerr << s[0]; s = s.substr(1);
    } while (s.size() and s[0] != ',');
    cerr << " = " << h;
    debug_out(s, line, t...);
}
#ifdef DEBUG
#define debug(...) debug_out(#__VA_ARGS__, __LINE__, __VA_ARGS__)
#else
#define debug(...) 42
#endif
```

8.5 template.cpp

```
#include <bits/stdc++.h>

using namespace std;

#define _ ios_base::sync_with_stdio(0);cin.tie(0);
#define endl '\n'

typedef long long ll;

const int INF = 0x3f3f3f3f;
const ll LINF = 0x3f3f3f3f3f3f3f3fll;

int main() { _
    exit(0);
}
```

8.6 makefile

```
CXX = g++
CXXFLAGS = -fsanitize=address,undefined -fno-omit-frame-pointer -g
    -Wall -Wshadow -std=c++17 -Wno-unused-result -Wno-sign-compare
    -Wno-char-subscripts #-fuse-ld=gold
```

8.7 timer.cpp

```
// timer T; T() -> retorna o tempo em ms desde que declarou
using namespace chrono;
struct timer : high_resolution_clock {
    const time_point start;
    timer(): start(now()) {}
    int operator()() {
        return duration_cast<milliseconds>(now() - start).count();
    }
};
```

8.8 fastIO.cpp

```
int read_int() {
    bool minus = false;
    int result = 0;
    char ch;
    ch = getchar();
    while (1) {
        if (ch == '-') break;
        if (ch >= '0' && ch <= '9') break;
        ch = getchar();
    }
    if (ch == '-') minus = true;
    else result = ch-'0';
    while (1) {
        ch = getchar();
        if (ch < '0' || ch > '9') break;
        result = result*10 + (ch - '0');
    }
    if (minus) return -result;
    else return result;
}
```

8.9 hash.sh

```
# Para usar (hash das linhas [l1, l2]):
# bash hash.sh arquivo.cpp l1 l2
sed -n $2','$3' p' $1 | sed '/^#w/d' | cpp -dD -P -fpreprocessed | tr
    -d '[:space:]' | md5sum | cut -c-6
```

8.10 rand.cpp

```
mt19937 rng((int)
    chrono::steady_clock::now().time_since_epoch().count());

int uniform(int l, int r){
    uniform_int_distribution<int> uid(l, r);
    return uid(rng);
}
```