RAFAEL VILLASMIL
ASSIGNMENT 2

PROOF ERROR PROPAGATION FOR

$$f(x,y) = \frac{x}{y}$$

$$f(x_{i+1}, y_{i+1}) = f(x_i, y_i) + \frac{\Delta f}{\Delta x}(x_{i+1} - x_i + \frac{\Delta f}{\Delta y}(y_{i+1} - y_i) \ldots$$

WHERE ALL PARTIAL DERIVATIVES ARE EVALUATED AT THE BASE POINT. IF ALL SECOND-ORDER AND HIGHER TERMS ARE DROPPED. WE GET

$$\Delta f(\hat{x}, \hat{y}) = \left|\frac{\Delta f}{\Delta x}\right| \Delta \hat{x} + \left|\frac{\Delta f}{\Delta y}\right| \Delta \hat{y}$$

$$\frac{\Delta f}{\Delta x} = \frac{x^2}{y}, \quad \frac{\Delta f}{\Delta y} = -\frac{x}{y^2}$$

$$\Delta f(x,y) = \frac{x^2}{y} \Delta \hat{x} + \left(-\frac{x}{y^2}\right) \Delta \hat{y}$$

$$\boxed{\Delta f(x,y) = \frac{x^2}{y} \Delta \hat{x} - \frac{x}{y^2} \Delta \hat{y}}$$

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PROBLEM 4.6

PREDICT $Ln(2.5)$ USING ZERO TO 4TH ORDER TAYLOR SERIES WITH A BASE POINT OF 1

$f(x) = Ln(x) \Rightarrow$

$f(x)^1 = \frac{1}{x} \Rightarrow$

$f(x)^2 = -\frac{1}{x^2} \Rightarrow$

$f(x)^3 = -1(-2)\frac{1}{x^3} = \frac{2}{x^3} \Rightarrow$

$f(x)^4 = 2(-3)\frac{1}{x^4} = \frac{-6}{x^4}$

$f(1) = Ln(1) = 0$

$f(1)^1 = 1/1 = 1$

$f(1)^2 = -1/1^2 = -1$

$f(1)^3 = 2/1^3 = 2$

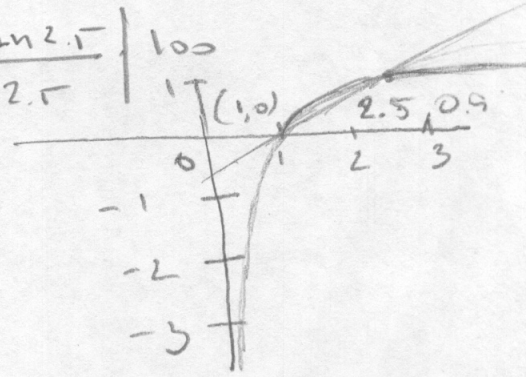
$f(1)^4 = -6/1^4 = -6$

$$Ln(2.5) \cong f(1) + \frac{f(1)^1 \cdot (2.5-1)}{1!} + \frac{f(1)^2 (1.5)^2}{2!} + \frac{f(1)^3 (1.5)^3}{3!} + \frac{f(1)^4 (1.5)^4}{4!}$$

$$Ln(2.5) \cong 0 + \frac{1(1.5)}{1} + \frac{(-1)(1.5)^2}{2} + \frac{(2)(1.5)^3}{6} + \frac{(-6)(1.5)^4}{24}$$

$$Ln(2.5) \cong 0 + 1.5 - 1.125 + 1.125 - 1.265625$$

EXPANSION	APPROXIMATE	$\varepsilon_T = \left\lvert \frac{V_A - Ln 2.5}{Ln 2.5} \right\rvert 100$
0	0	100%
1	1.5	63.7%
2	0.375	59.1%
3	1.5	63.7%
4	0.234375	74.4%



THE FUNCTION $f(x) = Ln(x)$ IS NON-LINEAR AT $X = 2.5$ AND THE STEP OF 1 IS TOO HIGH

RAFAEL VILLASMIL ASSIGNMENT 2 PROB. 4.11

$T = 6 \quad g = 9.81 \quad m = 50 \quad c = 12.5 \quad \tilde{c} = \pm 1.5$

$$V(T) = \frac{gm}{c}\left(1 - e^{-(c/m)T}\right)$$ DO FIRST ERROR ANALYSIS FOR SPEED WITH $\tilde{c} = \pm 1.5$

IF T, g & m ARE CONSTANT, THEN

$$V(c) = \frac{gm}{c}\left(1 - e^{-(c/m)T}\right) \Rightarrow$$

$$V'(c) = \frac{gm}{c^2}\left(e^{-(c/m)T} - 1\right) + \frac{gm}{c}\left(-e^{-(c/m)T} \times \left(-\frac{T}{m}\right)\right)$$

$$= \frac{9.81 \cdot 50}{12.5^2} \times \left(e^{-(12.5/50)6} - 1\right) + \frac{9.81 \cdot 50}{12.5}\left(-e^{-(12.5/50)6} \times \left(\frac{-6}{50}\right)\right)$$

$$= -2.43874 + 1.05067 \Rightarrow \boxed{V'(c) = -1.38806}$$

$$\Delta V(\tilde{c}) = |V'(c)| \times \Delta\tilde{c} = -1.38806 \times 1.5 \Rightarrow \boxed{\Delta V(1.5) = 2.082}$$

$$V(12.5) = \frac{9.81 \cdot 50}{12.5}\left(1 - e^{-\left(\frac{12.5}{50}\right)6}\right) \Rightarrow \boxed{V(12.5) = 30.484}$$

$$V(12.5) = V(12.5) \pm \Delta V(1.5) \Rightarrow \boxed{V(12.5) = 30.484 \pm 2.082}$$

CHECK

C	V
11	32.67
12.5	30.48
14	28.50