Trabalho Pratico 1 - Horario

Grupo 27

LCC 2024/2025

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Explicação do problema

Este projeto tem como objetivo construir um horario semanal para planear reuniões de projeto de uma "StartUp", seguindo as seguintes condições:

- Cada reunião só pode ocupar uma sala durante um determinado tempo e dia
- Cada reunião esta associada a um projeto
- Cada Projeto tem um conjunto de colaboradores
- Um dos colaboradores é o líder
- Cada projeto tem um numero de reuniões semanais
- O lider tem de participar em todas as reuniões do seu projeto
- Os colaboradores podem ou nao participar nas reuniões
- Em cada reunião tem de ter pelo menos 50% dos colaoradores

Variaveis

Inputs

- S - Sala
- D - Dia
- T - Tempo
- P - Projeto
- C - colaboradores
- $Constprojeto_{c,l,r}$ - Representa os colaboradores C associados a cada projeto, com um líder L designado e um número de reuniões semanais R.
- $Dispcolab_{c,d,t}$ - Representa a disponibilidade do colaborador C no dia D durante o tempo T.

Auxiliares

- $sl_{d,t,p,s}$ - Representa a atribuição de uma sala S a um projeto P, que ocorre no dia D e no tempo T
- $slc_{c,d,t,p,s}$ - representa a alocação de um dado colaborador C, num projeto P, a decorrer no dia D, no tempo T e na sala S

Implementação

```python
# Exemplo 1
S  = ["S1", "S2", "S3"]
P = ["P1", "P2", "P3", "P4"]
D = [1, 2, 3, 4, 5]
T = [1, 2, 3, 4, 5]
C = ["C1", "C2", "C3", "C4", "C5", "C6", "C7", "C8", "C9", "C10"]


Constprojeto = {
    "P1": (["C1", "C2", "C3", "C7"], "C1", 5),
    "P2": (["C4", "C5", "C6", "C8"], "C4", 6),
    "P3": (["C3", "C6", "C9", "C10"], "C3", 4),
    "P4": (["C2", "C5", "C7", "C9", "C10"], "C2", 5)
}


Dispcolab = {
    "C1": [(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4)],
    "C2": [(1, 1), (1, 2), (1, 4), (1, 5), (2, 1), (2, 3), (2, 4), (2, 5), (3, 2), (3, 4), (4, 1)],
    "C3": [(1, 1), (1, 3), (1, 4), (1, 5), (2, 1), (2, 4), (3, 1), (3, 3), (3, 4), (4, 2)],
    "C4": [(1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 5), (3, 1), (3, 4), (4, 1), (4, 3)],
    "C5": [(1, 1), (1, 3), (1, 4), (2, 1), (2, 4), (2, 5), (3, 2), (3, 3), (4, 1), (4, 2)],
    "C6": [(1, 2), (1, 3), (1, 4), (1, 5), (2, 1), (2, 2), (2, 4), (3, 1), (3, 4), (4, 1), (5, 1)],
    "C7": [(1, 1), (1, 3), (1, 4), (2, 1), (2, 2), (2, 4), (3, 1), (3, 4), (4, 1), (4, 3), (5, 2)],
    "C8": [(1, 2), (1, 4), (1, 5), (2, 2), (2, 4), (3, 1), (3, 3), (4, 1), (4, 3), (5, 1), (5, 3)],
    "C9": [(1, 1), (1, 3), (1, 4), (2, 1), (2, 2), (3, 1), (3, 4), (4, 1), (4, 3), (5, 1)],
    "C10": [(1, 2), (1, 4), (1, 5), (2, 1), (2, 3), (3, 1), (3, 3), (4, 2), (4, 4), (5, 1)]
}
```

```
h(S, P, D, T, C, Constprojeto, Dispcolab)
  Dia    Hora  Projeto   Sala    Participantes
  -----  ----  --------- ------  ------------------------------
      1     1  P4        S2      ['C2', 'C5', 'C7', 'C9']
      1     2  P4        S2      ['C2', 'C5', 'C7', 'C9', 'C10']
      1     3  P2        S3      ['C4', 'C6', 'C8']
      1     4  P4        S3      ['C2', 'C5', 'C7', 'C9']
      2     1  P1        S2      ['C1', 'C2']
      2     1  P2        S1      ['C4', 'C6', 'C8']
      2     2  P1        S1      ['C1', 'C7']
      2     2  P2        S3      ['C4', 'C6', 'C8']
      2     3  P1        S1      ['C1', 'C2']
      2     4  P3        S3      ['C3', 'C10']
      3     1  P1        S2      ['C1', 'C3', 'C7']
      3     1  P2        S1      ['C4', 'C5', 'C6']
      3     2  P1        S3      ['C1', 'C2']
      3     3  P3        S3      ['C3', 'C10']
      3     4  P2        S1      ['C4', 'C8']
      4     1  P4        S2      ['C2', 'C5', 'C7', 'C9', 'C10']
      4     2  P4        S3      ['C2', 'C5', 'C7']
      4     3  P3        S2      ['C3', 'C6', 'C9']
      5     1  P3        S1      ['C3', 'C6', 'C10']
      5     2  P2        S3      ['C4', 'C5']
```

In [79]:
```python
# Exemplo 2

S  = ["S1", "S2","S3"]
P = ["P1", "P2", "P3", "P4"]
D = [1, 2, 3, 4, 5]
T = [1,2,3,4,5]
C = ["C1","C2","C3","C4","C5","C6", "C7"]

Constprojeto = {
    "P1": (["C1", "C3","C5"], "C1", 4),
    "P2": (["C1","C2","C4","C7"], "C2", 5),
    "P3": (["C2","C3","C5", "C6","C7"], "C3", 7),
    "P4" :(["C1","C2","C3","C4","C7"], "C4", 9)
}

Dispcolab ={
"C1":[(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 7), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (2, 7), (3, 1),
(3, 2), (3, 3), (3, 4), (3, 5),(3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (4, 7), (5, 1),
(5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (5, 7)],
"C2":[(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (2, 7),
(3, 1), (3, 2), (3, 3), (3, 5), (3, 6), (3, 7), (4, 1), (4, 3), (4, 4), (4, 5), (4, 6), (4, 7), (5, 1),
(5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (5, 7)],
"C3":[(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 1), (2, 3), (2, 4), (2, 5), (2, 6), (2, 7), (3, 1),
(3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (3, 7), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (4, 7),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)],
"C4":[(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (2, 7),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (3, 7), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(4, 7), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (5, 7)],
"C5":[(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (2, 7),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (3, 7), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(4, 7), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (5, 7)],
"C6":[(1, 1), (1, 2), (1, 3), (1, 5), (1, 6), (1, 7), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (2, 7), (3, 2),
(3, 3), (3, 4), (3, 5), (3, 6), (3, 7), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (4, 7), (5, 1), (5, 2),
(5, 3), (5, 4), (5, 5), (5, 6), (5, 7)],
"C7":[(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 7), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (2, 7), (3, 1), (3, 2),
(3, 3), (3, 4), (3, 5), (3, 6), (3, 7), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (4, 7), (5, 1),
(5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (5, 7)]
}


h(S, P, D, T, C, Constprojeto, Dispcolab)
```

```
Dia    Hora   Projeto   Sala   Participantes
-----  -----  --------  -----  -----------------------------
    1      1  P3        S1     ['C2', 'C3', 'C5', 'C6', 'C7']
    1      2  P3        S1     ['C2', 'C3', 'C5', 'C6']
    1      3  P3        S1     ['C2', 'C3', 'C5', 'C7']
    1      4  P3        S1     ['C2', 'C3', 'C5', 'C7']
    1      5  P3        S1     ['C2', 'C3', 'C5', 'C6', 'C7']
    2      1  P2        S1     ['C1', 'C2', 'C4', 'C7']
    2      2  P4        S1     ['C1', 'C2', 'C4', 'C7']
    2      3  P2        S1     ['C1', 'C2', 'C4', 'C7']
    2      4  P4        S1     ['C1', 'C3', 'C4', 'C7']
    2      5  P2        S1     ['C1', 'C2', 'C4', 'C7']
    3      1  P2        S1     ['C1', 'C2', 'C4', 'C7']
    3      2  P2        S1     ['C1', 'C2', 'C4', 'C7']
    3      3  P3        S1     ['C2', 'C3', 'C5', 'C6', 'C7']
    3      4  P1        S1     ['C1', 'C3', 'C5']
    3      5  P3        S1     ['C2', 'C3', 'C5', 'C6', 'C7']
    4      1  P4        S1     ['C1', 'C2', 'C3', 'C4', 'C7']
    4      2  P4        S1     ['C1', 'C2', 'C3', 'C4', 'C7']
    4      3  P4        S2     ['C1', 'C3', 'C4', 'C7']
    4      4  P4        S1     ['C1', 'C2', 'C3', 'C4', 'C7']
    4      5  P4        S1     ['C1', 'C2', 'C3', 'C4', 'C7']
    5      1  P4        S1     ['C1', 'C2', 'C3', 'C4', 'C7']
    5      2  P4        S1     ['C1', 'C2', 'C3', 'C4', 'C7']
    5      3  P1        S1     ['C1', 'C3', 'C5']
    5      4  P1        S1     ['C1', 'C3', 'C5']
    5      5  P1        S1     ['C1', 'C3', 'C5']
```

In [80]:
```python
# Exemplo 3
S  = ["S1", "S2", "S3", "S4"]
P  = ["P1", "P2", "P3", "P4", "P5"]
D  = [1, 2, 3, 4, 5]
T  = [1, 2, 3, 4, 5, 6]
C  = ["C1", "C2", "C3", "C4", "C5", "C6", "C7", "C8", "C9", "C10", "C11", "C12"]

Constprojeto = {
    "P1": (["C1", "C2", "C3", "C6", "C9"], "C1", 3),
    "P2": (["C4", "C5", "C7", "C8", "C10"], "C4", 4),
    "P3": (["C2", "C3", "C5", "C6", "C11"], "C2", 3),
    "P4": (["C9", "C10", "C11", "C12"], "C9", 4),
    "P5": (["C1", "C3", "C7", "C8", "C12"], "C3", 4)
}

Dispcolab = {
    "C1": [(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2)],
    "C2": [(1, 1), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 2), (3, 3), (4, 1), (4, 4)],
    "C3": [(1, 2), (1, 3), (1, 4), (2, 2), (2, 4), (3, 1), (3, 3), (4, 2), (4, 3), (5, 1)],
    "C4": [(1, 2), (1, 4), (1, 5), (2, 1), (2, 2), (2, 4), (3, 1), (3, 3), (4, 1), (5, 1), (5, 2)],
    "C5": [(1, 1), (1, 3), (1, 4), (1, 5), (2, 2), (3, 1), (3, 2), (3, 4), (4, 2), (5, 2)],
    "C6": [(1, 1), (1, 3), (1, 5), (2, 1), (2, 4), (2, 5), (3, 1), (3, 3), (4, 1), (5, 2), (5, 3)],
    "C7": [(1, 1), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 2), (3, 3), (4, 1), (4, 2), (5, 1)],
    "C8": [(1, 2), (1, 3), (1, 5), (2, 2), (2, 4), (3, 1), (3, 3), (4, 1), (4, 3), (5, 1), (5, 2)],
    "C9": [(1, 1), (1, 2), (1, 3), (2, 1), (2, 3), (2, 4), (3, 1), (3, 3), (4, 1), (5, 2)],
    "C10": [(1, 2), (1, 4), (1, 5), (2, 1), (2, 4), (3, 1), (3, 3), (4, 2), (4, 4), (5, 2)],
    "C11": [(1, 1), (1, 3), (1, 4), (2, 2), (2, 4), (3, 1), (3, 2), (4, 1), (5, 1), (5, 3)],
    "C12": [(1, 1), (1, 3), (1, 4), (2, 1), (2, 4), (3, 1), (3, 2), (4, 1), (5, 1), (5, 3)]
}

h(S, P, D, T, C, Constprojeto, Dispcolab)
```

```
Dia    Hora   Projeto   Sala   Participantes
-----  -----  --------  -----  -----------------------------
    1      1  P3        S4     ['C2', 'C5', 'C6', 'C11']
    1      2  P2        S2     ['C4', 'C7', 'C10']
    1      3  P4        S2     ['C9', 'C10', 'C11', 'C12']
    1      4  P1        S3     ['C1', 'C2', 'C6', 'C9']
    1      4  P2        S4     ['C4', 'C7', 'C8']
    1      5  P5        S4     ['C3', 'C7', 'C8', 'C12']
    2      1  P1        S1     ['C1', 'C3', 'C9']
    2      1  P2        S3     ['C4', 'C8', 'C10']
    2      2  P5        S4     ['C1', 'C3', 'C7', 'C8']
    2      3  P3        S1     ['C2', 'C5', 'C11']
    2      4  P5        S4     ['C1', 'C3', 'C7']
    2      5  P4        S4     ['C9', 'C10']
    3      1  P5        S2     ['C1', 'C3', 'C7', 'C8', 'C12']
    3      2  P1        S4     ['C1', 'C2', 'C9']
    3      3  P4        S3     ['C9', 'C10']
    4      1  P3        S4     ['C2', 'C3', 'C5', 'C11']
    4      2  P4        S2     ['C9', 'C10', 'C11', 'C12']
    5      1  P2        S1     ['C4', 'C5', 'C8', 'C10']
```

Matriz de alocação de reuniões

A matriz sl serve para alocar reuniões no dia D no tempo T, do projeto P na sala S, logo temos

$$\forall_{d \in D},\ \forall_{t \in T},\ \forall_{p \in P},\ \forall_{s \in S}\ \text{sl}_{d,t,p,s} = 1$$

se existir no dia D e no tempo T uma reunião P na sala S

Matriz de alocação de colaboradores a uma reunião

A matriz slc serve para alocar colaboradores a reuniões, logo temos

$$\forall_{c \in C},\ \forall_{d \in D},\ \forall_{t \in T},\ \forall_{p \in P},\ \forall_{s \in S},\ \text{slc}_{c,d,t,p,s} = 1$$

se um colaborador tem no dia D, tempo T, uma reunião P na sala S

Restrições

1. Se o colaborador não tiver disponibilidade, não lhe é atribuída essa opção

$$\forall_{p \in P} \forall_{c \in \text{Constprojeto}[p][0]} \forall_{d \in D} \forall_{t \in T} \forall_{s \in S},\ (t,d) \notin \text{Dispcolab}_c \Rightarrow \text{slc}_{c,d,t,p,s} = 0$$

2. Cada projeto tem de ter o seu lider presente em todas as reuniões

$$\forall_{p \in P},\ \forall_{d \in D},\ \forall_{t \in T},\ \forall_{s \in S},\ (t,d) \notin \text{Dispcolab}[\text{Constprojeto}[p][1]] \Rightarrow \text{sl}_{d,t,p,s} = 0$$

3. Cada projeto tem um certo número de reuniões semanais

$$\forall_{p \in P},\ \sum_{d \in D,\, t \in T,\, s \in S} \text{sl}_{d,t,p,s} = \text{Constprojeto}[p][2]$$

4. Cada reunião tem de ter, no minimo, 50% de assiduidade

$$\forall_{p \in P},\ \forall_{d \in D},\ \forall_{t \in T},\ \forall_{s \in S},\ \sum_{c \in \text{Colaboradores do Projeto } p} slc[c,d,t,p,s] \geq \max\left(\left\lceil \frac{\text{len}(\text{Constprojeto}[p][0])}{2} \right\rceil, 2 \right) \cdot sl[d,t,p,s]$$

5. Cada sala só pode ter uma reunião num certo dia e horário

$$\forall_{d \in D},\ \forall_{t \in T},\ \forall_{s \in S}\ \sum_{p \in P} sl[d,t,p,s] \leq 1$$

6. Cada colaborador só pode participar numa reunião por horário

$$\forall_{c \in C},\ \forall_{d \in D},\ \forall_{t \in T}\ \sum_{p \in P} \sum_{s \in S}\ sl[d,t,p,s] \cdot \text{se } (c \in \text{Constprojeto}[p][0]) \leq 1$$

In [81]:
```python
from ortools.linear_solver import pywraplp
import math
from tabulate import tabulate

def h(S, P, D, T, C, Constprojeto, Dispcolab):

    solver = pywraplp.Solver.CreateSolver('SCIP') # Bom para problemas de otimização inteira mista

    if not solver:
        print("Solver SCIP não disponível.")
        return

    # Criar variáveis de reuniões por dia, horário, projeto e sala
    sl = {}
    for d in D:
        for t in T:
            for p in P:
                for s in S:
                    sl[d, t, p, s] = solver.BoolVar(f'sl[{d}][{t}][{p}][{s}]')

    # Criar variáveis de disponibilidade de colaboradores para as reuniões
    slc = {}
    for p in P:
        for c in Constprojeto[p][0]:
            for d in D:
                for t in T:
                    for s in S:
                        slc[c, d, t, p, s] = solver.BoolVar(f'slc[{c}][{d}][{t}][{p}][{s}]')

    # 1. Adicionar restrição de disponibilidade de colaboradores
    for p in P:
        for c in Constprojeto[p][0]:
            for d in D:
                for t in T:
                    for s in S:
                        if (t, d) not in Dispcolab[c]:
                            solver.Add(slc[c, d, t, p, s] == 0)

    # 2. Adicionar restrição de que o líder do projeto deve estar presente
    for p in P:
        for d in D:
            for t in T:
                for s in S:
                    lider = Constprojeto[p][1]
                    if (t, d) not in Dispcolab[lider]:
                        solver.Add(sl[d, t, p, s] == 0)
```

```python
    # 3. Adicionar restrição onde existem as reuniões semanais necessárias para cada projeto
    for p in P:
        solver.Add(
            solver.Sum(sl[d, t, p, s] for d in D for t in T for s in S) == Constprojeto[p][2]
        )

    # 4. Ajustar restrição onde pelo menos 50% dos colaboradores e mínimo de 2 devem estar presentes
    for p in P:
        for d in D:
            for t in T:
                for s in S:
                    necessarios = math.ceil(len(Constprojeto[p][0]) / 2)
                    solver.Add(
                        solver.Sum(slc[c, d, t, p, s] for c in Constprojeto[p][0]) >= max(necessarios,2) * sl[d, t, p, s]
                    )

    # 5. Evitar conflitos de sala: apenas uma reunião por sala, dia e horário
    for d in D:
        for t in T:
            for s in S:
                solver.Add(
                    solver.Sum(sl[d, t, p, s] for p in P) <= 1
                )

    # 6. Garantir que cada colaborador participe de no máximo uma reunião por horário
    for c in C:
        for d in D:
            for t in T:
                solver.Add(
                    solver.Sum(sl[d, t, p, s] for p in P for s in S if c in Constprojeto[p][0]) <= 1
                )

status = solver.Solve()

if status == pywraplp.Solver.OPTIMAL:
    cabeça = ['Dia', 'Hora', 'Projeto', 'Sala', 'Participantes']
    tabela = []
    for d in D:
        for t in T:
            for p in P:
                for s in S:
                    if sl[d, t, p, s].solution_value() > 0:
                        participantes = [c for c in Constprojeto[p][0] if slc[c, d, t, p, s].solution_value() > 0]
                        tabela.append([d, t, p, s, participantes])
    print(tabulate(tabela, headers=cabeça))
```