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Cluster growth modeling of plateau erosion

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Abstract. The pattern of erosion of a plateau along an escarpment may be modeled using cluster growth techniques, recently popularized in models of drainage network evolution. If erosion on the scarp takes place in discrete events at rates subject to local substrate strength, the whole range of behavior is described by a combination of three cluster growth mechanisms: invasion percolation, Eden growth and diffusion-limited aggregation (DLA). These model the relative importance of preexisting substrate strength, background weathering, and seepage weathering and erosion respectively. The rate of seepage processes is determined by the efflux of groundwater at the plateau margin, which in turn is determined by the pressure field in the plateau aquifer. If this process acted alone, it would produce erosion patterns in the form of Laplacian fractals, with groundwater recharge from a distant source, or Poissonian fractals, with groundwater recharge uniform over the plateau. DLA is used to mimic the Laplacian or Poissonian potential field and the corresponding seepage growth process. The scaling structure of clusters grown by pure DLA, invasion percolation, or Eden growth is well known; this study presents a model which combines all three growth mechanisms for the first time. Mixed growth processes create clusters with different scaling properties and morphologies over distinct length scale ranges, and this is demonstrable in natural examples of plateau erosion.

Introduction

The purpose of this study is to find a way to characterize and model the morphology of the erosion of plateaus. Usually, erosion takes place along an irregular front, which moves across the plateau leaving a zone of high relief behind it (Figure 1). The principal mechanisms for this erosion are scarp degradation, often controlled by seepage and sapping, and surface runoff and mass wasting on the zone of high relief below the scarp. The rate of erosion is a function of both substrate weakness and the erosion mechanisms. However, the role of the pattern of substrate weakening in determining the pattern of erosion is often underestimated. The emphasis of the model outlined below is to treat erosion on an escarpment as a discrete process, operating at a rate proportional to local weakness; the variety of erosion and weathering processes are subsumed into model weakening mechanisms.

The rates of sapping and seepage weathering are both controlled by groundwater flux. Thus the pattern of plateau erosion dominated by these processes is controlled by the pattern of instabilities and focusing of groundwater. However, seepage processes do not act alone: the variation of lithological strength will also influence the pattern of erosion, as will any background weakening processes, such as earthquakes, precipitation along the escarpment, and so on. The difficult problem to solve is how to quantify the relevance of preexisting strength, seepage weathering/weakening, and background weathering/weakening in determining the geometry of an erosion front. The approach used here is to identify scaling exponents (fractal dimensions) for a variety of geometrical properties of cluster model erosion patterns and for a range of rates of competing erosion processes and to identify relative length scales over which such scaling operates. It may be possible to use these scaling exponents to characterize plateau erosion patterns.

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Seepage Processes

Rainfall on a plateau must (1) be returned to the atmosphere by evapotranspiration (2) form surface runoff and flow off the plateau edges, or (3) enter the groundwater and flow along an aquifer and out of the plateau edge or escarpment. The latter efflux drives sapping and seepage erosion and seepage weathering. In many cases, rainfall on the plateau does not lead to significant erosion by runoff over the escarpment [*Dunne*, 1990], for example, where upland playa lakes demonstrate that most plateau rain enters the groundwater, or where upland topography routes surface flow away from the escarpment (Figure 1). In any case, it is difficult so see how plateau runoff can control the morphology of escarpment retreat unless such runoff either leads

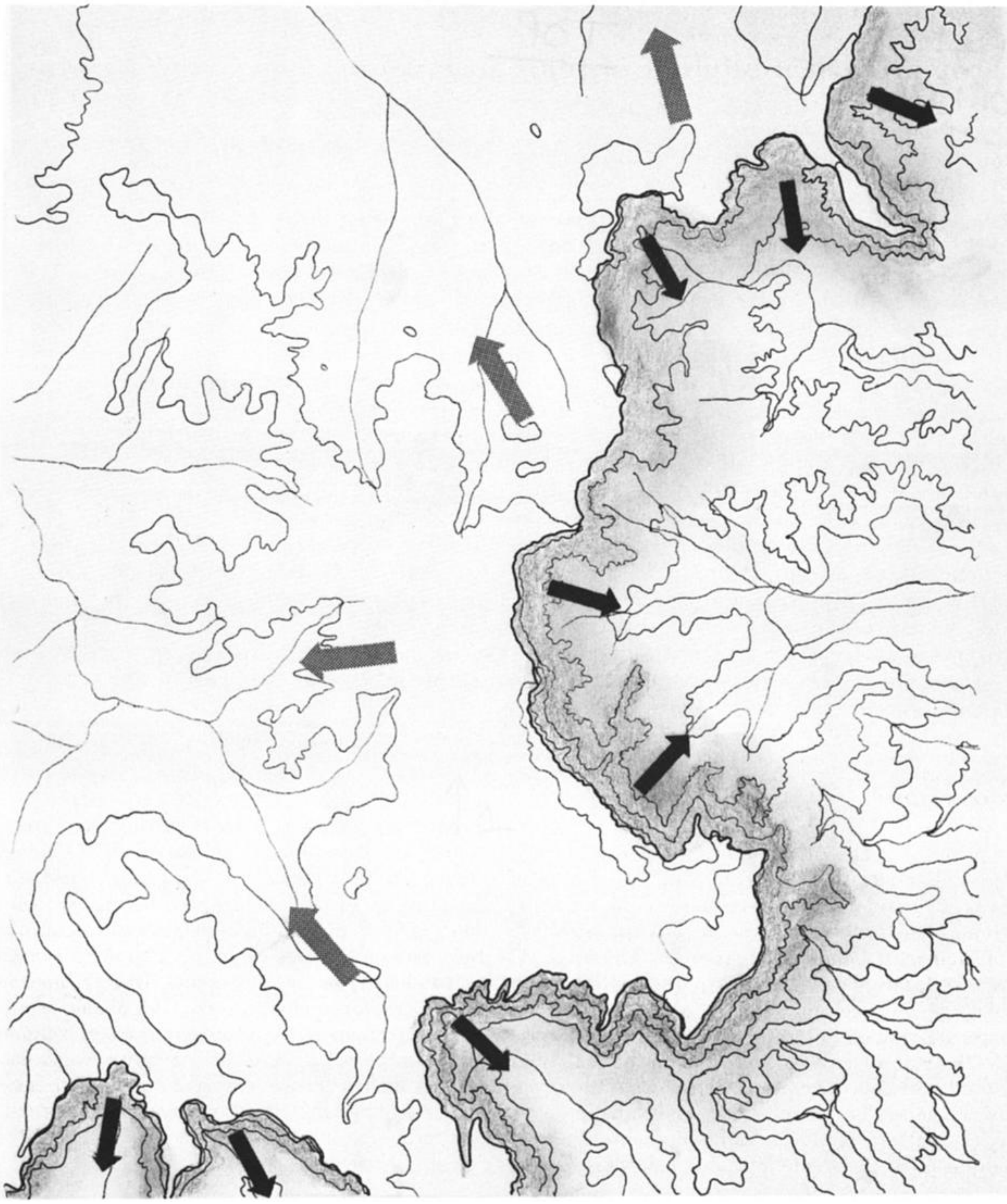


Figure 1. An area of Bryce Canyon, part of the Colorado Plateau, Utah. Precipitation on the plateau causes west to northwest directed surface runoff and streamflow (stippled arrows) and recharge of groundwater that effluxes along the escarpment to the east and south (solid arrows). Seepage erosion is observable along the escarpment, and seepage weathering of the scarp bedrock is likely to be widespread. The dominant erosion process along the escarpment is surface runoff on the steep scarp hillslopes. Note the scarp erosion front is indented by a series of broad, short, rounded "fingers" which will focus groundwater efflux (solid arrows). The observation that the plateau runoff is largely routed away from the escarpment allows us to infer that the upland runoff patterns are decoupled from the plateau front erosion patterns.

directly to undercutting of the scarp itself, or if it indirectly controls the seepage pattern by focusing infiltration of surface water and hence the pattern of groundwater recharge. Cases of undercutting by plateau surface runoff are not dealt with in this study. The case of infiltration focusing deserves further mention and is discussed in the section on Poissonian fingering. However, this study is intended to deal with cases where plateau runoff is not the dominant factor in determining the pattern of scarp retreat.

Sapping and seepage erosion are frequently important geomorphic processes along escarpments [*Baker*, 1990; *Dunne*, 1990; *Higgins et al.*, 1990; *Laity and Malin*, 1985], although the physical mechanisms involved are understood in only particular cases [*Laity*, 1983]. Emergence of groundwater through a rock surface can lead to surface runoff and erosion, either over a whole seepage face if subsurface flow is Darcy, or locally and more intensely if the flow of groundwater is great enough to form a pipe or follow a fracture, thereby allowing thin turbulent flow. The progressive seepage erosion of a cliff face, frequently strongest near its base, leads to undercutting and sapping collapse of the cliff.

Seepage weathering is more difficult to observe but may be more important in determining patterns of weakness along an escarpment. Such weakening occurs when groundwater outflow from a rock or soil surface causes dissolution and leaching or salt or ice crystal growth. The rate of these chemical and physical processes depends on the rate of seepage of water, and so it is sensitive to focusing of outflow of groundwater at points along a scarp. If weathered material is removed from the scarp as soon as it is formed, or if such material is eroded at a rate which depends on its weakness, then the behavior of seepage weathering will mimic that of seepage erosion and sapping. Hence the rate of each individual process will be very hard to assess in the field, and such separation will not be useful. It is simpler and perhaps more meaningful to treat seepage processes together: this is the approach taken in the cluster growth model.

Groundwater outflow tends to concentrate along relatively more permeable layers. So an escarpment formed in a sand-shale stratigraphy eroding by seepage mechanisms will tend to experience sapping and enhanced retreat along sandy, more permeable exposures [*Laity and Malin*, 1985]. It may form a staircase morphology, with steep sand cliffs and shallow gradient shale steps. This study will deal with a more simple permeability structure, in which flow is assumed to concentrate along one thin, horizontal aquifer. The focusing of flow in such a system is sufficiently complex for initial treatment. It will doubtless be worthwhile to consider more complicated permeability structures in future.

Groundwater pressure can also control scarp retreat by affecting slope stability, rather than by influencing seepage patterns. Fluctuations in subsurface water pressure following or during storms may be sufficient to increase the deviatoric stress below the hillslope beyond the yield stress, leading to hillslope failure and a slump.

Runoff Erosion and Mass Wasting on the Escarpment

While seepage erosion can weaken and dislodge material from a steep escarpment, it is clearly important that this material be removed so that the sapping process may continue unabated. Rock slope and hillslope erosion processes below a scarp perform this task [*Howard and Selby*, 1993]. In many cases, such erosion rates are more significant than the rate of seepage erosion, and so the pattern of scarp retreat is often controlled by the pattern of slope erosion. However, slope erosion rate is a function of the weakness of the slope material. As discussed in the previous section, seepage can lead directly to weakening of the substrate, but many other weathering and weakening processes operate. The pattern of weakness along the scarp slopes will also be a function of lithology and other geological properties. Hence the rate and pattern of runoff erosion and mass wasting events will depend on three forms of substrate weakness and weakening: seepage related, lithology related, and a third background mechanism. As with seepage processes, we can treat a fluctuating slope erosion rate as a uniform erosion rate in the presence of a fluctuating background weakening process. It is important to separate disorder (randomness) in slope erosion and weathering rates from preexisting disorder in the substrate, where the latter are due to lithological variations. The presence of each leads to characteristic patterns of evolution of the erosion front.

Cluster Growth and Erosion Processes

Cluster Growth Model

Rather than constructing partial differential equations to describe the competing erosion processes, I have instead designed a cluster growth algorithm which mimics those phenomena. Furthermore, both groundwater efflux sapping and background, externally driven erosion are modeled as weakening processes (seepage weathering and background weathering). The erosion algorithm takes the following form:

1. A square lattice is initialized with each site assigned a (Gaussian white noise) random weakness.
2. Erosion initiates from the lower boundary (scarp); the left and right boundaries are periodic (continuous).
3. Fluid flow and erosion propagation may proceed in each of the eight nearest neighbor directions possible on a square site lattice; lattice-distortion effects are minimized by modulating growth and flow diagonally by a factor of $\sqrt{2}-1$.
4. At each simulation cycle, erosion is performed in four steps: (1) A random walker is released either from "infinity" (Laplacian growth) or from a randomly chosen site on the model plateau (Poissonian growth). (2) When this random walker steps onto the cluster, the previous, external perimeter site is weakened by a factor γ, which determines the dominance of groundwater seepage weathering. This is a slight modification of the standard diffusion-limited aggregation (DLA) boundary condition (Figure 2). (3) Next, an external perimeter site is chosen at random, and this site is weakened by a factor η, which reflects the dominance of background weathering processes. (4) Finally, the weakest site on the external perimeter is identified and is eroded, in other words it is added to the cluster.
5. When the upper boundary is reached, erosion simulation terminates.

When $\eta \to 0$ and $\gamma \to 0$, we obtain cluster growth which is sensitive only to the initial distribution of lattice weakness values, or in model terms, we obtain an erosion process that is determined by the (unchanging)

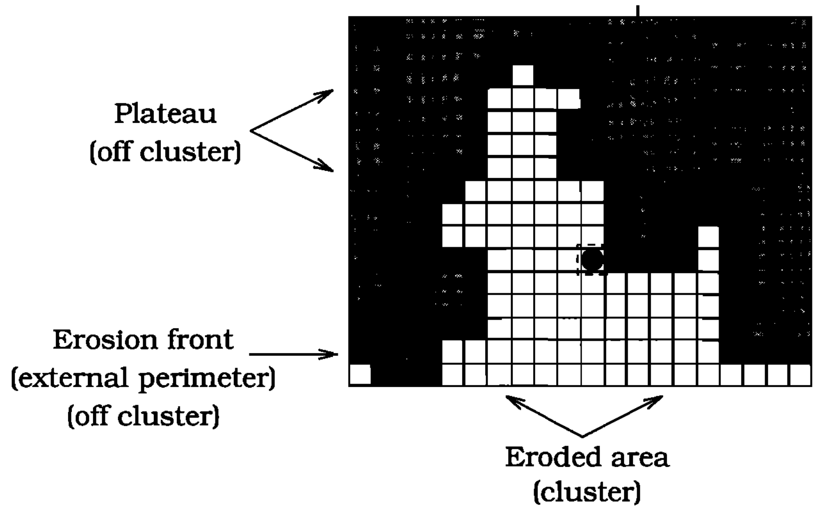


Figure 2. The cluster growth model incorporates diffusion-limited aggregation (DLA) in order to simulate Laplacian and Poissonian groundwater pressure fields and fluxes. In general, DLA is the process of initiating random walkers at some distance from the growth zone and adding a new cluster site when and where each random walker hits the cluster. The precise collision-and-growth condition can take two forms. The arrival of a random walker at the cluster may be defined as either stepping onto the external perimeter (dark grey sites, off but immediately adjacent to the cluster) or stepping onto the internal perimeter (white cluster sites) from the external perimeter. This study uses the latter boundary condition in order to model the effect of groundwater fluxing out of the escarpment (external perimeter).

substrate geology; this end-member process is known as invasion percolation (Figure 3). For $\eta \gg \gamma \gg 0$, background weathering processes dominate, and cluster growth takes the form of an Eden process, or quasi-uniform retreat, because model erosion takes place in a temporally and spatially random fashion. If $\gamma \gg \eta \gg 0$, DLA or model seepage weathering dominates, and the pattern of erosion is determined by simulated fluctuations in groundwater efflux and focusing of seepage: pseudo-viscous (Laplacian or Poissonian) fingering is this end-member process. Natural erosion patterns are the result of all three competing processes.

The model employs some minor changes to the standard DLA mechanism. The arrival of a random walker modifies the weakness parameter of the arrival site, and growth occurs after each arrival event at the weakest site on the external perimeter. If each site weakness is determined purely by the arrival of a random walker, then pure DLA growth results. However, spatial and temporal variations in the site strengths incorporate invasion percolation and Eden growth mechanisms respectively. The cluster growth model thus simulates seepage weathering of a variable strength substrate, undergoing erosion which is sensitive to the substrate strength. Seepage erosion and sapping are implicitly combined with seepage weathering.

Another alteration worth emphasizing is the inclusion of diagonal growth, in order to reduce the effect of lattice geometry on the clusters. The rate of diagonal growth is equal to the growth rate parallel to a lattice axis modulated by a factor of $\sqrt{2}-1$. It has been found that large standard DLA clusters are sensitive to the lattice geometry and tend to form fingers that parallel the lattice axes. Modulated diagonal growth, where the modulation factor is simply a function of the extra distance between cells diagonally versus orthogonally, appears to remove this lattice sensitivity [*Ball et al.*, 1989] for any lattice size within the present range of computation. For our purposes, growth rate is as close to isotropic as the lattice geometry will allow.

The most significant change to the DLA growth process is to treat the arrivals of random walkers as weakening events: a lattice site is added (fails or erodes) after each arrival, and the particular site that is chosen is the weakest of the set of external perimeter sites. Without any weakening process other than the flux of random walkers, the cluster must grow as a DLA cluster. However, unlike with standard DLA we can include preexisting weakness patterns, and a background weakening mechanism, and thus model spatial and temporal variation in substrate weakness. By treating certain erosion mechanisms such as seepage erosion and sapping as weakening processes in the presence of a continual background erosion rate, we can simulate the spectrum of erosion and weakening processes along a plateau edge.

Invasion Percolation

If the pattern of erosion of a plateau were determined solely by the distribution of strength and weakness of its soil and rock and if the distribution were random and uncorrelated, that pattern would not develop randomly, as one might expect. Instead, it would evolve in a manner analogous to invasion percolation [*Wilkinson and Willemsen*, 1983]. This is a growth process which belongs to the same universality class as normal percolation theory [*Stauffer*, 1979], and it originated as a model for the behavior of the slow displacement of one fluid

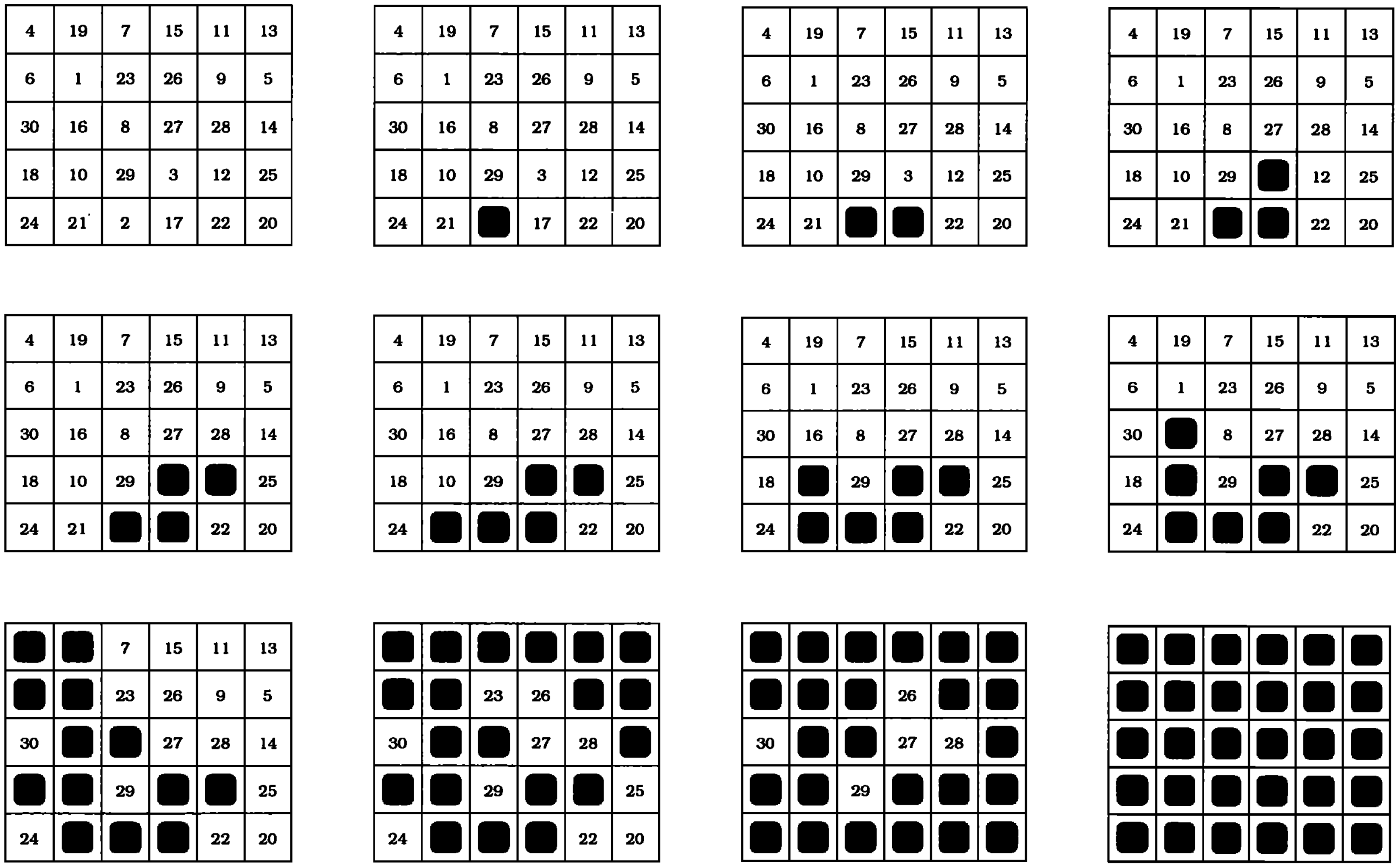


Figure 3. A summary of the invasion percolation mechanism. The grid of numbers represents the substrate strength at each site on a lattice. Growth of a cluster is performed from the lower edge and, in this figure, in four directions (north, east, south and west, if available) only. In the cluster growth model, growth is also permitted diagonally. At each step, the weakest (lowest value) site along the cluster perimeter is invaded or added to the cluster. The first growth event occurs from the bottom edge onto the site with value 2 and progresses onto successively neighboring sites with values 17, 3, 12, 21, and so on. Note that the growth of the cluster picks out a path of weakness, which is highly irregular (fractal) in the presence of spatially uncorrelated disorder, as illustrated here. Growth can either be terminated when the opposite edge is reached, which is the case for true invasion percolation, or when all sites have become occupied, as here.

by another in a heterogeneous porous medium, termed capillary fingering [*Lenormand*, 1989; *Lenormand et al.*, 1983; *Lenormand and Zarcone*, 1985].

Figure 3 illustrates the mechanics of invasion percolation, in a simpler form than that used in the cluster growth erosion model. In this illustration, as in the erosion model, growth begins from the bottom edge by occupying the lattice site with the lowest value: the weakest site. The external perimeter of the cluster is now slightly different, and in the next growth step the weakest site on this new perimeter is occupied and added to the cluster. Each growth event deforms the external perimeter (the set of empty lattice sites surrounding the cluster) so that new growth directions become possible. If the lattice weakness values are randomly distributed, spatially uncorrelated, and fixed for the duration of growth, an invasion percolation cluster will be formed. The scaling properties of a structure well known: the fractal dimension of the whole pattern is $d_f = 1.896$, the invasion cluster perimeter, which is the model erosion front, scales as $d_p = 1.333$ [*Grossman and Aharony*, 1986; *Ziff*, 1986], the shortest paths across a cluster, which are tentatively correlated with model streams below the escarpment, have fractal dimension $d_{min} = 1.130$ [*Barma*, 1985; *Herrmann et al.*, 1984; *Herrmann and Stanley*, 1988].A model of drainage network evolution based on invasion percolation has been proposed to explain the Hack relation between stream length and basin area [*Stark*, 1991a, b; *L. Pietronero and W. Schneider*, preprint, 1991]. It is fundamentally important to note that the lattice geometry is irrelevant to these scaling properties: the choice of a square, triangular, or hexagonal lattice, whether site or bond type, will not affect the scaling exponents as long as the growth process belongs to the same universality class.

The progress of a pure invasion percolation erosion front is shown in Plate 1. The four stages of growth also show two important properties of invasion percolation. One is the phenomenon of bursting: phases of slow growth (failure of relatively strong points) alternate with phases of rapid growth, or bursts, when the perimeter grows into a weak area of the lattice. So the movement of the erosion front occurs in pulses of invasion into small weak areas. The second important property is the persistence of outliers of noninvaded lattice behind the moving front and thus inside the cluster. Plate 1 shows a variety of outlier sizes and numbers: percolation theory tells us that such outliers have no inherent length scale and are thus power law size distributed, and the scaling of this distribution is a corollary of the fractal dimension of the cluster itself. Each outlier is surrounded by lattice sites whose strength is such that they are never invaded (eroded). In fact, once an invasion percolation cluster is sufficiently large, growth only occurs into lattice sites below a critical weakness. This value is known as the percolation threshold; for example, for percolation on a square site lattice in the plane, this value is about 0.593, given weakness values uniformly distributed over the unit interval. A model erosion front would thus have a critical strength above which erosion rates are zero.

In reality, however, all points along an erosion front will eventually, regardless of the relative hardness, erode. Fortunately, this realization does not destroy the applicability of percolation, since the observations above apply to an infinite cluster. For a finite size cluster, the condition that a set of sites will never erode is reduced to the weaker condition that there is a set of sites which will not erode during the growth of the percolation front across the finite lattice. Thus, in general, the time it takes for the stronger points to erode will introduce a time scale into the invasion percolation process and hence a length scale into the pattern of erosion. Actually, we can be more specific. If we calculate the percolation threshold for a growing erosion front, we can predict that the erosion rates of the sites with weakness value close to the threshold will determine the effective width of the zone behind the erosion front which will show percolation scaling. Similarly, along the erosion front a break in percolation scaling will occur at an equivalent, upper length scale.

Laplacian and Poissonian Fingering

Groundwater outflow at the margin of a plateau determines the rate of erosion by sapping-related processes. Fluctuations in water pressure produce instabilities in the pattern of efflux, which combine with discrete erosion events to produce a fractal pattern of erosion. If the groundwater flow is largely restricted to a subhorizontal aquifer, then the erosion pattern develops in a similar manner to viscous fingering [*Hinrichsen et al.*, 1989; *Lenormand*, 1989; *Lenormand et al.*, 1983; *Måløy et al.*, 1985; *Oxaal et al.*, 1987]. This phenomenon occurs where a viscous fluid is displaced from a porous medium by an inviscid fluid, at a rate where capillary forces are negligible. Laplacian fractal erosion fronts are obtained [*Pietronero et al.*, 1986] if groundwater recharge occurs at some distance from the plateau scarp (Plate 2). The overall (cluster) fractal dimension for such patterns is well known to be about $d_f \approx 1.67$, and model streams are Euclidean ($d_{min} = 1.0$). If instead recharge occurs uniformly over the uneroded plateau, Poissonian fractal fronts develop (Plate 3). These are likely to have a limiting cluster dimension of $d_f \rightarrow 2.0$, with Euclidean streams. Note that these model streams are simply short paths across the cluster, and they represent the "trail" of streams that are left behind an advancing cluster front. It is most unlikely that the pattern and scaling of true streams below an escarpment will follow such trails: the inclusion of the scaling properties of these short paths is for completeness sake. The color coding of the eroded cluster sites is according to the timing of their erosion: the pattern of colors is a qualitative indication of the relative elevation that may develop behind the moving escarpment but does not denote any model topography.

The diffusion-limited aggregation (DLA) component of the cluster growth model studied here simulates the pressure field and flux of groundwater [*Ball*, 1986; *Meakin*, 1987b; *Meakin and Tolman*, 1989; *Sahimi and Yortsos*, 1985; *Witten and Sander*, 1981]. Random walkers act as tracers for the relevant potential field

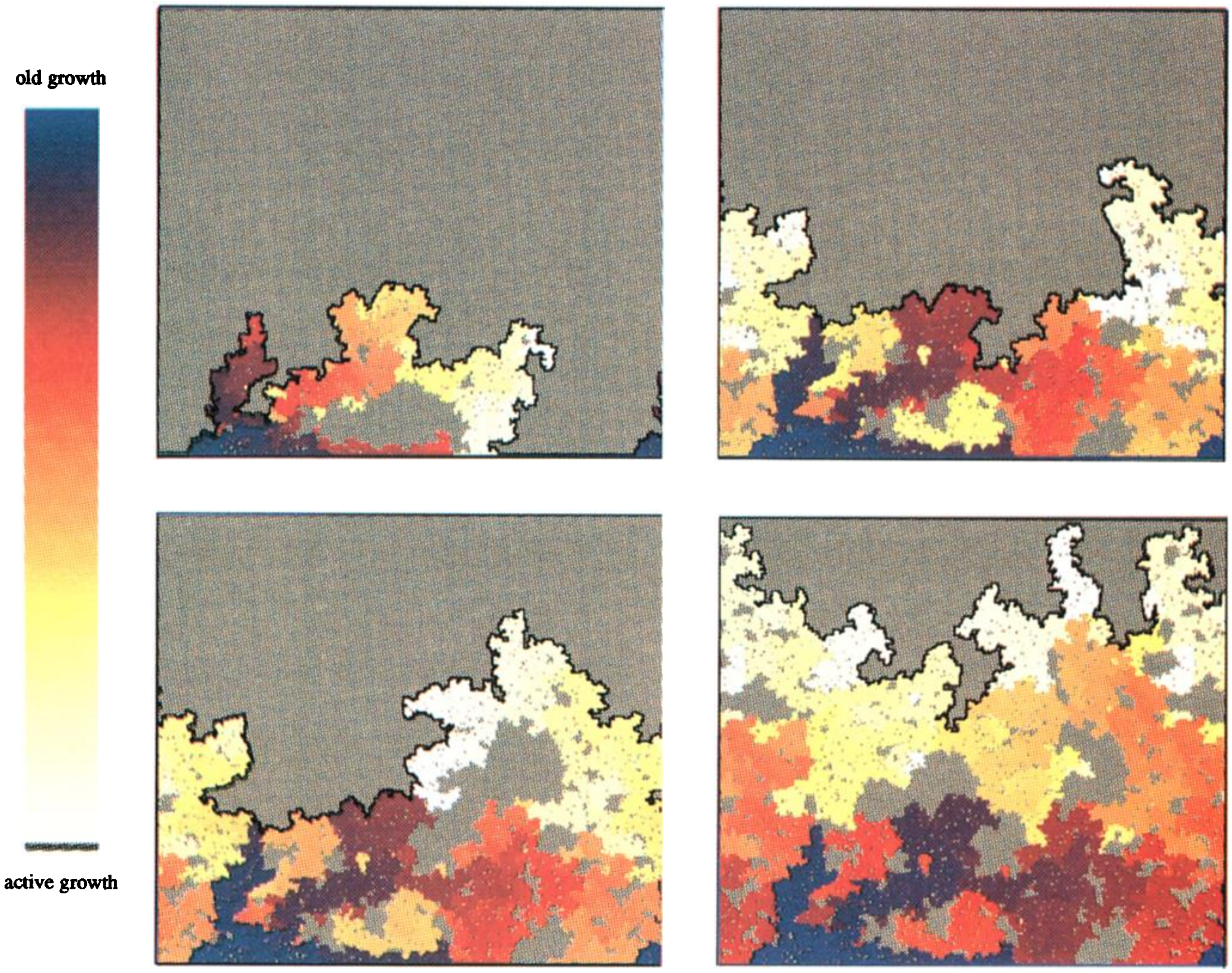


Plate 1. A simulation of pure invasion percolation retreat of an erosion front. As in Figure 3, growth occurs from the bottom edge, but in this case the left and right edges are wrapped around to join, forming a periodic boundary condition. Growth can occur along the lattice axes and diagonally (eight nearest neighbors). The upper grey area is empty lattice (uneroded plateau), the black sites are the external perimeter of the cluster (erosion front) and the cluster sites (eroded areas) are color coded to reflect the relative timing of their addition (erosion) to the cluster. Four stages of invasion are illustrated. A variety of size of outliers are left behind the moving erosion front, and these give the invasion percolation cluster its fractal structure, with mass dimension $d_f = 1.896$.

by stepping randomly from infinity until they hit the growth boundary (Figure 2). The density of random walkers represents the potential (pressure) field and the flux of random walkers represents the fluid flux and thus the gradient of the potential field.

The combination of seepage weakening and erosion in the presence of variable degrees of inherent substrate strength are modeled in Plate 2. As the dominance of flow weakening decreases, the regularity of the Laplacian fingers is reduced, and the invasion percolation process becomes more evident.

Poissonian Versus Laplacian Fingering

The distribution of recharge of the plateau groundwater is crucially important in determining the scaling of erosion fingers. We can easily distinguish two extremes:

1. Rainfall occurs effectively at infinity, i.e., at some great distance from the plateau scarp. This leads to a groundwater pressure field that we may model using Laplace's equation $\nabla^2\phi = 0$.

2. Recharge occurs uniformly over the whole plateau. The groundwater pressure field may then be modeled using Poisson's equation in its simplest form of $\nabla^2\phi = k$, where k is some constant reflecting the rainfall rate.

Plates 3 and 4 show the process of Poissonian fingering. Uniform recharge over the plateau is easily modeled with the cluster growth model by selecting random points off the cluster from which to release random walkers. Poissonian fingering initially develops as Laplacian fingering, but as the fingers grow they tend to fill the available space until Euclidean, space-filling cluster is formed. Strictly speaking, Laplacian growth can be treated as a subset of Poissonian growth.

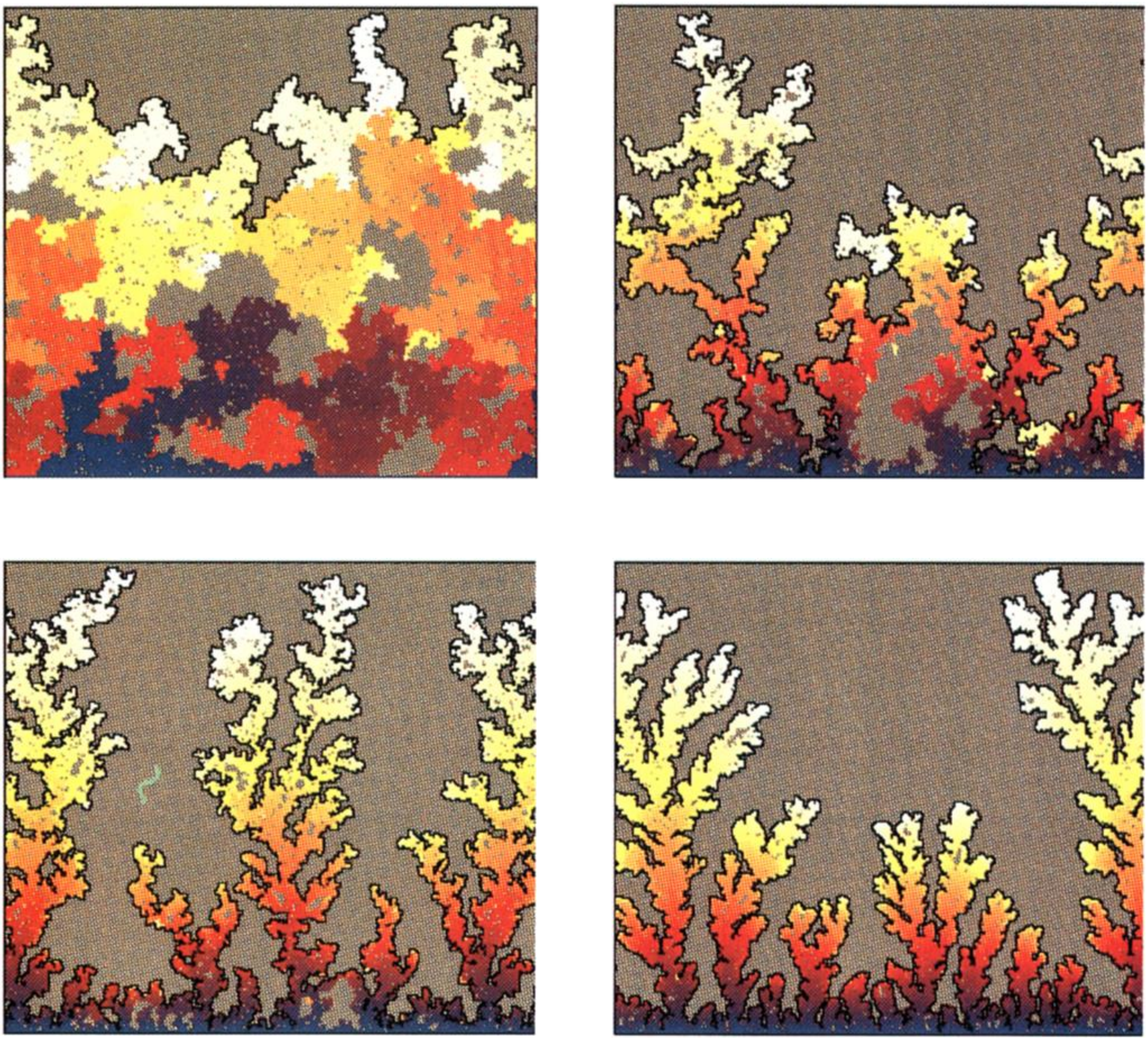


Plate 2. Laplacian fingering merging with invasion percolation, modeled using the substrate weakening form of DLA. The four examples shown here range from pure DLA, which produces DLA clusters which are equivalent to Laplacian fingers, with mass dimension $d_f = 1.67$, to pure invasion percolation. This range is produced by varying the intensity of weakening by arrival of random walkers (seepage weathering) versus the magnitude of fixed, quenched random spatial weakness (original substrate weakness). **Color scale as in Plate 1.**

Eden Growth

Background erosion and weathering of the plateau scarp, which does not depend on either groundwater flux or preexisting substrate strength, can be treated as random, uncorrelated weakening subject to continual erosion. The discrete nature of erosion events means that we cannot model this erosion process as simply uniform retreat, because a purely continuous background process masks any scaling of the erosion zone width. The correct cluster growth analog for this process, which includes the punctuated nature of background erosion, is Eden growth [*Devillard and Stanley,* 1989; *Freche et al.,* 1985; *Jullien and Botet,* 1985; *Meakin,* 1987a]. There are a number of variants of this model, but we consider the case where a site is randomly chosen on the external perimeter of the cluster (which is the set of uneroded sites immediately adjacent to the model erosion front). In pure Eden growth, this site fails and is eroded. In our cluster growth model, this site is weakened by a factor that reflects the relative rate of background weathering. Normal Eden growth is obtained if this mechanism alone is employed. However, when the DLA weakening and initial weakness patterns are included, we can model the combined processes of Poissonian fingering, invasion percolation and Eden growth.

The scaling of Eden clusters is generally simple: the whole model erosion pattern is Euclidean ($d_f = 2.0$), as

Plate 3. Poissonian fingering simulated by the DLA cluster growth model. A Poissonian potential field is mimicked by initiating random walkers at random points off the cluster (on the plateau) rather than from infinity, and this models a groundwater pressure field recharged uniformly over the plateau rather than from some distant source. Poissonian fingers form an irregular structure, which is Euclidean with mass dimension $d_f = 2$ in the plane. However, if growth is terminated when the upper edge is reached, the model erosion pattern is space filling in only the scaling sense, since much of the model plateau remains, as here, uneroded. **Color scale as in Plate 5.**

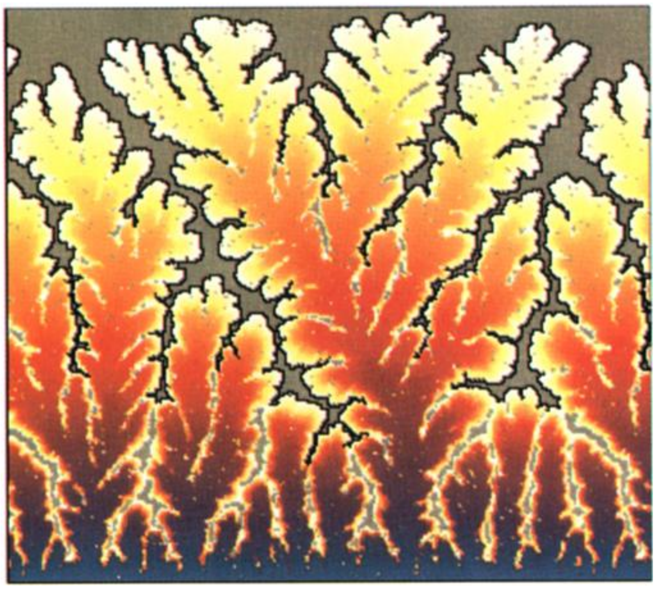


Plate 4. Poissonian fingering with strong noise reduction. The fine texture of Plate 3 is replaced by smoother fingers. The introduction of noise reduction supplies an effective inner (short) length scale below which all scaling and structure is trivial. Thus noise reduction models the effect of real damping of groundwater efflux and seepage weathering pattern fluctuations and also the presence of a geomorphic length scale such as the scarp slope length, which determines the length scale of mass wasting events. **Color scale as in Plate 5.**

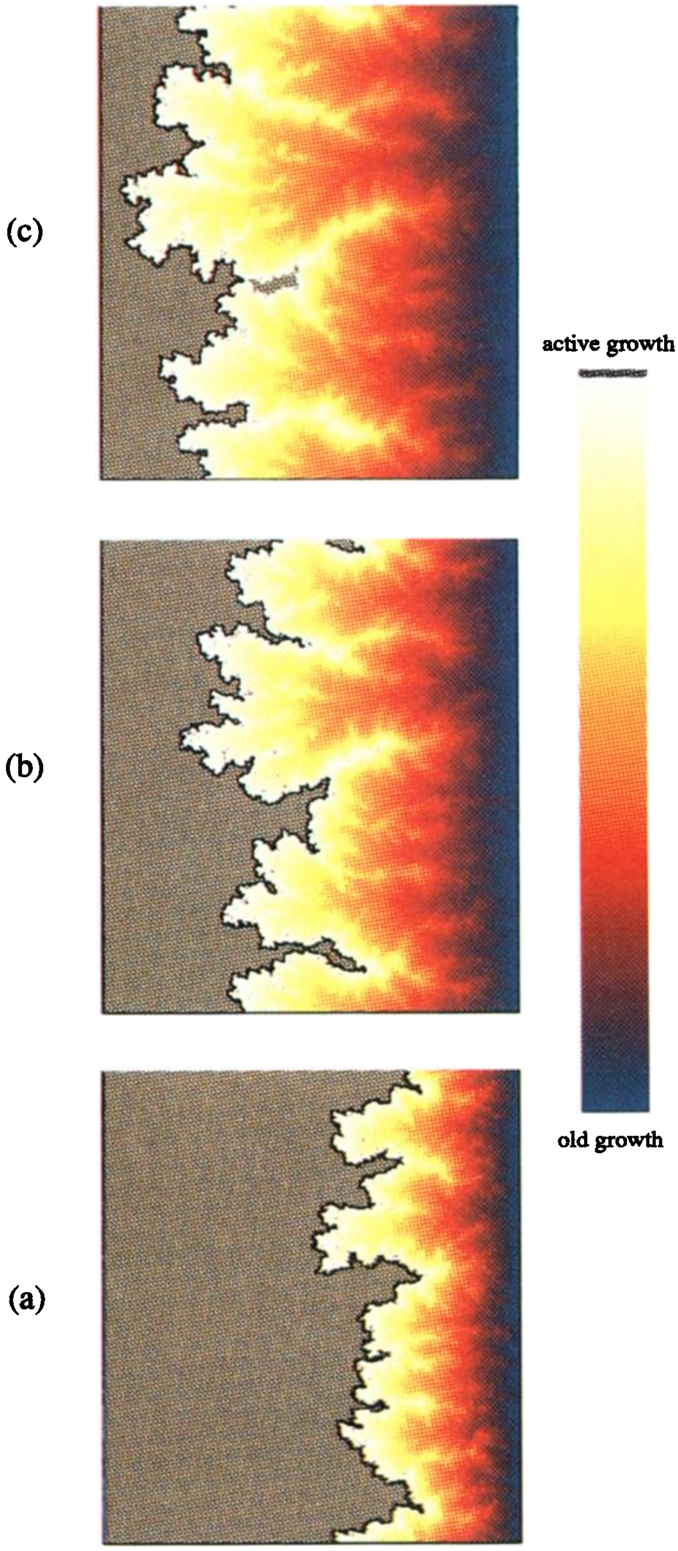
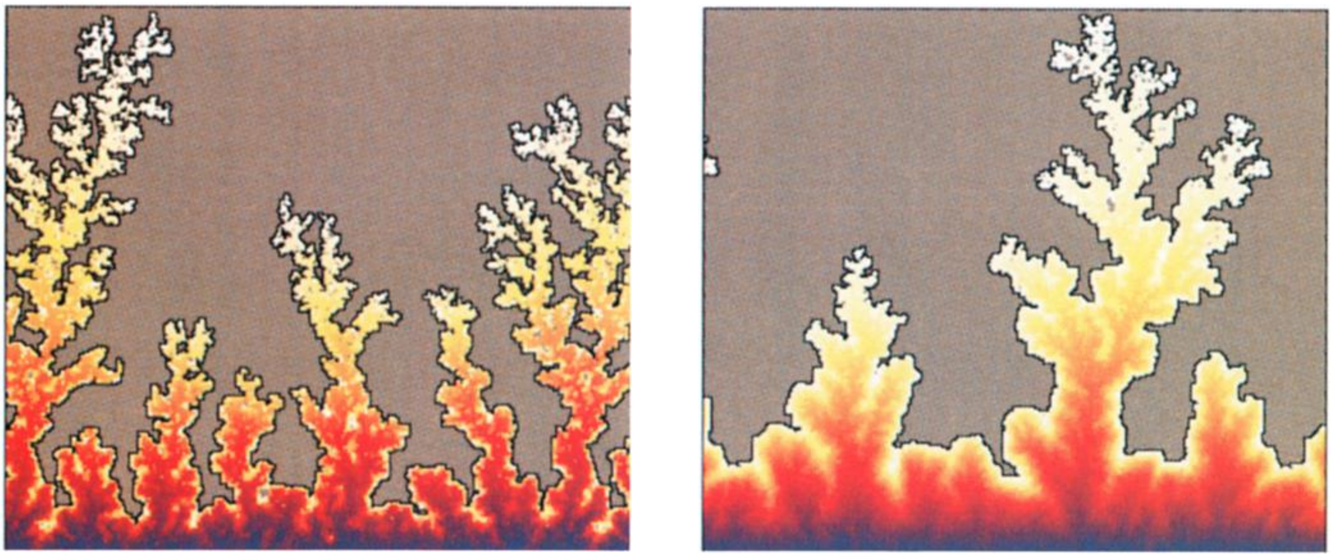


Plate 5. Eden growth dominant over Poissonian fingering. This case is analogous to that of Bryce Canyon (Figure 1). The three stages of growth of the model cluster illustrate the development and disappearance of short, broad fingers of faster erosion. This morphology arises from model parameters which simulate seepage weathering and erosion of a plateau subject to uniform groundwater recharge, but where the background weakness and weathering of the plateau bedrock is more significant. The background, Eden process puts an upper (long) length scale, above which no fingering is observed. Note the presence of an outlier in Plate 5(c), which is a model mesa or butte. This will erode fairly rapidly as the erosion front progresses.



(a) (b)

Plate 6. Illustrations of the effect of combining Eden growth, invasion percolation, and DLA. The bias in each, greater in Plate 6(a), is in favor of DLA, which here is modeling Laplacian growth. **Color scale as in Plate 5.**

are the model streams ($d_{min} = 1.0$). Only the erosion front width itself is scaling: it is self-affine with Hurst exponent $H = 0.5$.

Plate 5 demonstrates model escarpment retreat dominated by background erosion with weak seepage erosion and weathering. In this example, the seepage morphology is suppressed on all but the longest wavelengths. Eden growth here has the effect of introducing a length scale, above which the effect of seepage mechanisms are filtered out. In this sense, there is a degree of similarity with the effect of noise reduction, which is discussed below.

Mixed Behavior

All natural systems of plateau erosion are combined processes, involving a mixture of Eden, invasion percolation and Poissonian fingering mechanisms. The morphology of erosion fronts for this three-member cluster model changes quite strikingly with small changes in the rates of these three processes. Further complexity arises from the range of damping of fluctuations in both groundwater pressure and background weathering. In the cluster growth model, this is simulated by a procedure called noise reduction. It seems that an increase in background weathering rate (Eden process) over the seepage weathering rate tends to dampen the effect of flow instabilities. For flux-dominant erosion, well-defined canyons form; as the background weathering rate increases, the minimum wavelength of flow instabilities, and the concomitant pseudo-viscous finger wavelength, increases. When the background process dominates, only the largest wavelength fingering is evident, and erosion takes place roughly as a uniform retreat with a sinuous erosion front. Plate 6 shows the morphology of cluster growth by the combination of all three processes.

A Phase Diagram for Erosion Patterns

The cluster growth model presented here separates plateau erosion into three distinct mechanisms: invasion percolation, Poissonian or Laplacian fingering and Eden growth. The crossover between each process is illustrated in a triangular "phase" diagram in Figure 4. The relative sensitivity of erosion rates to preexisting substrate strength, background weathering and seepage weathering will determine the importance of each of those three processes respectively. Such crossover behavior has been studied in specific cases [*Lenormand*, 1989; *Mártin et al.*, 1984; *Nadal et al.*, 1986; *Oxaal et al.*, 1987; *Smith and Collins*, 1989]. However, it is not yet clear whether there are any distinct boundaries between different mechanisms and their related scaling properties or whether the crossover between mechanisms is always diffuse. It is certain that different scaling properties change at different rates across the phase diagram. For example, there is an identifiable combined process of invasion percolation and Laplacian fingering midway between the two respective end-members (Plate 2). At this point, the model streams have the scaling behavior of invasion percolation ($d_{min} = 1.130$), but the overall cluster/model erosion pattern has the viscous fingering exponent ($d_f = 1.67$).

Noise Reduction

One important parameter for the cluster growth model that is not included on the phase diagram is the noise reduction factor [*Ball et al.*, 1989; *Meakin*, 1987b].

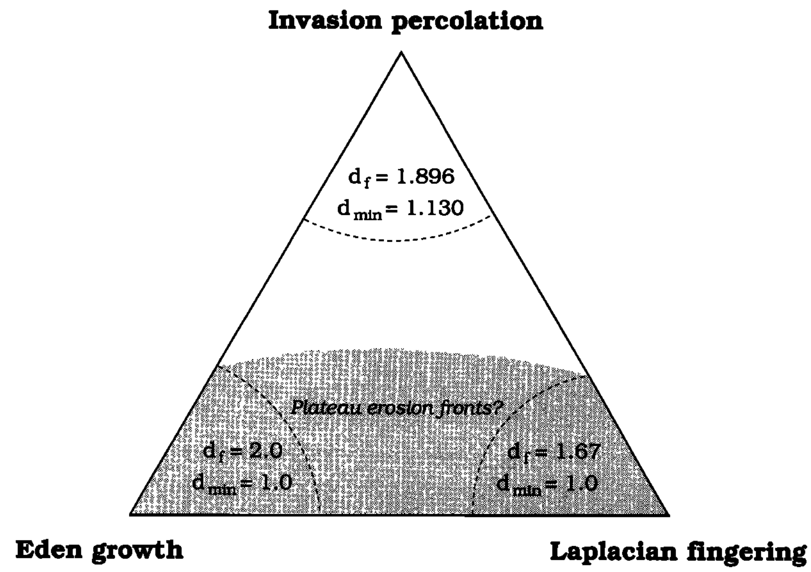


Figure 4. This is a kind of phase diagram intended to illustrate the relative importance of the model weathering and erosion processes. Invasion percolation, which reflects preexisting variability in substrate strength and erosion rates, is unlikely to dominate erosion front morphology. Laplacian, or Poissonian, fingering arising from a DLA growth mechanism in the model reflects seepage weathering as well as seepage erosion and sapping and is thought to be a significant control on plateau front morphology. Background weathering processes and mean substrate weakness are modeled through the Eden mechanism and are thought to be as important. Values of scaling exponents for the three end-member processes include d_{min}, which is the fractal dimension of the shortest path across such structures, and which may affect the scaling of streams formed below the escarpment. Any mixed growth process may exhibit each scaling behavior over distinct length scale ranges.

This is a measure of the damping of fluctuations over time in the rates of background weathering and seepage weathering. The latter occurs because of fluctuations in the groundwater pressure at the erosion front caused largely by the discrete nature of erosion events (e.g., slumps, storm events, freeze-thaw fractures). The degree of damping in some cases is a function of the ratio of the rate of fluctuation versus the rate of erosion. Fluctuations in groundwater efflux or background weathering on a time scale much shorter than the erosion itself will not be "seen" by the erosion process and will thus be damped out.

In the cluster growth model, noise reduction is implemented by requiring a weakening event at a site to wait until a threshold number of visits has been passed [*Meakin,* 1987b]. Remember that on each model cycle, flow weakening is modeled by the arrival of random walkers at the perimeter of the cluster, and background weakening is simulated by randomly choosing a site on the cluster perimeter. If we count the number of times a perimeter site has been visited by a random walker, and the number of times it has been randomly chosen from the set of perimeter sites, we can require the multiplicative weakening of that site to occur only when each of the respective counters has passed a noise threshold total. The illustrations of Poissonian fingering in plates 3 and 4 show the results of simulations with high and low noise thresholds, and it is clear that this mechanism of noise reduction leads to significant smoothing of the cluster perimeter. The degree of noise reduction evidently controls the range of scaling of the model erosion front, and despite its abstract nature it may prove to be a useful parameter in describing real erosion fronts.

Discussion

Permeability and Weakness Heterogeneity

One important factor that has been ignored in this study is the role of permeability heterogeneity in determining the pattern of seepage weathering and sapping. Spatially correlated, possibly multifractal permeability and initial strength distributions have a fundamental effect on the scaling patterns produced by percolation and DLA mechanisms. This sensitivity needs further examination.

Recharge Focusing

The recharge of plateau aquifers can also be nonuniform. In areas of the Colorado Plateau, for example, in Zion National Park, it is evident that small upland, plateau streams often flow directly into the large amphitheatre canyon heads. However, the canyon morphology in Zion is almost certainly the result of sapping. It is perplexing that the low relief (of the order of meters) arid plateau and its small, sometimes ephemeral streams can apparently influence the growth direction of the enormous canyons (relief of a few hundred meters). It is probable that despite their small scale, these streams strongly affect the pattern of groundwater recharge, by localizing infiltration through localizing surface flow. I propose that the morphology of plateau retreat where upland streams flow over the escarpment is determined by such recharge focusing. In places this conjecture is difficult to confirm, because fault patterns suggest a structural control on patterns of upland stream, aquifer permeability and bedrock weakness. In general, however, recharge focusing appears to be the most significant control on the evolution of plateau margins not dealt with by the cluster growth model presented in this study.

Length Scales in Mixed Processes

While the scaling of the three individual growth mechanisms in this model are well known, as discussed above, the behavior of mixed mechanisms is not. The main area of uncertainty is the question of whether or not mixed processes exhibit a continous range of scaling and growth geometries between each end member. Some models of combined growth mechanisms show a finite size scaling effect: in the asymptotic limit of an infinitely large cluster, scaling that is apparently due to a mixture of two processes converges to the scaling of one process alone. This is equivalent to saying that there must exist some length scale above which one process controls the scaling. When a background process such as Eden growth is present, it is inevitable that fractal scaling will be destroyed above a length scale determined by the balance between the time scales of cluster movement and background erosion of strong areas. This is because fractal clusters, such as in Laplacian fingering or invasion percolation, only arise where a large proportion of the model plateau is never eroded. This point was discussed in the section dealing with invasion percolation, but it applies equally to DLA: a DLA cluster has an outer zone of active growth and an inner region of frozen growth, from which random walkers, or model groundwater flux, is screened off and cannot penetrate. Model plateau areas in this frozen region are never eroded, whereas in reality this cannot happen. So, we can conjecture that the relative dominance of background weathering and erosion will put an upper length scale on fractal scaling of plateau erosion fronts, although an Eden mechanism suggests that a self-affine scaling may be present instead.

For mixed DLA and invasion percolation, in the form presented here, it is likely that no fixed point exists, in the renormalization sense, to which the growth behavior converges in the asymptotic limit of an infinite cluster. Put more simply, the weakening model of combined DLA and invasion percolation probably exhibits a continuous change in scaling behavior between each end-member for all cluster sizes.

In summary, up to some length scale the geometry of a plateau margin may exhibit fractal scaling which is a function of the relevance of inherent weakness variation versus the weakening and erosion rates due to seepage. Above this length scale, either Euclidean or self-affine geometry will be observed; the magnitude of this crossover length will be determined by the relative magnitude of background weathering and erosion of the more resistant plateau bedrock.

Conclusions

The aim of this paper has been to introduce a cluster growth model for the pattern of erosion of plateaus. Three key factors have been identified as the principal controls on the scaling and morphology of erosional escarpments. These are the rate of seepage weathering and sapping, the rate of background weathering, and the sensitivity of erosion rates to existing substrate strength.

The pattern of seepage erosion is directly controlled by the groundwater flow distribution and the development of instabilities in the efflux along the escarpment. Erosion patterns controlled by this process alone develop as pseudo-viscous fingers, and the cluster growth model simulated this process using a modified diffusion-limited aggregation model. Laplacian fingering arises when the groundwater is recharged at some significant distance from the erosion front, and Poissonian fingering evolves when recharge is uniform over the plateau.

Background weathering processes are assumed to be spatially and temporally uncorrelated and essentially random. The cluster growth analog is likely to be Eden growth.

Sensitivity to the weakness of the eroding substrate alone leads to an erosion process that is equivalent to invasion percolation, up to some length scale which is a function of the erosion rate of resistant substrate. No spatial correlation of substrate strength was considered.

Natural erosion is likely to be a mixed process involving each of the above mechanisms to a greater or lesser degree. A phase diagram was constructed to differentiate each mechanism and to provide a basis for recognizing the relative importance of each process in natural examples. It is hoped that further work will give us a good idea of the variation in scaling properties across the phase diagram, and that such fractal dimensions and characteristic length scales will help to characterize plateau escarpment erosion mechanisms.

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