Scoring, Term Weighting and the Vector Space Model

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Most of these slides comes from the course:

Information Retrieval and Web Search,

Christopher Manning and Prabhakar Raghavan

Content

- Ranked retrieval
- Scoring documents
- Term frequency
- Collection statistics
- Tf-idf
- Weighting schemes
- Vector space scoring

Boolean retrieval

- Thus far, our queries have all been Boolean:
  - documents either **match** or **don't**
- Good for expert users with precise understanding of their needs and the collection
- Also good for applications: applications can easily consume 1000s of results
- **Not good for the majority of users**
  - Most users incapable of writing Boolean queries (or they are, but they think it's too much work)
  - Most users **don't want to wade through 1000s of results**
    - This is particularly true of web search.

Problem with Boolean search: feast or famine

- Boolean queries often result in **either too few** (=0) **or too many** (1000s) **results**
- Query 1: “*standard user dlink 650*” → 200,000 hits
- Query 2: “*standard user dlink 650 no card found*”: 0 hits
- It takes a lot of skill to come up with a query that produces a manageable number of hits
- AND gives too few; OR gives too many.

Ranked retrieval models

- Rather than a set of documents satisfying a query expression, in **ranked retrieval models**, the system returns an ordering over the (top) documents in the collection with respect to a query
- **Free text queries:** Rather than a query language of operators and expressions, the user's query is just one or more words in a human language
- In principle, there are two separate choices here – the **query language** and the **retrieval model** - but in practice, ranked retrieval models have normally been associated with free text queries.

Feast or famine: not a problem in ranked retrieval

- When a system produces a ranked result set:
  - The size of the result set is not an issue
  - We just show the top *k* (≈ 10) results
  - We don’t overwhelm the user

  - ***Premise: the ranking algorithm works***

Scoring as the basis of ranked retrieval

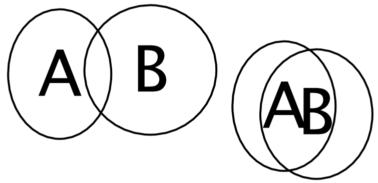
- We wish to return in order the documents most likely to be useful to the searcher
- How can we rank-order the documents in the collection with respect to a query?
- Assign a score – say in [0, 1] – to each document
- This score measures how well document and query "match".

Query-document matching scores

- We need a way of assigning a score to a query/document pair
- Let's start with a one-term query
- If the query term does not occur in the document:
  - The score should be 0
  - *Why? Can we do better?*
- The more frequent the query term in the document, the higher the score (should be)
- We will look at a number of alternatives for this.

Take 1: Jaccard coefficient

- A commonly used measure of overlap of two sets *A* and *B*
- jaccard$(A,B) = |A \cap B| \,/\, |A \cup B|$
- jaccard$(A,A) = 1$
- jaccard$(A,B) = 0$ if $A \cap B = 0$
- *A* and *B* don't have to be the same size
- Always assigns a number between 0 and 1



Jaccard coefficient: Scoring example

- What is the query-document match score that the Jaccard coefficient computes for each of the two documents below?
- Query: *ides of march*
- Document 1: *caesar died in march*
- Document 2: *the long march*

- jaccard$(Q,D) = |Q \cap D| / |Q \cup D|$

- $jaccard(Query, Document1) = 1/6$
- $jaccard(Query, Document2) = 1/5$

Issues with Jaccard for scoring

- It doesn't consider *term frequency* (how many times a term occurs in a document)
- Rare terms in a collection are more informative than frequent terms - Jaccard doesn't consider this information
- We need also a more sophisticated way of normalizing for length
- We can use $|A \cap B| / \sqrt{|A \cup B|}$
- . . . instead of $|A \cap B|/|A \cup B|$ (Jaccard) for length normalization.

Recall (Part 2): Binary term-document incidence matrix

Antony	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
	1	1	0	0	0	1
Brutus	1	1	0	1	0	0
Caesar	1	1	0	1	1	1
Calpurnia	0	1	0	0	0	0
Cleopatra	1	0	0	0	0	0
mercy	1	0	1	1	1	1
worser	1	0	1	1	1	0

Each document is represented by a binary vector $\in \{0,1\}^{|V|}$.

Term-document count matrices

- Consider the number of occurrences of a term in a document:
  - Each document is a count vector in $\mathbb{N}^{v}$: a column below

Antony	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
	157	73	0	0	0	0
Brutus	4	157	0	1	0	0
Caesar	232	227	0	2	1	1
Calpurnia	0	10	0	0	0	0
Cleopatra	57	0	0	0	0	0
mercy	2	0	3	5	5	1
worser	2	0	1	1	1	0

Bag of words model

- Vector representation doesn't consider the ordering of words in a document
- *John is quicker than Mary* and *Mary is quicker than John* have the same vectors
- This is called the bag of words model
- In a sense, this is a step back: the positional index was able to distinguish these two documents

Term frequency tf

- The term frequency $\mathrm{tf}_{t,d}$ of term *t* in document *d* is defined as the number of times that *t* occurs in *d*
- We want to use tf when computing query-document match scores - but how?
- Raw term frequency is not what we want:
  - A document with 10 occurrences of the term **is more relevant** than a document with 1 occurrence of the term.
  - **But not 10 times** more relevant
- Relevance does not increase proportionally with term frequency.

NB: frequency = count, in IR

Log-frequency weighting

- The log frequency weight of term t in d is

$$w_{t,d} = \begin{cases} 1 + \log_{10} \mathrm{tf}_{t,d}, & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0, & \text{otherwise} \end{cases}$$

- 0 → 0, 1 → 1, 2 → 1.3, 10 → 2, 1000 → 4, etc.
- Score for a document-query pair: sum over terms *t* in both *q* and *d*:

$$score(d,q) = \sum_{t \in q \cap d} (1 + \log \mathrm{tf}_{t,d})$$

- The score is 0 if none of the query terms is present in the document

Document frequency

- Rare terms – in the whole collection - are more informative than frequent terms
  - Recall stop words
- Consider a term in the query that is rare in the collection (e.g., *arachnocentric*)
- A document containing this term is very likely to be relevant to the query *arachnocentric*
- We want a high weight for rare terms like *arachnocentric*.

Document frequency, *cont'd*

- Generally frequent terms are **less informative** than rare terms
- Consider a **query** term that is frequent in the collection (e.g., *high*, *increase*, *line*)
- A document containing such a term is more likely to be relevant than a document that doesn't
- **But** consider a query containing two terms – e.g.: high *arachnocentric*
- For a **frequent** term in a document, s.a., high, we want a positive weight but **lower** than for terms that are **rare** in the collection, s.a., *arachnocentric*
- We will use **document frequency** (df) to capture this.

idf weight

- df_t is the <u>document</u> frequency of *t*: the number of documents that contain *t*
  - df_t is an inverse measure of the informativeness of *t (the smaller the better)*
  - $\mathrm{df}_t \leq N$
- We define the idf (**inverse document frequency**) of *t* by

$$\mathrm{idf}_t = \log(N/\mathrm{df}_t)$$

Is a function of t only – does not depend on the document

  - We use log (N/df_t) instead of N/df_t to "dampen" the effect of idf.

idf example, suppose *N* = 1 million

term	df_t	idf_t
calpurnia	1	6
animal	100	4
sunday	1,000	3
fly	10,000	2
under	100,000	1
the	1,000,000	0

$$\mathrm{idf}_t = \log\left(N/\mathrm{df}_t\right)$$

There is one idf value for each term *t* in a collection.

Effect of idf on ranking

- Can idf have an effect on ranking for one-term queries? E.g. like:
  - iPhone
- idf has no effect on ranking one term queries - since there is one idf value for each term in a collection
  - idf affects the ranking of documents for queries with at least two terms
  - For the query capricious person, idf weighting makes occurrences of capricious count for much more in the final document ranking than occurrences of person.

Collection vs. Document frequency

- The collection frequency of *t* is the number of occurrences of *t* in the collection, counting multiple occurrences
- Example:

Word	Collection frequency	Document frequency
insurance	10440	3997
try	10422	8760

- Which word is a better search term (and should get a higher weight)?

tf-idf weighting

- The tf-idf weight of a term is the product of its tf weight and its idf weight:

$$w_{t,d} = (1 + \log \mathrm{tf}_{t,d}) \times \log(N / \mathrm{df}_t)$$

- Best known weighting scheme in information retrieval
  - Note: the "-" in tf-idf is a hyphen, not a minus sign!
  - Alternative names: tf.idf, tf x idf

1. **Increases** with the **number of occurrences within a document**
2. **Increases** with the **rarity of the term in the collection.**

Final ranking of documents for a query

$$\mathrm{Score}(d,q) = \sum_{t \in q \cap d} \mathrm{tf}_{\mathrm{t,d}}. \times \mathrm{idf}_t$$

We will see some other options for computing the score …

Binary → count → weight matrix

dimensions

Antony	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
	5.25	3.18	0	0	0	0.35
Brutus	1.21	6.1	0	1	0	0
Caesar	8.59	2.54	0	1.51	0.25	0
Calpurnia	0	1.54	0	0	0	0
Cleopatra	2.85	0	0	0	0	0
mercy	1.51	0	1.9	0.12	5.25	0.88
worser	1.37	0	0.11	4.15	0.25	1.95

Each document is now represented by a real-valued vector of tf-idf weights $\in \mathbb{R}^{|V|}$

Documents as vectors

- So we have a |V|-dimensional vector space
  - **Terms** are **axes** of the space
  - **Documents** are **points** or vectors in this space
- Very high-dimensional:
  - 400,000 in RCV1
  - tens of millions of dimensions when you apply this to a web search engine
- These are very sparse vectors - most entries are zero.

Queries as vectors

- Key idea 1: Do the same for queries: represent them as vectors in the space
- Key idea 2: Rank documents according to their proximity to the query in this space
- proximity = similarity of vectors
- proximity ≈ inverse of distance
- Recall: We do this because we want to get away from the you're-either-in-or-out Boolean model
- Instead: rank more relevant documents higher than less relevant documents.

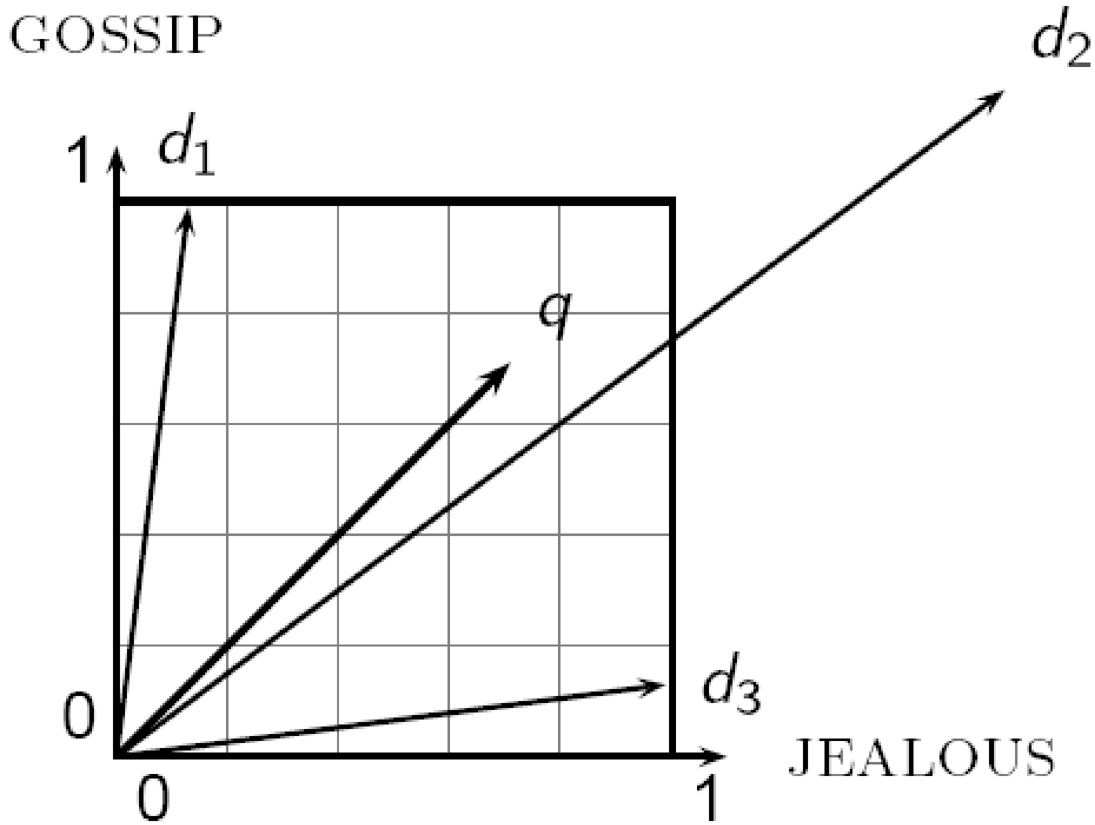
Sec. 6.3

Formalizing vector space proximity

- First cut: distance between two points
  - (= distance between the end points of the two vectors)
- Euclidean distance?
- Euclidean distance is a bad idea . . .
- . . . because Euclidean distance is large for vectors of different lengths.

Why distance is a bad idea

The Euclidean distance between q and d_2 is large even though the distribution of terms in the query q and the distribution of terms in the document d_2 are very similar.



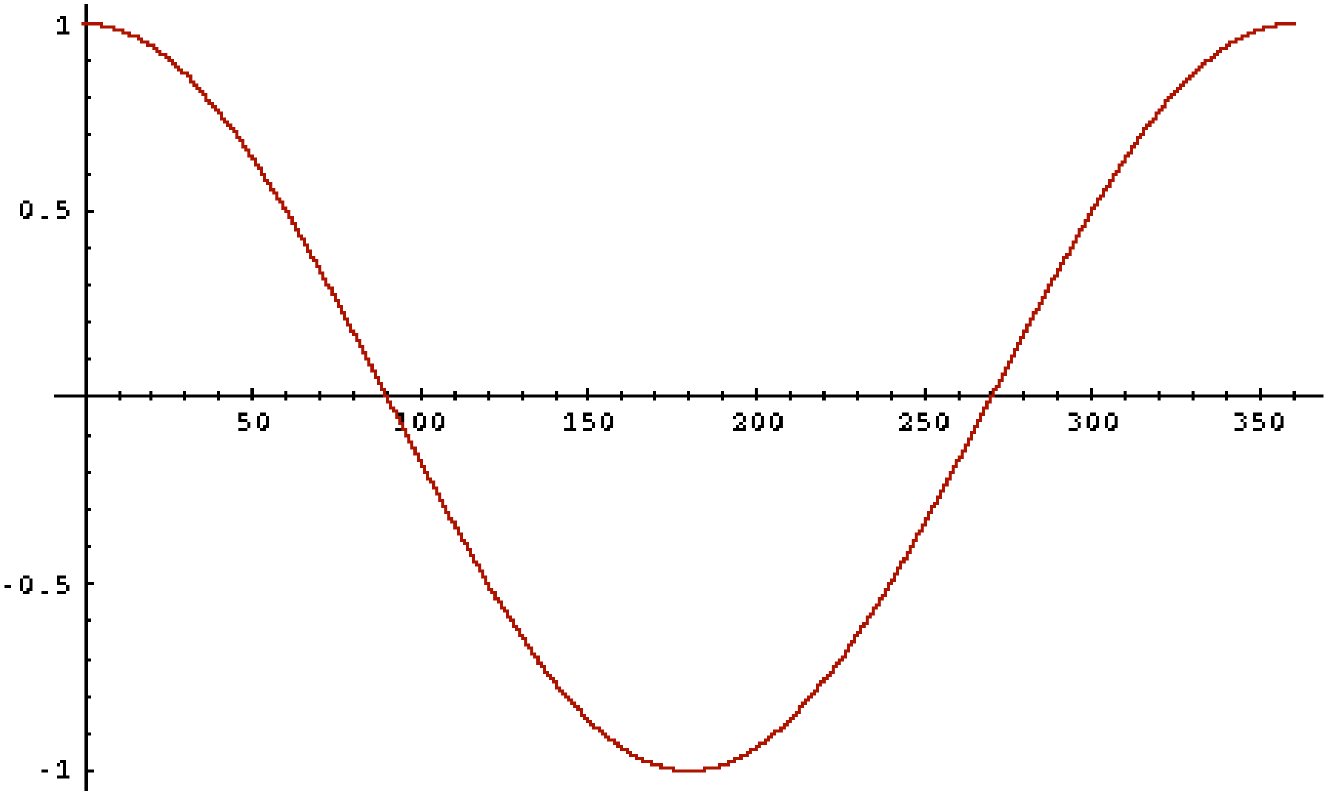
Use angle instead of distance

- Thought experiment: take a document *d* and append it to itself
- Call this document *d′*
- "Semantically" d and d′ have the same content
- The Euclidean distance between the two documents can be quite large
- If the $tf_{t,d}$ representation is used then the angle between the two documents is 0, corresponding to maximal similarity
- Key idea: Rank documents according to angle with query.

From angles to cosines

- The following two notions are equivalent:
  - Rank documents in increasing order of the angle between query and document
  - Rank documents in decreasing order of cosine(query,document)
- Cosine is a monotonically decreasing function for the interval [0^o, 180^o]

From angles to cosines



- But how – *and why* – should we be computing cosines?

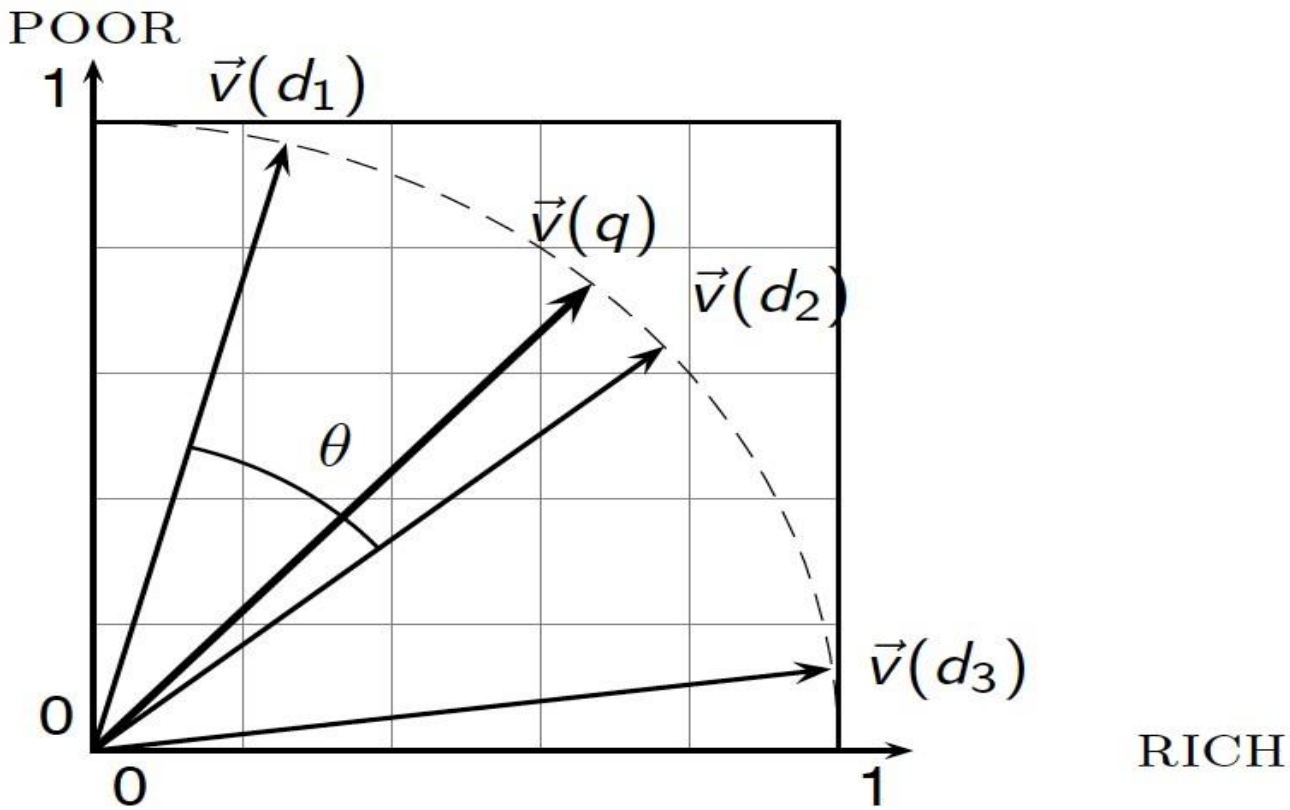
Length normalization

- A vector can be (length-) normalized by dividing each of its components by its length – for this we use the L_2 norm

- Dividing a vector by its L_2 norm makes it a unit (length) vector (on surface of unit hypersphere)
- Effect on the two documents d and d′ (d appended to itself) from earlier slide: they have identical vectors after length-normalization
  - Long and short documents now have comparable weights.

Cosine for length-normalized vectors

- For length-normalized vectors, cosine similarity is simply the dot product (or scalar product)



Cosine similarity amongst 3 documents

How similar are the novels

SaS: *Sense and Sensibility*

PaP: *Pride and Prejudice*, and

WH: *Wuthering Heights*?

term	SaS	PaP	WH
affection	115	58	20
jealous	10	7	11
gossip	2	0	6
wuthering	0	0	38

Term frequencies (counts)

Note: To simplify this example, we don't do idf weighting.

3 documents example contd.

Log frequency weighting

term	SaS	PaP	WH
affection	3.06	2.76	2.30
jealous	2.00	1.85	2.04
gossip	1.30	0	1.78
wuthering	0	0	2.58

After length normalization

term	SaS	PaP	WH
affection	0.789	0.832	0.524
jealous	0.515	0.555	0.465
gossip	0.335	0	0.405
wuthering	0	0	0.588

cos(SaS,PaP) ≈
$0.789 \times 0.832 + 0.515 \times 0.555 + 0.335 \times 0.0 + 0.0 \times 0.0$
≈ 0.94

cos(SaS,WH) ≈ 0.79

cos(PaP,WH) ≈ 0.69

tf-idf weighting has many variants

Term frequency		Document frequency		Normalization	
n (natural)	$\mathrm{tf}_{t,d}$	n (no)	1	n (none)	1
l (logarithm)	$1 + \log(\mathrm{tf}_{t,d})$	t (idf)	$\log \frac{N}{\mathrm{df}_t}$	c (cosine)	$\frac{1}{\sqrt{w_1^2 + w_2^2 + \ldots + w_M^2}}$
a (augmented)	$0.5 + \frac{0.5 \times \mathrm{tf}_{t,d}}{\max_t(\mathrm{tf}_{t,d})}$	p (prob idf)	$\max\{0, \log \frac{N - \mathrm{df}_t}{\mathrm{df}_t}\}$	u (pivoted unique)	$1/u$
b (boolean)	$\begin{cases} 1 & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0 & \text{otherwise} \end{cases}$			b (byte size)	$1/CharLength^{\alpha}$, $\alpha < 1$
L (log ave)	$\frac{1 + \log(\mathrm{tf}_{t,d})}{1 + \log(\mathrm{ave}_{t \in d}(\mathrm{tf}_{t,d}))}$				

Columns headed 'n' are acronyms for weight schemes.

Weighting may differ in queries vs documents

- Many search engines allow for different weightings for queries vs. documents
- SMART Notation: denotes the combination in use in an engine, with the notation *ddd.qqq*, using the acronyms from the previous table
- A very standard weighting scheme is: lnc.ltc
- Document: logarithmic tf (l as first character), no idf and cosine normalization
- Query: logarithmic tf (l in leftmost column), idf (t in second column), cosine normalization …

Summary – vector space ranking

- Represent the query as a weighted tf-idf vector
- Represent each document as a weighted tf-idf vector
- Compute the cosine similarity score for the query vector and each document vector
- Rank documents with respect to the query by score
- Return the top K (e.g., $K = 10$) to the user.