Calculus 2 Project:

"Integrating" Concepts from Abstract Mathematics and Computer-Aided Design

The purpose of this project is to encourage students to make connections between abstract mathematical concepts and real-world applications. This project is not intended to serve as a deep-dive into mathematically challenging exercises. Rather, it is designed to offer insights into and an exploration of the utility of (even relatively simple) mathematics in engineering applications. The project illustrates a connection between integration techniques and 3-D computer modeling. It is designed to be accessible to new learners and supplement early- to mid- semester Calculus 2 instructional materials.

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MATH 0230 Analytical Geometry & Calculus II

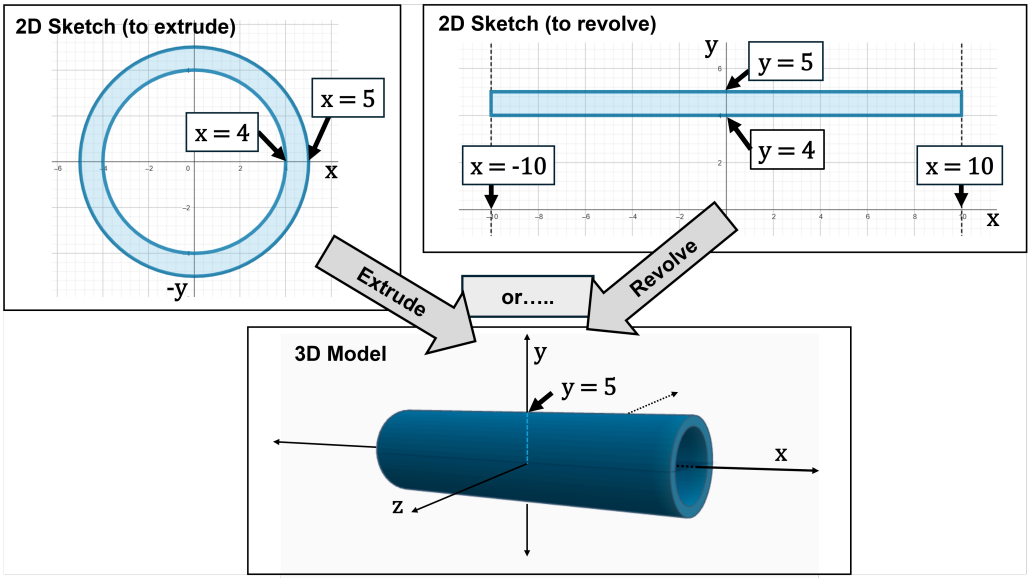
Project 1: "Integrating" Calculus and CAD

Background: Computer-Aided Design (CAD) software is often used by engineers who want to create detailed models of their designs and products. CAD modeling involves creating 2D sketches and converting them to 3D parts. Built-in CAD software tools allow for the efficient conversion from simple sketches to ready-to-manufacture models. Tools for *extrusion* allow modelers to extend a 2D sketch along a straight path, adding in the third dimension and, hence, transforming the sketch into a solid object or surface. Likewise, modelers can use tools to *revolve* their 2D sketch about an axis of rotation, creating a solid or surface that is symmetric about the chosen axis. The images below illustrate the conversion of 2D sketches to 3D parts via extrusion and revolution.

Our task: We are using CAD software to design a space shuttle. (Yes, this *is* rocket science!) We are tasked with modeling two components: the cylindrical body and the nose cone. These components must meet specified volumetric and financial design requirements.

Part I: Building the Body of a Space Shuttle

The cylindrical body of the space shuttle can be modeled equally easily via either extrusion or revolution. To demonstrate this for our company's new intern, we elect to create our model using both methods. The 2D sketches to be used for extrusion and revolution are shown below. In using the revolution tool, we select the **axis of rotation to be the line $y = 0$.** Note that lengths obtained from all sketches are in **units of meters** (m). Follow the steps below to confirm we have met the desired design specifications.



1. Observe the 2D sketch for revolving. Use this sketch and any information above to set up the appropriate integral to calculate the volume of the solid comprising the walls of the body of the space shuttle. **Use the Washer Method**, and evaluate the integral. In 1-2 sentences, justify the use of this method of integration. *Tip*: Begin by sketching the area of integration and the "washers" in your own xy-plane.

2. Observe the 2D sketch for extrusion. Use this sketch and any relevant details from the 2D sketch for revolving to **geometrically confirm the volume** you obtained above. Recall the formula

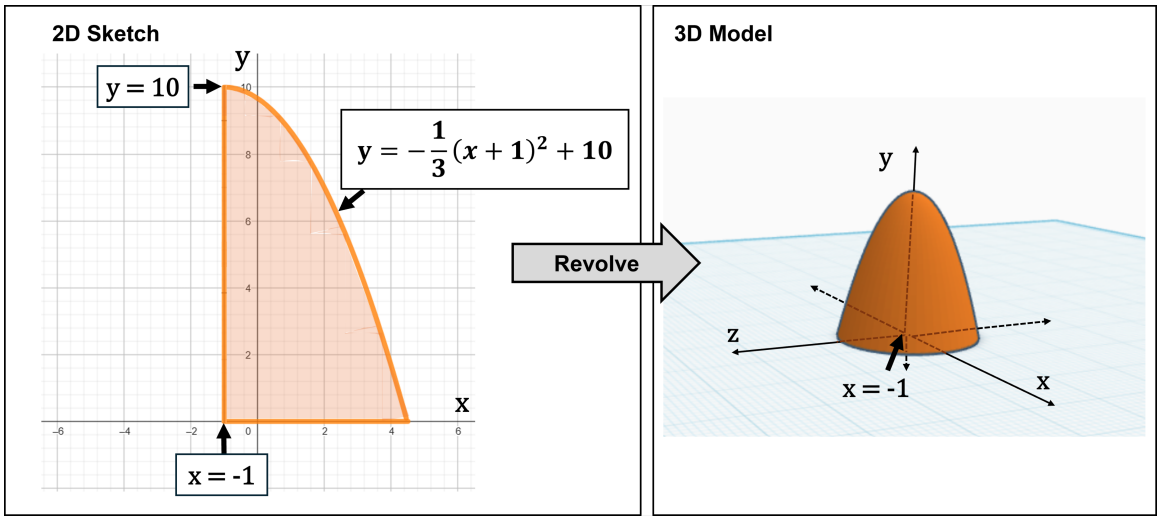
$$V_{cyl} = \pi r^2 h,$$

where r is the radius of the base of the cylinder and h is the height of the cylinder. Additionally, in 1-2 sentences, comment on whether you expect the use of integration or formulas to be more efficient for computing the volume of more complex shapes.

✓ *Checkpoint:* Confirm that the material required to construct the body of the space shuttle has volume within 560 ± 5.6 m^3 (i.e., an error tolerance of 1%). You do not need to submit any work for this. If confirmed, we have met the required design specifications and can move on to modeling the space shuttle's nose cone.

Part II: Crafting the Nose Cone

We will create a model of the space shuttle's nose cone using the revolution tool built into our CAD modeling software. To do this, we create a 2D sketch and select an axis of rotation. Our 2D sketch is shown below. We select the **axis of rotation to be the line $x = -1$.**



1. Observe the 2D sketch of the nose cone. Use this sketch and any information above to set up the appropriate integral to calculate the volume of material making up this solid 3D part. **Use the Disk Method**, and evaluate the integral. In 1-2 sentences, justify the use of this method of integration. *Tip*: Begin by sketching the area of integration and the "disks" in your own xy-plane.

2. We are given a budget for this project of \$19 million. The nose cone is to be constructed out of heat-resistant material that costs \$30,000 per m^3. The body of the space shuttle is to be constructed out of lightweight and structurally strong material that costs \$8,000 per m^3. Based on our current designs, will construction of the space shuttle body and nose cone fall within the budget? Show work and justify your answer.

✓ *Checkpoint:* If the cost for the two parts is within the budget, we have completed our project.