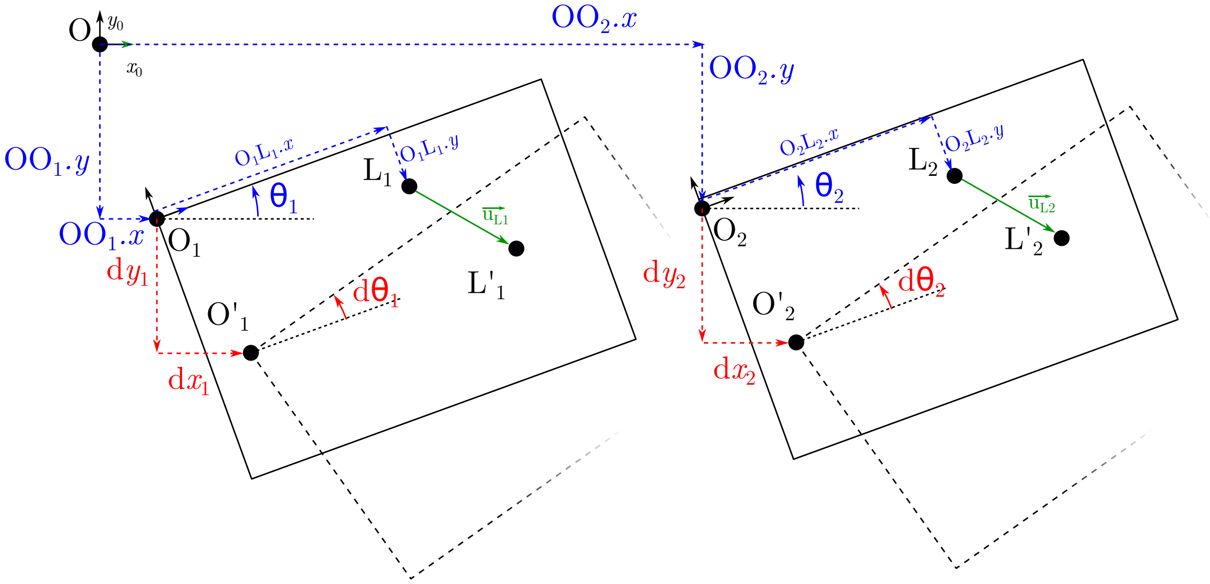

1 Force de L_2 sur L_1

$$
\begin{aligned}
\overrightarrow{\mathscr{R}_{(L_2\rightarrow L_1)}} =& k\overrightarrow{L_1'L_2'}\\
=& k\left(\overrightarrow{L_1'L_1}+\overrightarrow{L_1L_2}+\overrightarrow{L_2L_2'}\right)\\
=& k\left(-\overrightarrow{U_{(L_1\in 1/0)}}+\overrightarrow{L_1L_2}+\overrightarrow{U_{(L_2\in 2/0)}}\right)\\
=& k\left(-\left(\overrightarrow{U_{(O_1\in 1/0)}}+\overrightarrow{L_1O_1}\wedge\overrightarrow{d\theta_{(1/0)}}\right)+\overrightarrow{L_1L_2}+\left(\overrightarrow{U_{(O_2\in 2/0)}}+\overrightarrow{L_2O_2}\wedge\overrightarrow{d\theta_{(2/0)}}\right)\right)\\
=& k\left(-\begin{pmatrix}\mathrm{d}x_1\\ \mathrm{d}y_1\\ 0\end{pmatrix}_{\mathscr{B}_0}+\begin{pmatrix}O_1L_1.x\\ O_1L_1.y\\ 0\end{pmatrix}_{\mathscr{B}_0}\wedge\begin{pmatrix}0\\ 0\\ \mathrm{d}\theta_1\end{pmatrix}_{\mathscr{B}_0}\right.\\
&+\overrightarrow{L_1L_2}\\
&\left.+\begin{pmatrix}\mathrm{d}x_2\\ \mathrm{d}y_2\\ 0\end{pmatrix}_{\mathscr{B}_0}-\begin{pmatrix}O_2L_2.x\\ O_2L_2.y\\ 0\end{pmatrix}_{\mathscr{B}_0}\wedge\begin{pmatrix}0\\ 0\\ \mathrm{d}\theta_2\end{pmatrix}_{\mathscr{B}_0}\right)\\
=& k\left(-\begin{pmatrix}\mathrm{d}x_1\\ \mathrm{d}y_1\\ 0\end{pmatrix}_{\mathscr{B}_0}+\begin{pmatrix}\mathrm{d}\theta_1 O_1L_1.y\\ -\mathrm{d}\theta_1 O_1L_1.x\\ 0\end{pmatrix}_{\mathscr{B}_0}\right.\\
&+\overrightarrow{L_1L_2}\\
&\left.+\begin{pmatrix}\mathrm{d}x_2\\ \mathrm{d}y_2\\ 0\end{pmatrix}_{\mathscr{B}_0}-\begin{pmatrix}\mathrm{d}\theta_2 O_2L_2.y\\ -\mathrm{d}\theta_2 O_2L_2.x\\ 0\end{pmatrix}_{\mathscr{B}_0}\right)\\
=& k\begin{pmatrix}-\mathrm{d}x_1+\mathrm{d}\theta_1\left(O_1L_1.y\right)+\mathrm{d}x_2-\mathrm{d}\theta_2\left(O_2L_2.y\right)+L_1L_2.x\\ -\mathrm{d}y_1-\mathrm{d}\theta_1\left(O_1L_1.x\right)+\mathrm{d}y_2+\mathrm{d}\theta_2\left(O_2L_2.x\right)+L_1L_2.y\end{pmatrix}_{\mathscr{B}_0}
\end{aligned}
$$

2 Moment

$$
\begin{aligned}
\overrightarrow{\mathscr{M}_{O'_{1(2\to1)}}} &= \overrightarrow{O'_1L'_1} \wedge \overrightarrow{\mathscr{R}_{(2\to1)}} \\
&= \left(\overrightarrow{O'_1O_1} + \overrightarrow{O_1L_1} + \overrightarrow{L_1L'_1}\right) \wedge \overrightarrow{\mathscr{R}_{(2\to1)}} \\
&= \left(-\overrightarrow{U_{(O_1\in1/0)}} + \overrightarrow{O_1L_1} + \overrightarrow{U_{(L_1\in1/0)}}\right) \wedge \overrightarrow{\mathscr{R}_{(2\to1)}} \\
&= \left(\cancel{-\overrightarrow{U_{(O_1\in1/0)}}} + \overrightarrow{O_1L_1} + \left(\cancel{\overrightarrow{U_{(O_1\in1/0)}}} + \overrightarrow{L_1O_1} \wedge \overrightarrow{d\theta_{(1/0)}}\right)\right) \wedge \overrightarrow{\mathscr{R}_{(2\to1)}} \\
&= \left(\begin{pmatrix} O_1L_1.x \\ O_1L_1.y \\ 0 \end{pmatrix}_{\mathscr{B}_0} - \begin{pmatrix} O_1L_1.x \\ O_1L_1.y \\ 0 \end{pmatrix}_{\mathscr{B}_0} \wedge \begin{pmatrix} 0 \\ 0 \\ \mathrm{d}\theta_1 \end{pmatrix}_{\mathscr{B}_0}\right) \wedge \overrightarrow{\mathscr{R}_{(2\to1)}} \\
&= k \begin{pmatrix} O_1L_1.x - \mathrm{d}\theta_1 O_1L_1.y \\ O_1L_1.y + \mathrm{d}\theta_1 O_1L_1.x \end{pmatrix}_{\mathscr{B}_0} \wedge \begin{pmatrix} -\mathrm{d}x_1 + \mathrm{d}\theta_1\left(O_1L_1.y\right) + \mathrm{d}x_2 - \mathrm{d}\theta_2\left(O_2L_2.y\right) + L_1L_2.x \\ -\mathrm{d}y_1 - \mathrm{d}\theta_1\left(O_1L_1.x\right) + \mathrm{d}y_2 + \mathrm{d}\theta_2\left(O_2L_2.x\right) + L_1L_2.y \end{pmatrix}_{\mathscr{B}_0} \\
\overrightarrow{\mathscr{M}_{O'_{1(2\to1)}}} \cdot \overrightarrow{z} &= k \times [\\
& O_1L_1.x\left(-\mathrm{d}y_1 - \mathrm{d}\theta_1\left(O_1L_1.x\right) + \mathrm{d}y_2 + \mathrm{d}\theta_2\left(O_2L_2.x\right) + L_1L_2.y\right) - \mathrm{d}\theta_1 O_1L_1.y L_1L_2.y \\
& -\left(O_1L_1.y\left(-\mathrm{d}x_1 + \mathrm{d}\theta_1\left(O_1L_1.y\right) + \mathrm{d}x_2 - \mathrm{d}\theta_2\left(O_2L_2.y\right) + L_1L_2.x\right) + \mathrm{d}\theta_1 O_1L_1.x L_1L_2.x\right)] \\
&= k \times [\\
& \mathrm{d}x_1\left(O_1L_1.y\right) - \mathrm{d}y_1\left(O_1L_1.x\right) + \mathrm{d}\theta_1\left(-O_1L_1.x^2 - O_1L_1.y\ L_1L_2.y - O_1L_1.y^2 + O_1L_1.x\ L_1L_2.x\right) \\
& - \mathrm{d}x_2\left(O_1L_1.y\right) + \mathrm{d}y_2\left(O_1L_1.x\right) + \mathrm{d}\theta_2\left(O_1L_1.x\ O_2L_2.x + O_1L_1.y\ O_2L_2.y\right) \\
& + O_1L_1.x\ L_1L_2.y - O_1L_1.y\ L_1L_2.x]
\end{aligned}
$$

3 Système

Dans le système

$$K \cdot U = F$$

, on a :

$$
K = \begin{bmatrix}
-k & 0 & (kO_1L_1.y) & \dots \\
0 & -k & -(kO_1L_1.x) & \dots \\
(k\ O_1L_1.y) & -(k\ O_1L_1.x) & k(O_1L_1.x\ L_1L_2.x - (O_1L_1.x^2 + O_1L_1.y^2 + O_1L_1.y\ L_1L_2.y)) & \dots \\
\dots & k & 0 & -(kO_2L_2.y) \\
\dots & 0 & k & (kO_2L_2.x) \\
\dots & -(k\ O_1L_1.y) & (k\ O_1L_1.x) & k(O_1L_1.x\ O_2L_2.x + O_1L_1.y\ O_2L_2.y)
\end{bmatrix}
$$

$$
U = \begin{pmatrix} \mathrm{d}x_1 \\ \mathrm{d}y_1 \\ \mathrm{d}\theta_1 \\ \mathrm{d}x_2 \\ \mathrm{d}y_2 \\ \mathrm{d}\theta_2 \end{pmatrix}
$$

$$
F = \begin{pmatrix} -kL_1L_2.x \\ -kL_1L_2.y \\ k\left(O_1L_1.y\ L_1L_2.x - O_1L_1.x\ L_1L_2.y\right) \end{pmatrix}
$$