On a N solides, possédant l liaisons entre elles.

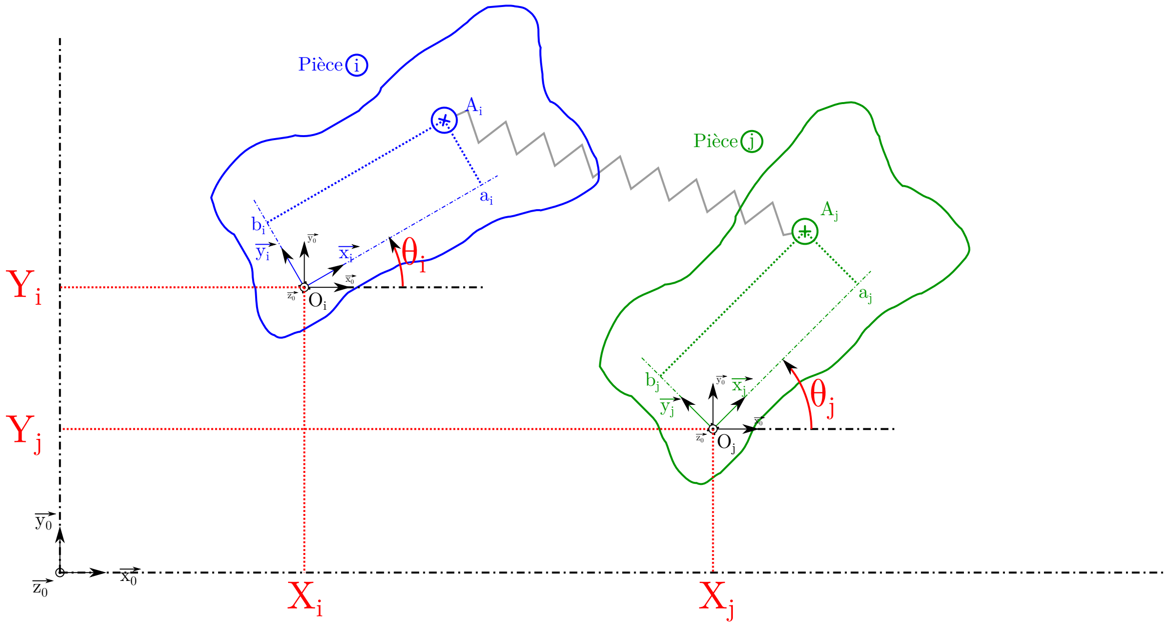
I. Principe des travaux virtuels :

$$\sum W^* = 0 \tag{1}$$

$$\sum_{k=1}^{l} W^*_{liaisons\ k} + W^*_{souris} + W^*_{Blocage} = 0 \tag{2}$$

II. Travail virtuel des liaisons

Soit une liaison pivot k entre des solides i et j, au point $A_i = A_j$, de raideur $\mathbf{K_L}$. On ne prend que le mouvement virtuel de i.



$$\begin{aligned}
W^* =& \overrightarrow{F_{j\mapsto i}} \cdot \overrightarrow{U^*_{A_i\in i/j}} && \forall \overrightarrow{U^*_{A_i\in i/j}} \quad (3)\\
=& \mathbf{K_L}\overrightarrow{A_iA_j} \cdot \left(\overrightarrow{U^*_{O_i\in i/j}} + \overrightarrow{\Theta^*_{i/j}} \wedge \overrightarrow{O_iA_i}\right) && (4)\\
=& \mathbf{K_L}\left(\overrightarrow{A_iO_i} + \overrightarrow{O_iO_0} + \overrightarrow{O_0O_j} + \overrightarrow{O_jA_j}\right) \cdot \left(X^*\overrightarrow{x_0} + Y^*\overrightarrow{y_0} + \overrightarrow{\Theta^*_{i/j}} \wedge (a_i\overrightarrow{x_i} + b_i\overrightarrow{y_i})\right) && (5)\\
=& \mathbf{K_L}\left((-a_i\overrightarrow{x_i} - b_i\overrightarrow{y_i}) + (-X_i\overrightarrow{x_0} - Y_i\overrightarrow{y_0}) + (X_j\overrightarrow{x_0} + Y_j\overrightarrow{y_0}) + (a_j\overrightarrow{x_j} + b_j\overrightarrow{y_j})\right) && (6)\\
& \cdot \left(X^*\overrightarrow{x_0} + Y^*\overrightarrow{y_0} + \Theta^*\overrightarrow{z_0} \wedge (a_i\overrightarrow{x_i} + b_i\overrightarrow{y_i})\right) && (7)\\
=& \mathbf{K_L}\left(-a_i\left(\cos(\Theta_i)\overrightarrow{x_0} + \sin(\Theta_i)\overrightarrow{y_0}\right) - b_i\left(-\sin(\Theta_i)\overrightarrow{x_0} + \cos(\Theta_i)\overrightarrow{y_0}\right)\right) && (8)\\
& - X_i\overrightarrow{x_0} - Y_i\overrightarrow{y_0} + X_j\overrightarrow{x_0} + Y_j\overrightarrow{y_0} && (9)\\
& + a_j\left(\cos(\Theta_j)\overrightarrow{x_0} + \sin(\Theta_j)\overrightarrow{y_0}\right) + b_j\left(-\sin(\Theta_j)\overrightarrow{x_0} + \cos(\Theta_j)\overrightarrow{y_0}\right) && (10)\\
& \cdot \left(X^*\overrightarrow{x_0} + Y^*\overrightarrow{y_0} + a_i\Theta^*\overrightarrow{y_i} - b_i\Theta^*\overrightarrow{x_i}\right) && \forall (X^*, Y^*\Theta^*) \in \mathbb{R}^3 \quad (11)\\
& && (12)
\end{aligned}$$

Linéarisation de la trigo :

Si $\left(X_i^p, Y_i^p, \Theta_i^p, X_j^p, Y_j^p, \Theta_j^p\right)$ sont les solutions au pas de temps précédent, et que l'on suppose que la solution est proche de la précédente, alors :

$$\cos(\Theta_i) \approx \cos(\Theta_i^p) - (\Theta_i - \Theta_i^p)\sin(\Theta_i^p) \tag{13}$$
$$\sin(\Theta_i) \approx \sin(\Theta_i^p) + (\Theta_i - \Theta_i^p)\cos(\Theta_i^p) \tag{14}$$
$$\cos(\Theta_j) \approx \cos(\Theta_j^p) - (\Theta_j - \Theta_j^p)\sin(\Theta_j^p) \tag{15}$$
$$\sin(\Theta_j) \approx \sin(\Theta_j^p) + (\Theta_j - \Theta_j^p)\cos(\Theta_j^p) \tag{16}$$

Reprenons le travail (virtuel)

$$W^* \approx \mathbf{K_L}\left[-a_i\left((\cos(\Theta_i^p) - (\Theta_i - \Theta_i^p)\sin(\Theta_i^p))\,\overrightarrow{x_0} + (\sin(\Theta_i^p) + (\Theta_i - \Theta_i^p)\cos(\Theta_i^p))\,\overrightarrow{y_0}\right)\right. \tag{17}$$
$$- b_i\left(-(\sin(\Theta_i^p) + (\Theta_i - \Theta_i^p)\cos(\Theta_i^p))\,\overrightarrow{x_0} + (\cos(\Theta_i^p) - (\Theta_i - \Theta_i^p)\sin(\Theta_i^p))\,\overrightarrow{y_0}\right) \tag{18}$$
$$- X_i\overrightarrow{x_0} - Y_i\overrightarrow{y_0} + X_j\overrightarrow{x_0} + Y_j\overrightarrow{y_0} \tag{19}$$
$$+ a_j\left(\left(\cos(\Theta_j^p) - (\Theta_j - \Theta_j^p)\sin(\Theta_j^p)\right)\overrightarrow{x_0} + \left(\sin(\Theta_j^p) + (\Theta_j - \Theta_j^p)\cos(\Theta_j^p)\right)\overrightarrow{y_0}\right) \tag{20}$$
$$\left.+ b_j\left(-\left(\sin(\Theta_j^p) + (\Theta_j - \Theta_j^p)\cos(\Theta_j^p)\right)\overrightarrow{x_0} + \left(\cos(\Theta_j^p) - (\Theta_j - \Theta_j^p)\sin(\Theta_j^p)\right)\overrightarrow{y_0}\right)\right] \tag{21}$$
$$\cdot \tag{22}$$
$$\left(X^*\overrightarrow{x_0} + Y^*\overrightarrow{y_0} + a_i\Theta^*\left(-\sin(\Theta_i)\overrightarrow{x_0} + \cos(\Theta_i)\overrightarrow{y_0}\right) - b_i\Theta^*\left(\cos(\Theta_i)\overrightarrow{x_0} + \sin(\Theta_i)\overrightarrow{y_0}\right)\right) \tag{23}$$

- **Projection sur $\overrightarrow{x_0}$:** Attension, on suppose que dans le mouvement virtuel, $\Theta_i \approx \Theta_i^p$

$$W^*_{/\overrightarrow{x_0}} \approx \mathbf{K_L}\left[-a_i\left(\cos(\Theta_i^p) - (\Theta_i - \Theta_i^p)\sin(\Theta_i^p)\right)\right. \tag{24}$$
$$+ b_i\left(\sin(\Theta_i^p) + (\Theta_i - \Theta_i^p)\cos(\Theta_i^p)\right) \tag{25}$$
$$- X_i + X_j \tag{26}$$
$$+ a_j\left(\cos(\Theta_j^p) - (\Theta_j - \Theta_j^p)\sin(\Theta_j^p)\right) \tag{27}$$
$$\left.- b_j\left(\sin(\Theta_j^p) + (\Theta_j - \Theta_j^p)\cos(\Theta_j^p)\right)\right] \tag{28}$$
$$\cdot\left(X^* - a_i\Theta^*\sin(\Theta_i^p) - b_i\Theta^*\cos(\Theta_i^p)\right) \tag{29}$$
$$\approx \mathbf{K_L}\left[-a_i\left(\cos(\Theta_i^p) + \Theta_i^p\sin(\Theta_i^p)\right)\right. \tag{30}$$
$$+ b_i\left(\sin(\Theta_i^p) - \Theta_i^p\cos(\Theta_i^p)\right) \tag{31}$$
$$+ a_j\left(\cos(\Theta_j^p) + \Theta_j^p\sin(\Theta_j^p)\right) \tag{32}$$
$$- b_j\left(\sin(\Theta_j^p) - \Theta_j^p\cos(\Theta_j^p)\right) \tag{33}$$
$$\left.+ a_i\Theta_i\sin(\Theta_i^p) + b_i\Theta_i\cos(\Theta_i^p) - X_i + X_j - a_j\Theta_j\sin(\Theta_j^p) - b_j\Theta_j\cos(\Theta_j^p)\right] \tag{34}$$
$$\cdot\left(X^* - a_i\Theta^*\sin(\Theta_i^p) - b_i\Theta^*\cos(\Theta_i^p)\right) \tag{35}$$

Posons :

$$D = -a_i\left(\cos(\Theta_i^p) + \Theta_i^p\sin(\Theta_i^p)\right) \tag{36}$$
$$+ b_i\left(\sin(\Theta_i^p) - \Theta_i^p\cos(\Theta_i^p)\right) \tag{37}$$
$$+ a_j\left(\cos(\Theta_j^p) + \Theta_j^p\sin(\Theta_j^p)\right) \tag{38}$$
$$- b_j\left(\sin(\Theta_j^p) - \Theta_j^p\cos(\Theta_j^p)\right) \tag{39}$$

et

$$cv_i = a_i\sin(\Theta_i^p) + b_i\cos(\Theta_i^p) \tag{40}$$
$$cv_j = a_j\sin(\Theta_j^p) + b_j\cos(\Theta_j^p) \tag{41}$$
$$\tag{42}$$

Approximation

$$W^*_{/\vec{x_0}} \approx \mathbf{K_L}\left[D + \Theta_i\left(a_i\sin(\Theta_i^p) + b_i\cos(\Theta_i^p)\right) - X_i + X_j - \Theta_j\left(a_j\sin(\Theta_j^p) + b_j\cos(\Theta_j^p)\right)\right] \tag{43}$$

$$\cdot\left(X^* - \Theta^*\left(a_i\sin(\Theta_i^p) + b_i\cos(\Theta_i^p)\right)\right) \tag{44}$$

$$\approx \mathbf{K_L}\left[D - X_i + 0Y_i + \Theta_i\left(a_i\sin(\Theta_i^p) + b_i\cos(\Theta_i^p)\right) + X_j + 0Y_j - \Theta_j\left(a_j\sin(\Theta_j^p) + b_j\cos(\Theta_j^p)\right)\right] \tag{45}$$

$$\cdot\left(X^* - \Theta^*\left(a_i\sin(\Theta_i^p) + b_i\cos(\Theta_i^p)\right)\right) \tag{46}$$

$$\approx \mathbf{K_L}\left[D - X_i + 0Y_i + \Theta_i cv_i + X_j + 0Y_j - \Theta_j cv_j\right] \tag{47}$$

$$\cdot\left(X^* - \Theta^* cv_i\right) \tag{48}$$

Sous forme matricielle :

$$\approx \left(\begin{bmatrix} -\mathbf{K_L} & 0 & \mathbf{K_L}cv_i & \mathbf{K_L} & 0 & -\mathbf{K_L}cv_j \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \mathbf{K_L}cv_i & 0 & -\mathbf{K_L}cv_i^2 & -\mathbf{K_L}cv_i & 0 & \mathbf{K_L}cv_icv_j \end{bmatrix} \cdot \begin{bmatrix} X_i \\ Y_i \\ \Theta_i \\ X_j \\ Y_j \\ \Theta_j \end{bmatrix} + \begin{bmatrix} D\mathbf{K_L} \\ 0 \\ -D\mathbf{K_L}cv_i \end{bmatrix}\right) \cdot \begin{bmatrix} X^* \\ Y^* \\ \Theta^* \end{bmatrix} \tag{49}$$

- **Projection sur $\vec{y_0}$:** Attention, on suppose que dans le mouvement virtuel, $\Theta_i \approx \Theta_i^p$

$$W^*_{/\vec{y_0}} \approx \mathbf{K_L}\left[-a_i\left(\sin(\Theta_i^p) + (\Theta_i - \Theta_i^p)\cos(\Theta_i^p)\right)\right. \tag{50}$$

$$- b_i\left(\cos(\Theta_i^p) - (\Theta_i - \Theta_i^p)\sin(\Theta_i^p)\right) \tag{51}$$

$$- Y_i + Y_j \tag{52}$$

$$+ a_j\left(\sin(\Theta_j^p) + (\Theta_j - \Theta_j^p)\cos(\Theta_j^p)\right) \tag{53}$$

$$\left.+b_j\left(\cos(\Theta_j^p) - (\Theta_j - \Theta_j^p)\sin(\Theta_j^p)\right)\right] \tag{54}$$

$$\cdot \tag{55}$$

$$\left(Y^* + a_i\Theta^*\left(\cos(\Theta_i)\right) - b_i\Theta^*\left(+\sin(\Theta_i)\right)\right) \tag{56}$$

$$\approx \mathbf{K_L}\left[-a_i\left(\sin(\Theta_i^p) - \Theta_i^p\cos(\Theta_i^p)\right)\right. \tag{57}$$

$$- b_i\left(\cos(\Theta_i^p) + \Theta_i^p\sin(\Theta_i^p)\right) \tag{58}$$

$$+ a_j\left(\sin(\Theta_j^p) - \Theta_j^p\cos(\Theta_j^p)\right) \tag{59}$$

$$+ b_j\left(\cos(\Theta_j^p) + \Theta_j^p\sin(\Theta_j^p)\right) \tag{60}$$

$$\left.-a_i\Theta_i\cos(\Theta_i^p) + b_i\Theta_i\sin(\Theta_i^p) - Y_i + Y_j + a_j\Theta_j\cos(\Theta_j^p) - b_j\Theta_j\sin(\Theta_j^p)\right] \tag{61}$$

$$\cdot \tag{62}$$

$$\left(Y^* + a_i\Theta^*\cos(\Theta_i) - b_i\Theta^*\sin(\Theta_i)\right) \tag{63}$$

$$\tag{64}$$

Posons :

$$E = -a_i\left(\sin(\Theta_i^p) - \Theta_i^p\cos(\Theta_i^p)\right) \tag{65}$$

$$- b_i\left(\cos(\Theta_i^p) + \Theta_i^p\sin(\Theta_i^p)\right) \tag{66}$$

$$+ a_j\left(\sin(\Theta_j^p) - \Theta_j^p\cos(\Theta_j^p)\right) \tag{67}$$

$$+ b_j\left(\cos(\Theta_j^p) + \Theta_j^p\sin(\Theta_j^p)\right) \tag{68}$$

et

$$cw_i = -a_i\cos(\Theta_i^p) + b_i\sin(\Theta_i^p) \tag{69}$$

$$cw_j = -a_j\cos(\Theta_j^p) + b_j\sin(\Theta_j^p) \tag{70}$$

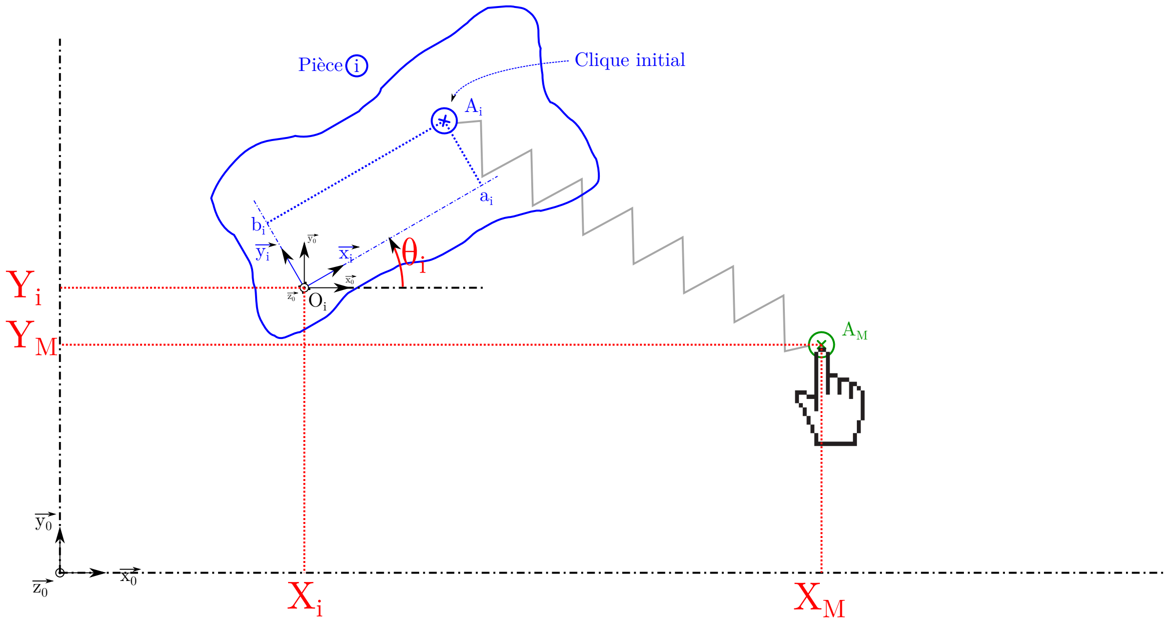
$$\tag{71}$$

$$
\begin{aligned}
W^*_{/\overrightarrow{y_0}} \approx & \mathbf{K_L} \left[E + \Theta_i \left(-a_i \cos(\Theta_i^p) + b_i \sin(\Theta_i^p)\right) - X_i + X_j + \Theta_j \left(a_j \cos(\Theta_j^p) - b_j \sin(\Theta_j^p)\right)\right] & (72)\\
& \cdot \left(Y^* + \Theta^* \left(a_i \cos(\Theta_i^p) - b_i \sin(\Theta_i^p)\right)\right) & (73)\\
\approx & \mathbf{K_L} \left[E + 0 - Y_i + \Theta_i cw_i + 0 + Y_j - \Theta_j cw_j\right] & (74)\\
& \cdot \left(Y^* - \Theta^* cw_i\right) & (75)\\
& & (76)
\end{aligned}
$$

Sous forme matricielle :

$$
\approx \left(\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\mathbf{K_L} & \mathbf{K_L} cw_i & 0 & \mathbf{K_L} & \mathbf{K_L} cw_j \\ 0 & \mathbf{K_L} cw_i & -\mathbf{K_L} cw_i^2 & 0 & -\mathbf{K_L} cw_i & -\mathbf{K_L} cw_i cw_j \end{bmatrix} \cdot \begin{bmatrix} X_i \\ Y_i \\ \Theta_i \\ X_j \\ Y_j \\ \Theta_j \end{bmatrix} + \begin{bmatrix} 0 \\ E\mathbf{K_L} \\ -E\mathbf{K_L} cw_i \end{bmatrix} \right) \cdot \begin{bmatrix} X^* \\ Y^* \\ \Theta^* \end{bmatrix} \qquad (77)
$$

III. Travail virtuel de la souris



$$
\begin{aligned}
W^*_M =& \overrightarrow{F_{M\mapsto i}} \cdot \overrightarrow{U^*_{A_i \in i/0}} & (78)\\
=& \mathbf{K_M} \overrightarrow{A_i A_M} \cdot \left(\overrightarrow{U^*_{O_i \in i/0}} + \overrightarrow{\Theta^*_{i/0}} \wedge \overrightarrow{O_i A_i}\right) & (79)\\
=& \mathbf{K_M} \left[\overrightarrow{A_i O_i} + \overrightarrow{O_i O_0} + \overrightarrow{O_0 M}\right] \cdot \left(X^* \overrightarrow{x_0} + Y^* \overrightarrow{y_0} + \overrightarrow{\Theta^*_{i/j}} \wedge (a_i \overrightarrow{x_i} + b_i \overrightarrow{y_i})\right) & (80)\\
=& \mathbf{K_M} \left[(-a_i \overrightarrow{x_i} - b_i \overrightarrow{y_i}) + (-X_i \overrightarrow{x_0} - Y_i \overrightarrow{y_0}) + (X_M \overrightarrow{x_0} + Y_M \overrightarrow{y_0})\right] & (81)\\
& \cdot \left(X^* \overrightarrow{x_0} + Y^* \overrightarrow{y_0} + \Theta^* \overrightarrow{z_0} \wedge (a_i \overrightarrow{x_i} + b_i \overrightarrow{y_i})\right) \qquad \forall (X^*, Y^* \Theta^*) \in \mathbb{R}^3 & (82)\\
=& \mathbf{K_M} \left[(-a_i \left(\cos(\Theta_i) \overrightarrow{x_0} + \sin(\Theta_i) \overrightarrow{y_0}\right) - b_i \left(-\sin(\Theta_i) \overrightarrow{x_0} + \cos(\Theta_i) \overrightarrow{y_0}\right)\right) & (83)\\
& + (-X_i \overrightarrow{x_0} - Y_i \overrightarrow{y_0}) + (X_M \overrightarrow{x_0} + Y_M \overrightarrow{y_0})] & (84)\\
& \cdot \left(X^* \overrightarrow{x_0} + Y^* \overrightarrow{y_0} + a_i \Theta^* \overrightarrow{y_i} - b_i \Theta^* \overrightarrow{x_i}\right) \qquad \forall (X^*, Y^* \Theta^*) \in \mathbb{R}^3 & (85)
\end{aligned}
$$

$$
\begin{aligned}
W_M^* = &\mathbf{K_M}\left[\left(-a_i\left(\cos(\Theta_i)\overrightarrow{x_0}+\sin(\Theta_i)\overrightarrow{y_0}\right)-b_i\left(-\sin(\Theta_i)\overrightarrow{x_0}+\cos(\Theta_i)\overrightarrow{y_0}\right)\right)\right. && (86)\\
&\left.+\left(-X_i\overrightarrow{x_0}-Y_i\overrightarrow{y_0}\right)+\left(X_M\overrightarrow{x_0}+Y_M\overrightarrow{y_0}\right)\right] && (87)\\
&\left(X^*\overrightarrow{x_0}+Y^*\overrightarrow{y_0}+a_i\Theta^*\left(-\sin(\Theta_i^p)\overrightarrow{x_0}+\cos(\Theta_i^p)\overrightarrow{y_0}\right)-b_i\Theta^*\left(\cos(\Theta_i^p)\overrightarrow{x_0}+\sin(\Theta_i^p)\overrightarrow{y_0}\right)\right) \quad \forall (X^*,Y^*\Theta^*)\in\mathbb{R}^3 && (88)\\
= &\mathbf{K_M}\left[-a_i\left(\left(\cos(\Theta_i^p)-(\Theta_i-\Theta_i^p)\sin(\Theta_i^p)\right)\overrightarrow{x_0}+\left(\sin(\Theta_i^p)+(\Theta_i-\Theta_i^p)\cos(\Theta_i^p)\right)\overrightarrow{y_0}\right)\right. && (89)\\
&-b_i\left(-\left(\sin(\Theta_i^p)+(\Theta_i-\Theta_i^p)\cos(\Theta_i^p)\right)\overrightarrow{x_0}+\left(\cos(\Theta_i^p)-(\Theta_i-\Theta_i^p)\sin(\Theta_i^p)\right)\overrightarrow{y_0}\right) && (90)\\
&\left.+\left(-X_i\overrightarrow{x_0}-Y_i\overrightarrow{y_0}\right)+\left(X_M\overrightarrow{x_0}+Y_M\overrightarrow{y_0}\right)\right] && (91)\\
&\left(X^*\overrightarrow{x_0}+Y^*\overrightarrow{y_0}+a_i\Theta^*\left(-\sin(\Theta_i^p)\overrightarrow{x_0}+\cos(\Theta_i^p)\overrightarrow{y_0}\right)-b_i\Theta^*\left(\cos(\Theta_i^p)\overrightarrow{x_0}+\sin(\Theta_i^p)\overrightarrow{y_0}\right)\right) \quad \forall (X^*,Y^*\Theta^*)\in\mathbb{R}^3 && (92)
\end{aligned}
$$

Projection sur $\overrightarrow{x_0}$:

$$
\begin{aligned}
W_M^*/\overrightarrow{x_0} = &\mathbf{K_M}\left[-a_i\left(\cos(\Theta_i^p)-(\Theta_i-\Theta_i^p)\sin(\Theta_i^p)\right)+b_i\left(\sin(\Theta_i^p)+(\Theta_i-\Theta_i^p)\cos(\Theta_i^p)\right)-X_i+X_M\right] && (93)\\
&\cdot\left(X^*-\Theta^*\left(a_i\sin(\Theta_i^p)+b_i\cos(\Theta_i^p)\right)\right) && (94)\\
= &\mathbf{K_M}\left[-X_i+\Theta_i\left(a_i\sin(\Theta_i^p)+b_i\cos(\Theta_i^p)\right)\right. && (95)\\
&\left.+X_M-a_i\left(\cos(\Theta_i^p)+\Theta_i^p\sin(\Theta_i^p)\right)+b_i\left(\sin(\Theta_i^p)+\Theta_i^p\cos(\Theta_i^p)\right)\right] && (96)\\
&\cdot\left(X^*-\Theta^*\left(a_i\sin(\Theta_i^p)+b_i\cos(\Theta_i^p)\right)\right) && (97)
\end{aligned}
$$

Posons :

$$
\begin{aligned}
D &= X_M-a_i\left(\cos(\Theta_i^p)+\Theta_i^p\sin(\Theta_i^p)\right)+b_i\left(\sin(\Theta_i^p)-\Theta_i^p\cos(\Theta_i^p)\right) && (98)\\
cv_i &= a_i\sin(\Theta_i^p)+b_i\cos(\Theta_i^p) && (99)\\
& && (100)
\end{aligned}
$$

Ainsi :

$$
\begin{aligned}
W_M^*/\overrightarrow{x_0} = &\mathbf{K_M}\left[-X_i+\Theta_i cv_i+D\right] && (101)\\
&\cdot\left(X^*-\Theta^* cv_i\right) && (102)
\end{aligned}
$$

Sous forme matricielle :

$$
W_M^*/\overrightarrow{x_0} = \left(\begin{bmatrix} -\mathbf{K_M} & 0 & \mathbf{K_M}cv_i \\ 0 & 0 & 0 \\ \mathbf{K_M}cv_i & 0 & -\mathbf{K_M}cv_i^2 \end{bmatrix}\begin{bmatrix} X_i \\ Y_i \\ \Theta_i \end{bmatrix}+\begin{bmatrix} D\mathbf{K_M} \\ 0 \\ -D\mathbf{K_M}cv_i \end{bmatrix}\right)\cdot\begin{bmatrix} X^* \\ Y^* \\ \Theta^* \end{bmatrix} \qquad (103)
$$

Projection sur $\overrightarrow{y_0}$:

$$
\begin{aligned}
W_M^*/\overrightarrow{y_0} = &\mathbf{K_M}\left[-a_i\left(\sin(\Theta_i^p)+(\Theta_i-\Theta_i^p)\cos(\Theta_i^p)\right)\right. && (104)\\
&-b_i\left(\cos(\Theta_i^p)-(\Theta_i-\Theta_i^p)\sin(\Theta_i^p)\right) && (105)\\
&\left.-Y_i+Y_M\right] && (106)\\
&\cdot\left(Y^*+\Theta^*\left(a_i\cos(\Theta_i^p)-b_i\sin(\Theta_i^p)\right)\right) && (107)\\
= &\mathbf{K_M}\left[-Y_i+\Theta_i\left(-a_i\cos(\Theta_i^p)+b_i\sin(\Theta_i^p)\right)\right. && (108)\\
&\left.Y_M-a_i\left(\sin(\Theta_i^p)-\Theta_i^p\cos(\Theta_i^p)\right)-b_i\left(\cos(\Theta_i^p)+\Theta_i^p\sin(\Theta_i^p)\right)\right] && (109)\\
&\cdot\left(Y^*+\Theta^*\left(a_i\cos(\Theta_i^p)-b_i\sin(\Theta_i^p)\right)\right) && (110)
\end{aligned}
$$

Posons :

$$E = Y_M - a_i \left(\sin(\Theta_i^p) - \Theta_i^p \cos(\Theta_i^p)\right) - b_i \left(\cos(\Theta_i^p) + \Theta_i^p \sin(\Theta_i^p)\right) \tag{111}$$
$$cw_i = -a_i \cos(\Theta_i^p) + b_i \sin(\Theta_i^p) \tag{112}$$
$$\tag{113}$$

Ainsi :

$$W_M^*/\overrightarrow{y_0} = \mathbf{K_M} \left[-Y_i + \Theta_i cw_i + E\right] \tag{114}$$
$$\cdot \left(Y^* - \Theta^* cw_i\right) \tag{115}$$

Sous forme matricielle :

$$W_M^*/\overrightarrow{y_0} = \left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & -\mathbf{K_M} & \mathbf{K_M} cw_i \\ 0 & \mathbf{K_M} cw_i & -\mathbf{K_M} cw_i^2 \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ \Theta_i \end{bmatrix} + \begin{bmatrix} 0 \\ E\mathbf{K_M} \\ -E\mathbf{K_M} cw_i \end{bmatrix} \right) \cdot \begin{bmatrix} X^* \\ Y^* \\ \Theta^* \end{bmatrix} \tag{116}$$