

Chapter 5

5. Josephson Effect

5.1 Josephson Equations

5.1.1 SIS Josephson Junction

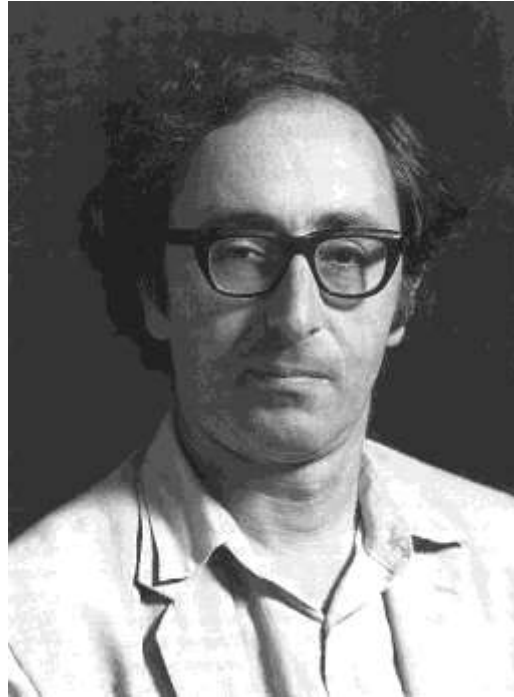
5.1.2 Ambegaokar-Baratoff relation

5.2 Josephson Coupling Energy

5.2.1 Josephson Junction with applied current

5.3 Applications of the Josephson Effect

5 Josephson Effect



Brian David Josephson (born 1940)

What happens if we weakly couple two superconductors ?

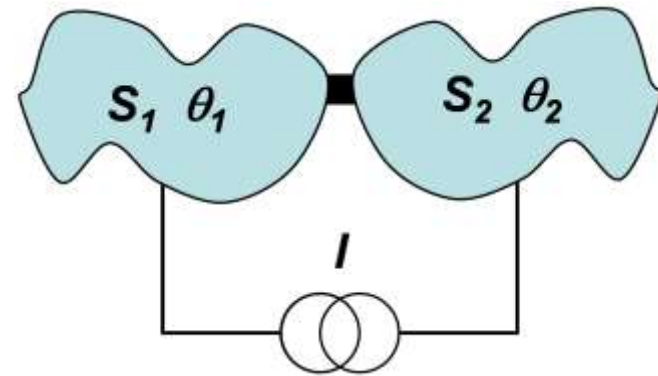
5.1 Josephson Effect (cf. 2.2.3)

- *what happens if we weakly couple two superconductors?*

- coupling by *tunneling barriers*, *point contacts*, *normal conducting layers*, etc.
- do they form a bound state such as a molecule?
- if yes, what is the binding energy?

- **B.D. Josephson** in 1962

(nobel prize with Esaki and Giaever in 1973)



→ Cooper pairs can tunnel through thin insulating barrier

naive expectation:

- tunneling probability for pairs $\propto (|T|^2)^2$
→ extremely small $\approx (10^{-4})^2$

Josephson:

- tunneling probability for pairs $\propto |T|^2$
- coherent tunneling of pairs („*tunneling of macroscopic wave function*“)

→ *finite supercurrent at zero applied voltage*

→ *oscillation of supercurrent at constant applied voltage*

} **Josephson effects**

→ *finite binding energy of coupled SCs = Josephson coupling energy*

5.1 Josephson Effect (cf. 2.2.3)

- coupling is weak \rightarrow supercurrent density is small $\rightarrow |\Psi|^2 = n_s$ is not changed
- supercurrent density depends on gauge invariant phase gradient γ :

$$J_s(\mathbf{r}, t) = \frac{q_s n_s \hbar}{m_s} \left[\nabla \theta(\mathbf{r}, t) - \frac{2\pi}{\Phi_0} \mathbf{A}(\mathbf{r}, t) \right] = \frac{q_s n_s \hbar}{m_s} \gamma(\mathbf{r}, t)$$

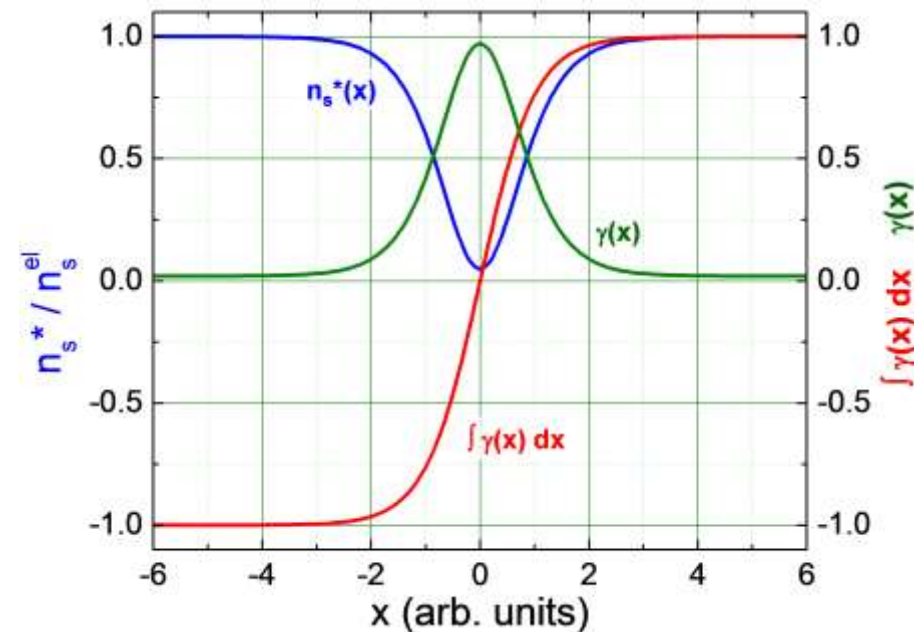
- **simplifying assumptions:**

- current density is homogeneous
- γ varies negligibly in electrodes
- J_s same in electrodes and junction area
 $\rightarrow \gamma$ in superconducting electrodes much smaller than in insulator I

- **then:**

- replace gauge invariant phase gradient γ by **gauge invariant phase difference**:

$$\begin{aligned} \varphi(\mathbf{r}, t) &= \int_1^2 \gamma(\mathbf{r}, t) \cdot d\mathbf{l} = \int_1^2 \left(\nabla \theta - \frac{2\pi}{\Phi_0} \mathbf{A} \right) \cdot d\mathbf{l} \\ &= \theta_2(\mathbf{r}, t) - \theta_1(\mathbf{r}, t) - \frac{2\pi}{\Phi_0} \int_1^2 \mathbf{A}(\mathbf{r}, t) \cdot d\mathbf{l} \end{aligned}$$



$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

$\int \gamma(x) dx$

$\gamma(x)$

n_s^*/n_s^{el}

$x \text{ (arb. units)}$

5.1 Josephson Effect (cf. 2.2.3)

first Josephson equation:

- we expect:

$$J_s = J_s(\varphi)$$

$$J_s(\varphi) = J_s(\varphi + n2\pi)$$

- for $J_s = 0$: phase difference φ must be zero:

$$J_s(0) = J_s(n \cdot 2\pi) = 0$$

therefore:

$$J_s(\varphi) = J_c \sin \varphi + \sum_{m=2}^{\infty} J_m \sin(m\varphi)$$

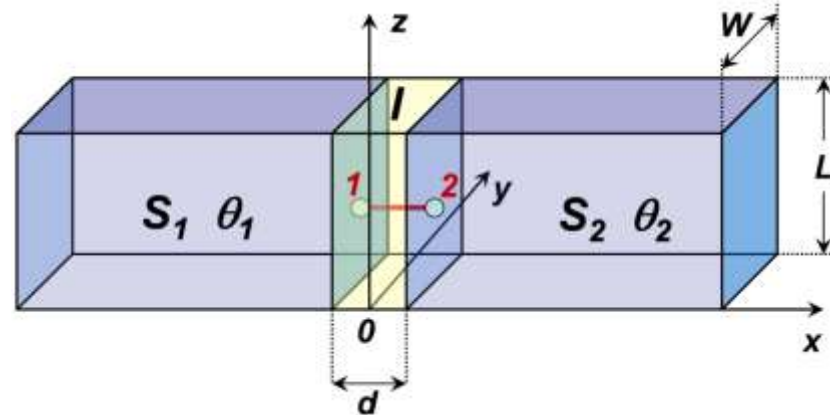
general formulation of 1st Josephson equation: **current-phase relation**

- in most cases: keep only 1st term (especially for weak coupling):

$$1. \text{ Josephson equation: } J_s(\varphi) = J_c \sin \varphi$$

- generalization to **spatially inhomogeneous** supercurrent density:

$$J_s(y, z, t) = J_c(y, z) \sin \varphi(y, z, t)$$



J_c : critical/maximum
Josephson current density

derived by
Josephson for
SIS junctions

supercurrent
density varies
sinusoidally with
 $\varphi = \theta_2 - \theta_1$ w/o
ext. potentials

5.1 Josephson Effect (cf. 2.2.3)

- other argument why there are only sin contributions to Josephson current

$$J_s(\varphi) = J_c \sin \varphi + \sum_{m=2}^{\infty} J_m \sin(m\varphi)$$

 *time reversal symmetry*

- if we reverse time, the Josephson current should flow in opposite direction
 - $t \rightarrow -t$, $J_s \rightarrow -J_s$
- the time evolution of the macroscopic wave functions is $\propto \exp[i\theta(t)] = \exp[i\omega t]$
 - if we reverse time, we have

$$\begin{aligned} \varphi(t) = \theta_2(t) - \theta_1(t) &\xrightarrow{t \rightarrow -t} \varphi(-t) = \theta_2(-t) - \theta_1(-t) \\ &= -[\theta_2(t) - \theta_1(t)] \\ &= -\varphi(t) \end{aligned}$$

- if the Josephson effect stays unchanged under time reversal, we have to demand

$$J_s(\varphi) = -J_s(-\varphi) \quad \Rightarrow \text{satisfied only by sin-terms}$$

5.1 Josephson Effect (cf. 2.2.3)

second Josephson equation:

- time derivative of the gauge invariant phase difference: $\varphi(\mathbf{r}, t) = \theta_2(\mathbf{r}, t) - \theta_1(\mathbf{r}, t) - \frac{2\pi}{\Phi_0} \int_1^2 \mathbf{A}(\mathbf{r}, t) \cdot d\mathbf{l}$

$$\frac{\partial \varphi}{\partial t} = \frac{\partial \theta_2}{\partial t} - \frac{\partial \theta_1}{\partial t} - \frac{2\pi}{\Phi_0} \frac{\partial}{\partial t} \int_1^2 \mathbf{A}(\mathbf{r}, t) \cdot d\mathbf{l}$$

- substitution of the energy-phase relation $-\hbar \frac{\partial \theta}{\partial t} = \frac{1}{2n_s} \Lambda \mathbf{J}_s^2 + q_s \phi$ gives:

$$\frac{\partial \varphi}{\partial t} = -\frac{1}{\hbar} \left(\frac{\Lambda}{2n_s} [\mathbf{J}_s^2(2) - \mathbf{J}_s^2(1)] + q_s [\phi(2) - \phi(1)] \right) - \frac{2\pi}{\Phi_0} \frac{\partial}{\partial t} \int_1^2 \mathbf{A} \cdot d\mathbf{l}$$

- supercurrent density across the junction is *continuous* ($\mathbf{J}_s(1) = \mathbf{J}_s(2)$):

$$\frac{\partial \varphi}{\partial t} = \frac{2\pi}{\Phi_0} \int_1^2 \left(-\nabla \phi - \frac{\partial \mathbf{A}}{\partial t} \right) \cdot d\mathbf{l} \quad (\text{term in parentheses} = \text{electric field})$$

2. Josephson equation:

$$\frac{\partial \varphi}{\partial t} = \frac{2\pi}{\Phi_0} \underbrace{\int_1^2 \mathbf{E}(\mathbf{r}, t) \cdot d\mathbf{l}}_{\text{voltage drop}}$$

**voltage – phase
relation**

5.1 Josephson Effect (cf. 2.2.3)

- for a constant voltage across the junction: $\frac{\partial \varphi}{\partial t} = \frac{2\pi}{\Phi_0} V$
 $\varphi(t) = \varphi_0 + \frac{2\pi}{\Phi_0} V \cdot t$

- I_s is oscillating at the Josephson frequency $f = V/\Phi_0$:

$$\begin{aligned} I_s(t) &= I_c \sin \varphi(t) \\ &= I_c \sin \left(\frac{2\pi}{\Phi_0} V \cdot t \right) \end{aligned}$$

$$\frac{f}{V} = \frac{\omega}{2\pi V} = \frac{1}{\Phi_0} \simeq 483.597898(19) \frac{\text{MHz}}{\mu\text{V}}$$

→ *voltage controlled oscillator*

- applications:
 - Josephson voltage standard
 - microwave sources
- derivation of Josephson equations for SIS junction from time-dependent Schrödinger equation:
 - see exercise sheet
 - see appendix G.4 in „Festkörperphysik“, 2. Auflage, R. Gross, A. Marx, de Gruyter (2014)

5.1.1 Special Topic: Superconducting Tunnel Junctions

- *Josephson effect in superconducting tunnel junctions*

- *insulating* tunneling barrier, thickness d

- what determines the *maximum Josephson current density* J_c ?

- calculation by **wave matching method**

- *energy-phase relation:*

$$-\hbar \frac{\partial \theta}{\partial t} = \frac{1}{2n_s} \Lambda \mathbf{J}_s^2 + q_s \phi + \mu$$

$$-\hbar \frac{\partial \theta}{\partial t} = \frac{1}{2n_s} \Lambda \mathbf{J}_s^2 \quad \Rightarrow \quad \frac{\partial \theta}{\partial t} = -\frac{E_0}{\hbar} \quad E_0 = \text{kinetic energy}$$

→ time dependent macroscopic wave function: $\psi(\mathbf{r}, t) = \psi(\mathbf{r}) e^{-i(E_0/\hbar)t}$

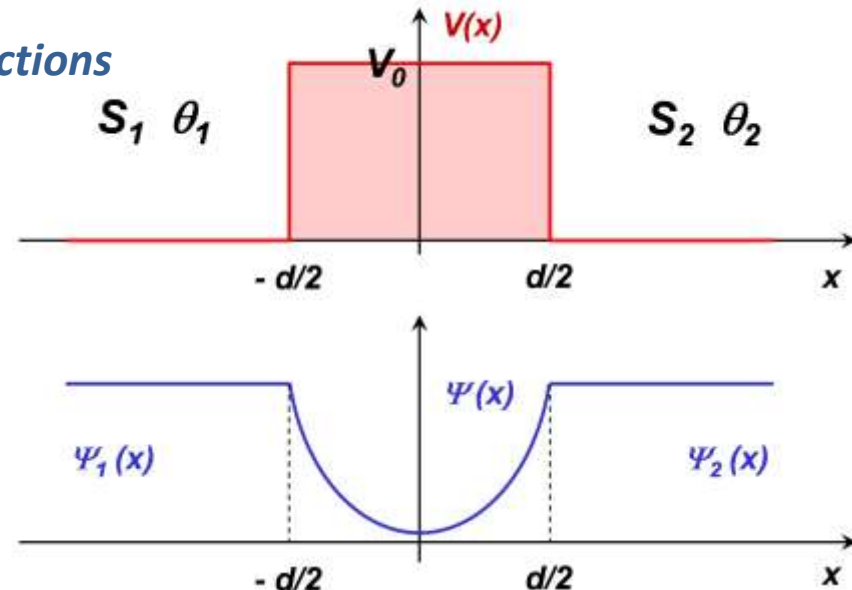
- wave function **within** barrier with height $V_0 > E_0$:

only **elastic** processes:

→ time evolution is the **same** outside and inside barrier

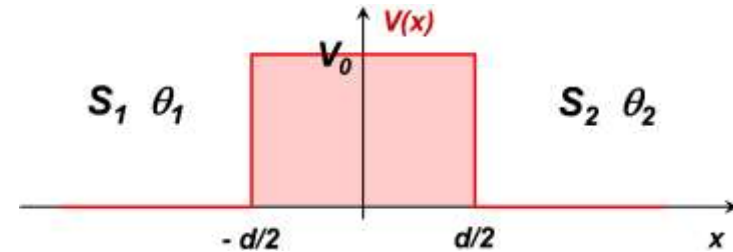
→ consider only **time independent** part

→ time independent Schrödinger(-like) equation for region of constant potential



5.1.1 Josephson Effect

$$-\frac{\hbar^2}{2m^*} \nabla^2 \psi(\mathbf{r}) = (E_0 - V_0) \psi(\mathbf{r})$$



- assumption:**

homogeneous barrier and supercurrent flow \rightarrow **1D problem**

- solutions:**

- in superconductors $\psi_{1,2} = \sqrt{n_{1,2}} e^{i\theta_{1,2}}$

- in insulator: sum of decaying and growing exponentials $\psi(x) = A \cosh(\kappa x) + B \sinh(\kappa x)$

- characteristic decay constant: $\kappa = \sqrt{\frac{2m_s(V_0 - E_0)}{\hbar^2}}$

barrier properties

- coefficients A and B are determined by the boundary conditions at $x = \pm d/2$:

$$\psi(-d/2) = \sqrt{n_1} e^{i\theta_1}$$

$$\psi(+d/2) = \sqrt{n_2} e^{i\theta_2}$$

$n_{1,2}, \theta_{1,2}$: Cooper pair density and wave function phase at the boundaries $x = \pm d/2$

$$\sqrt{n_1} e^{i\theta_1} = A \cosh(\kappa d/2) - B \sinh(\kappa d/2)$$

$$\sqrt{n_2} e^{i\theta_2} = A \cosh(\kappa d/2) + B \sinh(\kappa d/2)$$

5.1.1 Josephson Effect

- solving for A and B:
$$A = \frac{\sqrt{n_1} e^{i\theta_1} + \sqrt{n_2} e^{i\theta_2}}{2 \cosh(\kappa d/2)} \quad B = -\frac{\sqrt{n_1} e^{i\theta_1} - \sqrt{n_2} e^{i\theta_2}}{2 \sinh(\kappa d/2)}$$
 - supercurrent density:
$$\mathbf{J}_s = \frac{q_s}{m_s} \Re \left\{ \psi^* \left(\frac{\hbar}{i} \nabla \right) \psi \right\} = \frac{\hbar q_s}{m_s} \Im \{ \psi^* \nabla \psi \}$$
 - substituting the coefficients A and B:
$$\mathbf{J}_s = \frac{q_s}{m_s} \kappa \hbar \Im \{ A^* B \}$$
- $\mathbf{J}_s = \mathbf{J}_c \sin(\theta_2 - \theta_1)$ current-phase relation

→ **maximum Josephson current density J_c :**

$$J_c = -\frac{q_s}{m_s} \kappa \hbar \underbrace{\frac{\sqrt{n_1 n_2}}{2 \sinh(\kappa d/2) \cosh(\kappa d/2)}}_{=\sinh(2\kappa d)} = -\frac{q_s \hbar \kappa}{m_s} \frac{\sqrt{n_1 n_2}}{\sinh(2\kappa d)}$$

- **real junctions:**

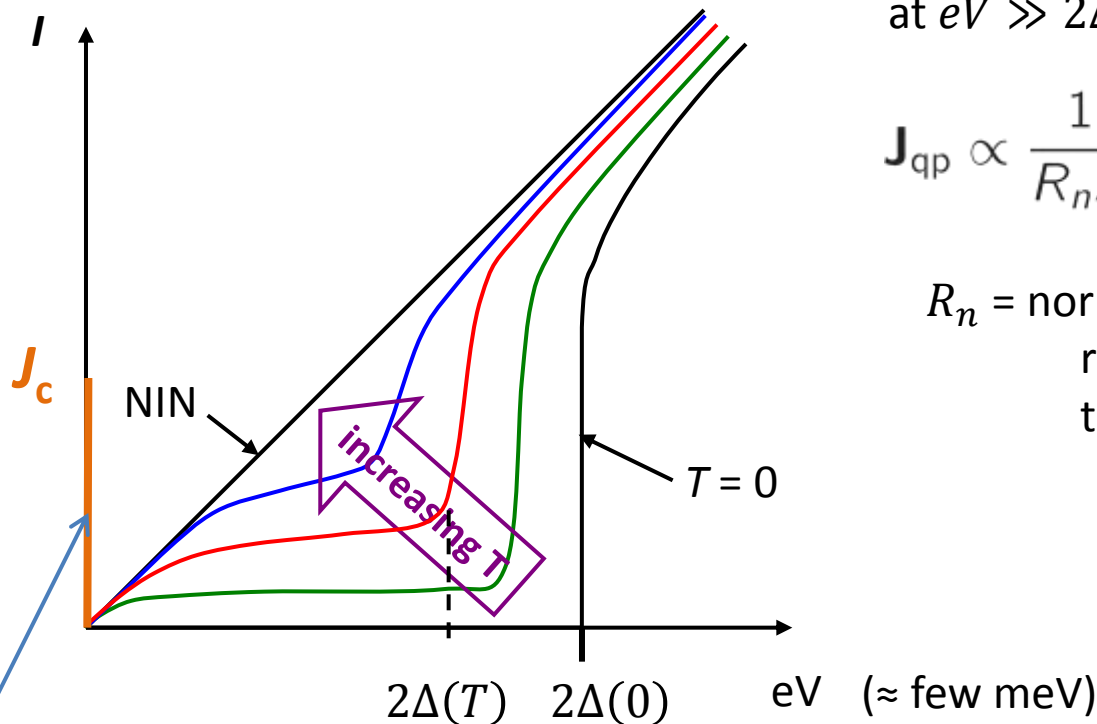
$V_0 \approx \text{few eV} \Rightarrow 1/\kappa < 1 \text{ nm}, d \text{ few nm} \Rightarrow \kappa d \ll 1$, then: $\sinh(2\kappa d) \simeq \frac{1}{2} \exp(2\kappa d)$

- maximum Josephson current **decays exponentially** with increasing barrier thickness d :

$$J_c = -\frac{q_s \hbar \kappa}{m_s} 2\sqrt{n_1 n_2} \exp(-2\kappa d)$$

5.1.2 Ambegaokar-Baratoff Relation

- quasiparticle tunneling:
- current-voltage characteristics



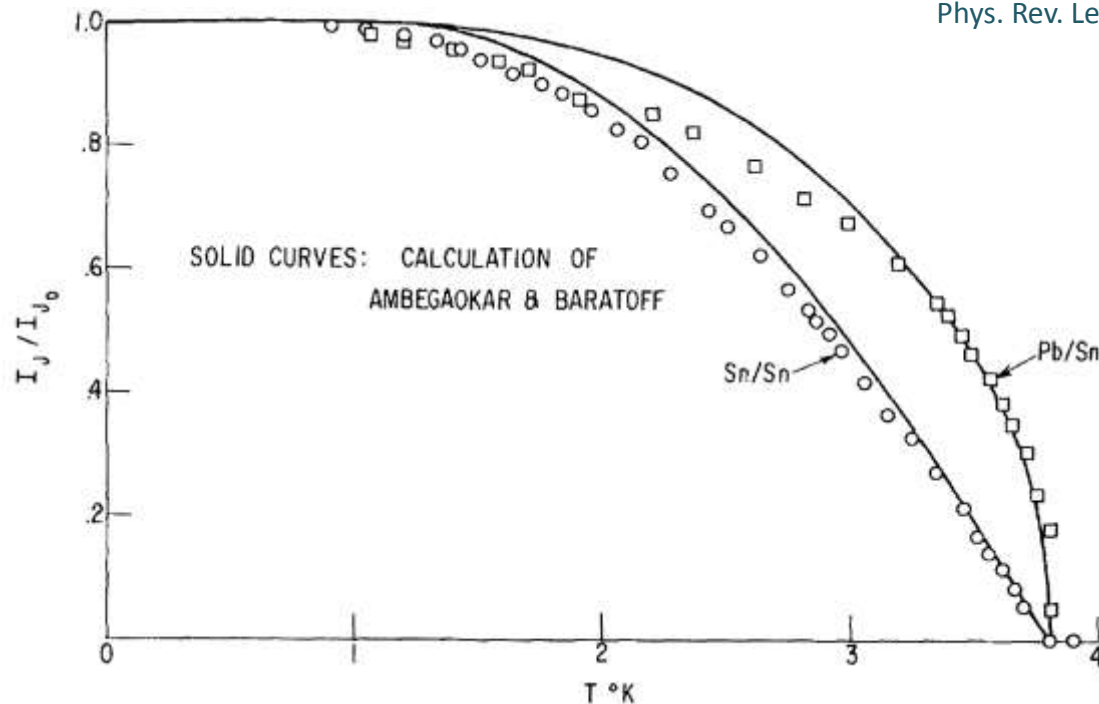
- Cooper pair tunneling:
$$J_c = \frac{e\hbar\kappa}{m} 2\sqrt{n_1 n_2} \exp(-2\kappa d)$$
- $V > 0$: time average of supercurrent vanishes: $\langle J_c \sin \frac{2eV}{\hbar} t \rangle = 0$

5.1.2 Ambegaokar-Baratoff Relation

- ratio of J_c and J_{qp} ($eV \gg 2\Delta$) = *const* $\rightarrow J_c R_n A = I_c R_n A = I_c R_n = \text{const}$
- exact calculation yields Ambegaokar-Baratoff relation:

$$I_c R_n = \frac{\pi}{2e} \Delta(T) \cdot \tanh \left(\frac{\Delta(T)}{2k_B T} \right)$$

V. Ambegaokar, A. Baratoff, *Tunneling Between Superconductors*, Phys. Rev. Lett. **10**, 486-489 (1963).



M.D. Fiske, Rev. Mod. Phys. **36**, 221–222
Temperature and Magnetic Field
Dependences of the Josephson Tunneling
Current

5.1 Summary

Macroscopic wave function Ψ :

describes ensemble of macroscopic number of superconducting pairs

$|\Psi|^2$ describes density of superconducting pairs

Current density in a superconductor:

$$\mathbf{J}_s = \frac{\hbar n_s q_s}{m_s} \left\{ \nabla \theta(\mathbf{r}, t) - \frac{q_s}{\hbar} \mathbf{A}(\mathbf{r}, t) \right\} = \frac{\hbar n_s q_s}{m_s} \left\{ \nabla \theta(\mathbf{r}, t) - \frac{2\pi}{\Phi_0} \mathbf{A}(\mathbf{r}, t) \right\}$$

Gauge invariant phase gradient:

$$\gamma(\mathbf{r}, t) = \nabla \theta(\mathbf{r}, t) - \frac{q_s}{\hbar} \mathbf{A}(\mathbf{r}, t) = \nabla \theta(\mathbf{r}, t) - \frac{2\pi}{\Phi_0} \mathbf{A}(\mathbf{r}, t)$$

Phenomenological London equations:

$$\frac{\partial}{\partial t} (\Lambda \mathbf{J}_s) = \mathbf{E} \qquad \nabla \times (\Lambda \mathbf{J}_s) = -\mathbf{B} \qquad (\Lambda = m_s / n_s q_s^2 = \mu_0 \lambda_L^2)$$

flux/fluxoid quantization:
$$\oint_C (\Lambda \mathbf{J}_s) \cdot d\mathbf{l} + \int_S \mathbf{B} \cdot d\mathbf{s} = n \Phi_0$$

5.1 Summary

Josephson equations:

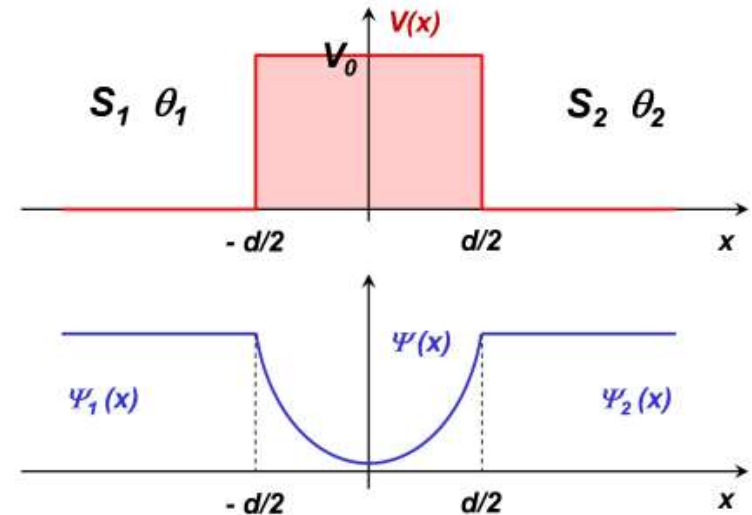
$$\mathbf{J}_s(\mathbf{r}, t) = \mathbf{J}_c(\mathbf{r}, t) \sin \varphi(\mathbf{r}, t)$$

$$\frac{\partial \varphi}{\partial t} = \frac{2eV}{\hbar} = \frac{2\pi}{\Phi_0} V \quad (\omega/2\pi = 483.6 \text{ GHz/mV})$$

maximum Josephson current density J_c :
wave matching method

$$\mathbf{J}_s = \mathbf{J}_c \sin(\theta_2 - \theta_1)$$

$$\mathbf{J}_c = -\frac{q_s \hbar \kappa}{m_s} 2\sqrt{n_1 n_2} \exp(-2\kappa d)$$



tunneling current of unpaired electrons (quasiparticles, cf. chapter 4.4.2):

$$\mathbf{J}_q = f(V) \cdot \exp(-2\kappa d)$$

5.2 Josephson Coupling Energy

- the two weakly coupled superconductors form “*molecule*” analogous to H₂ molecule
→ what is the *binding energy* of this molecule ?
- consider a JJ with initial current & phase difference equal to zero
then: **increase junction current from zero to finite value**
 - phase difference has to change
 - voltage-phase relation: finite junction voltage
 - external source has to supply energy (to accelerate the superelectrons)
 - stored in kinetic energy of moving superelectrons
 - integral of the supplied power $I \cdot V$ to increase current to $I(\varphi) = I_c \sin \varphi$
(voltage during increase of current):

$$E_J = \int_0^{t_0} I_s V \, dt = \int_0^{t_0} (I_c \sin \tilde{\varphi}) \left(\frac{\Phi_0}{2\pi} \frac{d\tilde{\varphi}}{dt} \right) dt$$

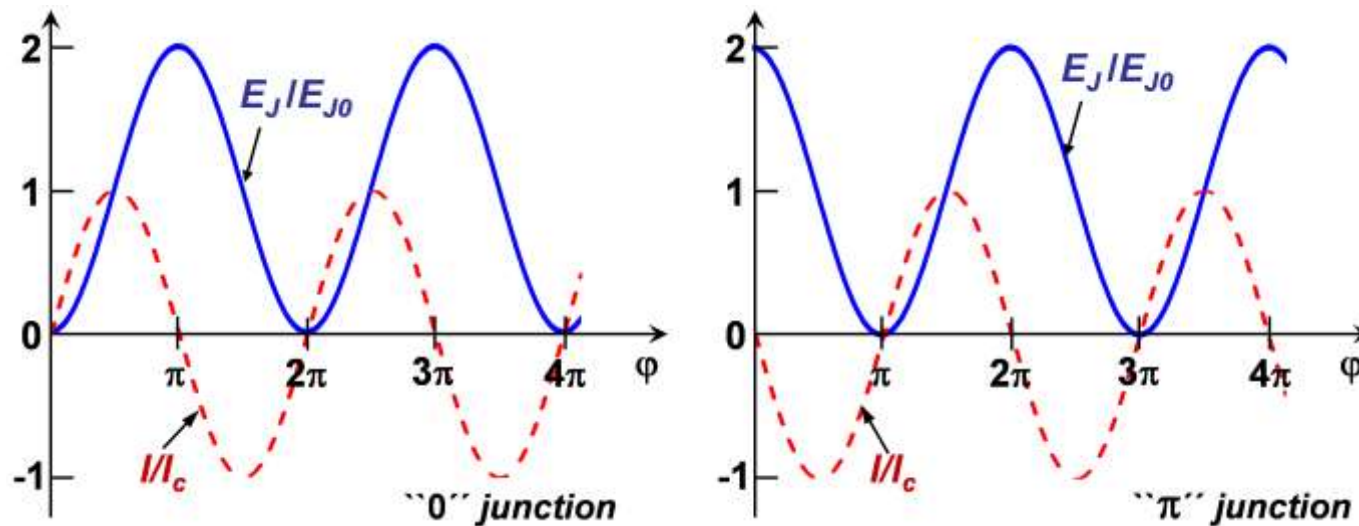
5.2 Josephson Coupling Energy

with $\varphi(0) = 0$ and $\varphi(t_0) = \varphi$:

$$E_J = \frac{\Phi_0 I_c}{2\pi} \int_0^\varphi \sin \tilde{\varphi} d\tilde{\varphi}$$

integration:
$$E_J = \frac{\Phi_0 I_c}{2\pi} (1 - \cos \varphi) = E_{J0} (1 - \cos \varphi)$$

Josephson coupling energy



- order of magnitude:**

- typically: $I_c \sim 1 \text{ mA} \Rightarrow E_{J0} \simeq 3 \times 10^{-19} \text{ J}$
- corresponds to thermal energy $k_B T$ for $T \simeq 20\,000 \text{ K}$
- junction with very small critical current: $I_c \simeq 1 \mu\text{A} \Rightarrow \text{thermal energy} \simeq k_B \times 20 \text{ K}$

5.2.1 Josephson Junction with Applied Current

- analysis of **stability** of (junction + current source) – system:

potential energy E_{pot} of the system under action of external force: $E_J - F \cdot x$

E_J : intrinsic free energy of the subsystem junction

F : generalized force ($F = I$)

x : generalized coordinate $\Rightarrow F \cdot \partial x / \partial t =$ power flowing into subsystem ($I \cdot V$):

$$x = \int V dt = \frac{\hbar}{2e} \varphi + c = \frac{\Phi_0}{2\pi} \varphi + c$$

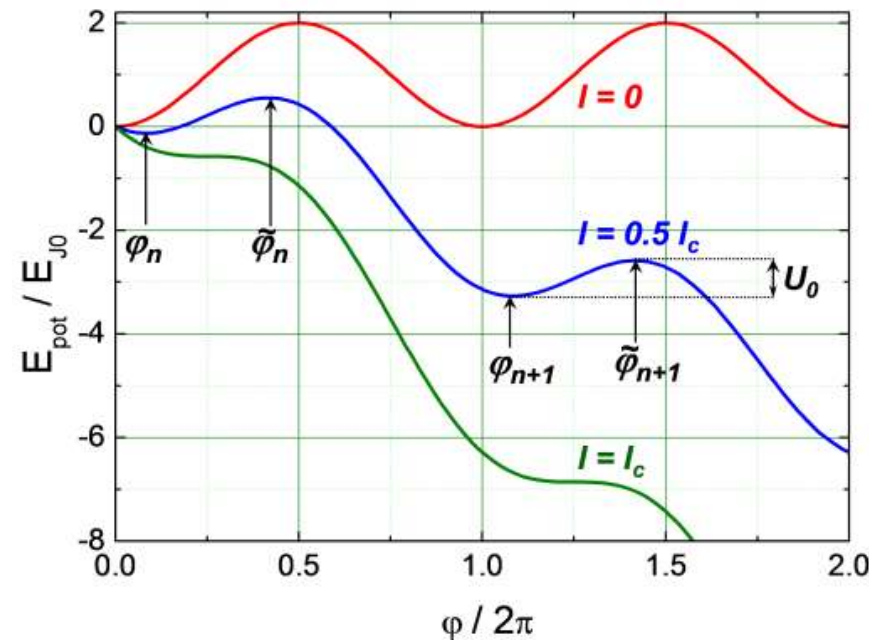
\rightarrow **potential energy**:

$$\begin{aligned} E_{\text{pot}}(\varphi) &= E_J(\varphi) - I \left(\frac{\Phi_0}{2\pi} \varphi + c \right) \\ &= E_{J0} \left[1 - \cos \varphi - \frac{I}{I_c} \varphi \right] + \tilde{c} \end{aligned}$$

tilted washboard potential

stable minima φ_n , unstable maxima $\tilde{\varphi}_n$,

states for different n : equivalent



- junction dynamics: motion of φ in tilted washboard potential (not discussed here)

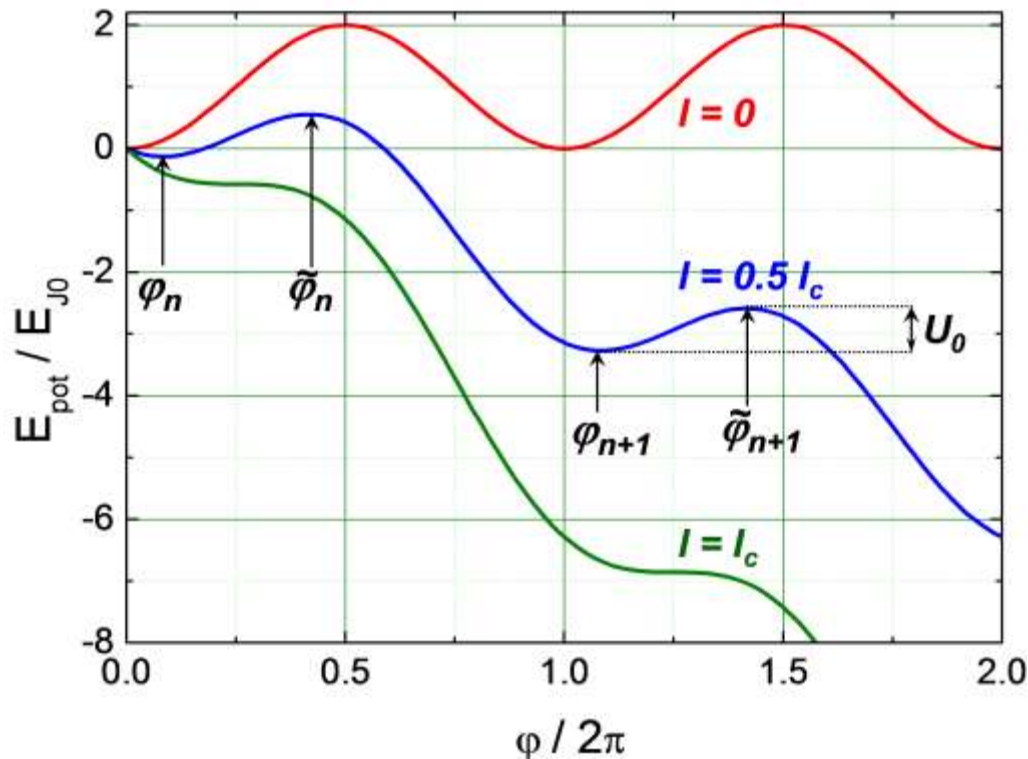
5.2.1 Josephson Junction with Applied Current

$-I_c < I < I_c \Rightarrow$ **constant phase difference:** $\varphi = \varphi_n = \arcsin\left(\frac{I}{I_c}\right) + 2\pi n$

\rightarrow zero junction voltage:

zero voltage state / ordinary (S) state

$$\varphi = \tilde{\varphi}_n = \pi - \arcsin\left(\frac{I}{I_c}\right) + 2\pi n$$

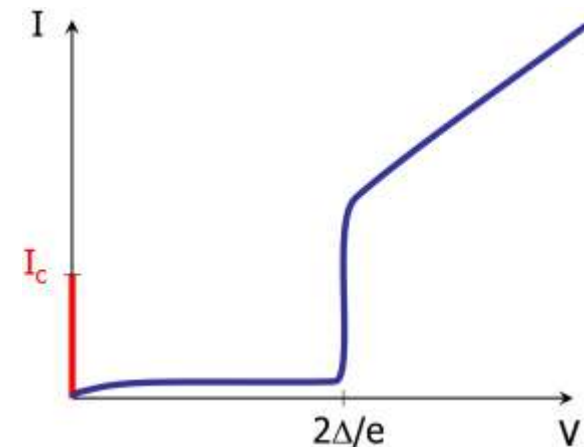
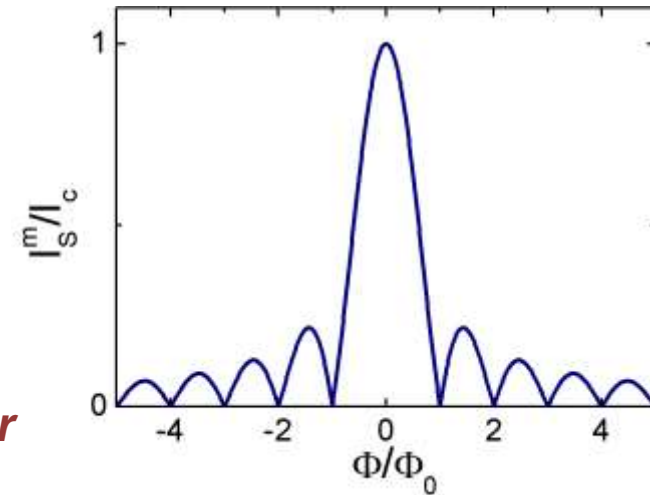


5.3 Applications of the Josephson Effect

large number of applications in **analog** and **digital** electronics

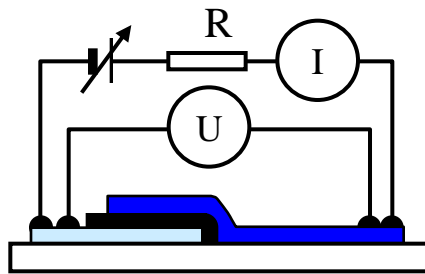
➔ detailed discussion in lecture „**Applied Superconductivity**“

- $I_S^m = I_S^m(B)$:
➔ **magnetic field sensors (SQUIDs)**
- $\beta_C \gg 1$
➔ **bistability**: zero/voltage state
➔ **switching devices, Josephson computer**
- 2nd Josephson equation
➔ **VCO, voltage standard**
- **nonlinear IVC**
➔ **mixers up to THz, oscillators**
- **macroscopic quantum behavior**
➔ **superconducting qubits**

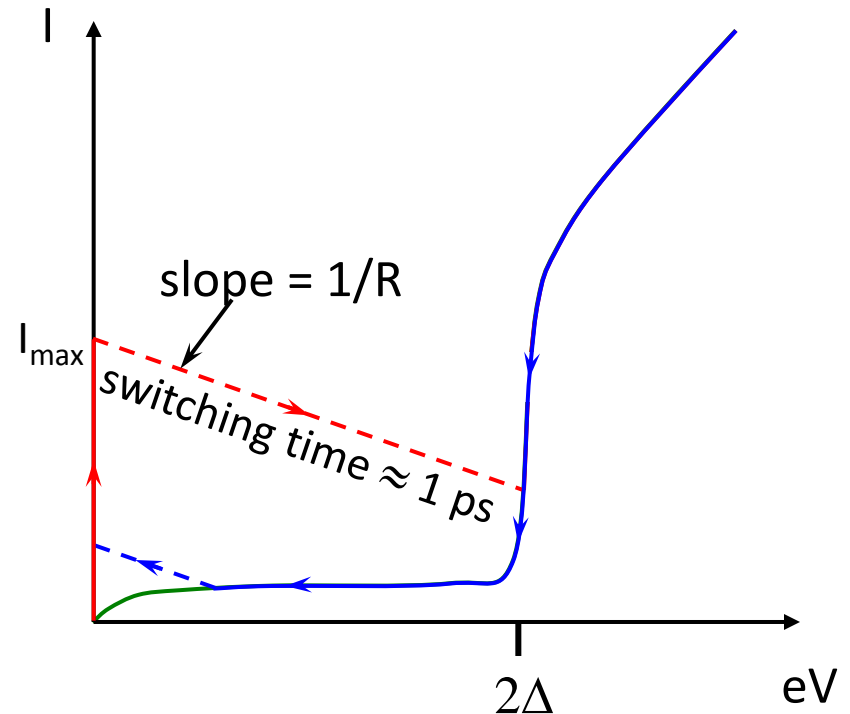


5.3 Applications of the Josephson Effect

- $V = 0$: Josephson current
- $V \neq 0$: quasiparticle current



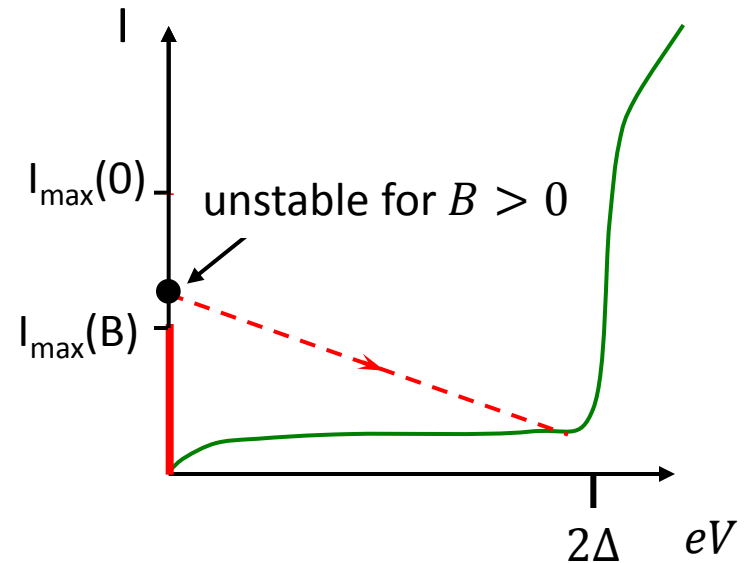
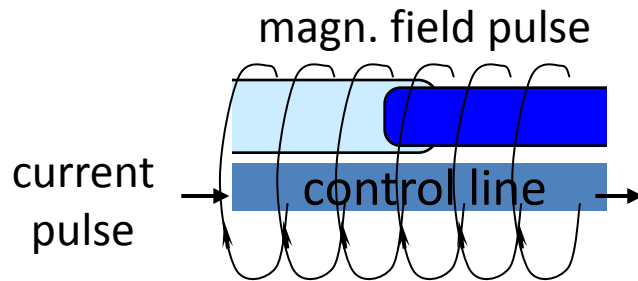
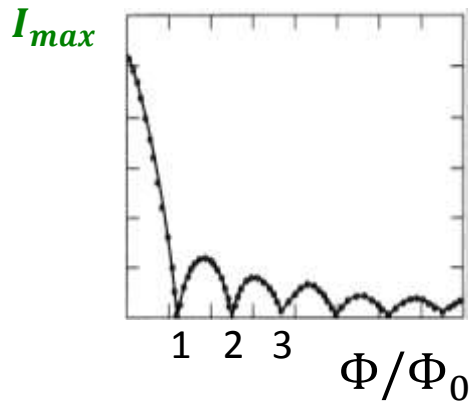
- hysteresis:
 - fast switching device
 - very low power consumption
 - \Rightarrow Josephson digital electronics



5.3 Applications of the Josephson Effect

principle of switching element:

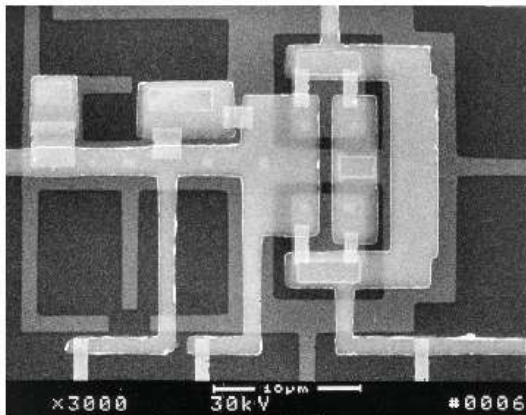
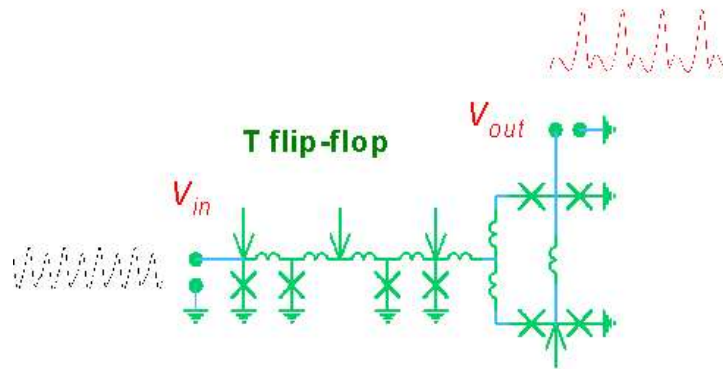
- magnetic field dependence of the maximum Josephson current



5.3 Applications of the Josephson Effect

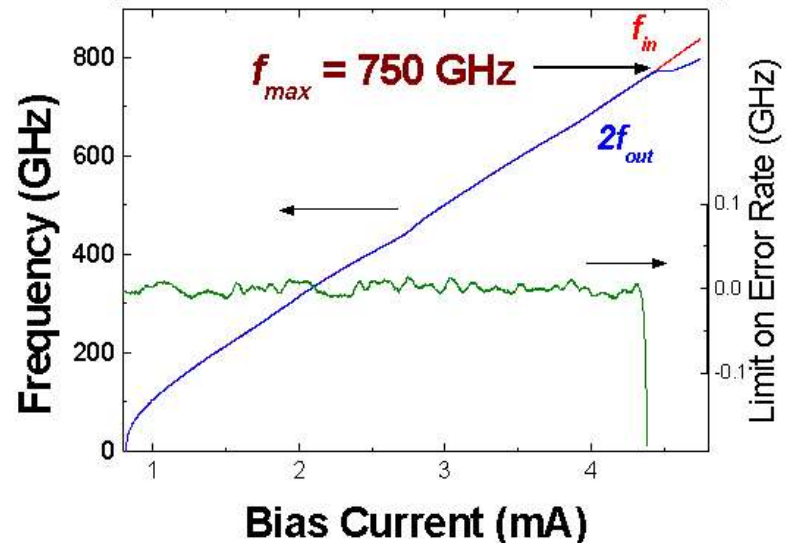
World's fastest digital IC - operates to 750 GHz

http://insti.physics.sunysb.edu/physics/news_fast_ic.htm



Dividers

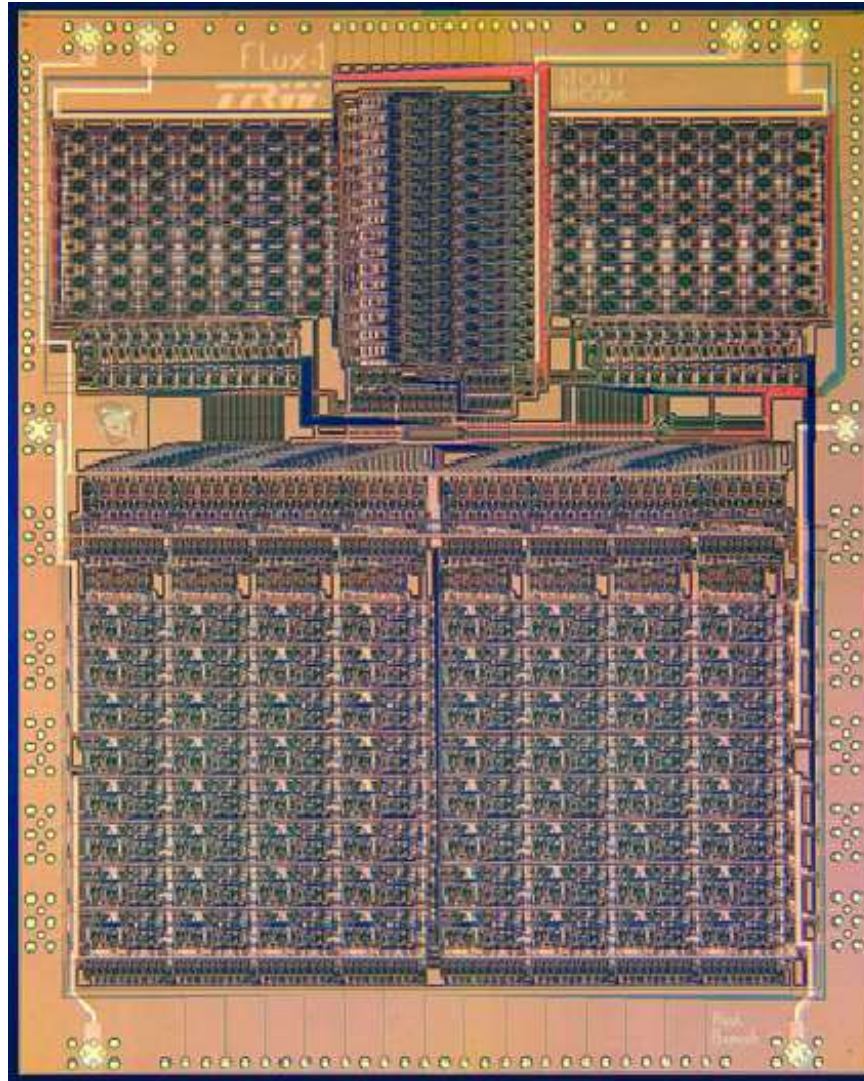
	RSFQ	Semi-conductor
Frequency	750 GHz	60 GHz
Power Dissipation	1.5 μ W	0.5 W



for details see: <http://gamayun.physics.sunysb.edu/RSFQ/>

- problem: integration of large number of JJs ($> 10^5$) with high yield and small parameter spread

5.3 Applications of the Josephson Effect



Stony Brook

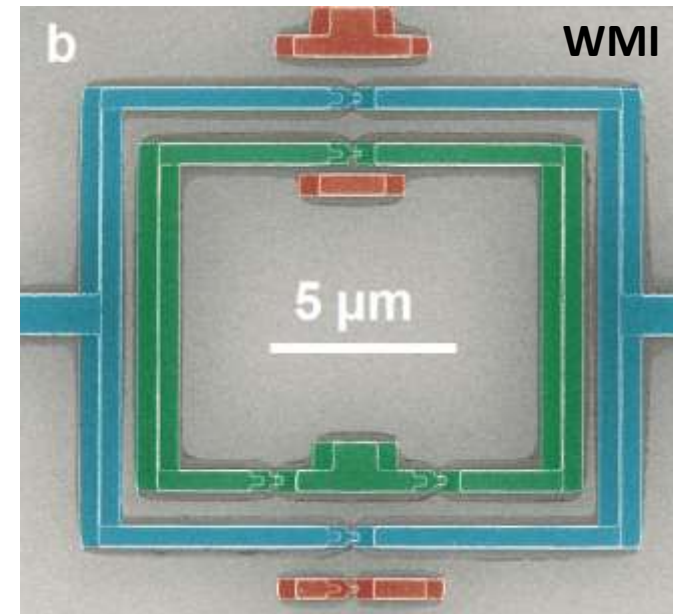
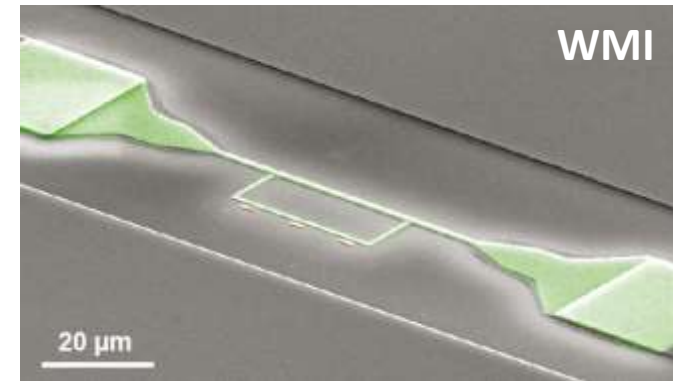
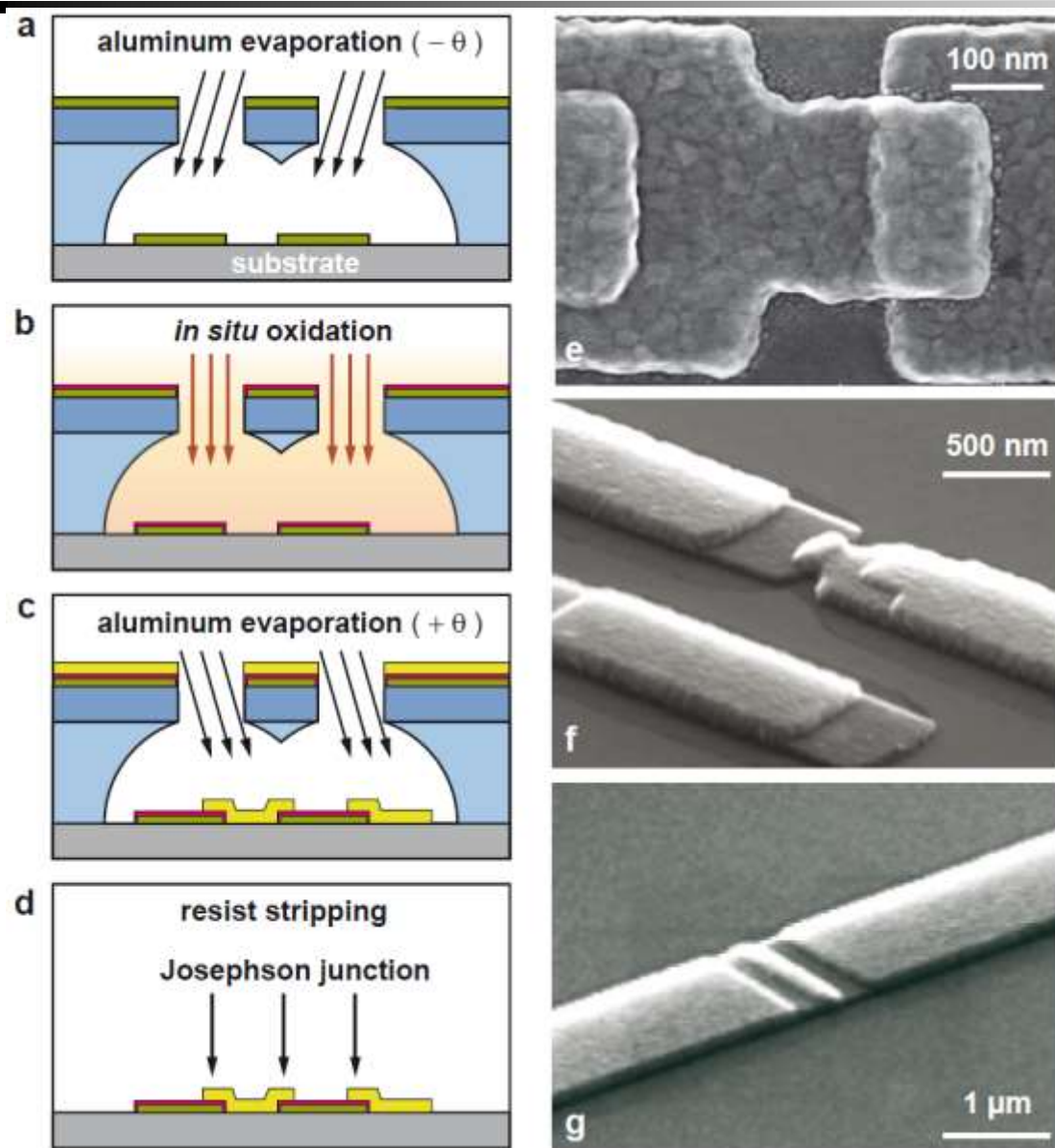
FLUX-1

- the first RSFQ MPU
- 8 bit ALU array
- 16 word instruction memory
- 70,000 JJs
- 14 mW
- 20-22 GHz @ $F = 2.0 \text{ } \mu\text{m}$
(\Rightarrow 120-140 GHz @ $0.3 \text{ } \mu\text{m}$)
- TRW's 4-metal process

<http://pavel.physics.sunysb.edu/RSFQ/>

5.3 Applications of the Josephson Effect

superconducting quantum bits



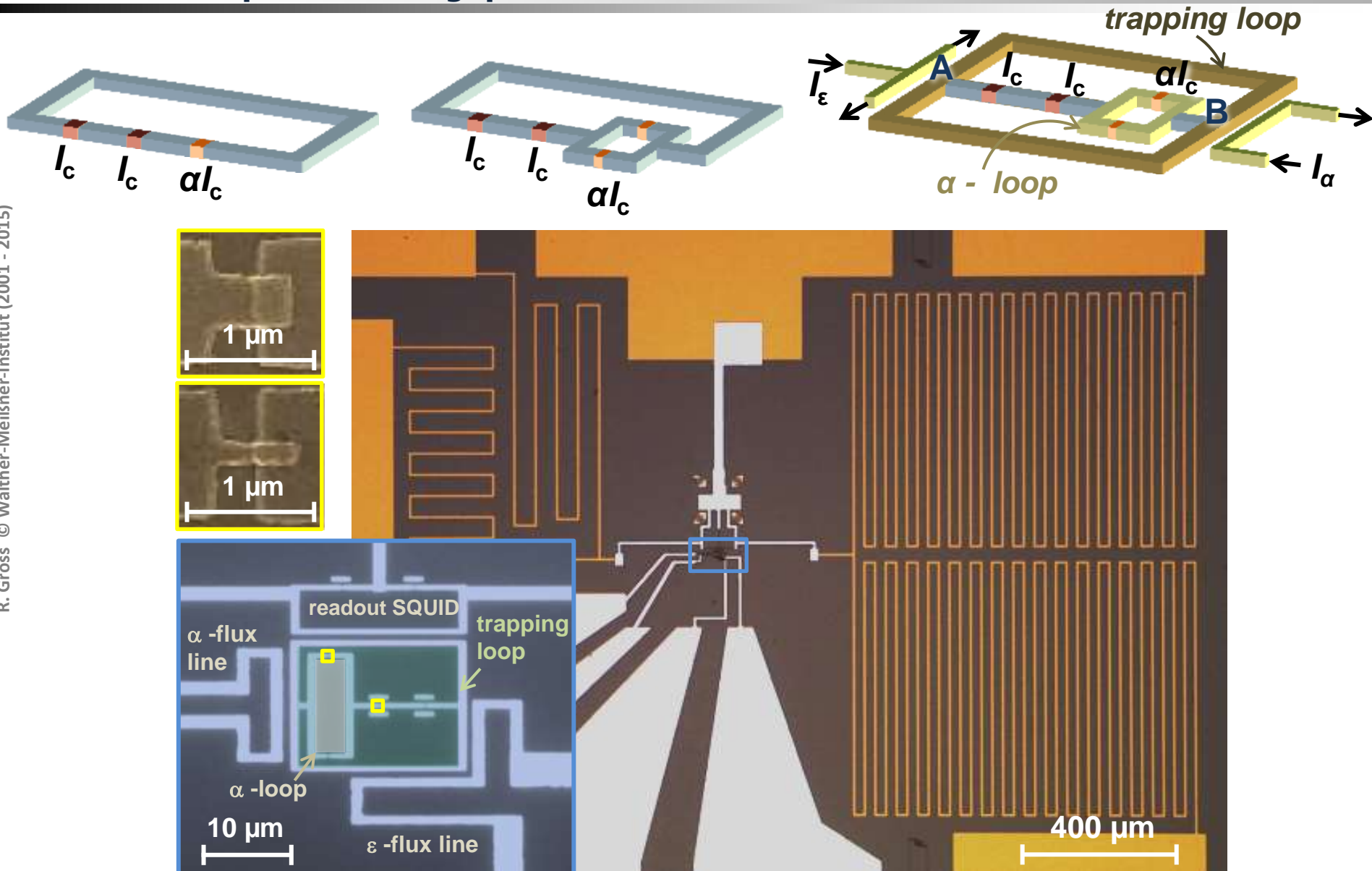
F. Deppe *et al.*, *PRB* 76, 214503 (2007)
T. Niemczyk *et al.*, *SUST* 22, 034009 (2009)

superconducting flux quantum bits fabricated at WMI

5.3 Applications of the Josephson Effect

superconducting quantum bits

R. Gross © Walther-Meißner-Institut (2001 - 2015)

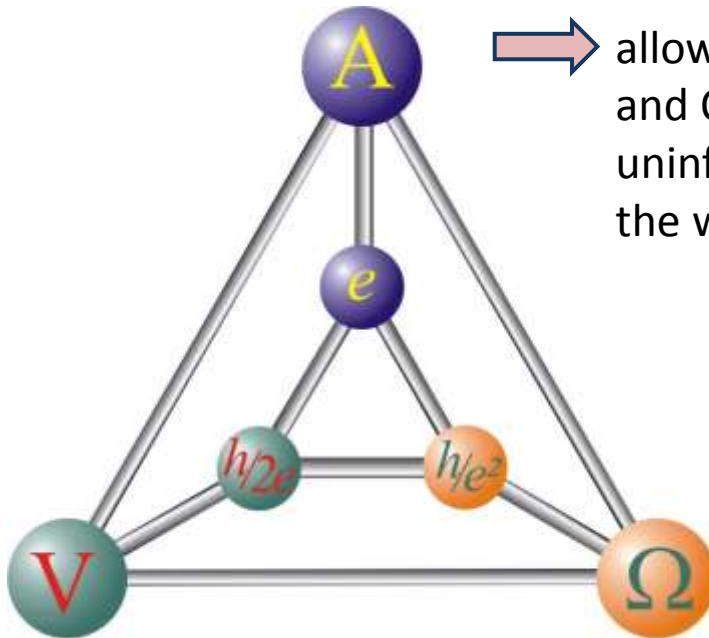


superconducting quantum circuits fabricated at WMI

5.3 Applications of the Josephson Effect: metrology

precise definition of the electrical voltage, current and the resistance by using fundamental quantum effects:

- **Josephson effect:** $V = \frac{h}{2e} \cdot f = \Phi_0 \cdot f$ (relation between voltage and time/frequency by flux quantum)
- **Single electron pump:** $I = e \cdot f$ (relation between current and time by charge quantum)
- **Quantum Hall effect:** $V = \frac{h}{e^2} \cdot I = R_K \cdot I$ (relation between voltage and current by quantum resistance, unit = 1 Klitzing)



allows the reproduction of the physical units Volt, Ampère and Ohm with a very high precision and largely uninfluenced by environmental parameters at any place in the world

realization of the Ampère by single electron pump not realized so far at sufficient precision
→ would allow an important experimental test of the consistency of the relations between the fundamental constants illustrated in the “**electrical triangle**”