• Haldane model on honeycomb lattice

Analogicznie jak w przypadku modelu SSH możemy zapisać hamiltonian w postaci macierzowej definiując transformaty Fouriera operatorów kreacji/anihilacji jako wektory:

$$a_A(k) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \; oraz \;\; a_B(k) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$a_A^+ a_B = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \; oraz \; a_B^+ a_A = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$a_A^+ a_B + a_B^+ a_A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \sigma_x$$

$$a_A^+ a_A - a_B^+ a_B = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \sigma_z$$

Wtedy można przekształcić Hamiltonian:

$$\begin{aligned}\mathcal{H}(\boldsymbol{k}) = t a_A^+ a_B\left(1 + e^{-ik_1} + e^{-ik_2}\right) + t a_B^+ a_A\left(1 + e^{ik_1} + e^{ik_2}\right) + V a_A^+ a_A - V a_B^+ a_B + \; V a_A^+ a_A - V a_B^+ a_B \\ + \; i\lambda(a_A^+ a_A - a_B^+ a_B)\left(e^{ik_1} - e^{-ik_1} - e^{ik_2} + e^{-ik_2} - e^{i(k_1-k_2)} + e^{-i(k_1-k_2)}\right) \\ - i\lambda(a_A^+ a_A - a_B^+ a_B)\left(e^{-ik_1} - e^{ik_1} - e^{-ik_2} + e^{ik_2} - e^{-i(k_1-k_2)} + e^{i(k_1-k_2)}\right)\end{aligned}$$

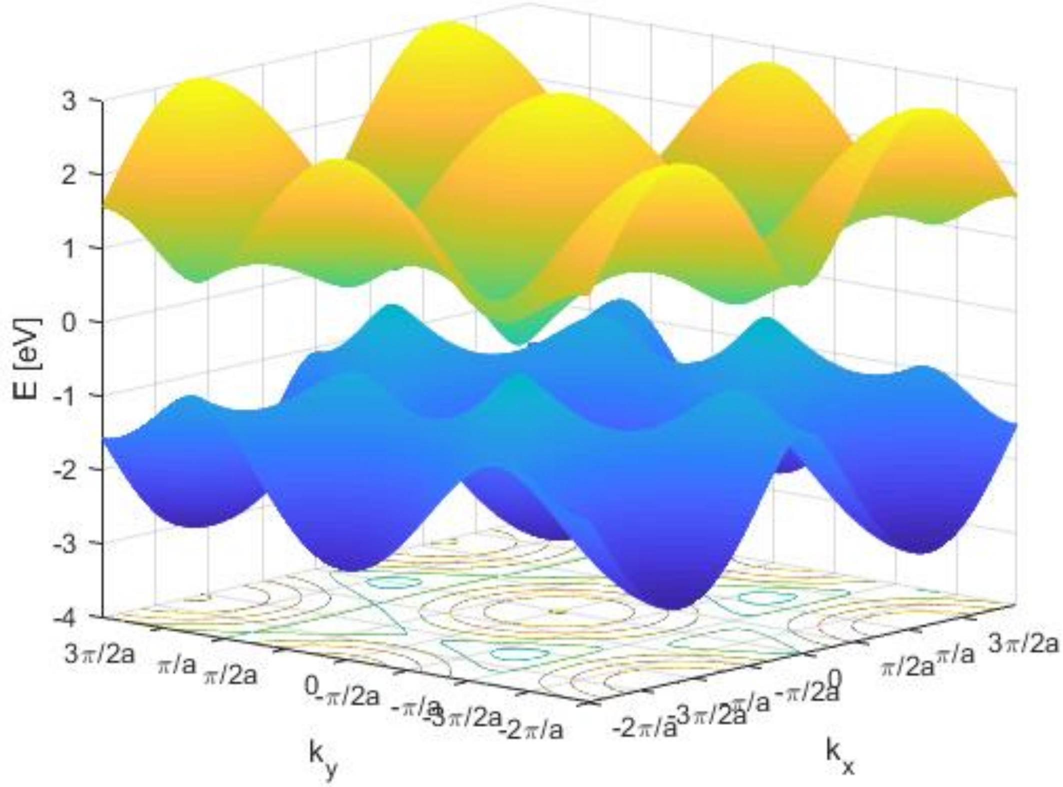
$$\begin{aligned}\mathcal{H}(\boldsymbol{k}) &= t[a_A^+ a_B + a_B^+ a_A] + t\left(e^{-ik_1} + e^{-ik_2}\right)a_A^+ a_B + t\left(e^{ik_1} + e^{ik_2}\right)a_B^+ a_A + 2V[a_A^+ a_A - a_B^+ a_B] \\ &\quad + \; i\lambda(a_A^+ a_A - a_B^+ a_B)\left(2isin(k_1) - 2isin(k_2) - 2isin(k_1 - k_2)\right) \\ &\quad - i\lambda(a_A^+ a_A - a_B^+ a_B)\left(-2isin(k_1) + 2isin(k_2) + 2isin(k_1 - k_2)\right) \\ &= t\sigma_x + t\left(e^{-ik_1} + e^{-ik_2}\right)a_A^+ a_B \sigma_x + t\left(e^{ik_1} + e^{ik_2}\right)a_B^+ a_A \sigma_x + 2V\sigma_z \\ &\quad + 2i\lambda\sigma_z\left(2isin(k_1) - 2isin(k_2) - 2isin(k_1 - k_2)\right)\end{aligned}$$

$$\begin{aligned}\mathcal{H}(\boldsymbol{k}) &= \begin{pmatrix} 2V - 4\lambda\left(sin(k_1) - sin(k_2) - sin(k_1 - k_2)\right) & t + t\left(e^{-ik_1} + e^{-ik_2}\right) \\ t + t\left(e^{ik_1} + e^{ik_2}\right) & -2V + 4\lambda\left(sin(k_1) - sin(k_2) - sin(k_1 - k_2)\right) \end{pmatrix} \\ &= \boldsymbol{D}(\boldsymbol{k}) \cdot \boldsymbol{\sigma}\end{aligned}$$

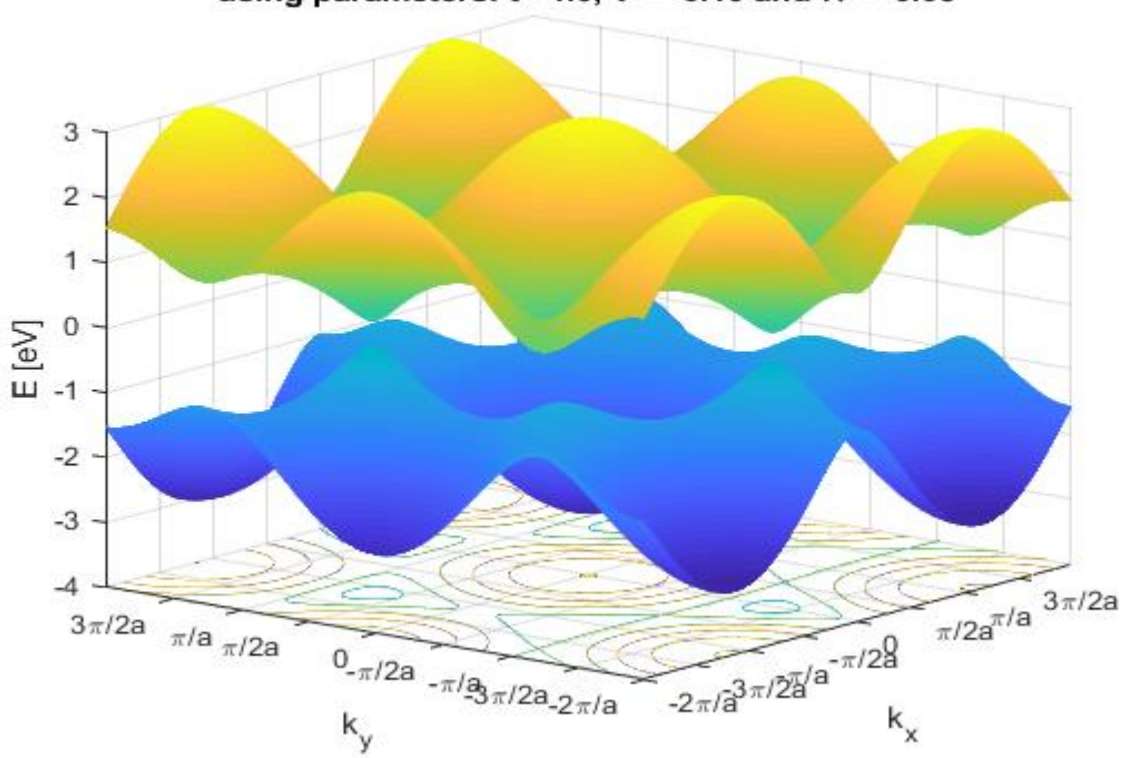
$$\boldsymbol{D}(\boldsymbol{k}) = \left[t(1 + \cos(k_1) + \cos(k_2)), t(\sin(k_1) + \sin(k_2)), 2V - 4\lambda\left(sin(k_1) - sin(k_2) - sin(k_1 - k_2)\right)\right]$$

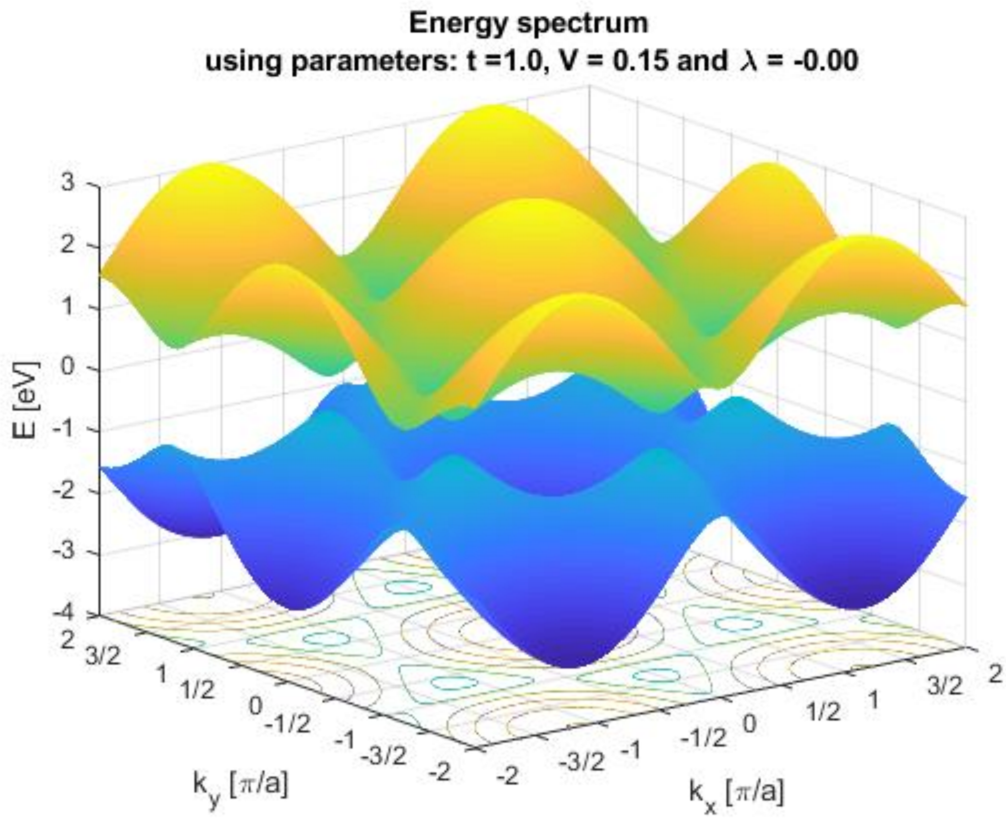
$$\boldsymbol{\sigma} = \left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix}\right)$$

Energy spectrum
using parameters: t =1.0, V = 0.10 and λ = -0.05

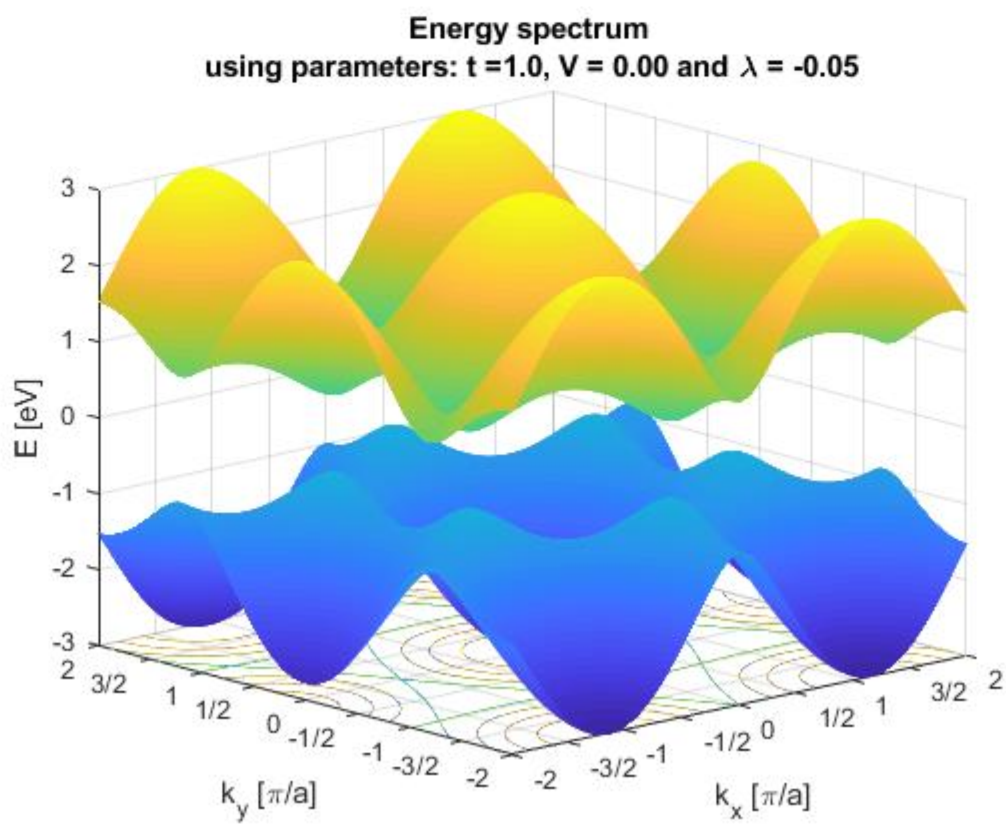


Energy spectrum
using parameters: t =1.0, V = -0.10 and λ = -0.05

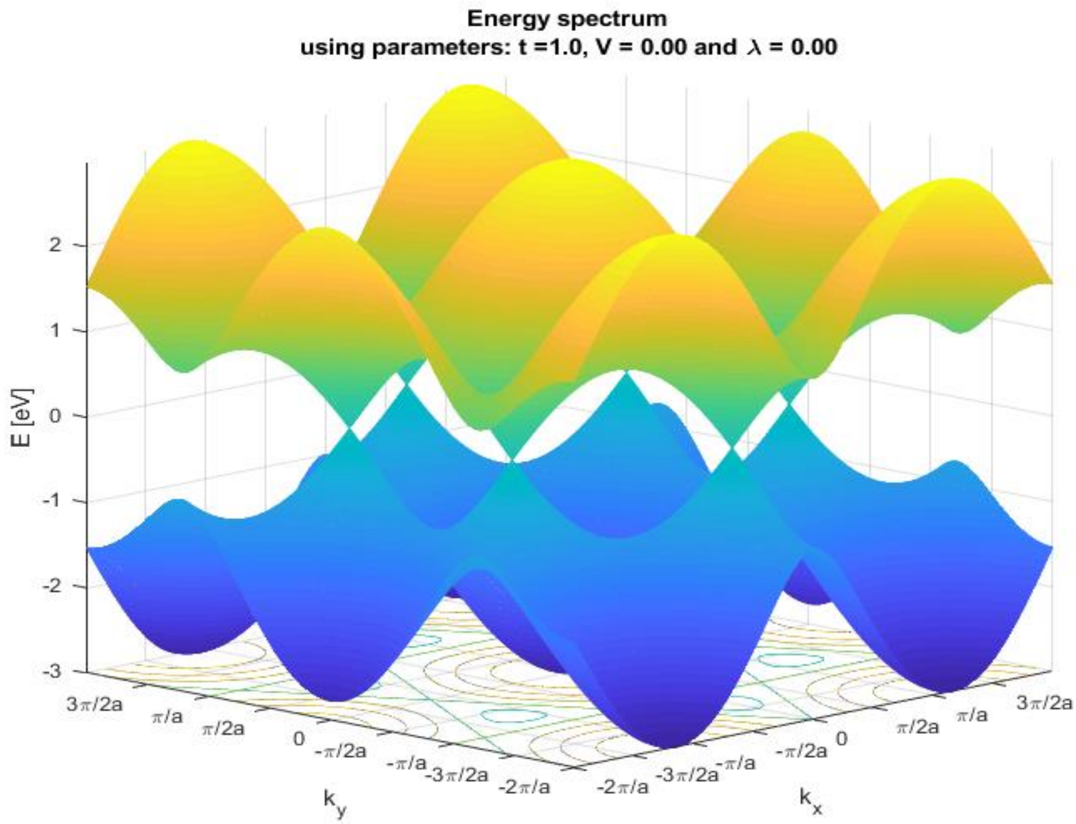


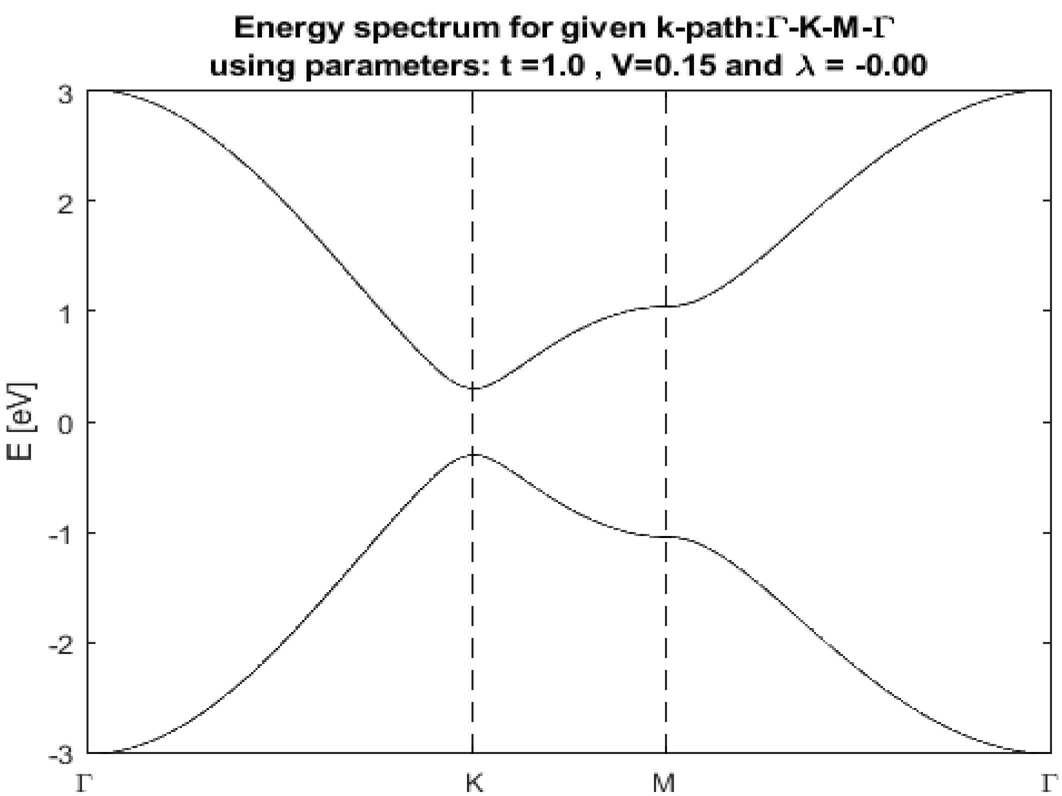


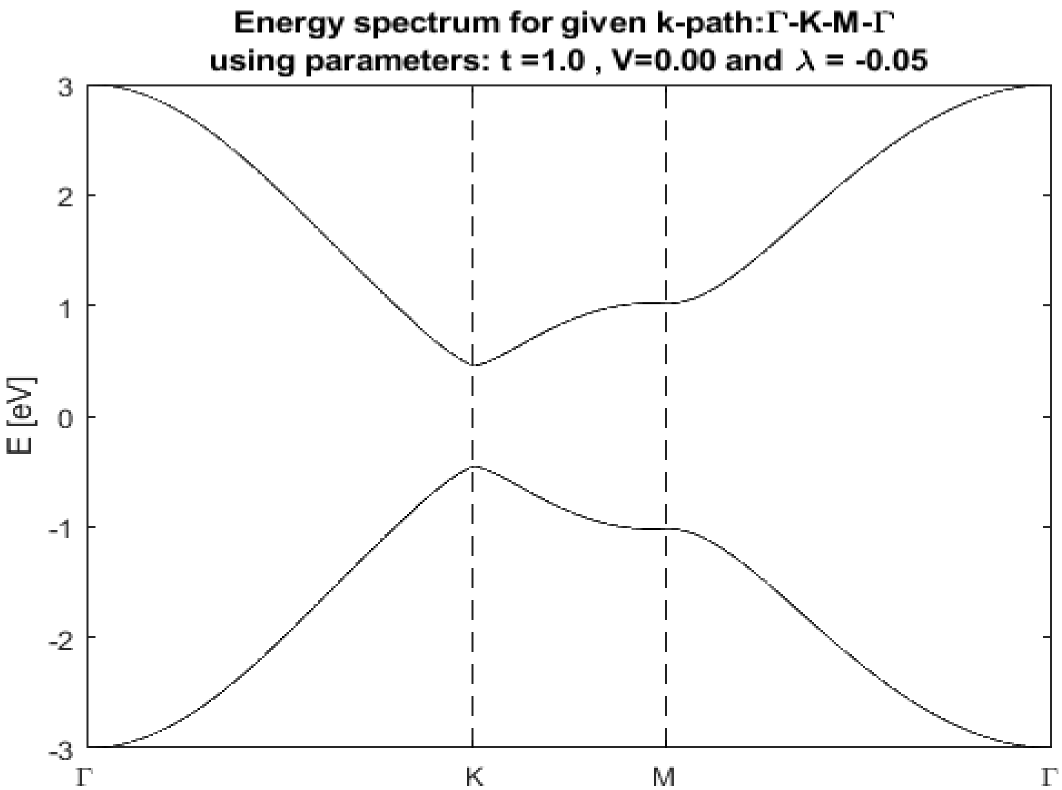
Energy spectrum
using parameters: t =1.0, V = 0.15 and λ = -0.00
E [eV]
k_y [π/a]
k_x [π/a]

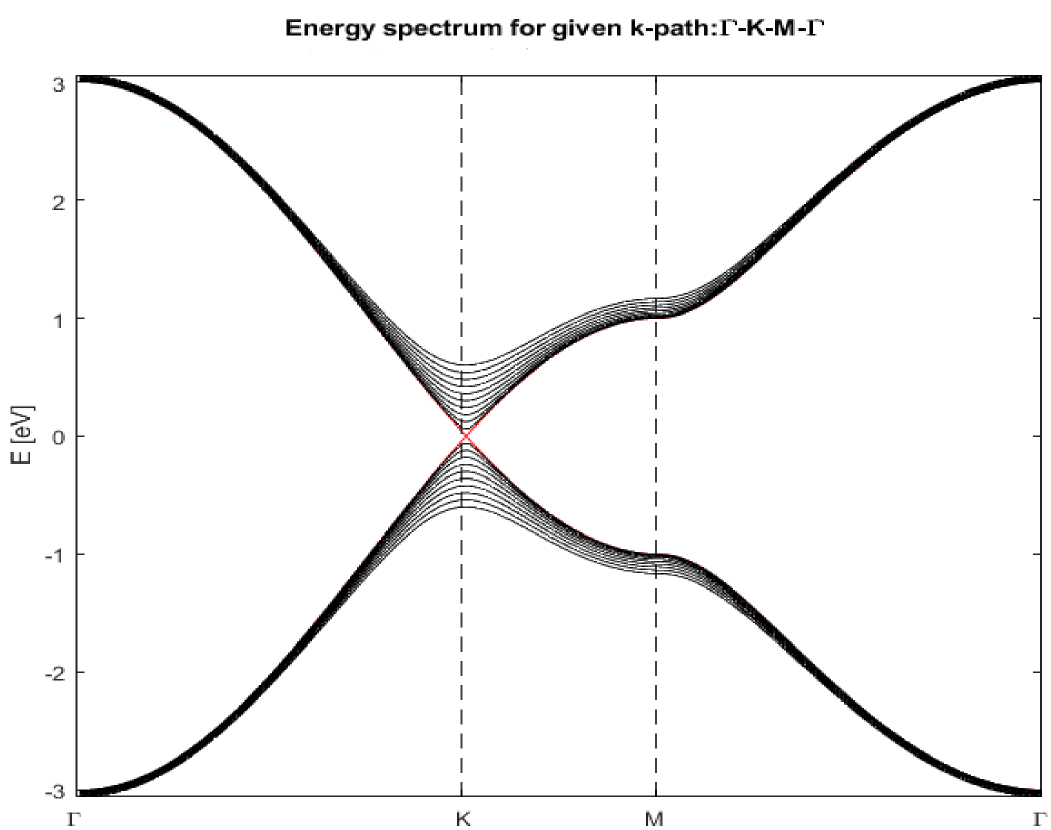


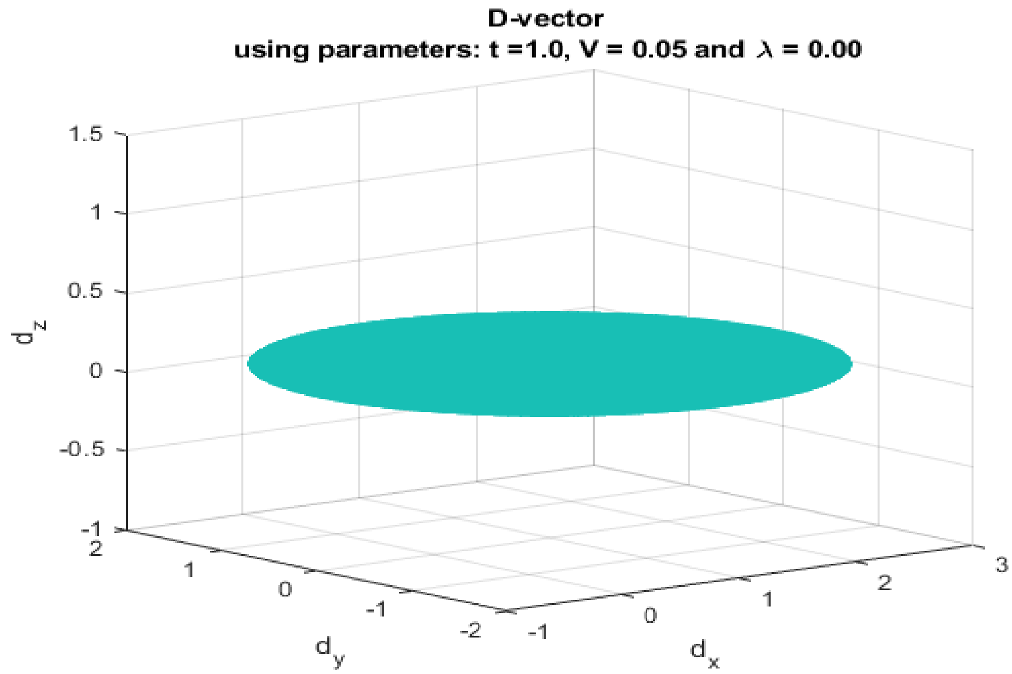
Energy spectrum
using parameters: t =1.0, V = 0.00 and λ = -0.05
E [eV]
k_y [π/a]
k_x [π/a]

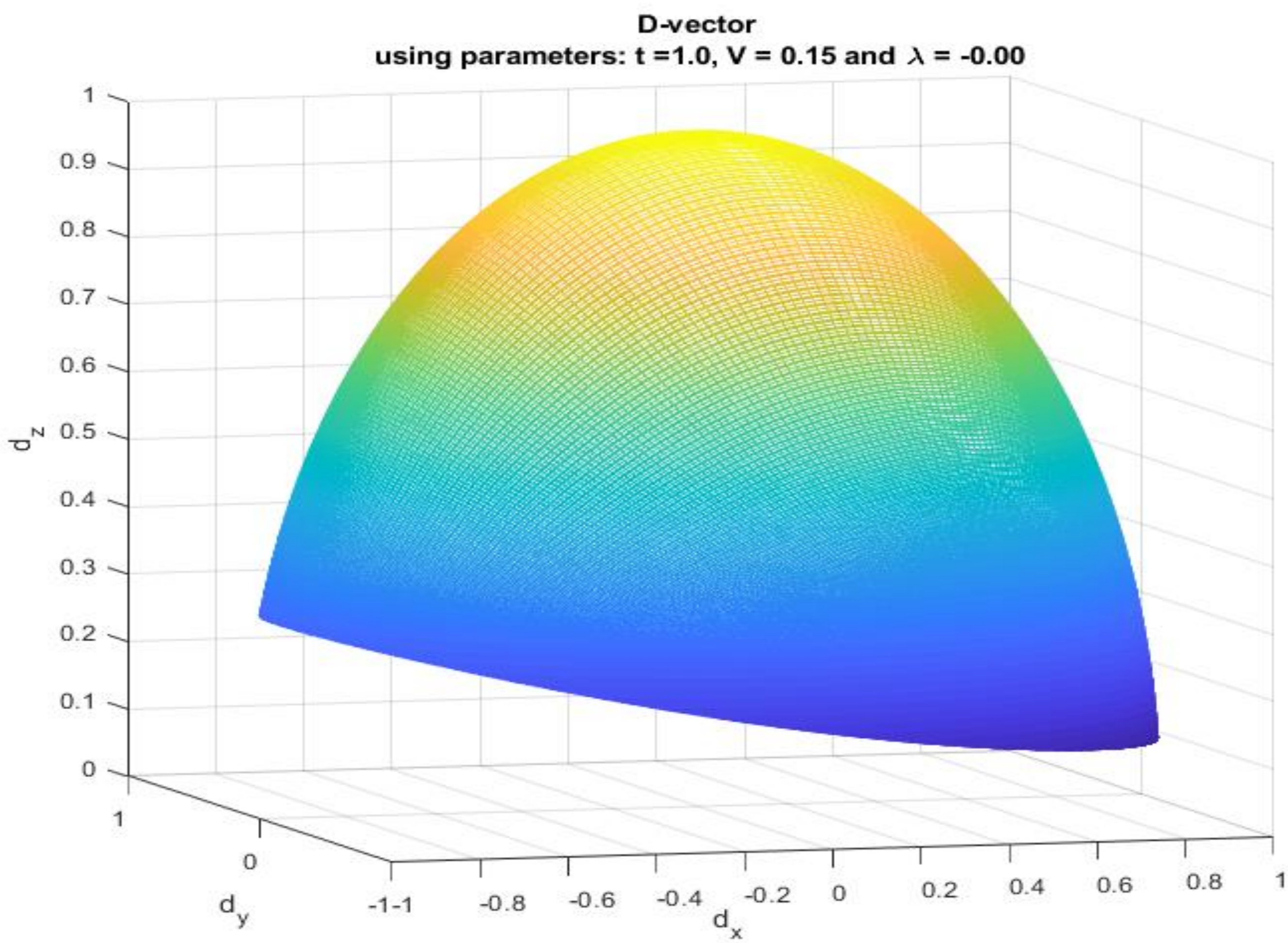

Energy spectrum
using parameters: t =1.0, V = 0.00 and λ = 0.00
E [eV]
k_y
k_x
3π/2a
π/a
π/2a
0
-π/2a
-π/a
-3π/2a
-2π/a

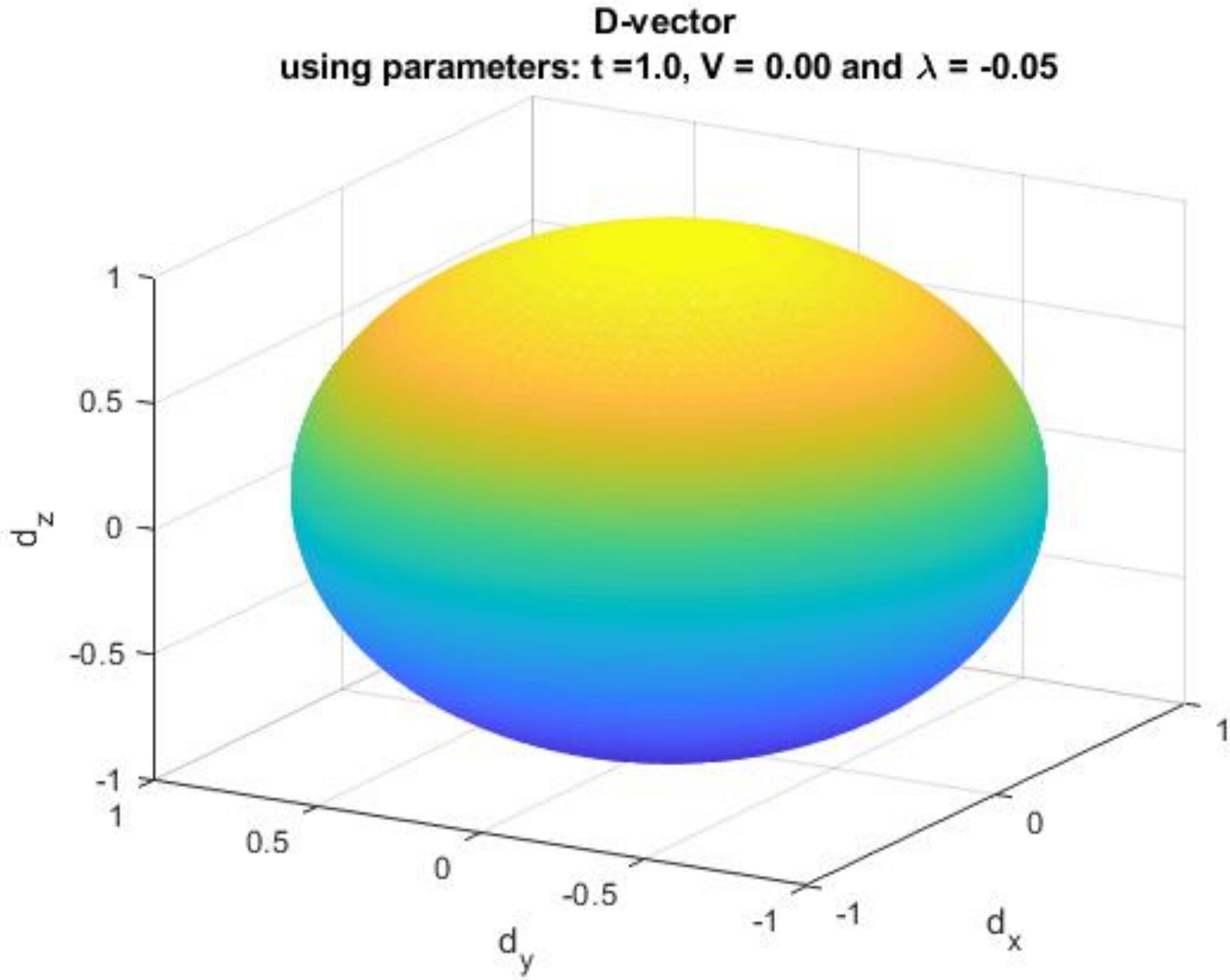

Energy spectrum for given k-path:Γ-K-M-Γ
using parameters: t =1.0 , V=0.15 and λ = -0.00
E [eV]
Γ
K
M
Γ

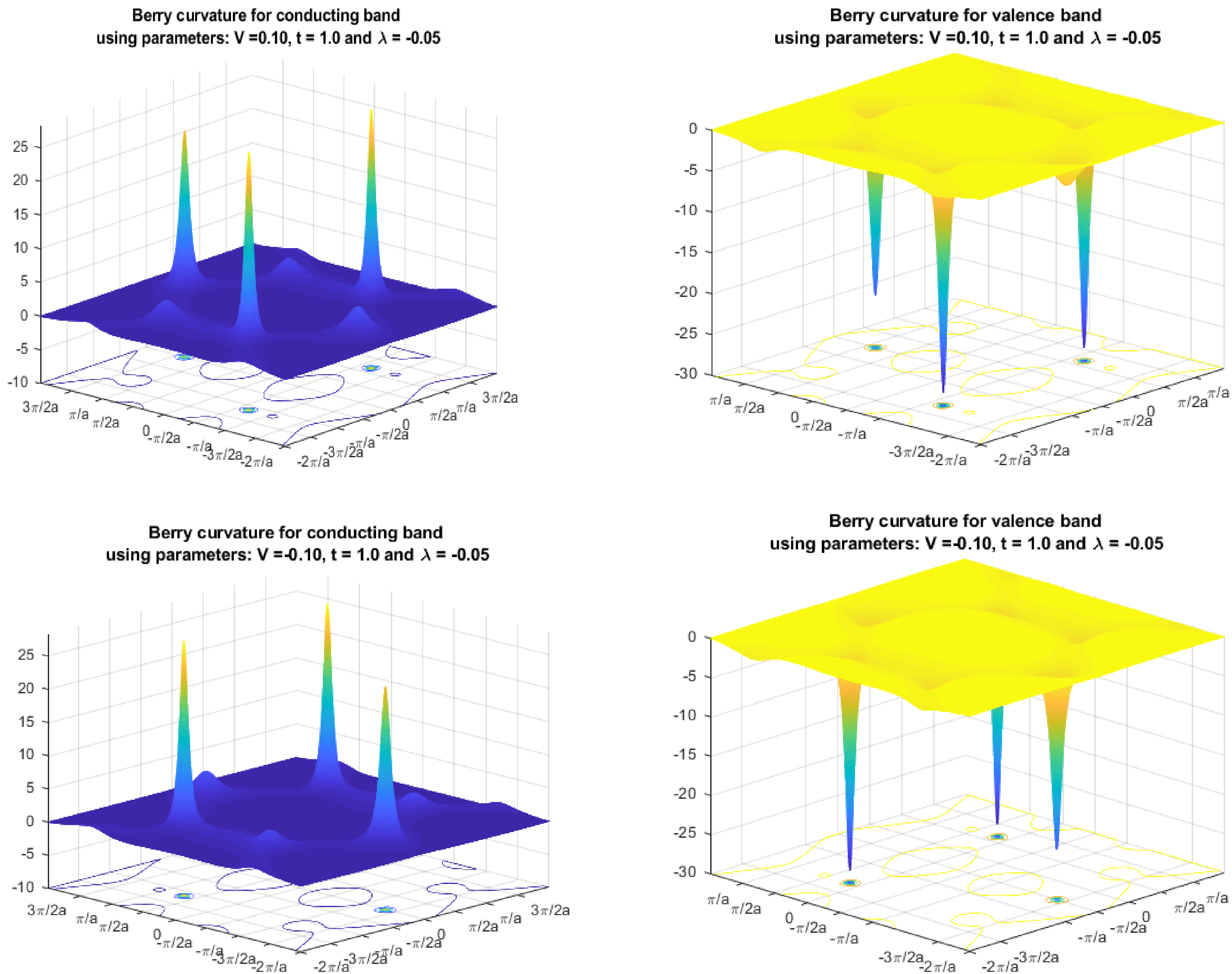

Energy spectrum for given k-path:Γ-K-M-Γ
using parameters: t =1.0 , V=0.00 and λ = -0.05
E [eV]
3
2
1
0
-1
-2
-3
Γ
K
M
Γ


Energy spectrum for given k-path:Γ-K-M-Γ
E [eV]
3
2
1
0
-1
-2
-3
Γ
K
M
Γ

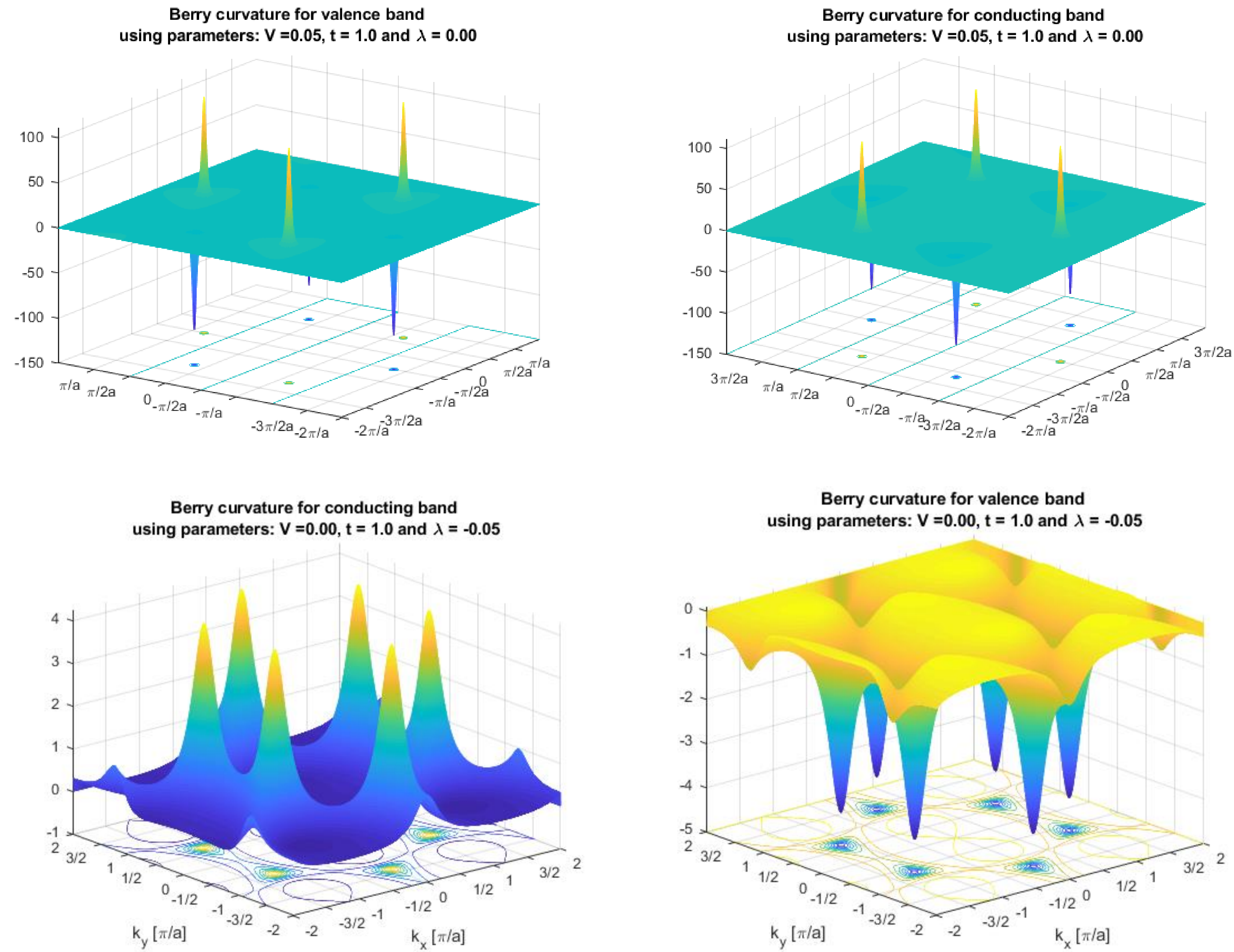

D-vector
using parameters: t =1.0, V = 0.05 and λ = 0.00
d_z
d_y
d_x


D-vector
using parameters: t =1.0, V = 0.15 and λ = -0.00
d_z
d_y
d_x


D-vector
using parameters: t =1.0, V = 0.00 and λ = -0.05
1
0.5
0
-0.5
-1
d_z
1
0.5
0
-0.5
-1
d_y
1
0
-1
d_x



Berry curvature for conducting band
using parameters: V =0.10, t = 1.0 and λ = -0.05
Berry curvature for valence band
using parameters: V =0.10, t = 1.0 and λ = -0.05
Berry curvature for conducting band
using parameters: V =-0.10, t = 1.0 and λ = -0.05
Berry curvature for valence band
using parameters: V =-0.10, t = 1.0 and λ = -0.05
25
20
15
10
5
0
-5
-10
-15
-20
-25
-30
3π/2a
π/a
π/2a
0
-π/2a
-π/a
-3π/2a
-2π/a


Berry curvature for valence band
using parameters: V =0.05, t = 1.0 and λ = 0.00
Berry curvature for conducting band
using parameters: V =0.05, t = 1.0 and λ = 0.00
Berry curvature for conducting band
using parameters: V =0.00, t = 1.0 and λ = -0.05
Berry curvature for valence band
using parameters: V =0.00, t = 1.0 and λ = -0.05
k_y [π/a]
k_x [π/a]

Pierwsza liczba Cherna w przypadku parametrów $\lambda \neq 0$ i $V = 0$ wynosi $c_n = \pm 1$, gdzie pasmo walencyjne i przewodnictwa mają liczbe Cherna o przeciwnym znaku (znak λ decyduje o ich znakach). Natomiast dla parametrów $\lambda = 0$ i $V \neq 0$ liczba Cherna wynosi $c_n = 0$ dla obu pasm. W programie otrzymano wartości $c_n = \pm 1.000126$ oraz $c_n = 0{,}000531$, gdzie miejsca po przecinku powstają w wyniku zbyt rzadkiej siatki w przestrzeni odwrotnej.

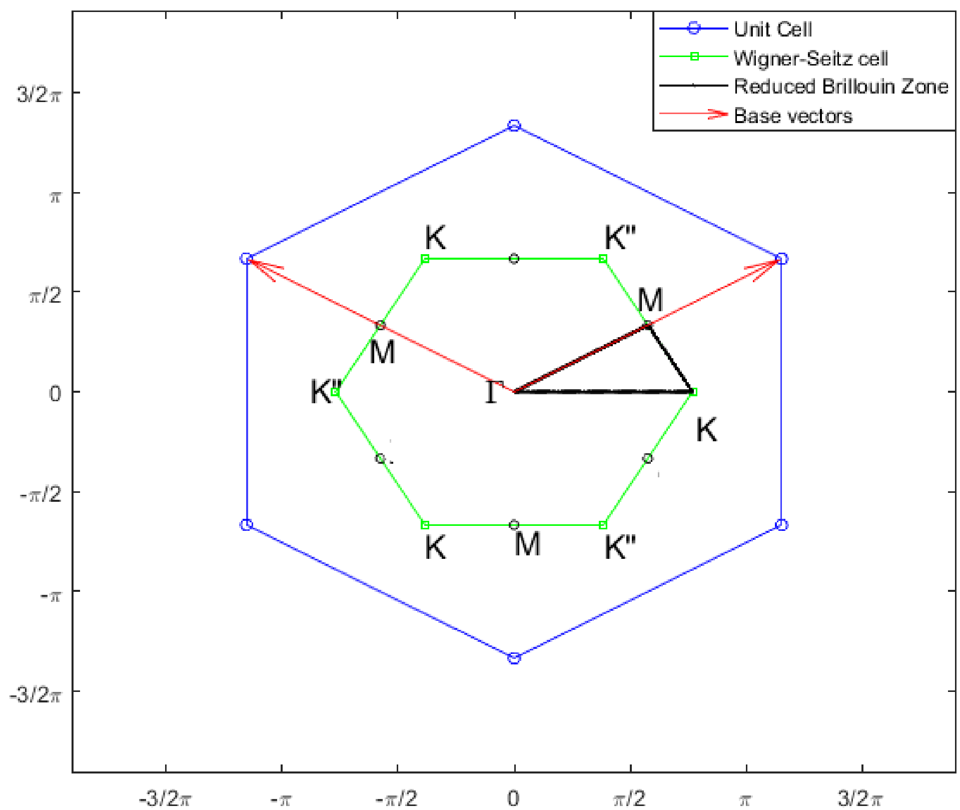
Wektory sieci odwrotnej to:

$$b_1 = \left[-\frac{2\pi}{\sqrt{3}}, \frac{2\pi}{3}\right] \quad oraz \quad b_2 = \left[\frac{2\pi}{\sqrt{3}}, \frac{2\pi}{3}\right]$$

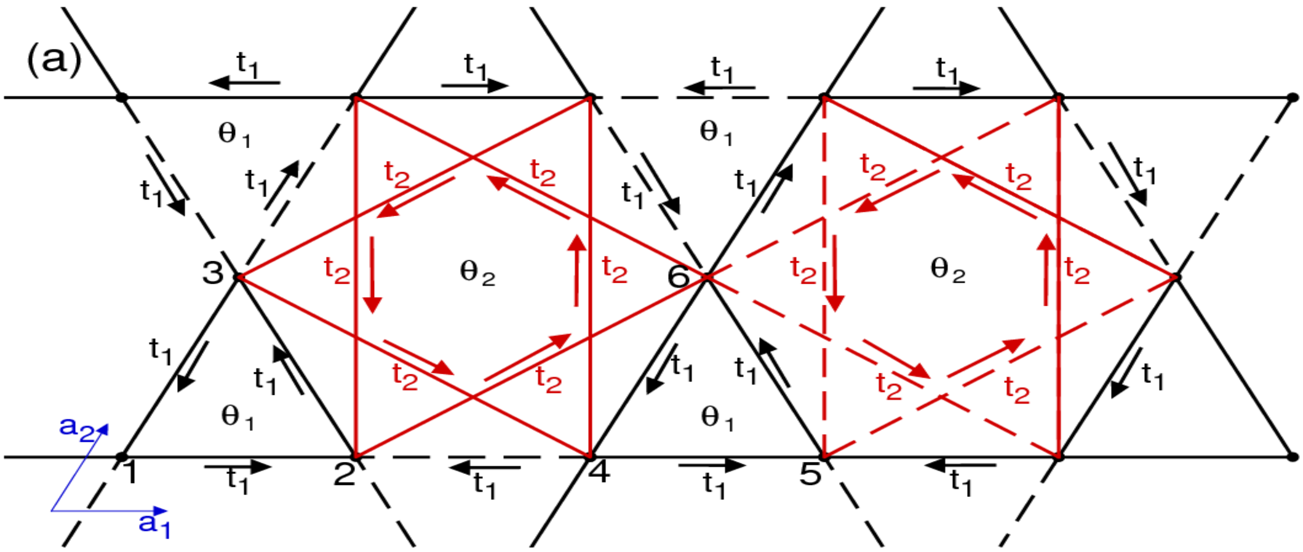
Punkty wysokiej symetrii to punkty:

$$K \ i \ K': \ \left[\frac{4\sqrt{3}\pi}{9}, 0\right], \left[\frac{2\sqrt{3}\pi}{9}, \frac{2\pi}{3}\right], \left[-\frac{2\sqrt{3}\pi}{9}, \frac{2\pi}{3}\right], \left[-\frac{4\sqrt{3}\pi}{9}, 0\right], \left[-\frac{2\sqrt{3}\pi}{9}, -\frac{2\pi}{3}\right], \left[\frac{2\sqrt{3}\pi}{9}, -\frac{2\pi}{3}\right]$$

$$M \ i \ M': \left[\frac{\pi}{\sqrt{3}}, \frac{\pi}{3}\right], \left[0, \frac{2\pi}{3}\right], \left[-\frac{\pi}{\sqrt{3}}, \frac{\pi}{3}\right], \left[-\frac{\pi}{\sqrt{3}}, -\frac{\pi}{3}\right], \left[0, -\frac{2\pi}{3}\right], \left[\frac{\pi}{\sqrt{3}}, -\frac{\pi}{3}\right]$$



- **Haldane model on kagome lattice**



Hamiltonian Haldane dla takiej sieci jest postaci (na rysunku powyżej $t_1 \to t_1 + i\lambda_1$ i analogicznie dla t_2):

$$H = -t_1 \sum_{\langle i,j\rangle} a_i^+ a_j + i\lambda_1 \sum_{\langle i,j\rangle} \nu_{ij} a_I^+ a_j - t_2 \sum_{\langle\langle i,j\rangle\rangle} a_i^+ a_j + i\lambda_1 \sum_{\langle\langle i,j\rangle\rangle} \nu_{ij} a_I^+ a_j$$

po transformacie Fouriera jest postaci:

$$\begin{aligned} H(k) = &-(t_1 - i\lambda_1) a_B^+ a_A \left(1 + e^{-ik_1}\right) - (t_1 + i\lambda_1) a_C^+ a_A (1 + e^{-1k_2}) - (t_1 + i\lambda_1) a_A^+ a_B \left(1 + e^{ik_1}\right) \\ &- (t_1 - i\lambda_1) a_C^+ a_B \left(1 + e^{i(k_1 - k_2)}\right) - (t_1 - i\lambda_1) a_A^+ a_C \left(1 + e^{ik_2}\right) \\ &- (t_1 + i\lambda_1) a_B^+ a_C \left(1 + e^{-i(k_1 - k_2)}\right) - (t_2 + i\lambda_2) a_B^+ a_A \left(e^{-ik_2} + e^{-i(k_1 - k_2)}\right) \\ &- (t_2 - i\lambda_2) a_C^+ a_A \left(e^{i(k_1 - k_2)} + e^{-ik_1}\right) - (t_2 - i\lambda_2) a_A^+ a_B \left(e^{-i(k_1 - k_2)} + e^{ik_1}\right) \\ &- (t_2 + i\lambda_2) a_C^+ a_B \left(e^{ik_1} + e^{-ik_2}\right) - (t_2 + i\lambda_2) a_A^+ a_C \left(e^{-i(k_1 - k_2)} + e^{ik_1}\right) - (t_2 - i\lambda_2) a_B^+ a_C \left(e^{ik_2} + e^{-ik_1}\right) \end{aligned}$$

Zapisując operatory kreacji macierzowo jako: $a_A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, $a_B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$, $a_C = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

otrzymamy hamiltonian:

$$H(k) = \begin{pmatrix} 0 & -(t_1 + i\lambda_1)\left(1 + e^{ik_1}\right) & -(t_1 - i\lambda_1)\left(1 + e^{ik_2}\right) \\ -(t_1 - i\lambda_1)\left(1 + e^{-ik_1}\right) & 0 & -(t_1 + i\lambda_1)\left(1 + e^{-i(k_1 - k_2)}\right) \\ -(t_1 + i\lambda_1)(1 + e^{-1k_2}) & -(t_1 - i\lambda_1)\left(1 + e^{i(k_1 - k_2)}\right) & 0 \end{pmatrix}$$
$$+ \begin{pmatrix} 0 & -(t_2 - i\lambda_2)\left(e^{-i(k_1 - k_2)} + e^{ik_1}\right) & -(t_2 + i\lambda_2)\left(e^{-i(k_1 - k_2)} + e^{ik_1}\right) \\ -(t_2 + i\lambda_2)\left(e^{-ik_2} + e^{-i(k_1 - k_2)}\right) & 0 & -(t_2 - i\lambda_2)\left(e^{ik_2} + e^{-ik_1}\right) \\ -(t_2 - i\lambda_2)\left(e^{i(k_1 - k_2)} + e^{-ik_1}\right) & -(t_2 + i\lambda_2)\left(e^{ik_1} + e^{-ik_2}\right) & 0 \end{pmatrix}$$

Wektory sieci prostej można w rożny sposób dobrać dla takiej sieci. W tym przypadku wybrano wektory:

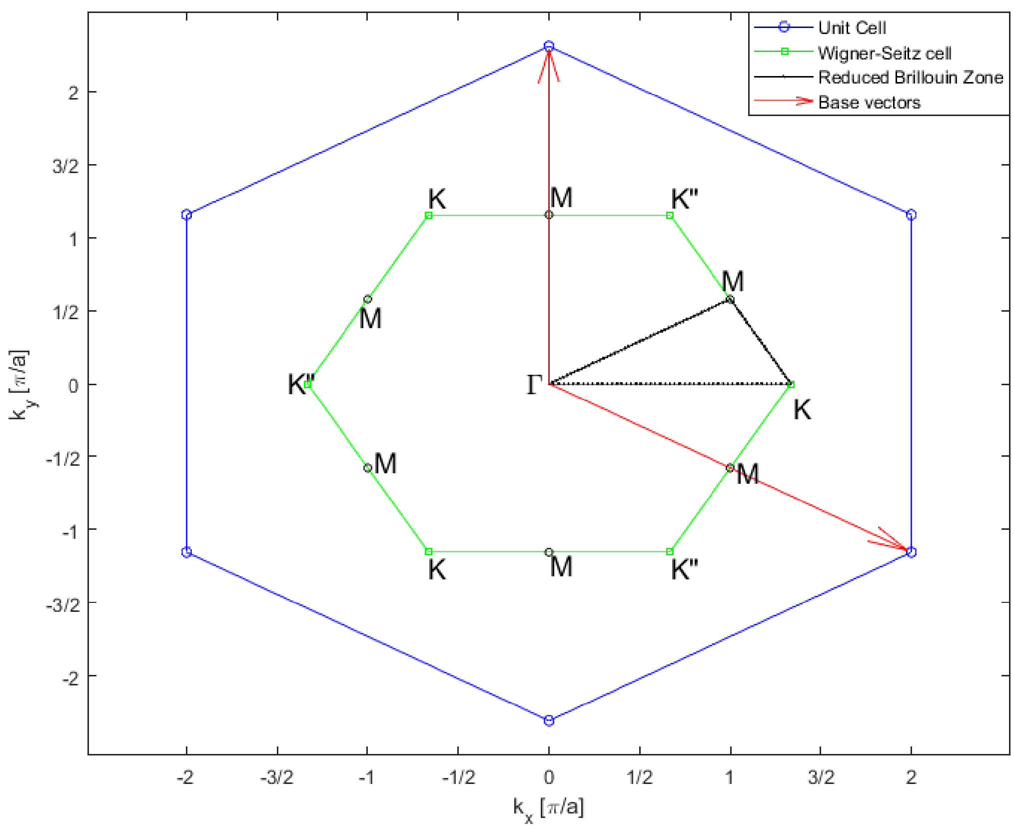
$a_1 = [a, 0]$ oraz $a_2\left[\frac{a}{2}, \frac{a\sqrt{3}}{2}\right]$, gdzie $a = 1$ to stała sieci. Wektory rozpinające sieć odwrotną są wtedy postaci:

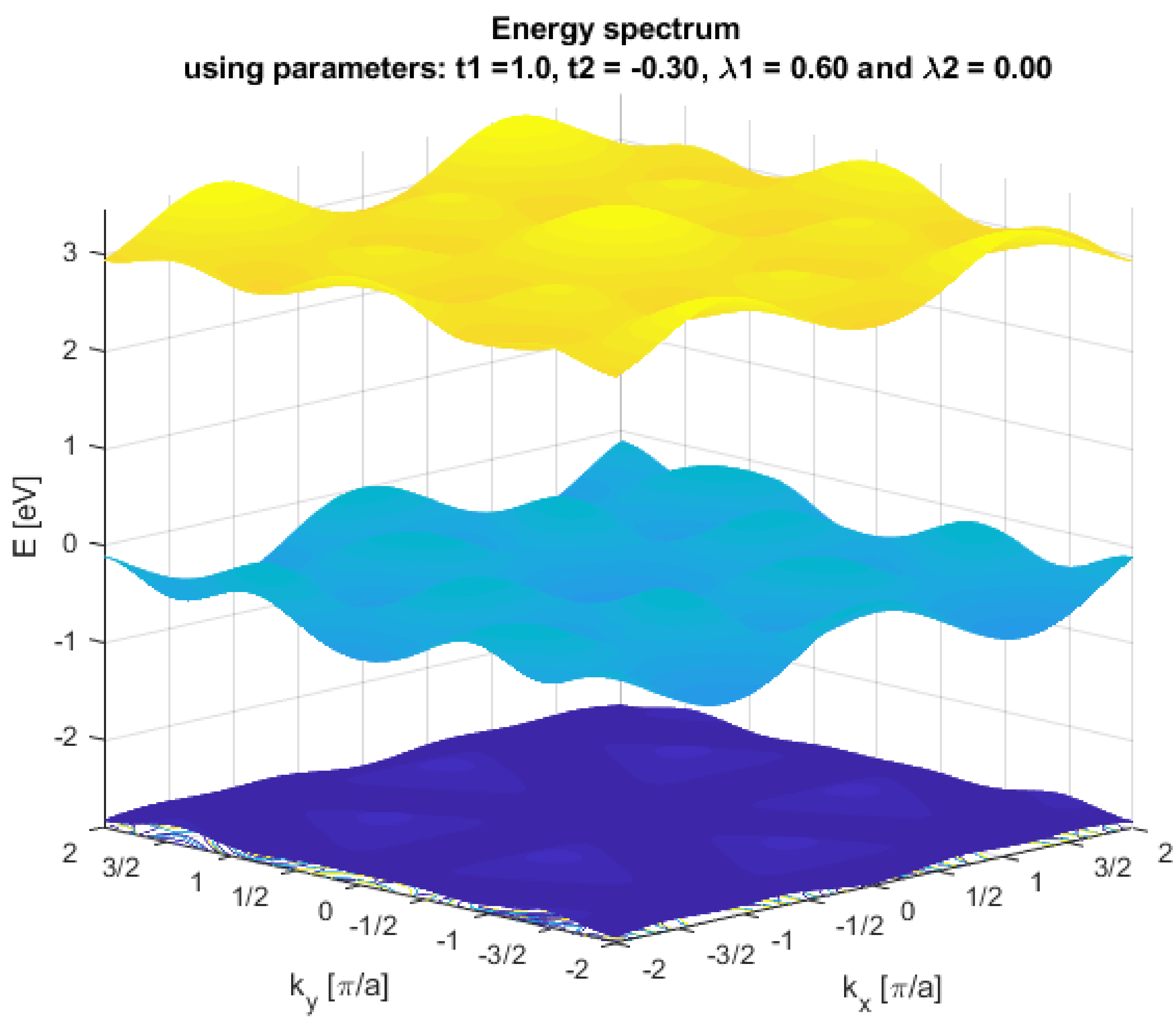
$$b_1 = \left[\frac{2\pi}{a}, -\frac{2\pi}{a\sqrt{3}}\right] \quad oraz \quad b_2 = \left[0, \frac{4\pi}{a\sqrt{3}}\right]$$

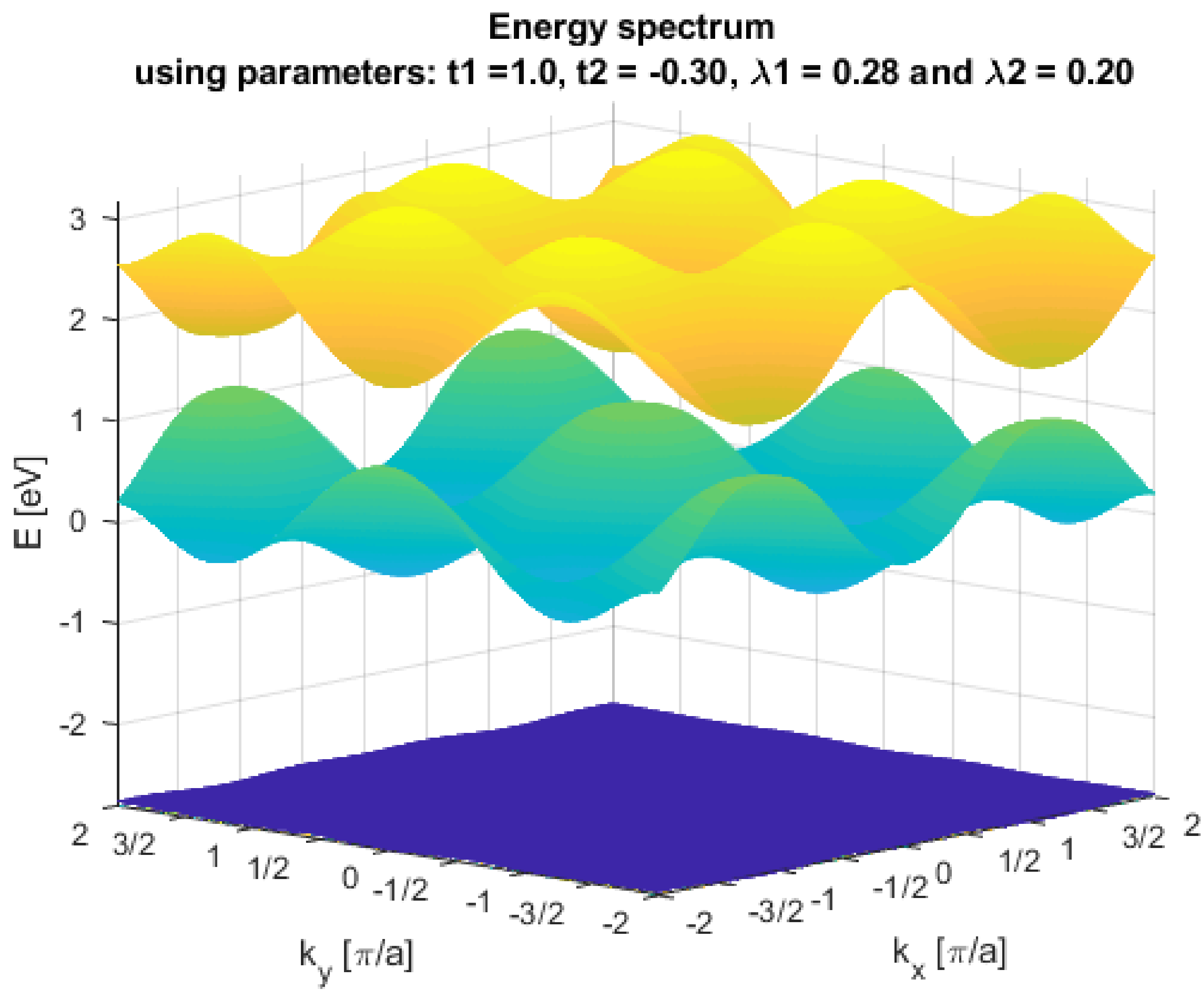
Punkty wysokiej symetrii są wtedy postaci:

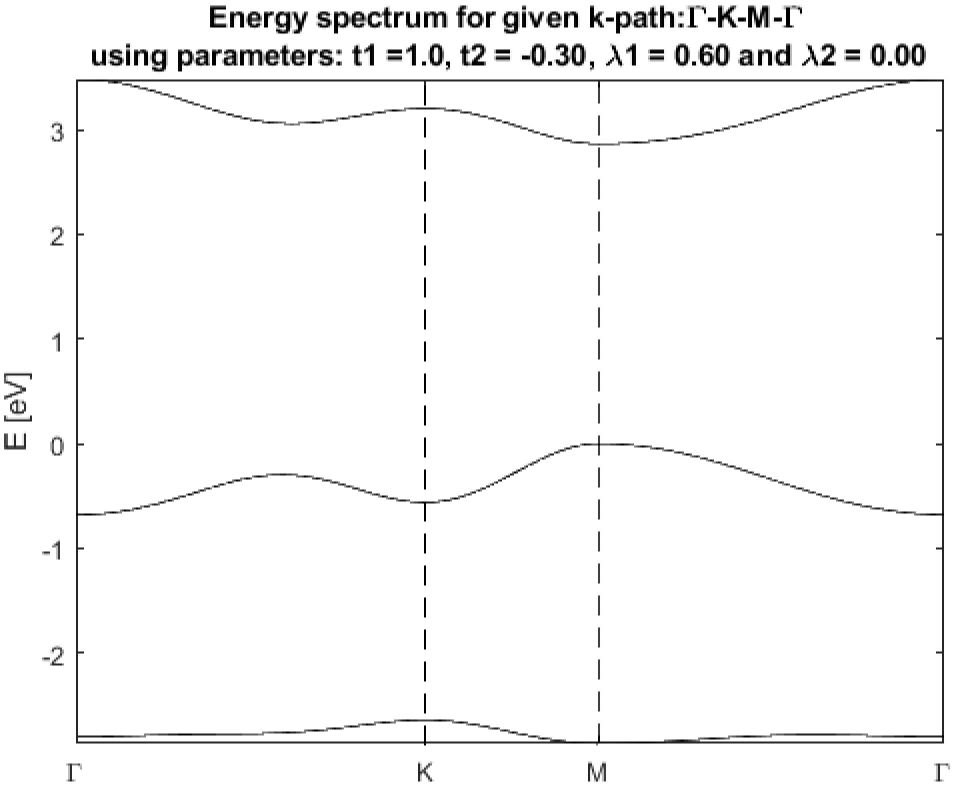
$$K\ i\ K': \left[\frac{4\pi}{3}, 0\right], \left[\frac{4\pi}{6}, \frac{2\pi}{\sqrt{3}}\right], \left[-\frac{4\pi}{6}, \frac{2\pi}{\sqrt{3}}\right], \left[-\frac{4\pi}{3}, 0\right], \left[-\frac{4\pi}{6}, -\frac{2\pi}{\sqrt{3}}\right], \left[\frac{4\pi}{6}, -\frac{2\pi}{\sqrt{3}}\right]$$

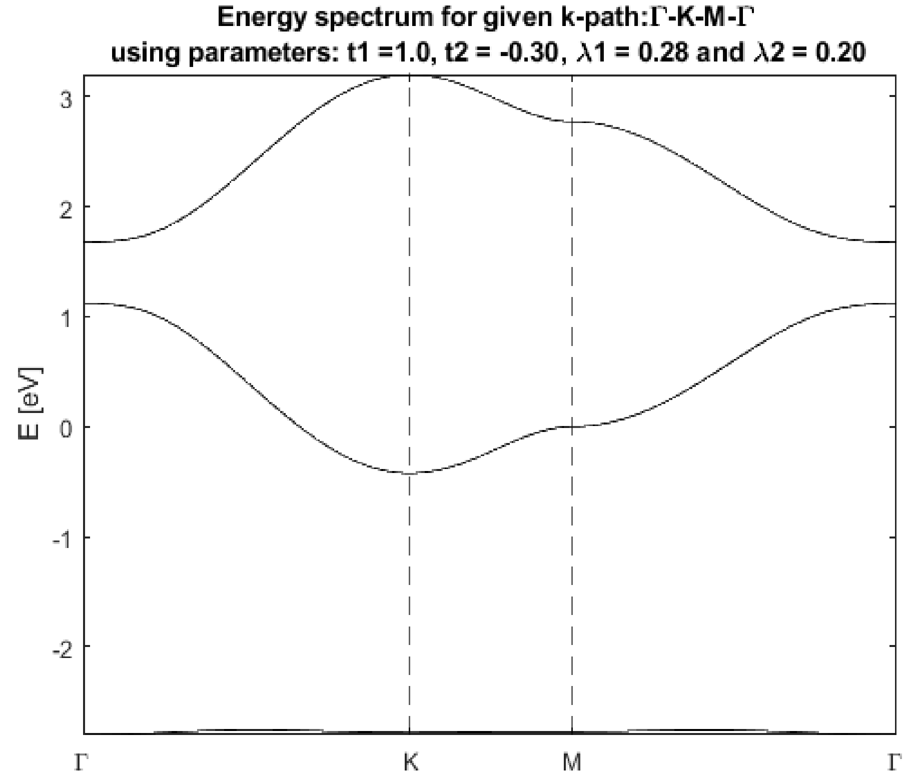
$$M\ i\ M': \left[\pi, \frac{\pi}{\sqrt{3}}\right], \left[0, \frac{2\pi}{\sqrt{3}}\right], \left[-\pi, \frac{\pi}{\sqrt{3}}\right], \left[-\pi, -\frac{\pi}{\sqrt{3}}\right], \left[0, -\frac{2\pi}{\sqrt{3}}\right], \left[\pi, -\frac{\pi}{\sqrt{3}}\right]$$

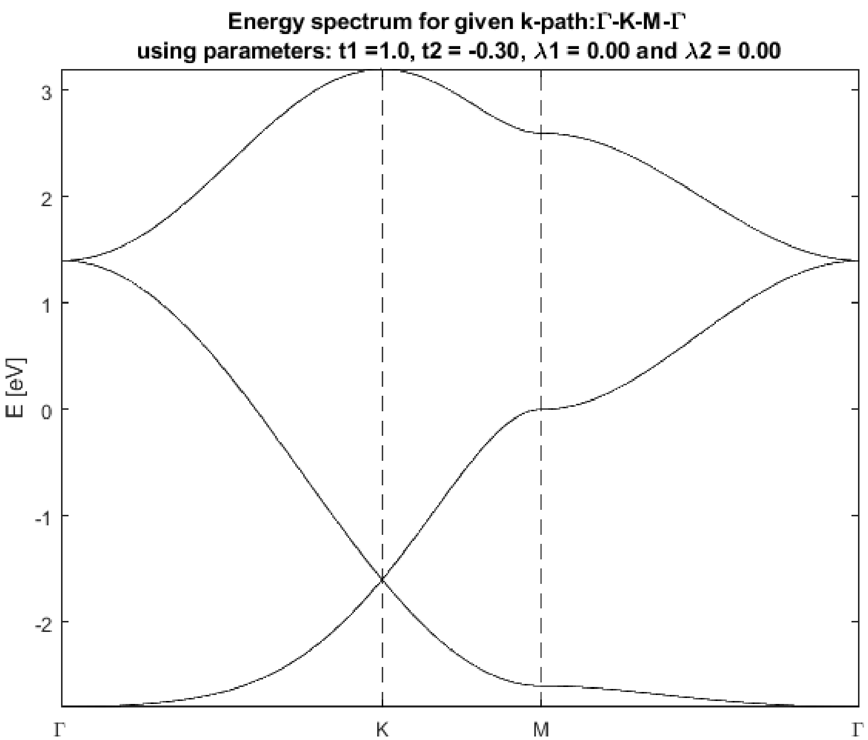


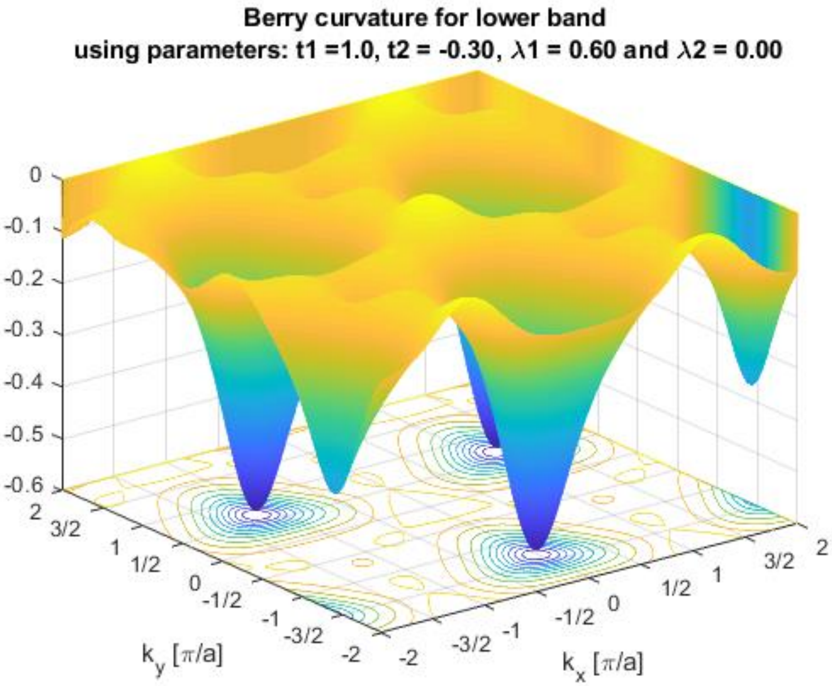

Energy spectrum
using parameters: t1 =1.0, t2 = -0.30, λ1 = 0.60 and λ2 = 0.00
E [eV]
3
2
1
0
-1
-2
2
3/2
1
1/2
0
-1/2
-1
-3/2
-2
k_y [π/a]
-2
-3/2
-1
-1/2
0
1/2
1
3/2
2
k_x [π/a]

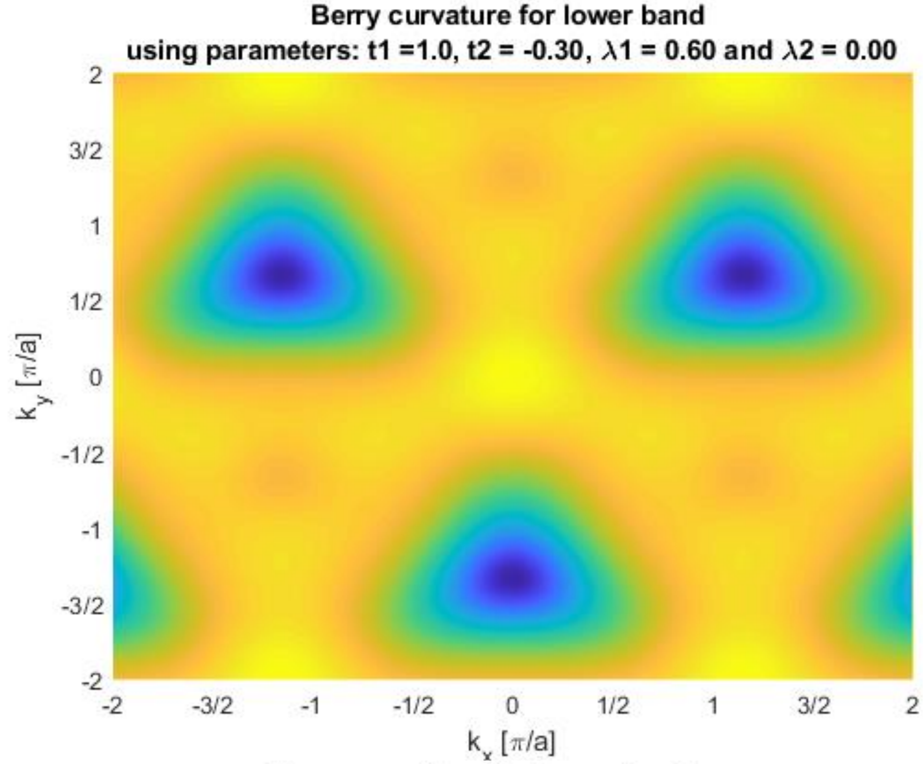

Energy spectrum
using parameters: t1 =1.0, t2 = -0.30, λ1 = 0.28 and λ2 = 0.20
E [eV]
3
2
1
0
-1
-2
k_y [π/a]
2 3/2 1 1/2 0 -1/2 -1 -3/2 -2
k_x [π/a]
-2 -3/2 -1 -1/2 0 1/2 1 3/2 2

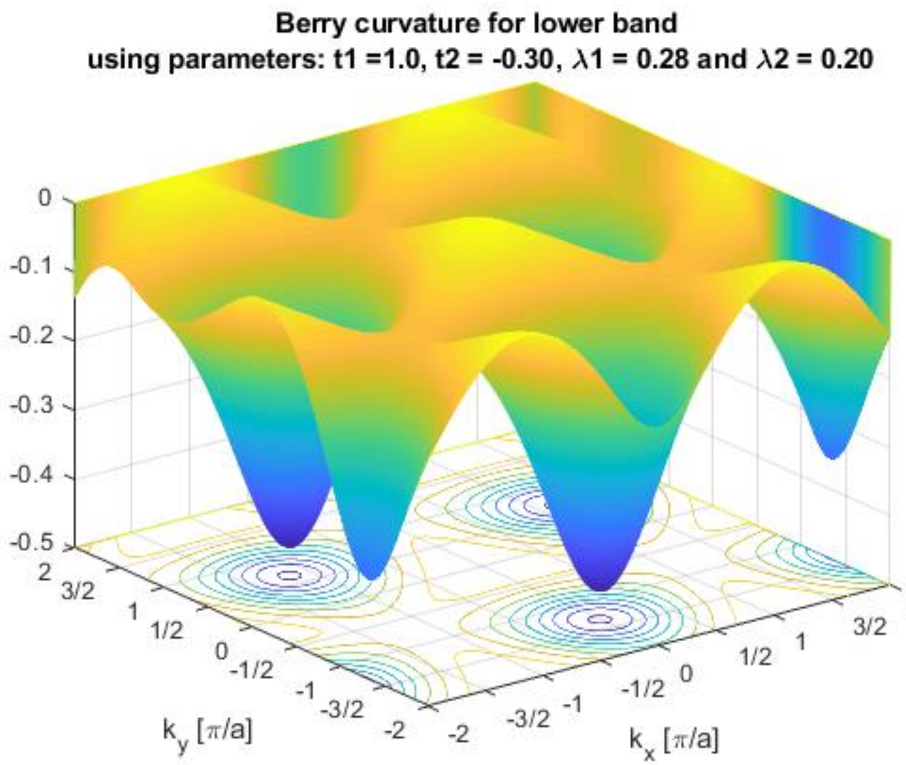

Energy spectrum for given k-path:Γ-K-M-Γ
using parameters: t1 =1.0, t2 = -0.30, λ1 = 0.60 and λ2 = 0.00
E [eV]
3
2
1
0
-1
-2
Γ
K
M
Γ

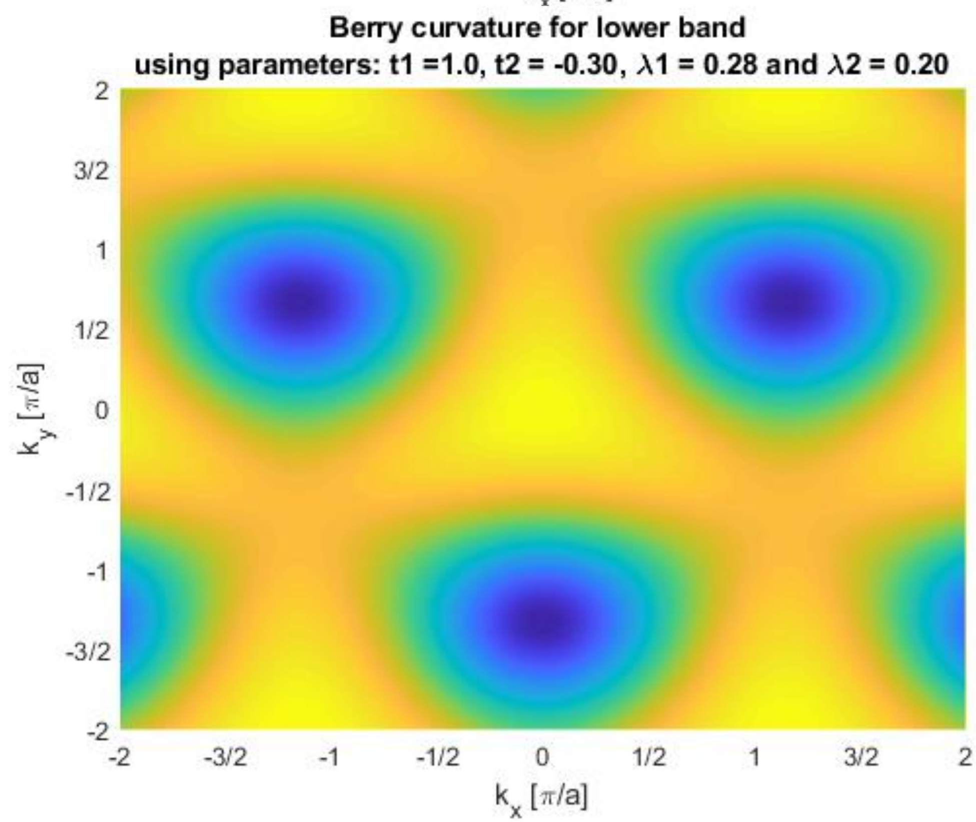

Energy spectrum for given k-path:Γ-K-M-Γ
using parameters: t1 =1.0, t2 = -0.30, λ1 = 0.28 and λ2 = 0.20
E [eV]
3
2
1
0
-1
-2
Γ
K
M
Γ

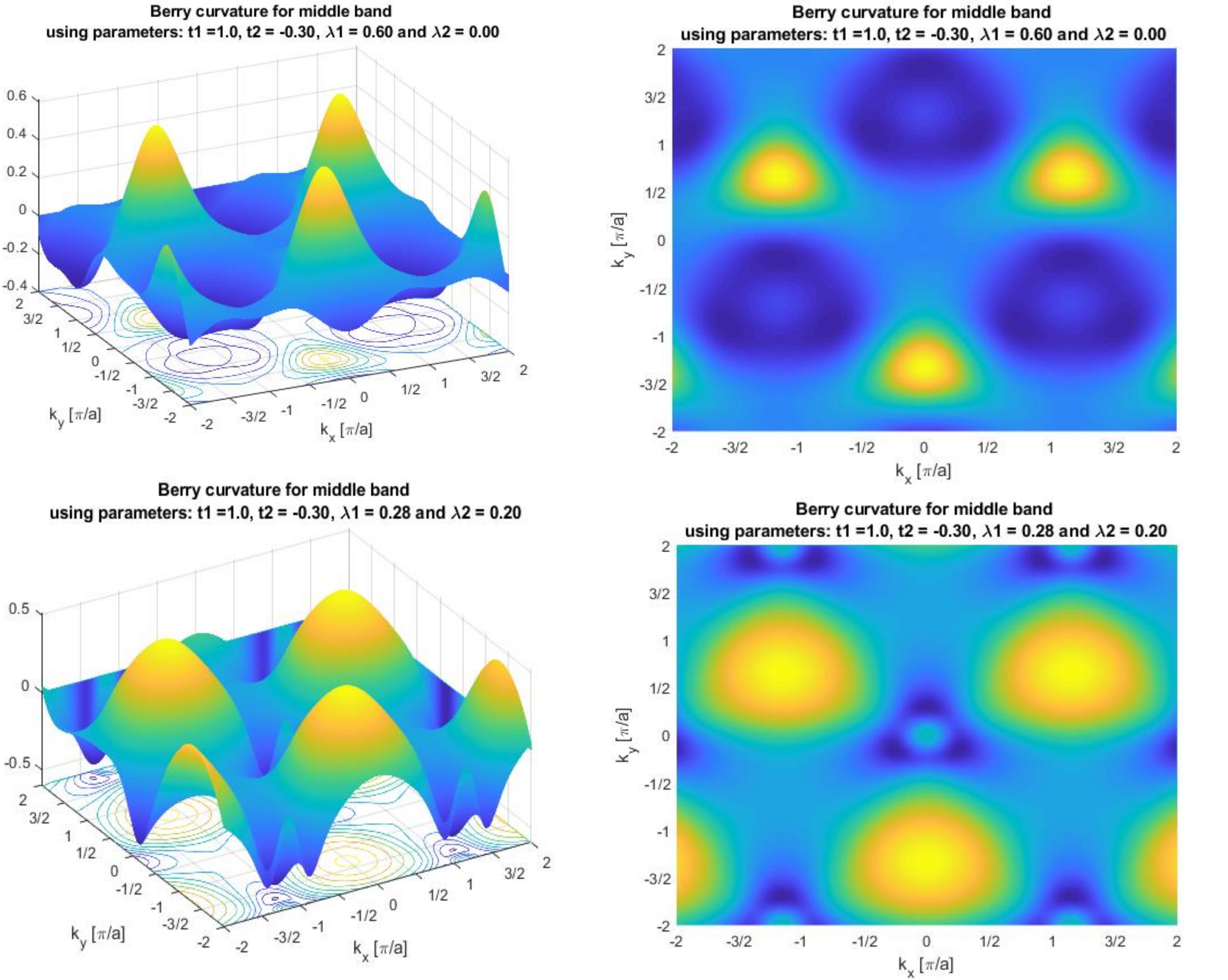

Energy spectrum for given k-path:Γ-K-M-Γ
using parameters: t1 =1.0, t2 = -0.30, λ1 = 0.00 and λ2 = 0.00
E [eV]
3
2
1
0
-1
-2
Γ
K
M
Γ


Berry curvature for lower band
using parameters: t1 =1.0, t2 = -0.30, λ1 = 0.60 and λ2 = 0.00
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
k_y [π/a]
k_x [π/a]
2
3/2
1
1/2
0
-1/2
-1
-3/2
-2

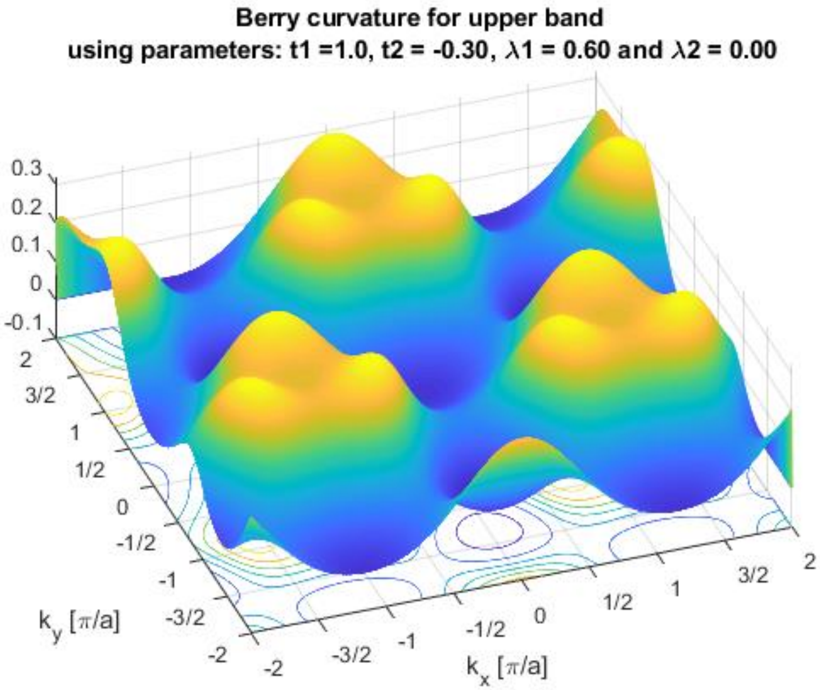

Berry curvature for lower band
using parameters: t1 =1.0, t2 = -0.30, λ1 = 0.60 and λ2 = 0.00
k_y [π/a]
k_x [π/a]
2
3/2
1
1/2
0
-1/2
-1
-3/2
-2

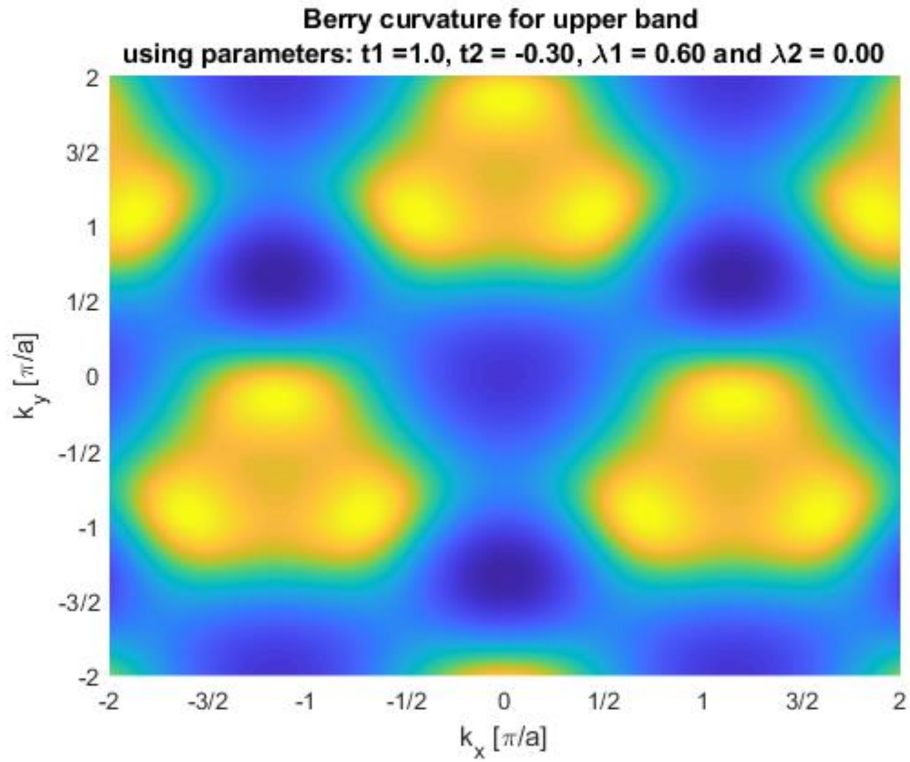

Berry curvature for lower band
using parameters: t1 =1.0, t2 = -0.30, λ1 = 0.28 and λ2 = 0.20
0
-0.1
-0.2
-0.3
-0.4
-0.5
k_y [π/a]
k_x [π/a]
2
3/2
1
1/2
0
-1/2
-1
-3/2
-2

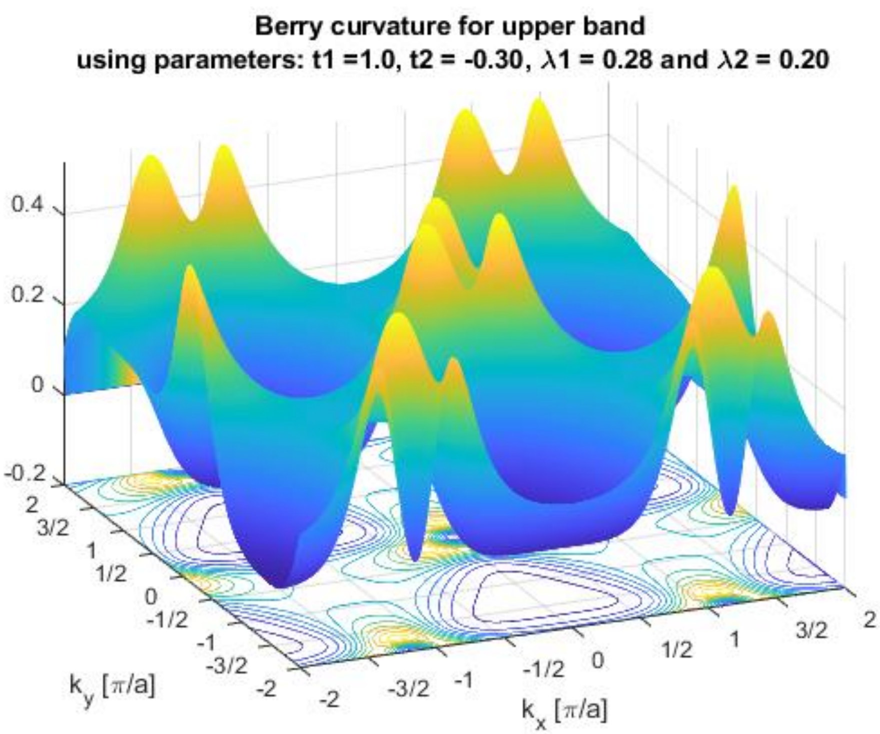

Berry curvature for lower band
using parameters: t1 =1.0, t2 = -0.30, λ1 = 0.28 and λ2 = 0.20
k_y [π/a]
k_x [π/a]
2
3/2
1
1/2
0
-1/2
-1
-3/2
-2

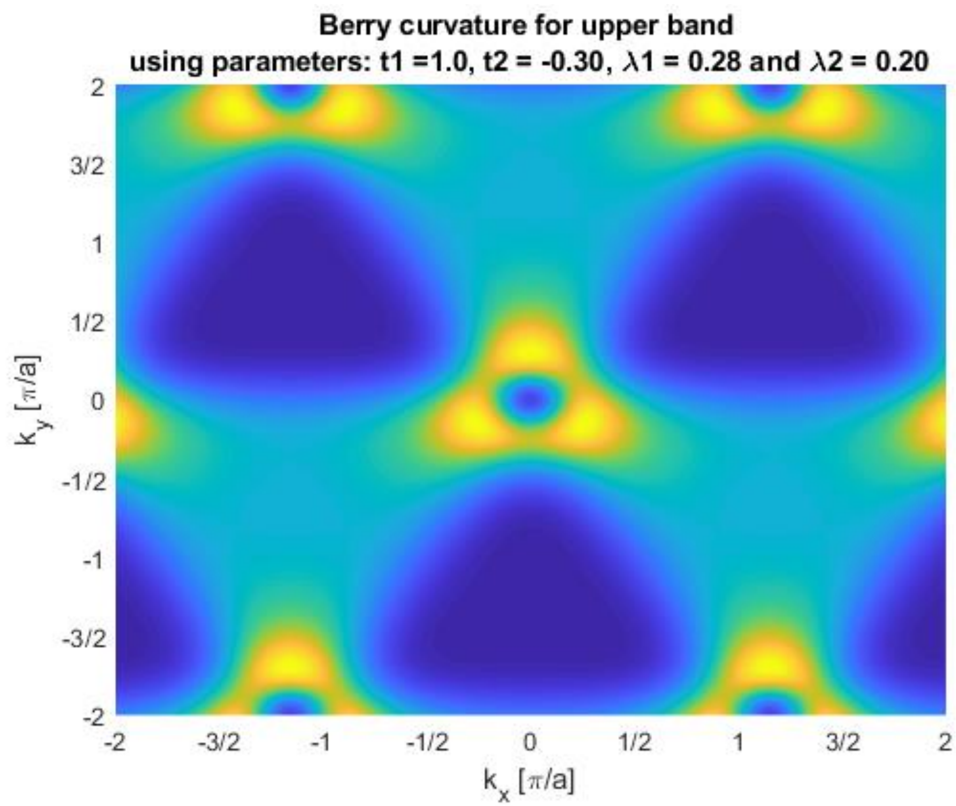


Berry curvature for middle band
using parameters: t1 =1.0, t2 = -0.30, λ1 = 0.60 and λ2 = 0.00
k_y [π/a]
k_x [π/a]
Berry curvature for middle band
using parameters: t1 =1.0, t2 = -0.30, λ1 = 0.60 and λ2 = 0.00
k_y [π/a]
k_x [π/a]
Berry curvature for middle band
using parameters: t1 =1.0, t2 = -0.30, λ1 = 0.28 and λ2 = 0.20
k_y [π/a]
k_x [π/a]
Berry curvature for middle band
using parameters: t1 =1.0, t2 = -0.30, λ1 = 0.28 and λ2 = 0.20
k_y [π/a]
k_x [π/a]


Berry curvature for upper band
using parameters: t1 =1.0, t2 = -0.30, λ1 = 0.60 and λ2 = 0.00
k_y [π/a]
k_x [π/a]


Berry curvature for upper band
using parameters: t1 =1.0, t2 = -0.30, λ1 = 0.60 and λ2 = 0.00
k_y [π/a]
k_x [π/a]


Berry curvature for upper band
using parameters: t1 =1.0, t2 = -0.30, λ1 = 0.28 and λ2 = 0.20
k_y [π/a]
k_x [π/a]


Berry curvature for upper band
using parameters: t1 =1.0, t2 = -0.30, λ1 = 0.28 and λ2 = 0.20
k_y [π/a]
k_x [π/a]

- Odchylenie standardowe krzywizny berryego