

# Introduction to Transportation Planning

## Route-choice, fixed point, assignment

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# Fixed-point problem in transportation

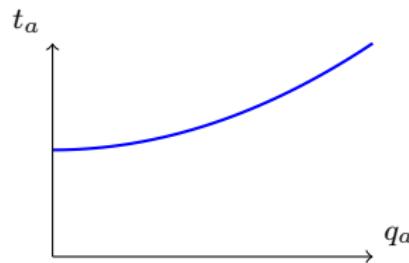
## Limited-capacity

Travel time is variable and changes with the demand.

### Waiting time

How long will I travel across the Aleje?

$$t_a = f(q_a)$$



Travel time **non-linearly** grows with number of cars at Aleje.



# Fixed-point problem in transportation

$$q_z^n = f(q_z^{n-1})$$

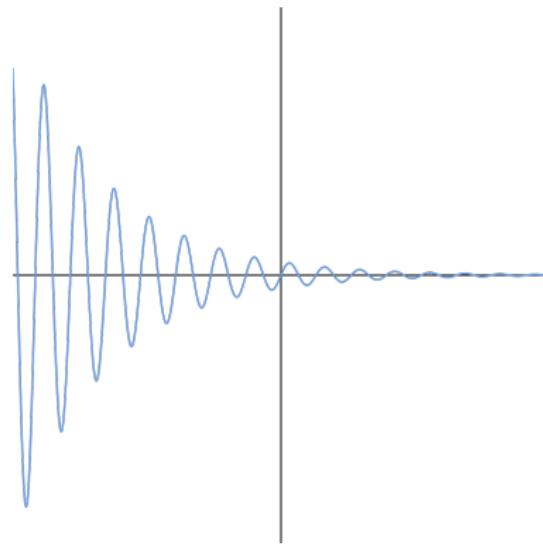
- ① how many drivers will select Aleje today (day  $n$ )?
- ② it depends on how satisfied they were from their decision yesterday (day  $n - 1$ ).
- ③ if number of drivers who selected Aleje today equals the number of drivers who selected Aleje yesterday - we are in the fixed-point - the system **stabilized/equilibrated**.
- ④ it also means that travel time at Aleje is exactly, like it was yesterday, and exactly like it was expected by the drivers who selected Aleje.



# Fixed-point problem in transportation

## Convergence

$$q_z^n = f(q_z^{n-1})$$



# Fixed-point problem in transportation

## Convergence

$$q_z^n = f(q_z^{n-1})$$

Days (iterations):

- 1st users on shortest paths - congested shortest paths
- 2-∞ users try to avoid congestion and find better path, which are however not as good as expected
- ∞ User Equilibrium

## User Equilibrium

- ① The journey times on all the routes actually used are equal and less than those which would be experienced by a single vehicle on any unused routes
- ② user-optimized equilibrium is reached when no user may lower his transportation cost through unilateral action.



## Path choice



## Road network path choice

For the transport network illustrated below let's determine the traffic flows on the bridge (dashed) and resulting travel time ( $t_a$ ) Numbers in zones show demand, i.e. number of cars willing to travel from the zone to the destination (factory) at the bottom.

Let's assume all links are of equal length and their free flow travel time is 1min. Let's further assume

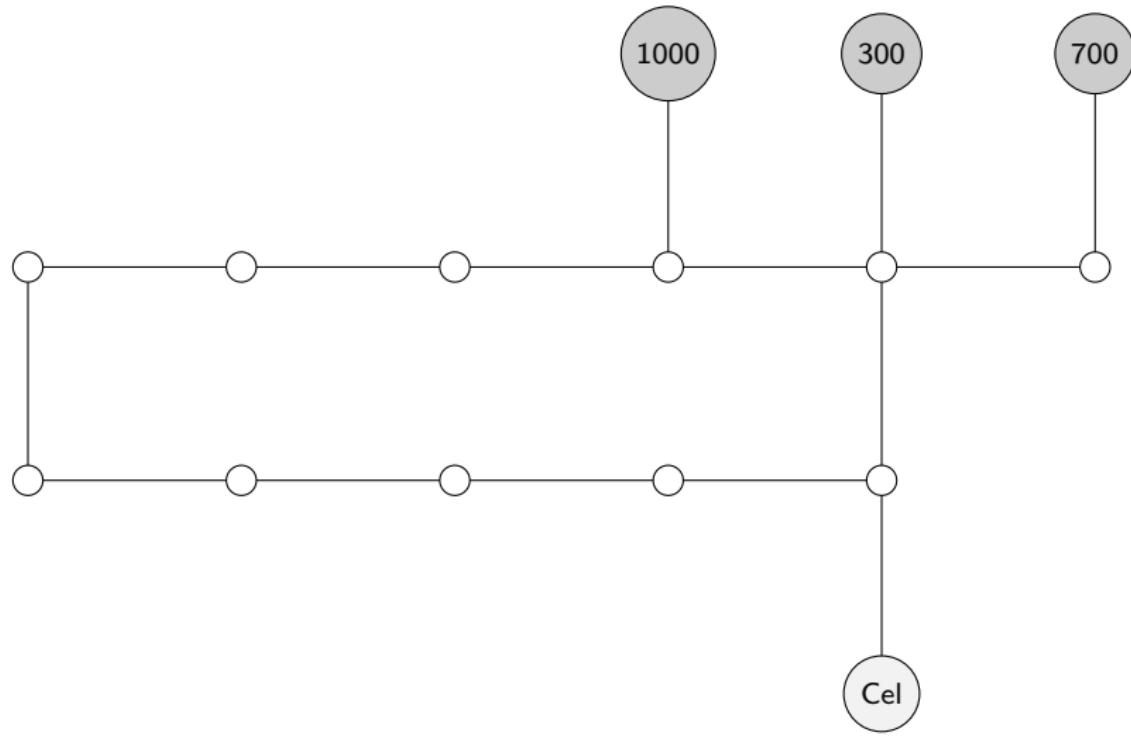
- ① unlimited capacity of all arcs
- ② bridge capacity ( $Q_a$ ) is 500 vehicles/hour, other links have unlimited capacity. To estimate travel time use this BPR formula:  $t_a = t_a^0 \cdot (1 + (q_a/Q_a)^2)$ . Give approximated value close to Wardrop conditions.



## Road network path choice

## numerical example

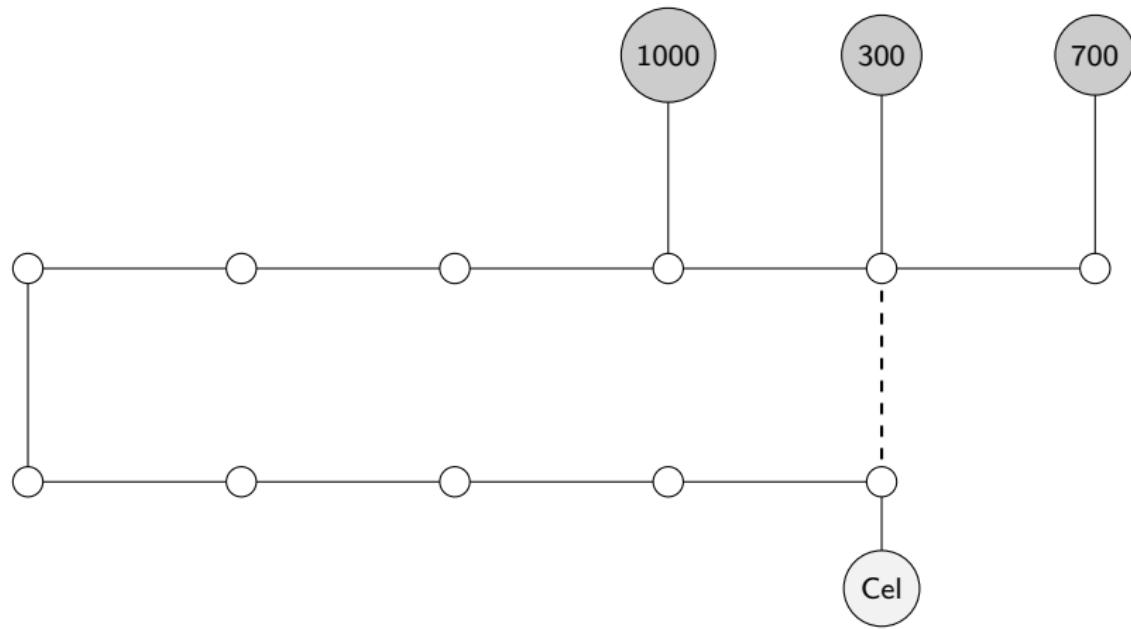
A: infinite capacity on all roads



## Road network path choice

## numerical example

B: bridge capacity ( $Q_a$ ) is 500 vehicles/hour, other links have unlimited capacity. To estimate travel time use this BPR formula:  $t_a = t_a^0 \cdot (1 + (q_a/Q_a)^2)$ . Give approximated value close to Wardrop conditions.



## Path choices



# Path/route choice

## Rationale

I want (demand) to travel from origin  $o$  to destination  $d$  on a directed network graph  $G(N, A)$  where both arcs  $a \in A$  and nodes  $n \in N$  have some associated travel costs  $c_a, c_n$ .

Cost is:

- user-perceived (subjective)
- typically related with time  $c_a \propto t_a$  or hugely driven by time
- may include other variables, like: length, comfort, toll (price), user-bias
- are often demand related  $c_a = f(q_a)$  which constitutes a fixed point.



# Path/route choice

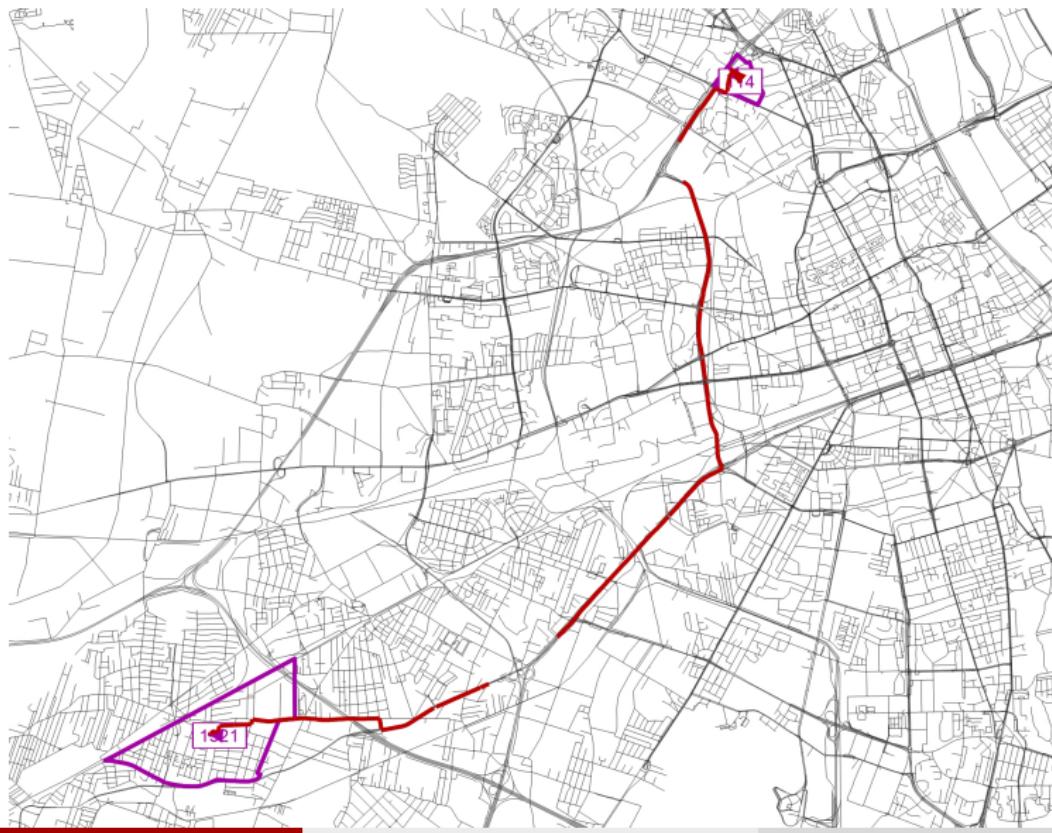
## Route choice problem

- Identify feasible paths  $K_{od}$  from  $o$  to  $d$ . Computationally hard, number of alternatives explode.
- Define probabilities of selecting a path based on given criteria.
- Select the most probable path (deterministic, all-or-nothing)
- or determine path probabilities (probabilistic, stochastic, multi-path).



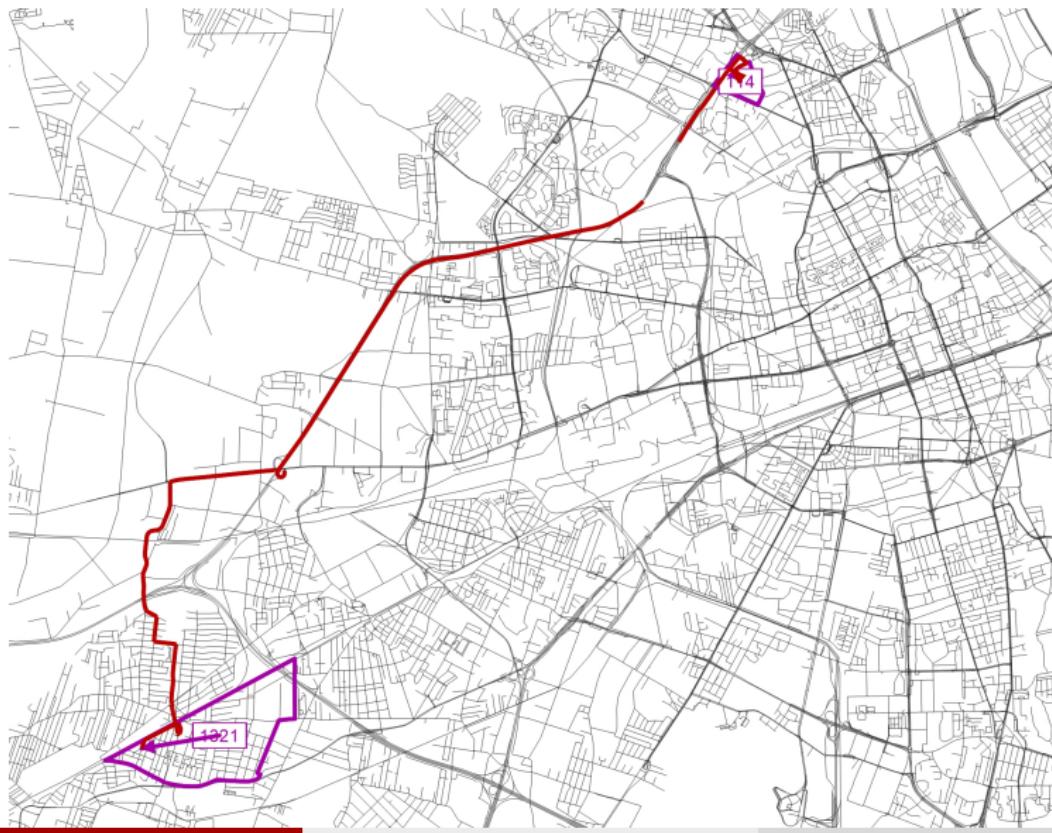
# Shortest paths

distance



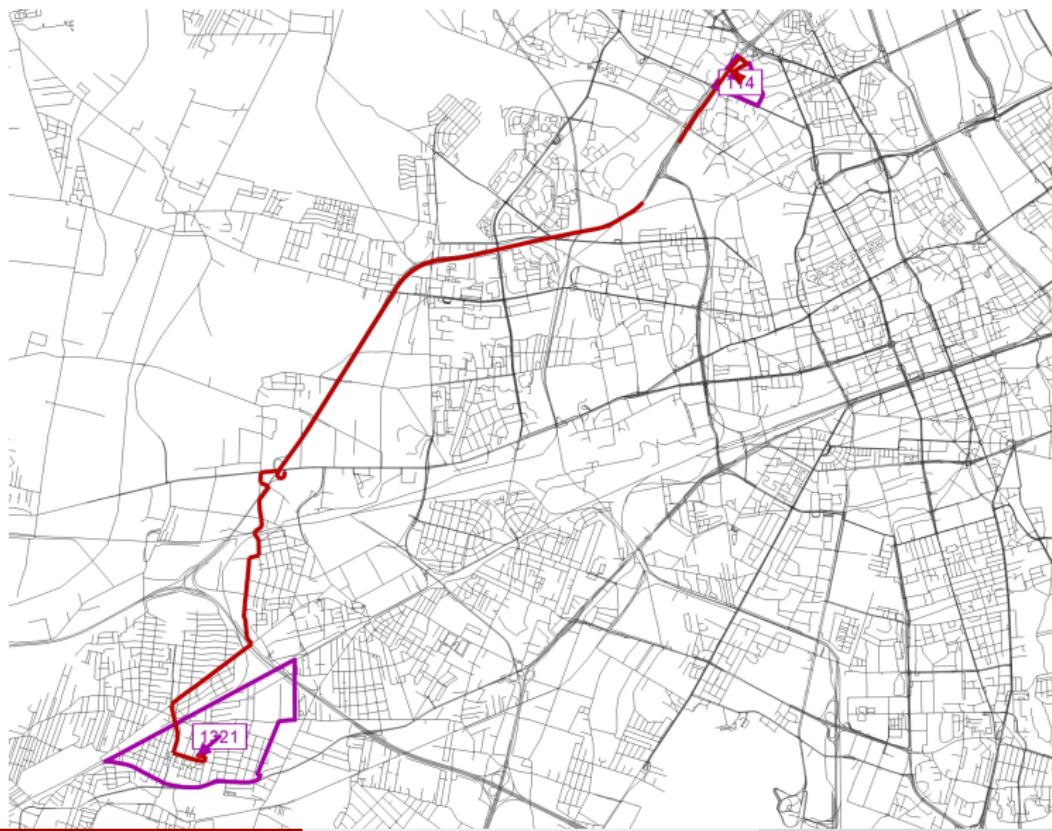
# Shortest paths

time



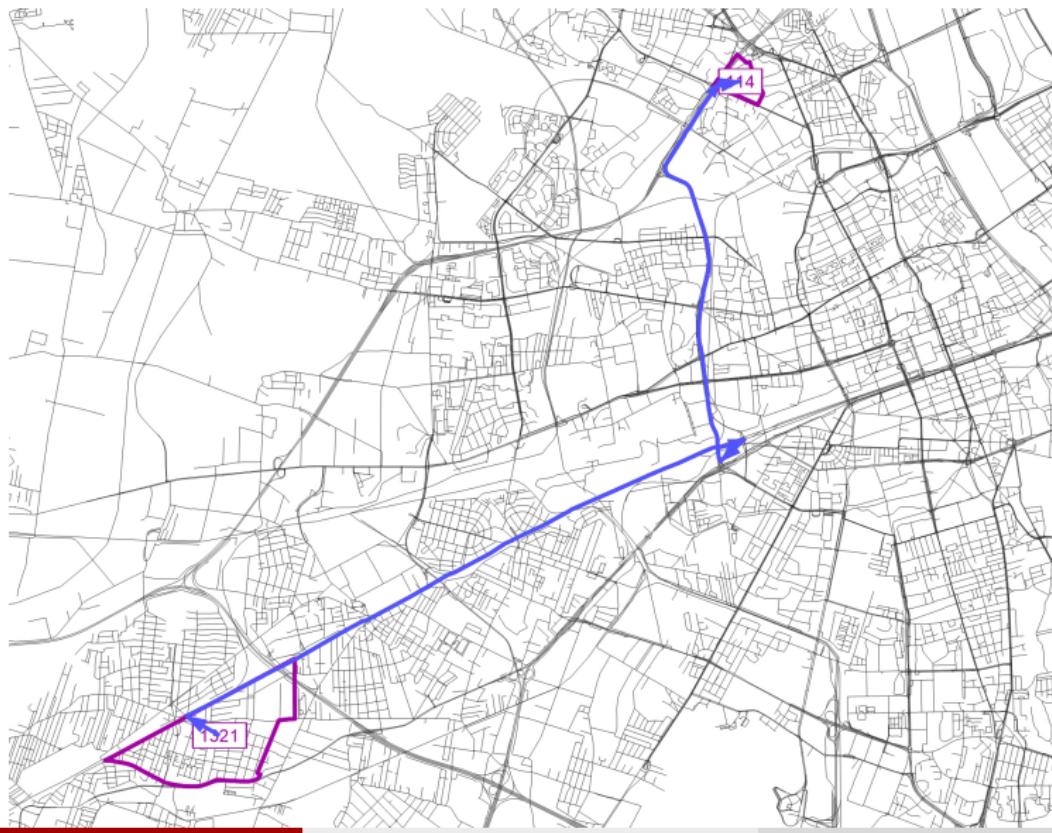
# Shortest paths

Congested time



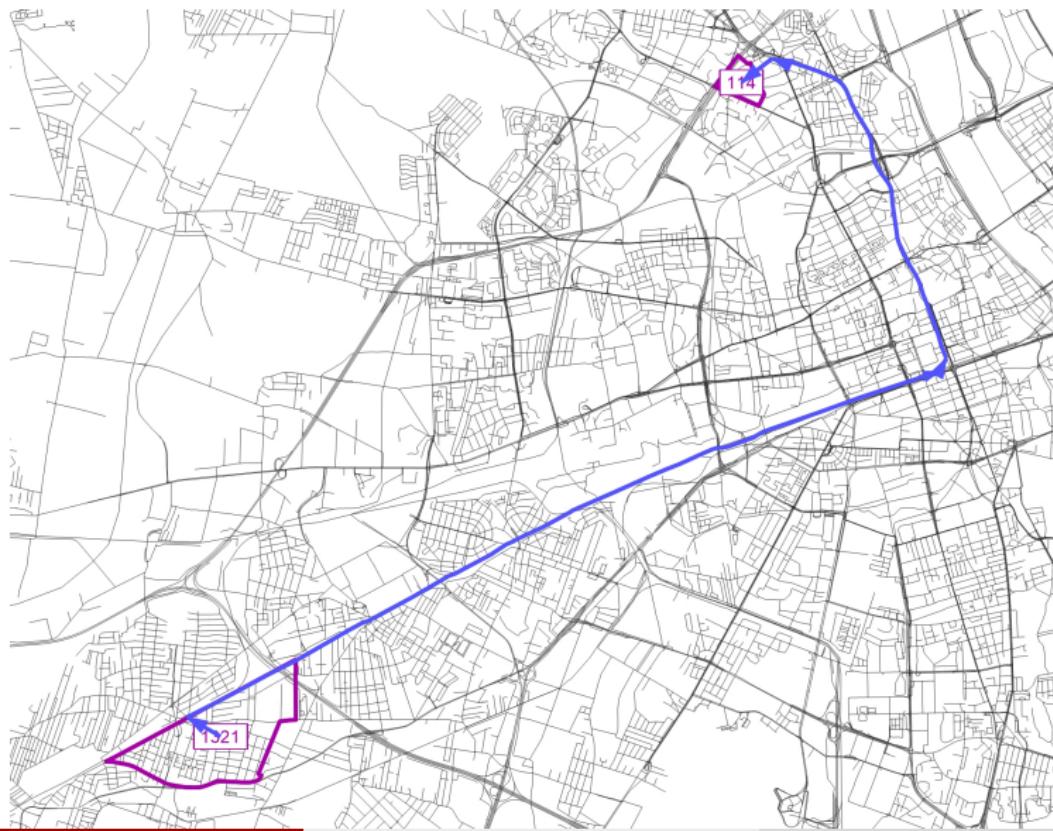
# Shortest paths

Transit AM peak



# Shortest paths

Transit late evening



# Summary

Thanks for attention

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