

Exercise 8. Traffic assignment, graph path searches.

Previous exercises covered generic steps of transport modeling. Starting from trip generation, through distribution up to mode choice. This resulted in the OD matrix for respective modes of transport, i.e. matrix where values are the estimated, expected trips quantities in a given time period (peak hour) using given mode of transport (walk, transit, car) between respective origin and destination.

In the final step of modelling procedure, the demand for trips is assigned onto routes connecting origin with destination using the transportation network. For each OD pair the optimal path in the network graph is found with optimality criteria typically being perceived journey time (ex. 6).

For car trips, the main factor in path searches is the travel time: drivers want to arrive at destination as soon as possible. There is, however, a feedback relation between travel times and their decision: the more drivers select given route, the more crowded it is, resulting in longer travel times. We exemplify travel time–flow relation with the classical BPR, given with the following formula:

$$t_a(q) = t_a^0 \cdot \left(1 + b \left(\frac{q_a}{q_a^{\max}} \right)^c \right) \quad (1)$$

, where:

t_a^0 free-flow travel time of arc a
 q_a traffic flow (number of vehicles) per hour on arc a
 q_a^{\max} capacity of arc a
 b, c parameters ($a=1$, $b=2$ by default).

Possibly, the path for which travel time was shortest, becomes so crowded that the longer routes become now optimal. For instance, travel time to cross the city center is short, yet usually it is so crowded, that it is reasonable to use longer by-passes. In such cases, drivers iteratively adapt their choices to find optimal solution.

Let's exemplify this with the following abstract exercise.

Exercise

Let's assume (generically), that the analyzed zone has only two connections with the road network: m direct, and n longer alternative (for instance a shopping mall with two ways out). Each of connections is parameterized with its length (l), free-flow speed (v^0) and capacity (q^{\max}).

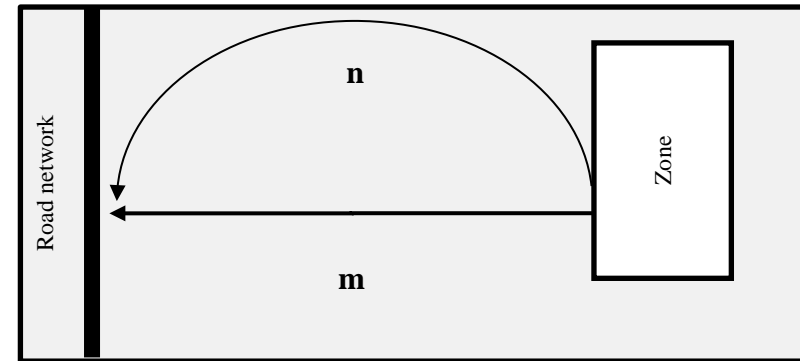
The total number of vehicles q departing from the zone in the afternoon peak-hour can be obtained from the results of previous exercises. It is the total daily trip generation P_i , multiplied by the share in the afternoon peak hour U_{PM} , share of private car p_{Car} and divided with the occupancy rate o_{Car} (1.2 person per vehicle on average):

$$q = P_i \cdot U_{PM} \cdot p_{Car}^i / o_{Car} \quad (2)$$

Daily trip generation	Afretnoon peak-hour share	Afternoon peak-hour share	Share of car trips in mode choice	Occupancy rate	Traffic flow departing from zone in the afternoon peak hour
[trips./day]	[%]	[trips/h]	[%]	[persons/vehicle]	[vehicles/hour]
				1.2	

We will use this value to simulate how vehicles exit from the zone in two following scenarios:

- all-or-nothing scenario, where we assume that everybody uses a shortest path m which will lead to congestion. Please estimate the travel time arising from loading the arc m with the flow q using formula 1 with default parameters. Free flow travel time can be estimated using the standard speed-distance-time relation
- In the second scenario we try to find the equilibrium, i.e. such division of traffic between two routes for which total travel times T_a are minimal. You can find exact solution analytically, numerically (e.g. Excel), or by trial and error (divide flow by two and then bisect into descending direction, at least four trials). The optimum (minimal vehicle kilometers) is obtained when travel times on the routes equalize.



parameters	m	n	
l [km] ¹			
v_0 [km/h]			
q_{\max} [veh/h]			
$t0$ [h]			
a) all-or-nothing²			
q [veh/h]			
$t_a(q)$ [h]			
$v_a(q)$ [km/h]			
$T_a = q_a \cdot t_a(q_a)$ [vehicle-hours]			
b) equilibrium			total
q [veh/h]			
$t_a(q)$ [h]			
$v_a(q)$ [km/h]			
$T_a = q_a \cdot t_a(q_a)$ [vehicle-hours]			

¹ gray fields are filled by lecturer

² fill the bold fields with value obtained with formula (2)

