Introduction to Transportation Planning - Lectures

Demand and Supply systems

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Demand and supply

introduction

Demand

the desire to purchase, coupled with the power to do so.
quantity of goods that buyers will take at a particular price.
service that people will or are able to buy at a certain price.

Supply

amount of something that economic agents are willing to provide to the marketplace. quantity available for purchase at a particular price.

Typically, a transport system is decomposed into:

supply
+
demand

Plan

We introduce supply & demand system with a real-life example

$\rightarrow$

we map this example on transport system.

Case of feeding hungry students

Hungry students demand to supply their hunger at lunch time.

Case of feeding hungry students

Supply

variety of companies offerring food

Case of feeding hungry students

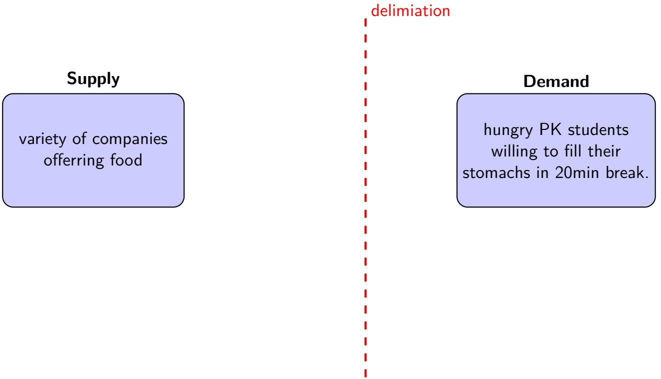
Supply

variety of companies offerring food

Demand

hungry PK students willing to fill their stomachs in 20min break.

Case of feeding hungry students



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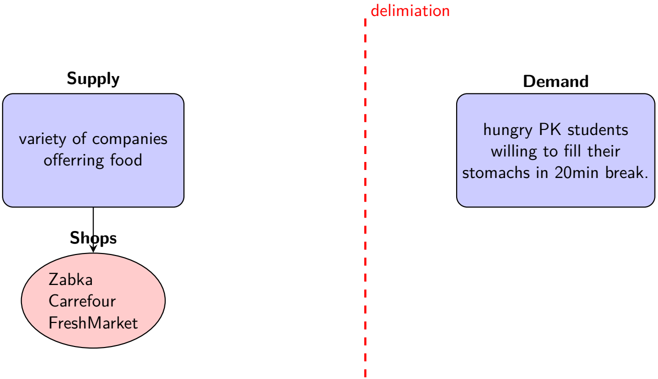

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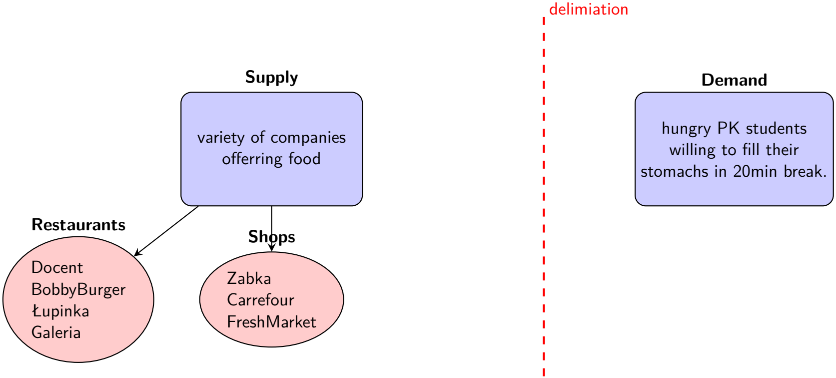

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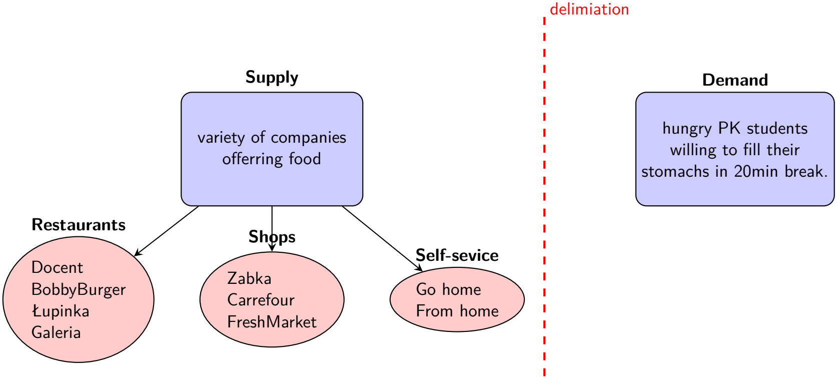

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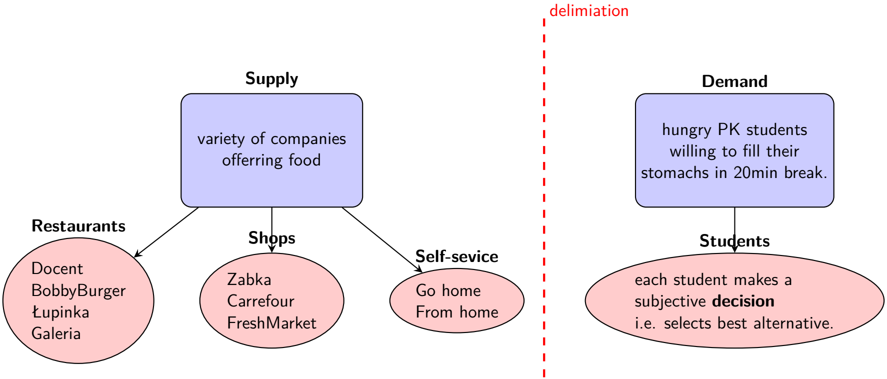

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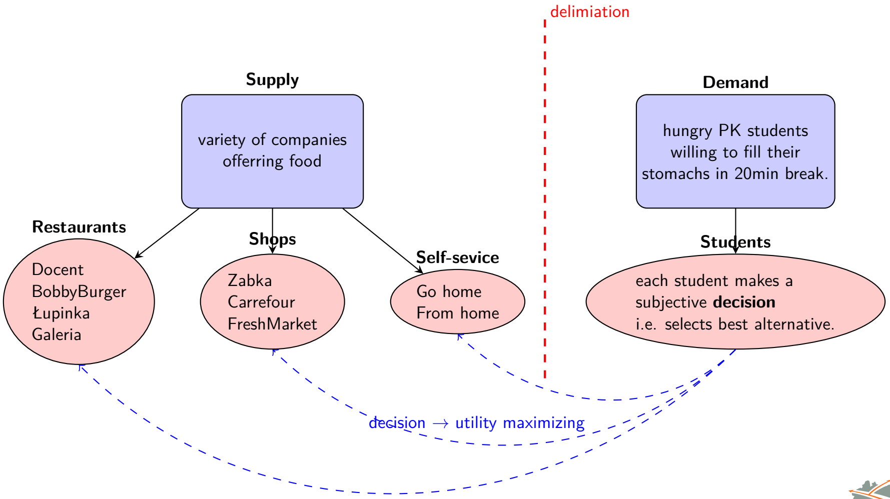

Case of feeding hungry students

Decision process

Each student s in a set of all students $\boldsymbol{S}$ selects makes a subjectively optimal decision, i.e. selects alternative a from set of available alternatives $\boldsymbol{A}$ such that his **perceived** utility of this alternative U_a^s is maximal in the set of all available alternatives:
$\forall s \in \boldsymbol{S} s_a = \operatorname{argmax}_{a \in \boldsymbol{A}} U_a^s$

Utility

Attractiveness of alternative (decision) for a given student.
Can be expressed as a function of parameters k_i of each alternative and weights w_i assigned to them by a student:
$U_a^s = \sum_{k_i \in K} w_i^s \cdot k_i$

Case of feeding hungry students

Describing alternatives

We describe each alternative $a \in \boldsymbol{A}$ by assigning (estimating) a value to each significant criteria. We assume that criteria significant for students are:

- price,
- quality,
- variety (how many options I have),
- how long do I need to walk and how long do I need to wait.

This list can be extended. Value of criteria shall be normalized, preferably to $[0, 1]$ range, with 0 being worst and 1 being best.

Example of values assigned to criteria of selected alternatives:

alternative	Żabka	Galeria	Expo
price	1	0.3	0.5
quality	0.3	0.7	0.7
variety	0.3	0.9	0.6
walk time	0.3	0.1	0.9
wait time	0.4	0.3	0.4

Case of feeding hungry students

Assigning values to criteria for alternatives is objective, but weighting those criteria is subjective. Some students may prefer quality over price, some can be a bit late some cannot, some care about quality, some do not.

Example of weights assigned to criteria by students:

w_i^s	a^1	b^2	random student[3]	random rich student[4]
price	0.3	0.9	$N(0.5, 0.1)$	$N(0.3, 0.05)$
quality	0.7	0.3	$N(0.5, 0.1)$	$N(0.7, 0.05)$
variety	0.7	0.3	$N(0.5, 0.1)$	$N(0.7, 0.0.5)$
walk time	0.7	0.3	$N(0.5, 0.1)$	$N(0.7, 0.05)$
wait time	0.8	0.1	$N(0.5, 0.1)$	$N(0.8, 0.1)$

Two important concepts:

heterogeneity population of student in heterogeneous, i.e. some students are different from another.

randomness weights assigned to criteria are random i.e. they can differ from day to day, and we cannot estimate them deterministically. They need to be treated as random variables.

[1] Student does not care about price, quality matters, want to be able to choose from variety of options, does not like to walk long distance, doesn't want to be late.

[2] Student cares about price a lot, quality and variety does not matter, he can walk long distance and can be a bit late.

[3] we are not sure what criteria he has, they are random and normal distributed with estimated mean and σ

[4] criteria are random but we know price is less important than quality.

Case of feeding hungry students

Estimating utility

Each students estimates his utility by evaluating the utility formula:

$$U_a^s = \sum_{k_i \in K} w_i^s \cdot k_i$$

Estimating ulity of 3 laternatives by student a:

alternative	Żabka	Galeria	Expo	w_i^s
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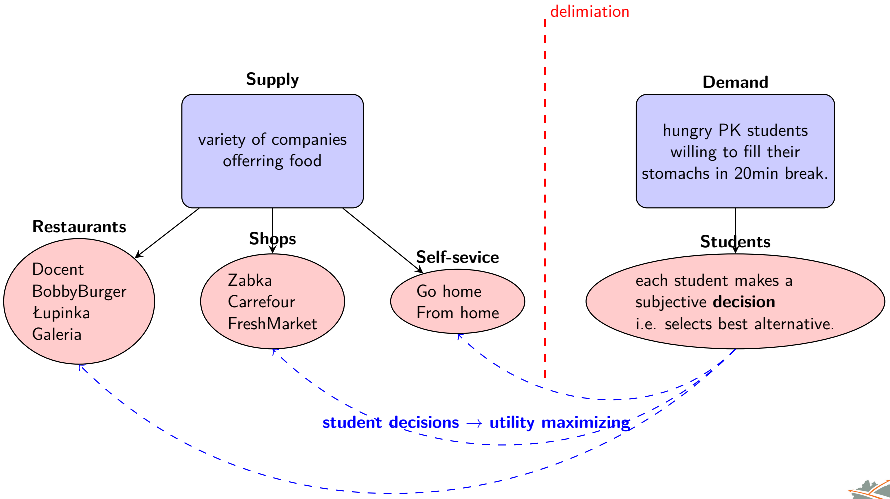
quality	0.3	0.7	0.7	0.7
variety	0.3	0.9	0.6	0.7
walk time	0.3	0.1	0.9	0.7
wait time	0.4	0.3	0.4	0.8
utility	12.5	15.2	**24.1**	

Expo is the **alternative** with **maximal** utility for **this student** - he will supply his demand at Expo.

Case of feeding hungry students



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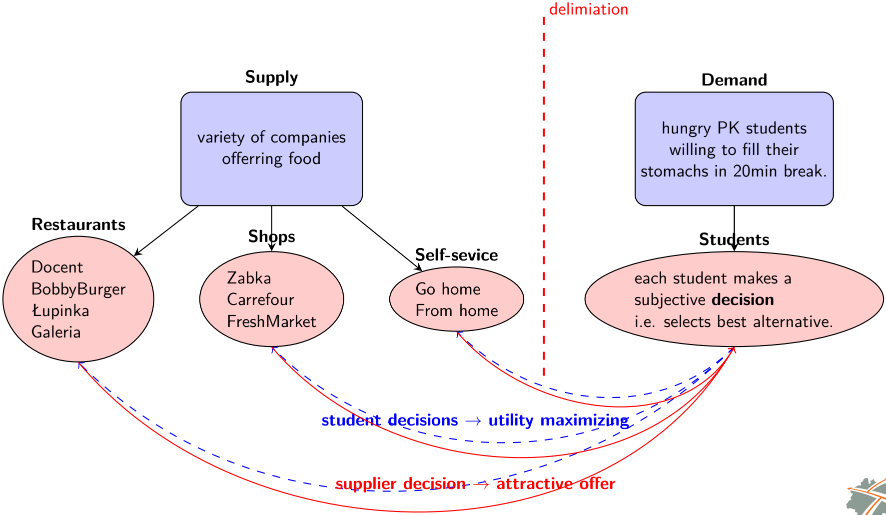


Case of feeding hungry students

Suppliers' decisions

Supplier

Prepares an offer, i.e.

- defines the price,
- variety,
- improves the quality, etc.

His objectives is to maximize number of students selecting his offer. Formally, each supplier proposes an offer which maximizes its perceives utility by students:

$$\boldsymbol{k^a} = \operatorname*{argmax}_{s \in \boldsymbol{S}} \left(U_a^s = \sum_{k_i^a \in K} w_i^s \cdot k_i \right)$$

Objectives for students and suppliers are similar:

- students wants to have the offer with the best utility,
- suppliers want to propose an offer which will be the best for maximal number of students.

Total welfare

We can express the total welfare of the system with:

$$W = \sum_{s \in \boldsymbol{s}} U^s_{a^*}$$

, where a^* is the alternative selected by student s, i.e. the one with highest utility.

System improvement

Will the system improve when new restaurant "U Babci Maliny" will open close to campus?
If some students will select this, i.e. it will have the highest utility. It will improve the total welfare W and thus improve the system.

Case of feeding hungry students

Limited capacity

Utility of some criteria can be variable and change with the demand.

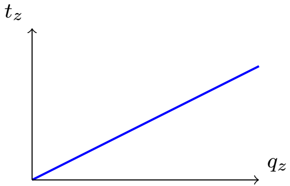
Waiting time

How long will I wait at Żabka until I get the food?

$$t_z = f(q_z)$$

t_z waiting time at Żabka.

q_z number of customers at Żabka.



waiting time linearly grows with number of students who has chosen Żabka.

Case of feeding hungry students

Limited capacity

1. Number of students in Żabka: $q_z = f(U_z)$ is a function if its utility;

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$$\begin{tikzcd}
\boxed{q_z} \arrow[d] & \boxed{U_z} \arrow[l] \\
\boxed{t_z} \arrow[ur] &
\end{tikzcd}$$

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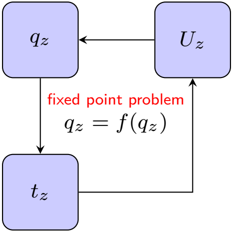


Fixed-point problem[5]

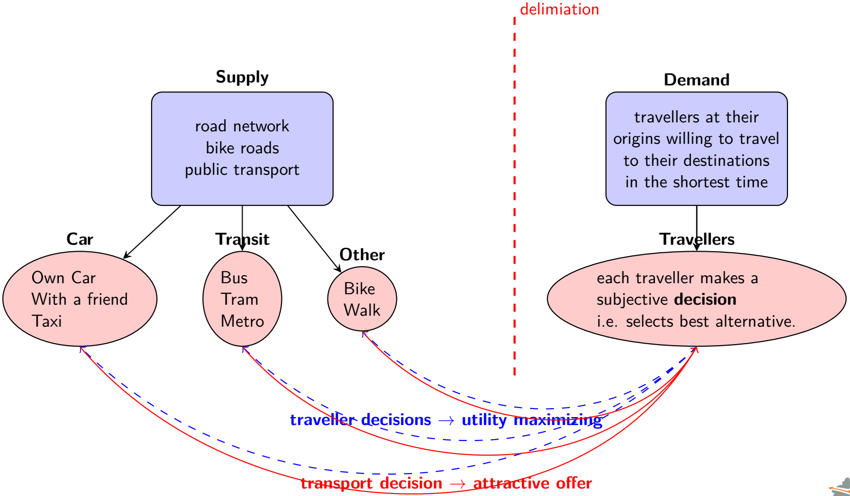
$$q_z^n = f(q_z^{n-1})$$

1. how many students will select Żabka today (day n)?
2. it depends on how satisfied they were from their decision yesterday (day $n-1$).
3. if number of students who selected Żabka today equals the number of students who selected Żabka yesterday - we are in the fixed-point - the system **stabilized/equilibrated**.
4. it also means that waiting time at Żabka is exactly, like it was yesterday, and exactly like it was expected by the students who selected Żabka.

[5] By Stefan Banach - Krakow/Lwow world famous mathematician

End of the case-study

Supply and demand in transportation



Fixed-point problem in transportation

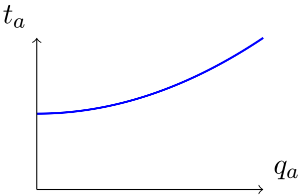
Limited-capacity

Travel time is variable and changes with the demand.

Waiting time

How long will I travel across the Aleje?

$$t_a = f(q_a)$$



Travel time **non-linearly** grows with number of cars at Aleje.

Fixed-point problem in transportation

$$q_z^n = f(q_z^{n-1})$$

1. how many drivers will select Aleje today (day n)?
2. it depends on how satisfied they were from their decision yesterday (day $n-1$).
3. if number of drivers who selected Aleje today equals the number of drivers who selected Aleje yesterday - we are in the fixed-point - the system **stabilized/equilibrated**.
4. it also means that travel time at Aleje is exaclty, like it was yesterday, and exactly like it was expected by the drivers who selected Aleje.

- Demand is supplied.
- Actors of the system maximize their subjective utility, select rationally.
- Suppliers try to offer something that will be chosen by maximal number of people.
- If the outcomes of our decisions are function of our decisions we are in a fixed-point problem. Solved iteratively.
- User Equilibrium is found when decisions do not vary from day-to-day, i.e. experience equals expectation.

Summary

Transportation as a Demand-Supply system

In transportation:

- Demand is the need to travel from origin to destination q_{od}.
- Supply is the transport system: road, public transport, walk, bike.
- Decisions are e.g. what mode of transport do I select, when do I go, what route do I choose?
- Travel time at roads and congestion in public transport are functions of decisions made by others in the system.
- User Equilibrium is found when no driver can change route to improve his/her travel time.

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