$$V_{\text{TRIP}} = \beta_1 CA + \beta_2 WRK + \beta_3 AGE + \beta_4 INL + \beta_5 WMN + \beta_6 ACC$$
$$V_{\text{NOTRIP}} = \beta_7 TOP + \beta_8 TOF + \beta_9 NT$$

Type of variable	Name of variable	
Socioeconomic	Car availability	*CA*
	Working status	*WRK*
	Age	*AGE*
	Income level	*INL*
	Woman	*WMN*
Location	Accessibility	*ACC*
Time availability	No. of other trips made by the person for other purposes	*TOP*
Individual–family relationships	No. of trips of made by other family members for the same purpose	*TOF*
Alternative specific attributes (ASA)	NOTRIP	*NT*
CA	Dummy variable: 0 = car not available; 1 car available	
WRK	Dummy variable: 0 = nonworker; 1 = worker	
AGE	Dummy variable: 0 = ≤35 years; 1 = ≥35	
INL	Income level in 6 points scale: 0 = low income; 5 = high income	
WMN	Dummy variable: 0 = man, 1 = woman	

	No trip			Trip					
	TOP	*TOF*	*NT*	*CA*	*WRK*	*AGE*	*INL*	*WMN*	*ACC*
Shopping	0.55	0.61	1.35	0.24	−2.69	−2.53	0.08	0.60	0.11
t	5.4	3.7	5.4	1.2	−9.7	−8.0	1.5	3.8	1.7
Other purposes	0.22	−1.18	2.66	–	−0.34	−0.34	0.20	0.53	–
t	2.2	−10.9	15.3		−2.0	−2.0	3.5	3.3	

	Goodness-of-fit statistics		
	ρ^2	*% right*	*LR*
Shopping	0.431	0.847	1904
Other purposes	0.689	0.933	3041

Fig. 4.5 Trip frequency model for the morning peak period

4.3.1.2 Distribution Models

Distribution models express the percentage (probability) $p^i[d/osh]$ of trips made by users of class i going to destination d, given the origin zone o, purpose s, and time period h. For simplicity of notation, the user class index is omitted here.

Distribution models can be divided into descriptive and behavioral models.

Descriptive Models One of the best-known descriptive distribution models is the gravity model, whose name derives from its resemblance to Newton's law of gravity. In its typical formulation, this model provides the actual demand flow $d_{od}[sh]$ rather than the destination shares $p[d/osh]$ for each od pair:

$$d_{od}[sh] = \alpha d_o \cdot [sh]d \cdot {}_d[sh] f(C_{od}) \tag{4.3.6a}$$

where α is a constant, $d_o \cdot [sh]$ and $d \cdot {}_d[sh]$ represent, respectively, the total trip production from o and total trip attraction to d for purpose s in period h,[13] C_{od} is a variable related to the generalized transportation cost, and $f(C_{od})$ is an impedance (sometimes called *friction*) function that decreases with C_{od}. Typical expressions for this function are:

$$f(C_{od}) = \exp(-\beta C_{od}) \tag{4.3.7a}$$

$$f(C_{od}) = C_{od}^{-\beta} \tag{4.3.7b}$$

$$f(C_{od}) = C_{od}^{-\beta} \exp(-\beta C_{od}) \tag{4.3.7c}$$

In order to satisfy (1.3.1) and (1.3.2) of Sect. 1.3.3, the constant α is usually replaced by two factors that depend on the origin and destination zones (a *doubly constrained* gravity model):

$$d_{od}[sh] = A_o B_d d_o \cdot [sh] d \cdot {}_d[sh] f(C_{od}) \tag{4.3.6b}$$

where

$$A_o = 1/\sum_{d'} B_{d'} d_{.d'} f(C_{od'}) \qquad B_d = 1/\sum_{o'} A_{o'} d_{o'.} f(C_{o'd})$$

The two equations above are mutually dependent and therefore constants A_0 and B_0 are unknown quantities of a nonlinear equation system that can be solved by an iterative procedure.

When only one of these two conditions is satisfied, that is, (1.3.1) (a *singly constrained* gravity model[14]) in (4.3.6b), then $B_d = 1$ and

$$d_{od}[sh] = \frac{d_{o.}[sh] \cdot d_{.d}[sh] f(C_{od})}{\sum_{d'} d_{.d'}[sh] f(C_{od'})} = d_{o.}[sh] \cdot p[d/osh] \tag{4.3.6c}$$

[13] See Sect. 1.3.3. Trip attractions $d \cdot {}_d[sh]$ can be computed as a function of the zonal characteristics using models similar to those used to calculate trip productions $d_o \cdot [sh]$: for example, a trip attraction classification table or linear regression model.

[14] Gravity models originally derived their name from their similarity with Newton's law of universal gravitation. Singly and doubly constrained gravity models were subsequently derived from entropy maximization principles. In this approach, the entropy measure of a given trip distribution is expressed as a function of the number of possible microstates (i.e., individual trips between each origin–destination pair) that satisfy the distribution. The entropy function is then maximized subject to constraints on the total number of trips produced by (and in some models attracted to) each zone, and to the total cost (distance) of transportation. Distribution models that maximize this entropy are referred to as singly (and doubly) constrained gravity models. Although these models are still commonly used, they do not provide the flexibility of random utility models (whether these are interpreted behaviorally or not), and also do not allow for the introduction of attributes that account for the perceived attractiveness of different destinations. It should be pointed out that more sophisticated destination choice models are still relatively unstudied. Indeed, because of the possibility of spatial autocorrelation, the multinomial logit model's assumption of i.i.d. disturbances is questionable for traffic zones near each other. In this case cross-nested logit or probit models should be used. Models should also take account of travelers' different degrees of familiarity with potential destinations through choice set modeling procedures.

with

$$p[d/osh] = \frac{d_{\cdot d}[sh]f(C_{od})}{\sum_{d'} d_{\cdot d'}[sh]f(C_{od'})} \tag{4.3.8}$$

It is easy to verify that model (4.3.8) is invariant with respect to the aggregation or disaggregation of traffic zones, given equal "distance" from the origin. In other words, with a specification such as (4.3.8) the probability $p[d]$ of choosing a zone d that is aggregated from two smaller zones d_1 and d_2 is equal to the sum of the probabilities $p[d_1]$ and $p[d_2]$. Indeed, if the cost is constant:

$$C_{od} = C_{od_1} = C_{od_2} \quad \Rightarrow \quad f(C_{od}) = f(C_{od_1}) = f(C_{od_2})$$

then because $d_{.d} = d_{.d1} + d_{.d2}$ it follows that

$$\begin{aligned} p[d] &= \frac{d_{\cdot d} f(C_{od})}{d_{\cdot d} f(C_{od}) + \sum_{d' \neq d} d_{\cdot d'} f(C_{od'})} \\ &= \frac{d_{\cdot d_1} f(C_{od_1})}{d_{\cdot d_1} f(C_{od_1}) + d_{\cdot d_2} f(C_{od_2}) + \sum_{d' \neq d} d_{\cdot d'} f(C_{od'})} \\ &\quad + \frac{d_{.d_2} f(C_{od_2})}{d_{\cdot d_1} f(C_{od_1}) + d_{\cdot d_2} f(C_{od_2}) + \sum_{d' \neq d} d_{\cdot d'} f(C_{od'})} \\ &= p[d_1] + p[d_2] \end{aligned}$$

The property of invariance with respect to zonal aggregation is very useful in application because it provides results that do not depend on the particular level of spatial disaggregation that is used.

Random Utility Models Random utility distribution models represent the probability $p^i[d/osh]$ that a user of class i chooses destination d, given the origin zone o, purpose s, and time period h.

Definition of Choice Alternatives It is generally assumed that the zones in the study area zone system represent elementary destination choice alternatives. In reality, the destination where one chooses to carry out an activity is not a traffic zone but rather a specific location or locations (i.e., an office or a shopping center) within a traffic zone, and it is these specific locations that are the elementary destination alternatives. Therefore, a traffic zone should be modeled as a compound alternative that results from the aggregation of its elementary destination alternatives.

Different model functional forms can be derived depending on whether the elementary alternatives are taken to be the traffic zone or the specific destination locations; therefore the two cases are discussed separately.