Supplementary material – Theoretical calculations of the dynamical magnetic susceptibility

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Our calculations of the dynamical magnetic susceptibility, $\chi''(\boldsymbol{Q}, E)$, plotted in Fig. 5 in the main manuscript, closely follow the renormalised mean field approach of Brinckmann and Lee [1], with some small differences in parameter choices which we detail in this section. In this approach the underlying Hamiltonian is assumed to be of the $t-J$ type:

$$H_{t-J} = P_s\big(-\sum_{ijs} t_{0,ij} c_{is}^{\dagger} c_{js} + J\sum_{\langle ij\rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j\big)P_s, \tag{1}$$

where P_s is an operator which projects out doubly occupied states. The core feature of the method in Ref. [1] is the decomposition of the electronic modes into fermonic (spinon) and bosonic (holon) degrees of freedom. In the low temperature limit the coherent single particle excitations are described by a Green's function for spinons

$$G(\omega, \boldsymbol{k}) = \frac{1}{\omega - \xi(\boldsymbol{k}) - \Sigma_R(\omega, \boldsymbol{k})}, \tag{2}$$

where $\xi(\boldsymbol{k})$ is a hopping dispersion (including terms up to next-next-nearest neighbor) renormalized by doping, x.

$$\xi(\boldsymbol{k}) = \xi_0(\boldsymbol{k}) + \xi'(\boldsymbol{k}), \tag{3a}$$

$$\xi_0(\boldsymbol{k}) = -2t(x)(\cos k_x + \cos k_y), \tag{3b}$$

$$\xi'(\boldsymbol{k}) = -4t'(x)\cos k_x \cos k_y - 2t''(x)(\cos 2k_x + \cos 2k_y) - \mu_p, \tag{3c}$$

and

$$\Sigma_R(\omega, \boldsymbol{k}) = |\Delta_R(\boldsymbol{k})|^2/[\omega + \xi(\boldsymbol{k})], \tag{4}$$

is a self energy term due to a d-wave spin gap (which also forms the superconducting gap at low temperatures when the holons are condensed)

$$\Delta_R(\boldsymbol{k}) = \Delta_0(x)(\cos k_x - \cos k_y). \tag{5}$$

We use a similar set of bandstructure parameters to our previous work, Ref. [2], which has its origin in fits of the YRZ ansatz to angle resolved photoemission (ARPES) studies of the curates [3]. In so doing, we depart slightly from the definitions in [1] and parametrize the doping dependence via Gutzwiller factors:

$$g_t = \frac{2x}{1+x}, \tag{6a}$$

$$g_s = \frac{4}{(1+x)^2}, \tag{6b}$$

TABLE I. The bare parameters.

t_0	t_0'	t_0''	J	χ	Δ_0
$3J/2.5$	$-0.3t_0$	$0.2t_0$	0.12 eV	0.338	$0.3t_0$

which renormalize the bare hopping and exchange parameters t_0, t_0', t_0'' and J:

$$t(x) = g_t t_0 + \frac{3}{8} g_s J\chi, \tag{6c}$$

$$t'(x) = g_t t_0', \tag{6d}$$

$$t''(x) = g_t t_0'', \tag{6e}$$

with $\chi = 0.338$ [4]. Note that due to the mean field factorisation of the spin interaction, the exchange J enters the dispersion as a spinon hopping term. Again following fits to ARPES data we choose a simple linearly decreasing phenomenological form for the gap:

$$\Delta_0(x) = \Delta_0(1 - x/x_{\mathrm{crit}}), \tag{7}$$

which disappears at $x_{\mathrm{crit}} = 0.20$, the overdoped edge of the superconducting dome [5]. The overall magnitude, Δ_0, is again chosen by fitting to the measured electronic structure [3]. We note that the final results of our calculations are not highly sensitive to the precise value of x_{crit}. Table I contains the bare parameter set from YRZ fits to ARPES data on Bi-2212 [5]. The chemical potential, μ_p is set based on Luttinger's sum rule as was the case in Ref. [6]. We refer the reader to Ref. [1] for further details of the method.

[1] J. Brinckmann and P. A. Lee, Phys. Rev. B **65**, 014502 (2001).

[2] M. P. M. Dean, A. J. A. James, R. S. Springell, X. Liu, C. Monney, K. J. Zhou, R. M. Konik, J. S. Wen, Z. J. Xu, G. D. Gu, V. N. Strocov, T. Schmitt, and J. P. Hill, Phys. Rev. Lett. **110**, 147001 (2013).

[3] H.-B. Yang, J. D. Rameau, Z.-H. Pan, G. D. Gu, P. D. Johnson, H. Claus, D. G. Hinks, and T. E. Kidd, Phys. Rev. Lett. **107**, 047003 (2011).

[4] F. C. Zhang, C. Gros, T. M. Rice, and H. Shiba, Superconductor Science and Technology **1**, 36 (1988).

[5] J. Rameau, (2012), private communication.

[6] K.-Y. Yang, T. M. Rice, and F.-C. Zhang, Phys. Rev. B **73**, 174501 (2006).