

2. Czy można tak dobrać stałą 
$$C$$
, aby funkcja  $f_{XY}(x,y)=Cxy+x+2y$ , dla  $0\leqslant x\leqslant 3$ ,  $1\leqslant y\leqslant 2$ , była gęstością dwuwymiarowej zmiennej losowej?

$$\int_{0}^{3} \int_{1}^{2} C \times y + x + 2y \, dy dx = \int_{0}^{3} \left( \times \left( \int_{2}^{2} cy + 1 \, dy \right) + 2 \int_{1}^{3} y \, dy \right) dx =$$

$$= \int_{0}^{3} \left( x \cdot \left( \frac{cy^{2}}{2} \right)^{2} + y \cdot y \cdot y \right) + 2 \cdot \left( \frac{1}{2} y^{2} \cdot y^{2} \cdot y \right) \right) dx =$$

$$= \int_{0}^{3} \left( x \cdot \left( \frac{c}{2} \cdot (4 - 1) + 1 \right) + 2 \cdot \frac{3}{2} \right) dx =$$

$$= \left( \frac{3}{2} \cdot C + 1 \right) \cdot \int_{1}^{3} x \, dx + 3 \cdot \int_{1}^{3} 1 \, dx =$$

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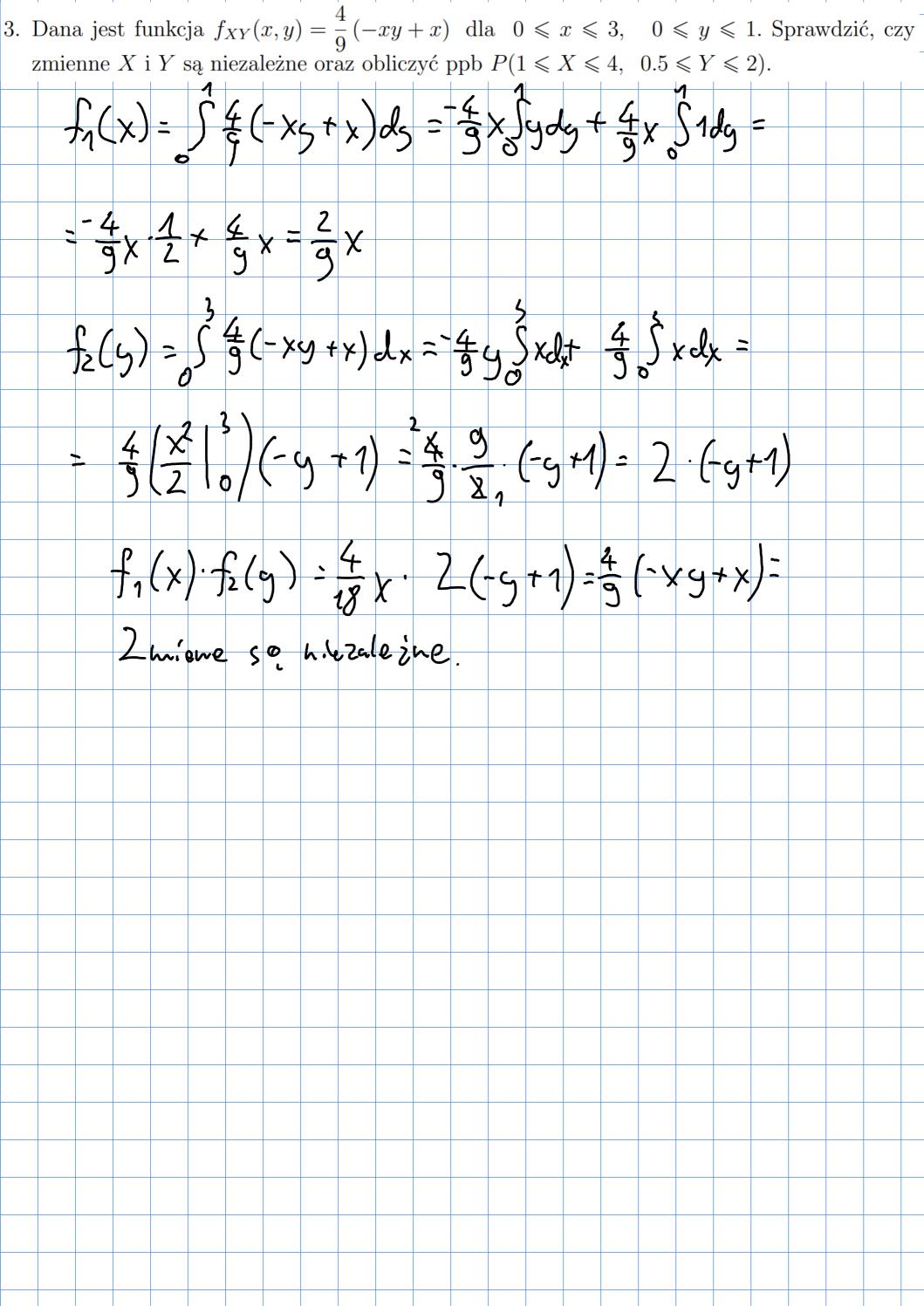
$$= \left( \frac{3}{2} \cdot C + 1 \right) \cdot \frac{9}{2} + 3 = \frac{2^{3}}{4} \cdot C + \frac{9}{2} + 9$$

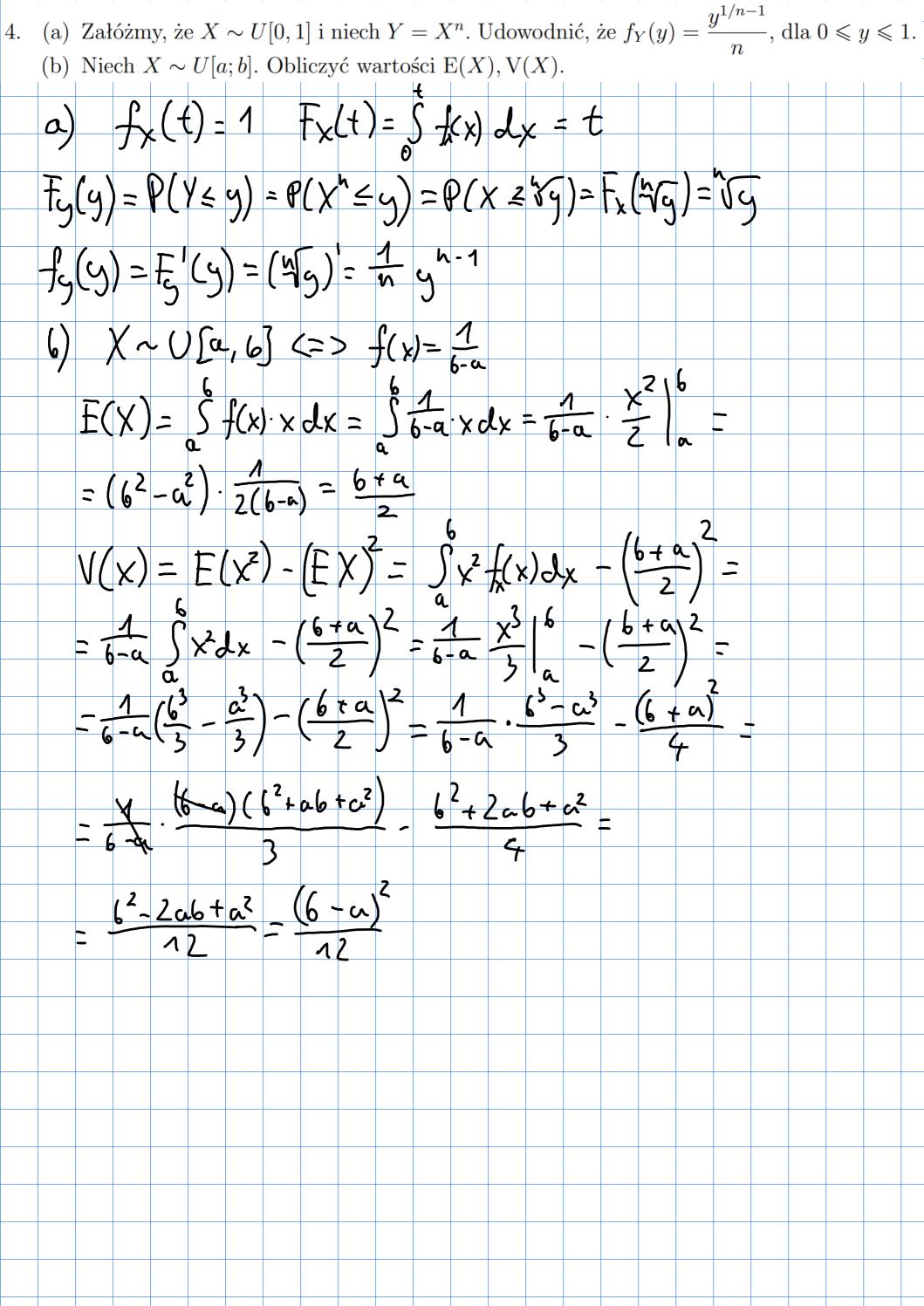
$$= \frac{3}{4} \cdot C + \frac{3}{4} \cdot C + \frac{9}{4} \cdot$$

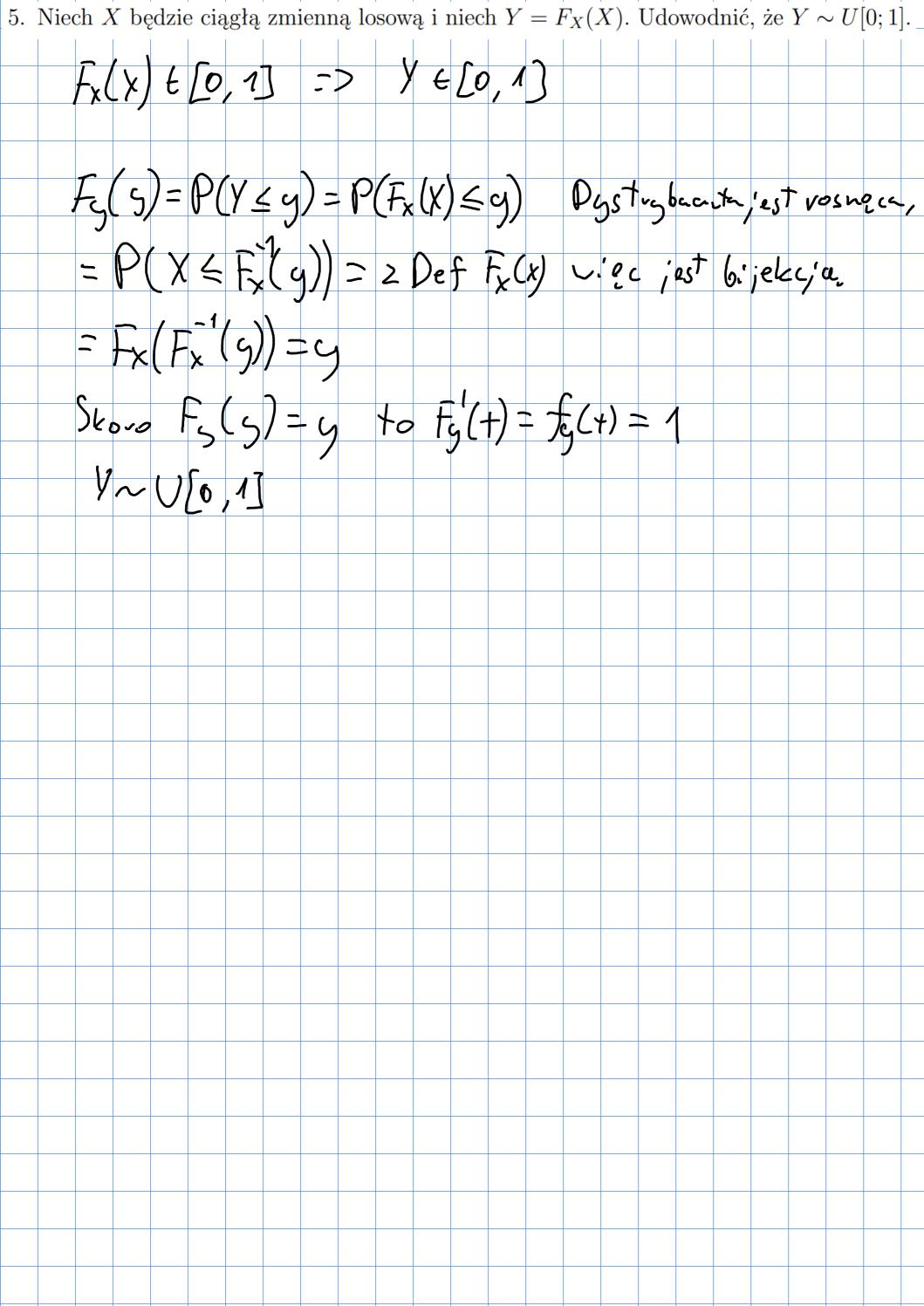
$$= (\frac{3}{2}c+1) \cdot \int_{\infty}^{\infty} dx + 3 \cdot \int_{0}^{1} 1 dx =$$

$$= (\frac{3}{2}c+1) \cdot \frac{9}{2} + 3 = \frac{27}{4}c + \frac{9}{2} + 9$$

$$= \frac{27}{4}c + \frac{9}{2} + 9 = 1 = 27c = -25c = -25c = -27c = -27c$$



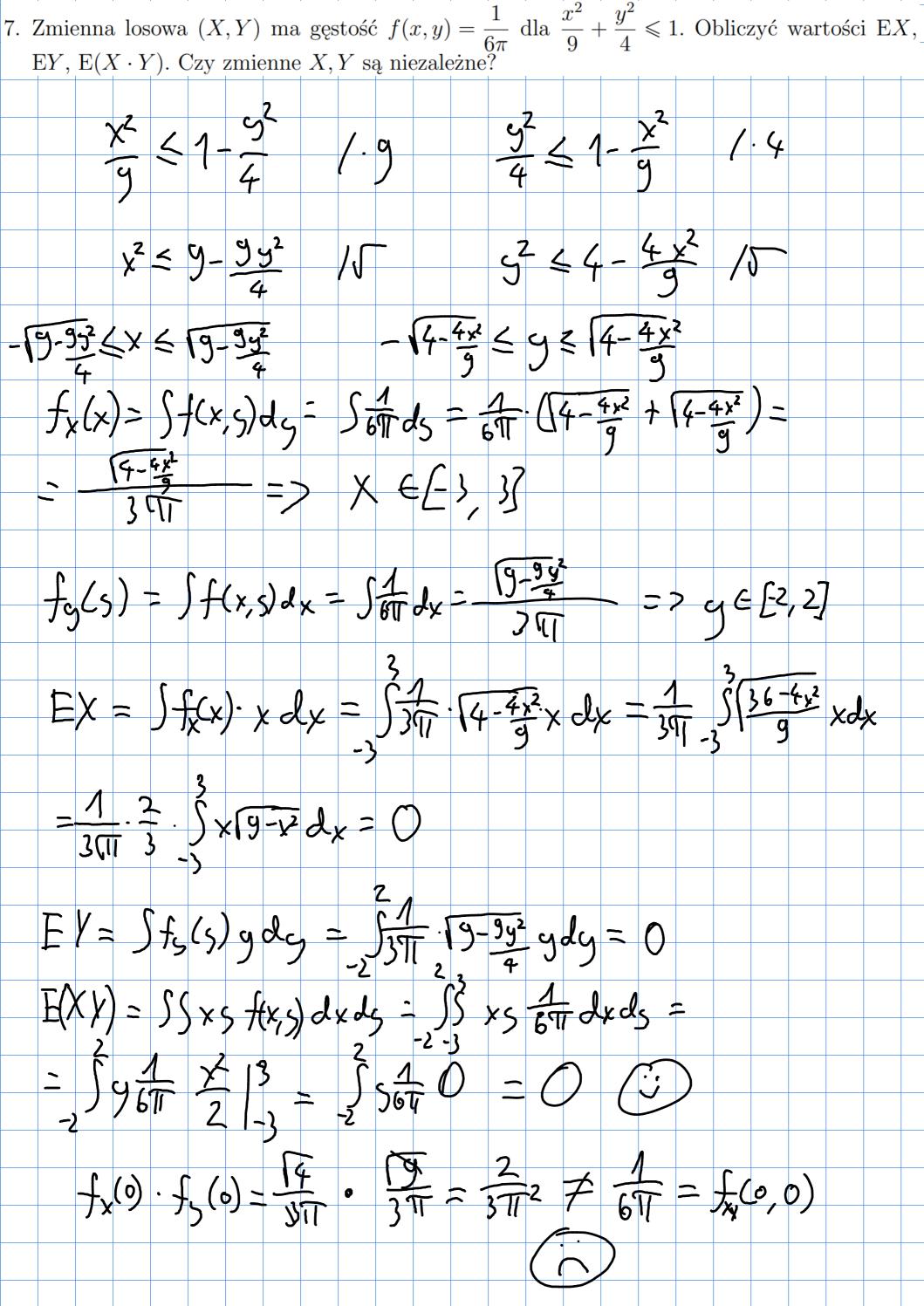




6. Niech  $Y=X^2$  (X określona na  $\mathbb R$ ). Wykazać, że

$$f_{Y}(y) = \frac{f_{X}(\sqrt{y}) + f_{X}(-\sqrt{y})}{2\sqrt{y}}, \text{ dla } y > 0.$$

$$f_{Z}(z) = P(X \le y) = P(X^{2} \le y) = P(x_{Z} \le X \le y) = \frac{1}{2} = P(X \le y) = \frac{1}{2} =$$



8. Niech X podlega standardowemu rozkładowi Cauchy'ego, 
$$f_X(x) = \frac{1}{\pi(1+x^2)}$$
,  $x \in \mathbb{R}$ . Udowodnić, że  $Y = \frac{1}{X}$  ma również standardowy rozkład Cauchy'ego.

$$F_{y}(g) = P(Y \leq y) = P(\frac{1}{x} \leq y) = P(\frac{1}{y} \leq x) = 1$$

$$1 - P(x \leq \frac{1}{y}) = 1 - F_{x}(\frac{1}{y}) = 1 - \int_{\pi}^{2} \frac{1}{\pi(1+x^{2})} dx = 1$$

$$= 1 - \int_{\pi}^{2} \frac{1}{(1+x^{2})} dx = 1 - \int_{\pi}^{2} \frac{1}{(1+x^{2})} dx + \int_{\pi}^{2} \frac{1}{\pi(1+x^{2})} dx = 1$$

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