Pergamon

PII: S0305-0548(97)00005-1

Computers Ops Res. Vol. 24, No. 10, pp. 945–953, 1997

0305-0548/97 $17.00+0.00

OPTIMIZATION OF ROLL CUTTING IN CLOTHING INDUSTRY

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(Received March 1996; in revised form January 1997)

Scope and Purpose—an important problem in a number of industries is unwanted trim loss as a result of one-dimensional stock cutting. Some good solutions to this problem are known when stock is of the same length or of a few different standard lengths. However, when the stock lengths are all different the generally acceptable solutions are not to be found in literature.

In the clothing industry there is the problem of reducing trim loss in one-dimensional roll cutting for different roll lengths. In order to solve this problem a sequential heuristic procedure is proposed, which leads to an almost optimal solution. With minor modifications this procedure can be used in other industries faced with similar problems. The computer program COLA based on the achieved algorithm was designed. Its PC version is released for public use.

Abstract—The article examines the sequential heuristic procedure for optimisation of roll cutting in the clothing industry. The issue of roll cutting can be defined as a bicriterial multidimensional knapsack problem with side constraints. To handle the bicriterial objective function a lexicographic approach is proposed. An item-oriented solution was found through a combination of approximations and heuristics. A sample problem is presented and solved. © 1997 Elsevier Science Ltd

1. INTRODUCTION

The problem of reducing trim loss in the one-dimensional stock cutting occurs in a wide variety of industries [3,7,8]. In the clothing industry the first phase of the stock cutting process is one-dimensional cutting of a small number of long pieces in the form of fabric rolls into a large number of short pieces named pattern shapes. The trim loss in one-dimensional roll cutting without optimisation is approximately 2%–3% and is treated as waste.

A pattern shape consists of pattern parts which make up a piece of clothing as illustrated in Fig. 1. It is rectangular and usually drawn on paper. The width of the rectangle equals the width of fabric rolls, so

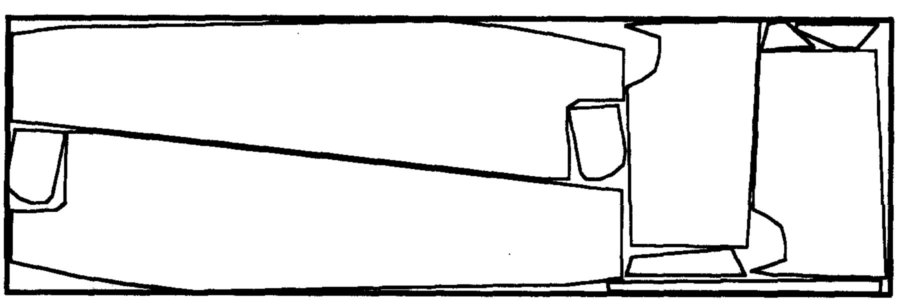

Fig. 1. Pattern shape.

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we can consider the cutting problem as one-dimensional. The width of a fabric roll is standardised and generally remains the same within one customer order. The length of the rectangle depends on the number and size of pattern parts which make up a piece of clothing.

Within the framework of one order it is necessary to make several pattern shapes. The number of different pattern shapes (hereinafter referred to as order lengths) generally ranges from 3 to 8. The roll lengths usually range between 10 and 100 m, whereas order lengths are mainly up to 5 m long. Most commonly 10–50 pieces of a particular order length are required.

Our objective is to create a plan of one-dimensional roll cutting into pattern shapes as illustrated in Fig. 2. which will minimise the overall trim loss considering practical conditions specific to the clothing industry. However, in practice rolls are often cut without a plan. This article describes the mathematical model of the cutting problem, solution development in the form of a computer program, and an example of the practical implementation of the program.

2. PROBLEM DESCRIPTION AND A FORMAL MODEL

Each customer order is planned individually. For each customer order there is a certain number of rolls differing in length, fabric and colour. In general the lengths of all rolls are different. The following notation is used;

m=Number of rolls.
d_j=Roll lengths; $j=1,...,m$.

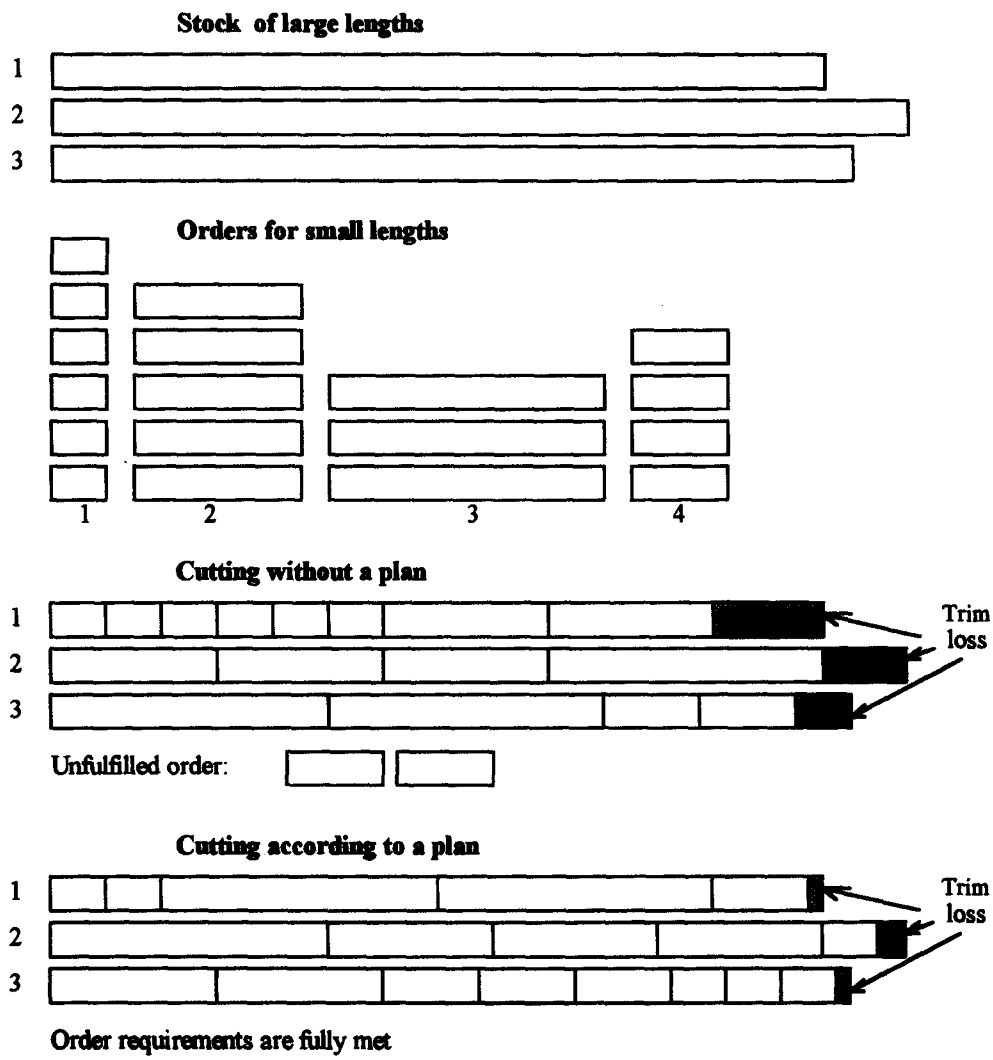

Fig. 2. Stock cutting.

We need a certain number of order lengths which consist of a particular number of pieces. We can denote the following;

n=Number of varying order lengths.
s_i=Order lengths; $i=1,...,n$.
b_i=Required number of pieces which are determined by length i, $i=1,...,n$.

We want to obtain;

x_{ij}=Number of pieces which are defined by order length i having been cut from roll j under the following conditions;

(1) min $\sum_{i=1}^{n} \Delta_i$ (minimize number of uncut order lengths, distribute evenly)

(2) min $\sum_{j=1}^{m} t_j$ (minimize trim loss which is smaller than UB)s.t.

(3) $\sum_{i=1}^{n} s_i \cdot x_{ij} + \delta_j = \delta_j \quad \forall j$ (knapsack constraints)

(4) $\sum_{j=1}^{m} x_{ij} = b_i - \Delta_i \quad \forall i$ (demand constraints)

(5) $\sum_{i=1} y_{ij} \leq Y \leq M \quad \forall j$ (maximum number of different order lengths for a roll)

(6) $x_{ij} \geq 0$, integer $\forall i,j$
$\delta_j \geq 0 \quad \forall j$
$t_j \geq 0 \quad \forall j$
$\Delta_i \geq 0 \quad \forall i$
$y_{ij} \epsilon 0,1 \quad \forall i,j$
$z_j \epsilon 0,1 \quad \forall j.$

For the above model the following functions are used;

$$z_j = \begin{cases} 0 \text{ if } x_{ij}=0 & \forall i \\ 1 \text{ otherwise} \end{cases}$$

to indicate whether roll j is used in the cutting plan;

$$y_{ij} = \begin{cases} 0 \text{ if } x_{ij}=0 \\ 1 \text{ otherwise} \end{cases}$$

to indicate whether order length i is cut from roll j,

$$t_j = \begin{cases} \delta_j \text{ if } z_j=1 \wedge \delta_j \leq UB \\ 0 \text{ otherwise.} \end{cases}$$

t_j indicates the extent of the trim loss relating to roll j.

Data: UB upper bound for the trim loss—it can be set e.g. to the smallest or longest order length.

In roll cutting it is necessary to pay attention to two limitations which arise from the practical conditions in the clothing industry. The first one is related to the requirement that, in the case of material shortage, the lack of fabric should be distributed evenly among the order lengths. If the shortage was accumulated with one order length this would cause deficit in clothes of one or two size groups. This would, in turn, bring about difficulties in sales. The second limitation refers to the fact that the number of different order lengths which are cut out of one roll Y should be not higher than four ($M=4$). In practice it was experienced that for $Y=4$, the trim loss becomes so low, that any further reduction would not outweigh the extra efforts of cutting.

The model makes a distinction between two groups of unutilized parts of rolls. First, there are unutilized parts which are long enough to be used again even though the requirements of the order were met. Second, there are unutilized parts which are too short to be used again. Hence, unutilized roll length which could be used further is not considered to be a trim loss. The question is, which unutilized length is so short that it could be referred to as trim loss. The answer depends on the quantity of available fabric. Overall trim loss can be defined in two ways.

(1) Overall trim loss is the sum of those trim losses in particular rolls which are shorter than the shortest order length.
(2) Overall trim loss is the sum of those trim losses in particular rolls which are shorter than the longest order length.

In the cases with enough required fabric available we refer to definition 1, whereas in other cases we refer to definition 2. If definition 1 was referred to in both cases this would—in cases of shortage of fabric—give rise to the following problem. As the aim of the algorithm is minimization of the overall trim loss, this could lead to unfulfilled requirements for the longest order lengths, even if the overall trim loss is small and the aim is achieved according to the logic of the algorithm. The trim losses which would be longer than the shortest order length but shorter than the longest order lengths could remain unutilized. The residual length which is larger than a certain lower bound (e.g. longest order length) is not considered as a trim loss. If there are sufficient rolls available, there will be cutting plans with "no trim loss" but ever growing stocks. To prevent this an additional condition must be set: only one residual length may be longer than the longest order length.

The problem described above can be classified according to Dyckoff [2]. Through a large variety of applications reported in the literature he developed a classification scheme for cutting stock problems. He classified problems using four characteristics as follows.

(1) *Dimensionality*
    (N) Number of dimensions
(2) *Kind of assignment*
    (B) All large objects and a selection of small items.
    (V) A selection of large objects and all small items.
(3) *Assortment of large objects*
    (O) One large object.
    (I) Many identical large objects.
    (V) Different large objects.
(4) *Assortment of small items*
    (F) Few items of different dimensions.
    (M) Many items of many different dimensions.
    (R) Many items of relatively few dimensions.
    (C) Many identical items.

Our integer cutting stock problem is one-dimensional. In the cases with enough required fabric available *kind of assignment* is V, *assortment of large objects* is V and *assortment of small items* is M. By using Dyckoff' s typology this can be described as 1/V/V/M. In other cases *assortment of large objects* is B and the problem can be described as 1/B/V/M.

3. SOLUTION DEVELOPMENT

Dyckoff [2] classifies the solution approaches to integer cutting stock problems into two groups: *object-* or *item-oriented* and *pattern oriented.* Approach to the integer cutting stock problem can be characterized as an *item-oriented,* where every item to be cut is treated individually. In the *pattern-oriented* approach, at first, order lengths are combined into cutting patterns, for which—in a succeeding step—the frequencies are determined that are necessary to satisfy the demands. The literature abounds in *pattern-oriented* solutions based on a hybrid algorithm, which was developed by Gilmore and Gomory [4,5]. However, a *pattern-oriented* approach is possible only when the stock is of the same length [4] or of the several standard lengths [5]. Our solution approach must be *item-oriented* because all stock lengths are different and the frequencies cannot be determined.

Item-oriented solution approach can be based on exact methods or on approximation algorithms [2]. For the 1/V/V/M or 1/B/V/M type of problem there is no exact method which could find an optimal *item-oriented* solution within reasonable time limits. Therefore, a solution in the form of approximation algorithm must be found.

The authors have not come across an approximation algorithm in literature which would be directly applicable to the described situation. Similar problems are solved by Sequential Heuristic Procedures (SHP). A similar kind of problem is the classical "bin packing problem", which can be solved for example by the "First Fit Decreasing" (FFD) or "Best Fit Decreasing" (BFD) SHP [1]. However, the basic feature of this type of problem is that all stock lengths are the same. An extensive review of the relevant literature up to 1992 is given in [9].

The primary advantage of SHP is its ability to control factors other than trim loss and to eliminate rounding problems by working only with integer values. The major disadvantage of an SHP is that it may generate a solution which has a greatly increased trim loss because of so-called ending conditions [6]. We will try to develop such a SHP, which will minimize the influence of ending conditions.

The cutting model we described could be termed a bicriterial multidimensional knapsack problem with side constraints. A basic simplification which was used to transform an exact algorithm into an approximate one with polynomial complexity is the limitation of processing one roll at a time until all order requirements are satisfied. The algorithm was developed on a step by step basis. The number of basic steps equals the number of rolls necessary for fulfilment of an order. At the beginning, all rolls belong to the set of unprocessed rolls. The set of processed rolls is empty. At each step, the set of unprocessed rolls is reduced by one and the set of processed rolls increases by one. Also, the number of cut pieces of particular order lengths changes, as well as the length of the processed roll, which becomes equal to trim loss.

While developing the algorithm, we had to address two basic questions. First, which roll is to be chosen from the set of the not yet processed rolls and second, which order lengths are to be cut out of it. The SHP is developed in such a way that it is based on the following assumptions:

(1) It is easier to find a good solution referring to one roll provided it is selected from the largest possible set of different rolls.
(2) It is easier to find a good solution for the selected roll provided it is selected from the largest possible set of feasible solutions. This happens when;
  (a) the roll is as long as possible,
  (b) the ordered lengths are as short as possible,
  (c) the number of different order lengths and the number of pieces is as large as possible.
(3) It is easier to find a good solution for the selected roll if differences in order lengths are as large as possible.

All the assumptions are proven statistically. However, we cannot be completely sure that the assumptions are valid in each individual case. The issue of roll selection can be tackled in accordance with the assumption 1 by calculating the solutions for all unprocessed rolls and selecting among them the one with the lowest trim loss. As it is possible that more rolls have the lowest trim loss, we apply the 2(a) assumption and choose the shortest roll. This can be achieved by the initial arranging of the rolls according to increasing lengths. By doing so, the longer rolls will be processed later when the conditions become more difficult because of the first assumption.

The remaining question to be dealt with is the selection of order lengths. According to condition (5) the number of order lengths cut up from the same roll is limited by a parameter Y whose value can be 1, 2, 3 or 4. Consistent with the assumption 2(c) and the condition (1) stipulating that the possible trim loss should be distributed among order lengths as evenly as possible, we select those order lengths which consist in the largest number of pieces. This can be achieved by arranging the order lengths according to the decreasing number of pieces. This selection also takes into consideration the assumption 3 because it prevents that at the end of the cutting process only one or two similar order lengths are left, but it is not consistent with the assumption 2(b); nevertheless, because of the condition (1) there is no other choice. For this reason this solution is reached only if there is not enough fabric.

When there is an abundance of fabric we face a dilemma when selecting the order lengths. This is which assumption should be given priority—either assumption 2(b) or 2(c). The dilemma can be resolved by developing three algorithmic variants and selecting the best result. The first variant is the same as the one which is applied when there is not enough fabric. The second variant is based on assumption 2(b). We select those order lengths which are the longest. This is achieved by arranging order lengths according to decreasing lengths. This means that shorter order lengths are processed later, i.e., when there is less room for manoeuvre and according to assumption 1 the cutting problem is becoming more complicated. The third variant is a compromise between assumptions 2(b) and 2(c). Order lengths are chosen from the arrangement which is made according to the decreasing value of the product obtained by multiplying order lengths by the number of necessary pieces.

As a result of the nature of the fabric the order lengths are defined precisely up to 1 cm, and the roll lengths up to 1 dm. Therefore all lengths can be expressed in centimetres and we refer fully to integer problem.

The algorithm for the optimisation of roll cutting in the clothing industry is shown with a flowchart in Fig. 3. In the flowchart there are additional indexes k and f which represent colour and fabric of the order lengths and rolls.

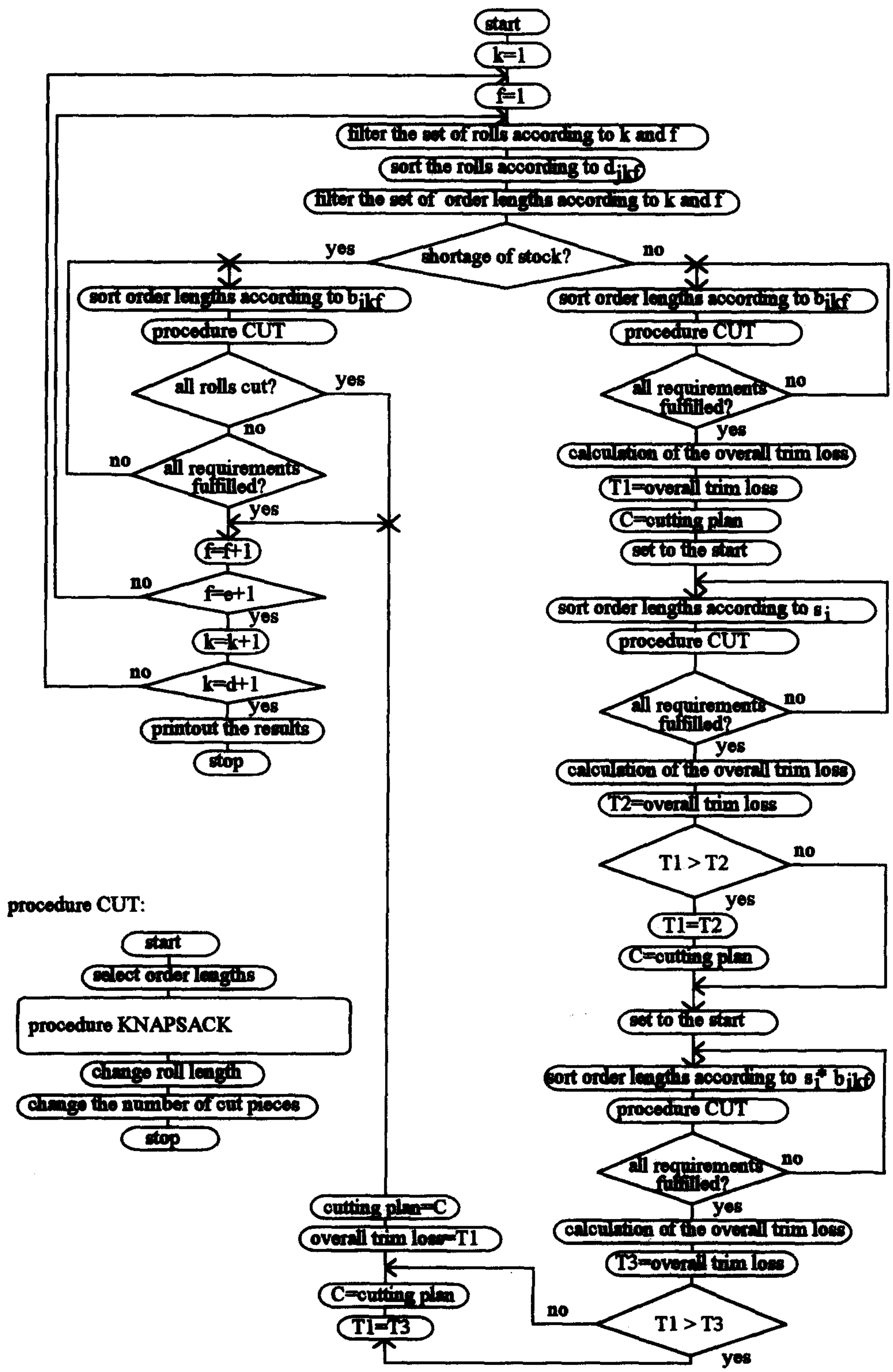

Fig. 3. Flowchart of algorithm of cutting.

d=Number of different colours.

e=Number of types of fabric.

b_{ikf}=Required number of pieces which are determined by length i, colour k, and fabric f; $i=1,...,n$; $k=1,...,d; f=1,...,e$.

d_{jkf}=Roll lengths; $j=1,...,m$; $k\epsilon\{1,...,d\}; f\epsilon\{1,...,e\}$.

The creation of a cutting plan is repeated for each combination k and f which represents an independent cutting problem.

The flowchart indicates that it is necessary to solve a series of knapsack problems, one for each basic step of algorithm. In the knapsack algorithm a sequence of vectors $(x_j)_Y$ is generated in a lexicographically decreasing order to find the first optimal combination. In case of more than one optimal combination, shorter order lengths are processed later, consistent with the assumption 1.

4. PROGRAM COLA

The proposed algorithm is written in FORTRAN programming language. The program consists of 2000 lines of code. The data input and the printout of the results are made in 4GL. The program can be run on a personal computer and it enables the achievement of 0.1% of the average planned trim loss per order. It was called COLA (COmputerized LAying out).

The program operates at great speed. Creating a cutting plan and the calculation of statistics for an extensive order consisting of 50 rolls and 1000 pattern pieces takes less than 10 s on a personal computer (486DX4, 100MHz). The time spent on creating the plan is negligible. This is especially evident when we compare it with the manual laying out of orders of average size, which can take about 4–8 h.

The speed enables the users to carry out a "what-if" analysis by changing the parameters of the order. For instance, a possible question could be "to what extent the solution would be worse if we cut only two different order lengths out of one roll?".

The example shown in Fig. 4 contains real data. However, the customer order related to is rather small, containing only three different order lengths and only 11 rolls in two colours and two fabrics. The devised cutting plan assumes a complete cutting of 10 rolls and partial cutting of 1 roll. The anticipated trim loss is only 44 cm, which makes 0.05% of total length. The saving of the fabric in this order is 10 m.

To clarify the printout of results, some further explanations need to be provided. The column "the following order length" (foll.o.l.) contains order length reference numbers which are about to be cut from the rolls given in the column "rolls". This item of information is helpful to the worker dealing with the rolls at the placing machine. The column "size numbers" refers to the size numbers of ready-made clothes, whose pieces make up a particular order length. Size numbers do not affect the creation of a cutting plan. They are added only to provide records and statistics. The "correction" column contains possible deviations between the planned and actually cut number of pieces.

We have carried out an analysis of 200 randomly generated samples with values, common in real life. This means that the average number of pieces which were cut from a roll was approximately 30. The order lengths differed in approximately 40%. The number of different order lengths which were cut out of a roll was 4. In 100 cases there was about 5% shortage of fabric, in other 100 cases there was about 5% surplus.

We have obtained the following results: in the first 100 cases the average trim loss was 0.03% and 0.17% in the worst case. In the other 100 cases the average trim loss was 0.01% and 0.06% in the worst case. In 98 out of first 100 cases the sum of all trim losses was lower than the shortest order length. This means that we have certainly found an optimal solution. In the remaining 100 cases all solutions were optimal.

5. CONCLUSION

We studied the problem of reducing trim loss in the one-dimensional stock cutting. In the clothing industry it arises as the problem of optimal utilisation of stock in roll cutting into pattern shapes. We established that an item oriented solution in the form of SHP must be found because of the practical conditions specific in clothing industry and we defined the issue of roll cutting as a bicriterial multidimensional knapsack problem with side constraints. We found a solution through a combination of approximation and heuristics. On the basis of the achieved algorithm we developed the computer program COLA.

The COLA program generates almost optimal solutions and facilitates a reduction in trim loss to 0.1%

OPTIMIZATION OF CUTTING date: 10-11-95 customer order: 635 model: jacket

DETAILS ABOUT ORDER LENGTHS

No.	order length	length	size numbers				pieces	colour	fabric
1	1	239	74	80	86	92	140	6449	BL-72
2	1	239	74	80	86	92	145	7209	BL-100
3	2	188	80	86	92		55	6449	BL-72
4	2	188	80	86	92		35	7209	BL-100
5	3	134	86	92			25	6449	BL-72
6	3	134	86	92			30	7209	BL-100

DETAILS ABOUT ROLLS

No.	roll	length	colour	fabric
1	15	10280	6449	BL-72
2	16	10220	6449	BL-72
3	17	10160	6449	BL-72
4	19	10180	6449	BL-72
5	20	10100	6449	BL-72
6	9	10200	7209	BL-100
7	10	10790	7209	BL-100
8	11	10100	7209	BL-100
9	13	10070	7209	BL-100
10	14	900	7209	BL-100
11	12	3000	7209	BL-100

RESULTS ARRANGED ACCORDING TO ORDER LENGTHS

order length	pieces	correction	roll	colour	fabric	foll.o.l.	size numbers			
1	32	.....	15	6449	BL-72	2	74	80	86	92
1	34	.....	16	6449	BL-72	2	74	80	86	92
1	36	.....	17	6449	BL-72	2	74	80	86	92
1	28	.....	19	6449	BL-72	2	74	80	86	92
1	10	.....	20	6449	BL-72	2	74	80	86	92
1	30	.....	9	7209	BL-100	2	74	80	86	92
1	42	.....	10	7209	BL-100	2	74	80	86	92
1	28	.....	11	7209	BL-100	2	74	80	86	92
1	40	.....	13	7209	BL-100	2	74	80	86	92
1	4	.....	12	7209	BL-100	2	74	80	86	92
2	14	.....	15	6449	BL-72	0	80	86	92	
2	9	.....	16	6449	BL-72	3	80	86	92	
2	4	.....	17	6449	BL-72	3	80	86	92	
2	10	.....	19	6449	BL-72	3	80	86	92	
2	18	.....	20	6449	BL-72	3	80	86	92	
2	4	.....	9	7209	BL-100	3	80	86	92	
2	4	.....	10	7209	BL-100	0	80	86	92	
2	11	.....	11	7209	BL-100	3	80	86	92	
2	2	.....	13	7209	BL-100	3	80	86	92	
2	4	.....	14	7209	BL-100	3	80	86	92	
2	10	.....	12	7209	BL-100	3	80	86	92	
3	3	.....	16	6449	BL-72	0	86	92		
3	6	.....	17	6449	BL-72	0	86	92		
3	12	.....	19	6449	BL-72	0	86	92		
3	4	.....	20	6449	BL-72	0	86	92		
3	17	.....	9	7209	BL-100	0	86	92		
3	10	.....	11	7209	BL-100	0	86	92		
3	1	.....	13	7209	BL-100	0	86	92		
3	1	.....	14	7209	BL-100	0	86	92		
3	1	.....	12	7209	BL-100	0	86	92		

TRIM LOSS

UTILIZED ROLLS

roll	trim loss	colour	fabric
15	0	6449	BL-72
16	0	6449	BL-72
17	0	6449	BL-72
19	0	6449	BL-72
20	3790	6449	BL-72
9	0	7209	BL-100
10	0	7209	BL-100
11	0	7209	BL-100
13	0	7209	BL-100
14	14	7209	BL-100
12	30	7209	BL-100

UNUTILIZED ROLLS

roll	length	colour	fabric

OVERALL TRIM LOSS ....: 44 cm (0,0512 %)
PROGRAMMED TRIM LOSS - fixed...............................: 0 cm
PROGRAMMED TRIM LOSS - variable...........................: 0 %
OVERALL TRIM LOSS WITHOUT OPTIMIZATION...: 1074 cm (1,2508 %)
SAVING OF STOCK...: 1030 cm (1,1996 %)

Fig. 4. Example of the COLA program printout.

on average. Processing an average customer order takes less than 10 s. The speed is of utmost importance in those cases when there should be the shortest possible lapse of time between the point of receiving the fabric in the warehouse and processing of the order. The speed also enables the users to carry out a "what-if" analysis by changing the parameters.

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