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Simulation of the stochastic one-dimensional cutting stock problem to minimize the total inventory cost

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Abstract

This paper utilizes Monte Carlo simulation to identify the distribution of the optimal values of the one-dimensional cutting stock problem to help decide the stock order quantity before the demand mix becomes known. Due to the lead time when ordering the raw material, it is not possible to wait and see what the demand mix is. Thus, a ‘‘here and now” approach of stochastic programming is used to hedge against future demand uncertainty. The LINGO software is used as a subroutine of a MATLAB code to set up and solve the model in each iteration by drawing random demand values. Once the raw material order is placed and later the demand mix becomes known, decisions concerning the cutting patterns and shortage or overage amounts are determined with a second mathematical model. This paper presents a novel solution approach (using the optimizer software LINGO as a subroutine in a MATLAB simulation code) to a classic problem encountered in production planning by proposing a new method to account for demand randomness. Two raw material ordering rules (mean plus ½ standard deviation and mean plus 1 standard deviation) were considered and implemented using three random cut demand cases (exponential, uniform, and normal). For the exponentially distributed cut length demands, both rules resulted in ordering an excessive number of rails suggesting a need for a different ordering rule. For the normally distributed cut length demands, in some cases shortages may result for either of the two rules. For uniformly distributed cut length demands both ordering rules were effective in meeting cut length demands.

Keywords: Manufacturing Economics; Production Planning & Control; Simulation & Mathematical Modeling of Manufacturing Decisions

1. Introduction

In operations research, the one-dimensional cutting-stock problem (ODCSP) describes the general problem of cutting standard length stock material into various specified sizes [1, 2] while minimizing the material waste (the

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remnant or drop in manufacturing terms). Minimizing the remnant is analogous to minimizing the manufacturing material cost. This computationally complex optimization model has many manufacturing applications. ODCSP arises in many industrial domains such as metal, paper, textile, and wood. To solve it, the problem can be formulated as an integer linear model first, and then solved using a common optimizer software.

In many cases, the demand mix (e.g. fifteen 10′ cuts, thirty 15′ cuts, etc.) may be contractually fixed. However, a more realistic case is that the demand mix is random [3, 4, 5] and possibly a correlated vector. In addition, the cut or the yield pattern matrix may have some randomness to reflect potential losses due to cutting errors and/or material defects. If any coefficient of the model is random with a known or assumed probability density function, the ODCSP becomes a stochastic ODCSP (SODCSP). With random demand mix for cuts, the decision maker may order more stock and risk excessive inventory and holding costs or order less and risk shortage costs.

ODCSP appears in a variety of industrial applications. This problem was first introduced by Gilmore and Gomory [1, 2]. A number of researchers in various branches of industry address ODCSP. For example, Dikili and Barlas [6] encounter it in a shipyard, Stadtler [7] in the aluminum industry, Lefrancois and Gascon [8] in a small manufacturing company, Atkin and Ozdemir [9] in coronary stent manufacturing, Benjaoran and Bhokha [10] in construction steel bar manufacturing, Zanarini [3] in the rubber mold industry, Sculli [4] in manufacturing of isolation tapes, etc. In the most cases, known or deterministic demand is assumed. However, there is still a notable number of cases where the demand is random or stochastic. As a result, the problem is often modified using additional objectives and/or constraints. Some examples include Filho *et al.* [5] and Sinuany-Stern and Weiner [11] who use two objectives. Zanarini [3], Alem *et al.* [12], and Beraldi *et al.* [13] assume stochastic demands. When addressing the problem solvability, Scheithauer and Terno [14] proved that the problem possesses the modified integer round-up property while Wongprakornkul and Charnsethiku [15] solved the problem with discrete demands and the capacitated planning objective. A different aspect of stochastic programming, distribution of the optimal values, is applied in this paper. The reader is referred to Ansaripour *et al.* [16] for a review of this rather obscure, but very practical approach. Feiring [17] considers production planning where demands are normally distributed with sequentially revised means and variances.

Equations 1, 2, and 3 present the simplest version of the ODCSP model using *m* cuts and *n* patterns. In the model, the input column vector D_i represents the demand for each cut size *i*; the input matrix a_{ij} represents the number of cuts size of type *i* that can be obtained from pattern *j*, while the output variables X_j represent the number of stocks that should be cut according to pattern *j*.

$$Min \sum_{j=1}^{n} X_j \tag{1}$$

$$\sum_{j=1}^{n} a_{ij} X_j \geq D_i \quad \forall\, i = 1, \ldots, m \tag{2}$$

$$X_j \in \ integer \ \ \forall\, j = 1, \ldots, n \tag{3}$$

2. Methodology

Raw material stock length, individual cut lengths, cut demand probability density functions, and the number of desired simulation iterations are interactively input into a MATLAB code. The code, then, sets up an applicable ODCSP by drawing random demand values for each cut length using the applicable random number variate generator equations. Next, each instance of ODCSP model is solved by submitting it to LINGO solver. The optimal value (number of raw stocks to order) for each iteration is stored in a data file. The process is repeated for the desired number of iterations. Finally, the output data file is analyzed to determine the probability density function of raw stocks (80 foot rails) that may be needed for all possible cut length demand combinations.

3. Deterministic example

The following deterministic example is adopted from Sarper and Jaksic [18]. The five rail cut lengths that the plant needs to construct railroad frogs are given: 24' 0" (A), 29' 10 ½" (B), 36' 6 5/8" (C), 38' 3" (D), and 54' 7" (E). The plant buys 80′-long steel rails from a steel mill. The demand levels (known or deterministic) for each cut length are as follows: A: 64, B: 38, C: 61, D: 54, E: 42. Fig. 1 shows an 80-foot steel rail being cut into various sizes for use in railroad frog manufacturing.

During the cutting operation, there is a loss of about 0.40" of length caused by the blade. Table 1 shows all 11 possible and feasible patterns that yield the number of cuts of each type. A pattern is feasible if the remnant length is less than the smallest cut length. Fig. 2 shows the LINGO [19] version of the optimization model (Equations 1 – 3) for the data in Table 1. Using the deterministic or known contractual demands in Table 1, the LINGO solution based on the column generation algorithm is: X2 = 11, X5 = 42, X6 = 13, X7 = 1, X9 = 3, and X10 = 54.

Fig. 1. A Rail being cut into required lengths from Sarper and Jaksic [18]

Table 1. Feasible cutting pattern for the sample deterministic problem

Patterns/ Cuts	24.04′ A	29.91′ B	36.59′ C	38.28′ D	54.61′ E	Remnant
1	3	0	0	2	0	7.88′
2	2	2	0	0	0	2.01′
3	1	0	1	0	0	19.37′
4	1	0	0	1	0	17.68′
5	1	0	0	0	1	13.50′
6	0	2	0	0	0	20.18′
7	0	1	1	0	0	13.50′
8	0	1	0	1	0	11.81′
9	0	0	2	0	0	6.82′
10	0	0	1	1	0	5.13′
11	0	0	0	2	0	3.44′
Demand :	64	38	61	54	42	

A total of (X2 + X5 + X6 + X7 + X9 + X10) 124 80′- long rails are needed to meet the demand with the total minimum waste or the remnant.

These 124 rails are to be cut according to patterns 2, 5, 6, 7, 9, and 10 shown in Table 1. Demands for each of the 5 cut lengths are met as follows:

- Number of 24.04′ long (A) cuts = 42 x 1 + 11 x 2 = 64,

- Number of 29.91′ long (B) cuts = 11 x 1 + 13 x 2 = 38,
- Number of 36.59′ long (C) cuts = 1 x 1 + 3 x 2 + 54 x 1 = 61,
- Number of 38.28′ long (D) cuts = 54 x 1 = 54.
- Number of 54.61′ long (E) cuts = 42 x 1 = 42.

The 124 rails have a total length of 9920 feet while the actual length of the total number of frogs is 24.04 x 64 + 29.91 x 38 + 36.59 x 61 + 38.28 x 54 + 54.61 x 42 = 9267.87 feet. The model in Fig. 2 has achieved a 93.4 % utilization. Utilization tends to approach 99% if more cut sizes are considered in the demand mix. Here, the demand mix was met exactly. The deterministic model sometimes results in extra cuts. The deterministic model is adequate when the plant operates on fixed long term contract. However, there are many cases where the demand is stochastic.

```
SET ECHOIN 1
MODEL:
[OBJROW] MIN=(X1 + X2 + X3 + X4 + X5 + X6 + X7 + X8 + X9 + X10 + X11);
3*X1 + 2*X2 + 1*X3 + 1*X4 + 1*X5 + 0*X6 + 0*X7 + 0*X8 + 0*X9 + 0*X10 + 0*X11 > 64;
0*X1 + 1*X2 + 0*X3 + 0*X4 + 0*X5 + 2*X6 + 1*X7 + 1*X8 + 0*X9 + 0*X10 + 0*X11 > 38;
0*X1 + 0*X2 + 1*X3 + 0*X4 + 0*X5 + 0*X6 + 1*X7 + 0*X8 + 2*X9 + 1*X10 + 0*X11 > 61;
0*X1 + 0*X2 + 0*X3 + 1*X4 + 0*X5 + 0*X6 + 0*X7 + 1*X8 + 0*X9 + 1*X10 + 2*X11 > 54;
0*X1 + 0*X2 + 0*X3 + 0*X4 + 1*X5 + 0*X6 + 0*X7 + 0*X8 + 0*X9 + 0*X10 + 0*X11 > 42;
@GIN (X1); @GIN (X2); @GIN (X3); @GIN (X4); @GIN (X5); @GIN (X6);
@GIN (X7); @GIN (X8); @GIN (X9); @GIN (X10); @GIN (X11);
DATA:
@TEXT('Optimal Values.TXT', 'A') = OBJROW;
ENDDATA
END
GO
QUIT
```

Fig. 2. ODCSP model for the sample problem (LINGO version)

4. Simulation of SODCSP

Manufacturers do not always know the demand exactly, but they have an idea on the random pattern based on order history and other market conditions. Raw material can be expensive and it can have a long lead time to procure as is the case with 80 foot long steel rails that must be ordered from a mill. Demand randomness for each cut length is expressed with a probability density function. Three random cases are considered for demand for each cut length, case 1 (exponential), case 2 (uniform), and case 3 (normal). Case 1 represents high uncertainty while cases 2 and 3 represent less uncertainty for each cut length demand. Sarper and Jaksic [18] applied goal programming to a similar problem. It is important to note that cut demands are derived from the demand for the finished product (rail frog). A MATLAB code was developed to simulate the general model given in equations 1-3 by using the example in Fig. 2. Deterministic demands in Fig. 2 (64, 38, 61, 54, 42) were replaced by the appropriate random number generator equations for the cases above. Each simulation iteration results in an optimal number of rails to order. The results are stored in an output file (Optimal Values.TXT). The MATLAB code calls LINGO software once a model with a randomly realized demand mix is set up. This Monte Carlo simulation process is modeled using the same methodology Sarper [20] used by calling the LINDO software within a FORTRAN code. Similar approach was used by Sarper [21, 22] in solving two distinct problems: optimization of metal cutting parameters and aircraft allocation to routes.

For case 1, deterministic means are now used as the mean demands for each exponentially distributed cut length demand: 64, 38, 61, 54, 42. For case 2, the demand for each cut length is uniformly distributed with the following lower and upper limits: 52 - 76; 20 – 56; 41 - 81; 32 - 76; 12 – 72. For case 3, the demand for each cut length is normally distributed with the following mean and standard deviation parameters: (64, 4), (38, 6), (61, 7), (54, 7), (42, 10). Fig. 3 shows the data input process of the simulation code for case 3.

```
Welcome to the One Dimensional Cutting Stock Model Simulator Program
Input Stock Length: 80
Please input cut lengths (no cut can be equal or greater than the stock length)
Input Cut Length (Input 0 to Terminate): 24.04
Input Cut Length (Input 0 to Terminate): 29.91
Input Cut Length (Input 0 to Terminate): 36.59
Input Cut Length (Input 0 to Terminate): 38.28
Input Cut Length (Input 0 to Terminate): 54.61
Input Cut Length (Input 0 to Terminate): 0
Cuts have been sorted in ascending order
Cut1=24.04 Cut2=29.91 Cut3=36.59 Cut4=38.28 Cut5=54.61
Select a Demand Probability Density Distribution for the Cuts: 1: Exponential 2: Uniform 3: Normal
Distribution: 3
How Many iterations do you want to run the simulation: 4000
Enter Mean for Cut 1 Demand (24.04 units): 64
Enter Standard Deviation of Cut 1 Demand (24.04 units): 4
Enter Mean for Cut 2 Demand (29.91 units): 38
Enter Standard Deviation of Cut 2 (29.91 units): 6
Enter Mean for Cut 3 Demand (36.59 units): 61
Enter Standard Deviation of Cut 3 Demand (36.59 units): 7
Enter Mean for Cut 4 Demand (38.28 units): 54
Enter Standard Deviation of Cut 4 Demand (38.28 units): 7
Enter Mean for Cut 5 Demand (54.61 units): 42
Enter Standard Deviation of Cut 5 Demand (54.61 units): 10
```

Fig. 3. Data input of the simulation code for case 3.

Fig. 4 shows the results of 2000 Monte Carlo simulation runs for case 1. Depending on all possible exponentially distributed demand combinations for the cuts shown in Table 1 and Fig. 3, the quantity of optimal number of rails determined by the models in equations 1 – 3 and Fig. 2 vary considerably from 26 to 466. The manufacturer must decide for an order quantity within the procurement lead time of the mill. Actual demands for each cut length will be known after the order has been placed. Ordering too few or too many rails results in shortage or excess inventory.

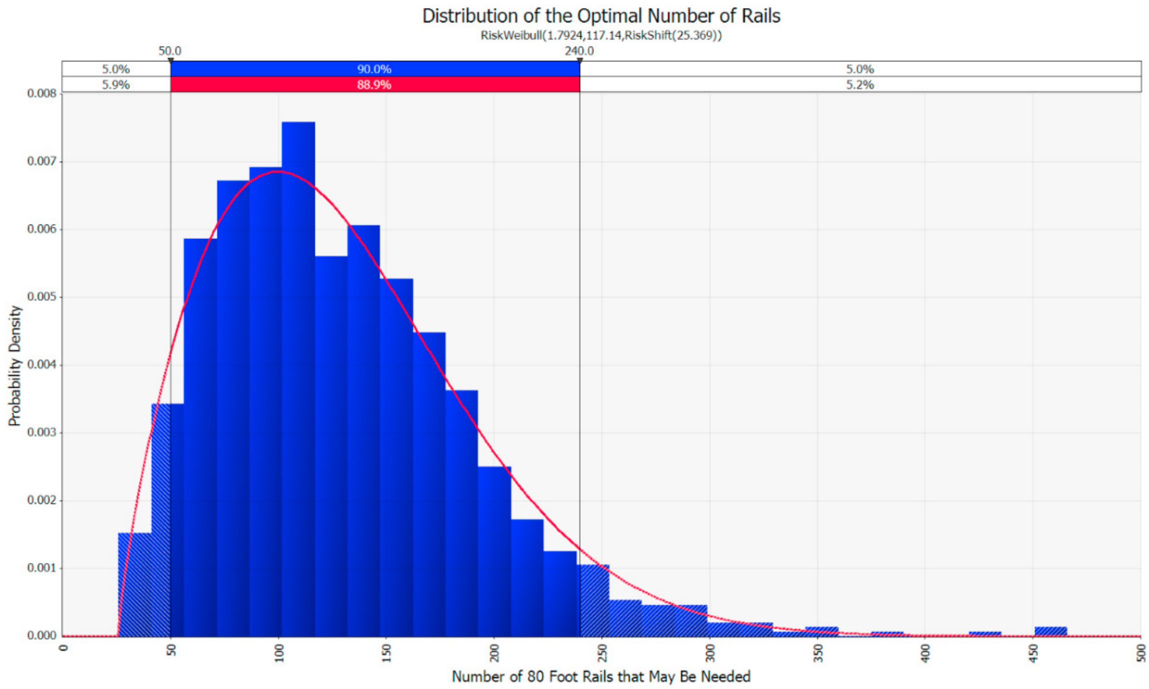

Fig 4. Probability density function of the optimal number of rails needed in case 1.

Table 2 shows simulation summaries for all three cases and raw order quantities based on two potential rules the management may try and adopt if deemed satisfactory. Rules 1 and 2 state ordering at the levels of mean + one-half std. deviation and mean + one std. deviation, respectively. Cases 2 and 3 have much less variation in comparison to case 1 as expected of the underlying uniformly and normally distributed demands for each cut length.

Table 2. Parameters of the optimal number of raw materials and rule-based order quantities

Demand Case	Mean	Std. Dev.	Minimum	Maximum	95% Percentile	Order (Rule 1)	Order (Rule 2)
1	130	60	26	466	240	160	190
2	126	17	75	169	153	135	143
3	126	10	88	167	142	131	136

5. Post-order deterministic solution

Equations 4-12 represent a deterministic inventory model that can be used in making final production decisions once a fixed quantity of rails have been ordered and actual demand mix is finally known. Table 3 shows the solution of the inventory model for sample realized demand levels for each of the cases. Fig. 6 shows an expanded model using case 3 in Table 3 and order rule 1 (131 rails ordered).

$$\text{Min} = \text{Unit\_Shortage\_Cost} * \text{TOTAL\_SHORTAGE} + \text{STOCKS\_USED} \tag{4}$$

$$\sum_{j=1}^{n} X_j - \text{STOCKS\_ORDERED} \leq 0 \tag{5}$$

$$\sum_{j=1}^{n} a_{ij} X_j + \text{Shortage\_i} - \text{Overage\_i} \geq D_i \quad \forall\, i = 1, \dots, m \tag{6}$$

$$\text{STOCKS\_USED} - \sum_{j=1}^{n} X_j = 0 \tag{7}$$

$$\text{TOTAL\_SHORTAGE} - \sum_{i=1}^{m} \text{Shortage\_i} = 0 \tag{8}$$

$$\text{TOTAL\_OVERAGE} - \sum_{i=1}^{m} \text{Overage\_i} = 0 \tag{9}$$

$$\text{STOCKS\_CARRIED} - \text{STOCKS\_ORDERED} + \text{STOCKS\_USED} = 0; \tag{10}$$

$$X_j \in \mathit{integer} \quad \forall\, j = 1, \dots, n \tag{11}$$

Table 3. Sample realized demands and summary solution of the deterministic inventory model

Demand Case	Realized Cut 1 Demand	Realized Cut 2 Demand	Realized Cut 3 Demand	Realized Cut 4 Demand	Realized Cut 5 Demand	Order Rule 1 Shortage	Rule 1 Stocks Used	Order Rule 2 Shortage	Rule 2 Stocks Used
1	18	114	20	7	38	0	109	0	109
2	57	49	44	71	51	0	135	0	135
3	61	41	75	62	54	19	131	12	136

In Table 3, shortage refers to total number unmet demand over all cut lengths.

LINGO [19] solution of case 3 in Table 3 (shown in Fig. 6) is as follows: X2 = 13, X5 = 35, X6 = 14, X9 = 7, and X10 = 62 for a total of 131 rails. As in the solution of the example in Fig. 2, each X indicates the number of 80-foot rails to be cut according to its cutting pattern number shown in Table 1. In addition, the model determines Shortage_5 = 19. All rails are used and 19 of the longest cuts (54.61′) cannot be delivered.

```
MODEL:
Min = Unit_Shortage_Cost * TOTAL_SHORTAGE +  STOCKS_USED;
STOCKS_ORDERED = 131;
Unit_Shortage_Cost  = 3;
X1 + X2 + X3 + X4 + X5 + X6 + X7 + X8 + X9 + X10 + X11 - STOCKS_ORDERED <= 0;
3*X1 + 2*X2 + 1*X3 + 1*X4 + 1*X5 + 0*X6 + 0*X7 + 0*X8 + 0*X9 + 0*X10 + 0*X11 + Shortage_1 -
Overage_1 > 61;
0*X1 + 1*X2 + 0*X3 + 0*X4 + 0*X5 + 2*X6 + 1*X7 + 1*X8 + 0*X9 + 0*X10 + 0*X11 + Shortage_2 -
Overage_2 > 41;
0*X1 + 0*X2 + 1*X3 + 0*X4 + 0*X5 + 0*X6 + 1*X7 + 0*X8 + 2*X9 + 1*X10 + 0*X11 + Shortage_3 -
Overage_3 > 75;
0*X1 + 0*X2 + 0*X3 + 1*X4 + 0*X5 + 0*X6 + 0*X7 + 1*X8 + 0*X9 + 1*X10 + 2*X11 + Shortage_4 -
Overage_4 > 62;
0*X1 + 0*X2 + 0*X3 + 0*X4 + 1*X5 + 0*X6 + 0*X7 + 0*X8 + 0*X9 + 0*X10 + 0*X11 + Shortage_5 -
Overage_5 > 54;
STOCKS_USED - X1 - X2 - X3 - X4 -X5 - X6 - X7 - X8 - X9 - X10 -X11 = 0;
TOTAL_SHORTAGE - Shortage_1 - Shortage_2 - Shortage_3 - Shortage_4 - Shortage_5 = 0;
TOTAL_OVERAGE - Overage_1 - Overage_2 - Overage_3 - Overage_4 - Overage_5  = 0;
STOCKS_CARRIED - STOCKS_ORDERED + STOCKS_USED = 0;
@GIN (X1);@GIN (X2);@GIN (X3);@GIN (X4);@GIN (X5);@GIN (X6);
@GIN (X6);@GIN (X7);@GIN (X8);@GIN (X9);@GIN (X10);@GIN(X11);
END
```

Fig. 6. Expanded inventory model for case 3 in Table 3.

6. Discussion of Example Results

Case 2 in Table 3 illustrates that both ordering rules are effective in meeting cut length demands with low variations (uniform distribution). The Rule 1 resulted in meeting the needs exactly (135 80-foot rails ordered and 135 rails used). However, the Rule 2 resulted in ordering 8 more 80-foot rails than needed (143 rails ordered while only 135 were used).

Case 1 shows the difficulties associated with cut length demands with high variation (exponentially distributed). Both rules resulted in excessive raw material ordering (160 80-foot rails for Rule 1 and 190 rails for Rule 2) to meet the demand for all cut lengths. Only 109 80-foot rails were needed. This indicates that additional ordering rules should be considered for highly variable demand cases.

For case 3 in Table 3 it can be observed that both ordering rules were not effective in preventing shortages when the realized cut length demands significantly exceeded the means in four out of five cuts. It should be noted that this demand vector is highly unlikely to occur, but it was chosen to illustrate the limitations of the method.

7. Conclusions and future work

This paper has proposed a simulation methodology to help solve SODCSP. The methodology can evaluate effects of various raw material order levels to account for demand randomness at the time of order. This is done by first minimizing shortage and then minimizing the number of rails used. The paper presents a solution approach using the optimizer software LINGO as a subroutine in a MATLAB simulation code, and by accounting for inherent random demand. Two raw material ordering rules were addressed and implemented using three random cut demand cases (exponential, uniform, and normal). For the highly variable cut length demand case (exponential distribution), additional ordering rules should be generated to prevent overstocking. Furthermore, this methodology can be used to evaluate various other order rules depending on the preference of the decision makers. The procedure favors carrying uncut stock over having unneeded cuts. Future work may include the effect of holding cost of unused stock that was ordered. In addition, dependent cut length demands may be considered. This would involve either a correlation coefficient matrix input or a joint probability distribution function for the demands.

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