2.1 $w_{n+1} = w_n - \alpha \frac{\partial J}{\partial w_n}$ $\qquad \frac{\partial J}{\partial w_n} = \sum_{i=1}^{N} -\left(y_i - \frac{1}{1+e^{-x_i w_n}}\right)\cdot x_i$

$$w_{n+1} = w_n - \alpha \sum_{i=1}^{N} -\left(y_i - \frac{1}{1+e^{-x_i w_n}}\right)\cdot x_i$$

Cannot be done in close form due to the non-linearity of the Sigmoid function.

2.2 $y = wx + w_0$ $\qquad \tilde{x} = \begin{bmatrix} 1 \\ x \end{bmatrix} \quad \tilde{w} = \begin{bmatrix} w_0 \\ w \end{bmatrix} \quad y = \tilde{w}\tilde{x}$

$$X_i = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 3 & 3 \\ 1 & -4 & -2 \\ 1 & -2 & -6 \end{bmatrix}$$

$$w^{(1)} = w^{(0)} - \alpha \sum_{i=1}^{N} -\left(y_i - \frac{1}{1+e^{-x_i w^{(0)}}}\right) x_i^T$$

$$w^{(1)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - 0{,}5\left(\overset{-1/2}{-\left(1-\tfrac{1}{2}\right)} \overset{\begin{smallmatrix}-0{,}5\\-1\\-2\end{smallmatrix}}{\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}} \overset{-1/2}{-\left(1-\tfrac{1}{2}\right)} \overset{\begin{smallmatrix}-\frac{1}{2}\\-\frac{3}{2}\\-\frac{3}{2}\end{smallmatrix}}{\begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}} \overset{1/2}{-\left(0-\tfrac{1}{2}\right)} \overset{\begin{smallmatrix}\frac{1}{2}\\-2\\-1\end{smallmatrix}}{\begin{bmatrix} 1 \\ -4 \\ -2 \end{bmatrix}} \overset{1/2}{-\left(0-\tfrac{1}{2}\right)} \overset{\begin{smallmatrix}0{,}5\\-1\\-3\end{smallmatrix}}{\begin{bmatrix} 1 \\ -2 \\ -6 \end{bmatrix}} \right)$$

$$w^{(1)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - 0{,}5\left(\begin{bmatrix} -0{,}5 \\ -1 \\ -2 \end{bmatrix} + \begin{bmatrix} -0{,}5 \\ -1{,}5 \\ -1{,}5 \end{bmatrix} + \begin{bmatrix} 0{,}5 \\ -2 \\ -1 \end{bmatrix} + \begin{bmatrix} 0{,}5 \\ -1 \\ -3 \end{bmatrix} \right)$$

$$w^{(1)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - 0{,}5 \begin{bmatrix} 0 \\ -5{,}5 \\ -7{,}5 \end{bmatrix} \Rightarrow w^{(1)} = \begin{bmatrix} 0 \\ 2{,}75 \\ 3{,}75 \end{bmatrix} \quad //$$

Prediction on next page

Prediction:

$$h(x) = \frac{1}{1+e^{-xw}} \rightarrow$$

$$x \cdot w = \begin{bmatrix} 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2,75 \\ 3,75 \end{bmatrix} = 0 - 2,75 + 3,75$$

$$1$$

$$h(x) = \frac{1}{1+e^{-1}} = 0,732$$