



Daily Coding Problem #178

Problem

This problem was asked by Two Sigma.

Alice wants to join her school's Probability Student Club. Membership dues are computed via one of two simple probabilistic games.

The first game: roll a die repeatedly. Stop rolling once you get a five followed by a six. Your number of rolls is the amount you pay, in dollars.

The second game: same, except that the stopping condition is a five followed by a five.

Which of the two games should Alice elect to play? Does it even matter? Write a program to simulate the two games and calculate their expected value.

Solution

Counterintuitively, the first game costs less to play on average.

Consider first rolling a 5 in both games. The probability of this is the same in both, which is $1/6$. But at the next roll, you can land either a 6 (to win) or another 5, which will keep you in the running. Rolling a 1, 2, 3, or 4 will require you to restart the sequence.

In the second game, you need to roll a 5. If you roll a 1, 2, 3, 4, or 6, you have to restart the rolls.

We can simulate the expected cost of each game:

```
from random import randint

NUM_TRIALS = 1000

def d6():
    return randint(1, 6)
```

```

def game_one():
    prev, curr = None, None
    cost = 0
    while prev != 5 or curr != 6:
        prev = curr
        curr = d6()
        cost += 1
    return cost

def ev_game_one():
    games = []
    for i in range(NUM_TRIALS):
        games.append(game_one())
    return sum(games) / len(games)

def game_two():
    prev, curr = None, None
    cost = 0
    while prev != 5 or curr != 5:
        prev = curr

        curr = d6()
        cost += 1
    return cost

def ev_game_two():
    games = []
    for i in range(NUM_TRIALS):
        games.append(game_two())
    return sum(games) / len(games)

```

After running each game for 1000 trials, we get:

\$36.226 for game 1

\$41.856 for game 2

So game 1 is indeed the better one to play.

