Habib University

Algorithms: Design & Analysis
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The Hamiltonian Cycle Problem

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Overview

We'll be tackling the Hamiltonian Cycle problem, a fundamental challenge in graph theory and computer science. This problem involves finding a cycle in an undirected graph where each vertex is visited once, and the cycle ends where it begins. *For example, visualize a graph where cities are vertices and roads are edges. Solving the Hamiltonian Cycle problem here means finding a path that goes through each city once and brings you back to the starting city, following the roads.* We plan to solve this using different algorithms in Python and compare their results as well as their time complexities.

Problem Description

The Hamiltonian Cycle Problem finds a Hamiltonian cycle in graph G. Let's formally define the Hamiltonian Cycle:

Let $G(V, E)$ be an undirected graph, where V is the set of vertices and E is the set of edges., Hamiltonian Cycle in the graph V is a cycle that visits every node in the graph once, returning to the starting node. In the Hamiltonian Cycle problem, the output is a sequence of vertices/nodes: $\{v_1, v_2, v_3, \ldots, v_n, v_1\}$ where:

* Each v_i is a unique element of V, for $1 \leq i \leq |V|$, ensuring that every vertex in the cycle is distinct and part of the graph's set of vertices, except for v_1, which is visited twice to indicate the start and end of the cycle.

* For each consecutive pair of vertices in the sequence (v_i, v_{i+1}), there exists an edge $\{v_i, v_{i+1}\} \in E$, guaranteeing that each pair of successive vertices in the cycle is directly connected by an edge in the graph. Additionally, there must be an edge $\{v_i, v_1\} \in E$ connecting the last vertex in the sequence back to the first vertex, thereby completing the cycle. For example, consider the following graph:

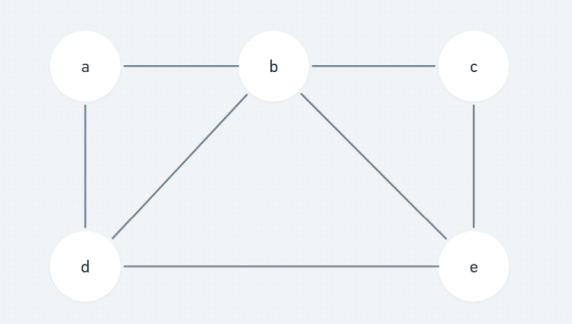

Figure 1: Undirected Graph

In the graph shown above in Figure 1 the sequence (a, b, c, d, e, a) is a Hamiltonian Cycle for the given graph. The Hamiltonian cycle path (colored in red) is shown in the figure 2 below:

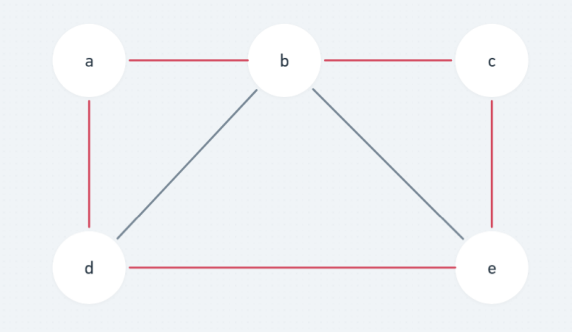

Figure 2: Hamiltonian Path Graph

Algorithms

We will be implementing three or possibly all four of the following algorithms:

- Brute Force

The brute force algorithm for solving the Hamiltonian Cycle problem involves systematically checking all possible permutations of vertices to find a cycle that visits each vertex exactly once.

- Backtracking Algorithm

The backtracking algorithm prunes search paths that cannot lead to a valid Hamiltonian cycle, avoiding unnecessary computations by backtracking when a dead end is encountered.

- Dynamic Programming

Dynamic programming solves it by breaking it down into smaller sub-problems and storing the solutions to these subproblems.

- Heuristic Algorithm

Heuristic algorithms use rules or strategies to guide the search process and often prioritize paths that are likely to lead to a Hamiltonian cycle.

References

- `https://www.geeksforgeeks.org/hamiltonian-cycle/`
  GeeksforGeeks article on the Hamiltonian Cycle problem.
- ChatGPT. Prompts used:
  - What is Hamiltonian Cycle Problem?
  - What are the various Algorithm Design Techniques to solve Hamiltonian cycle problem