

# Rossby Wellen



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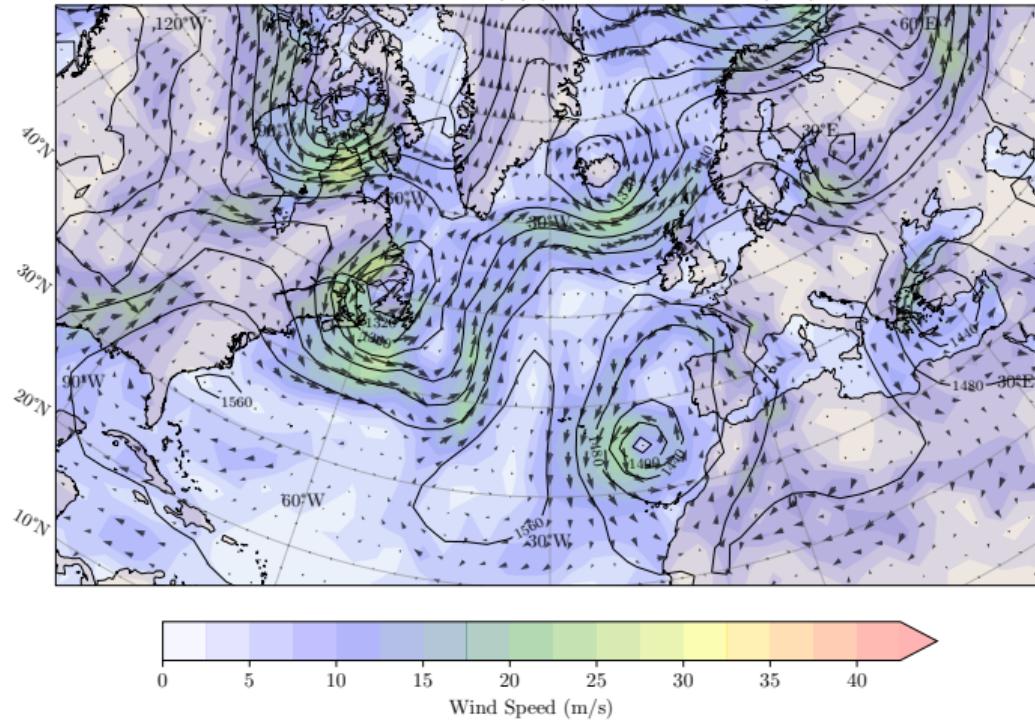
June 5, 2025

# Karte

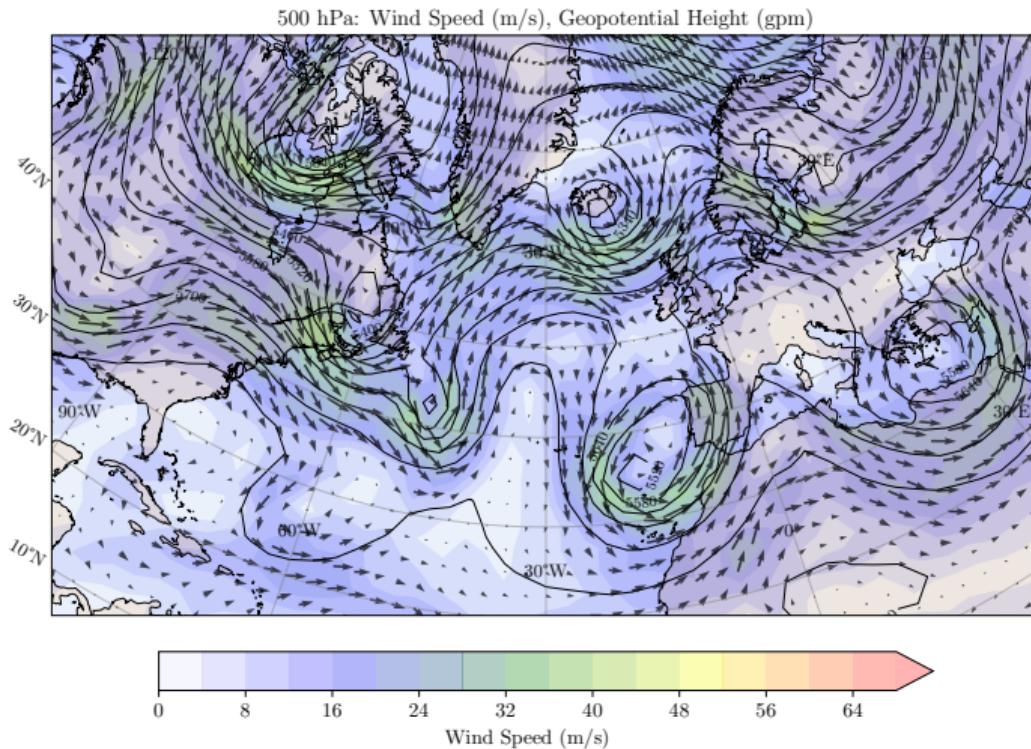


# Forecast: 01-May-2025 at 00:00, Level 850 hPa

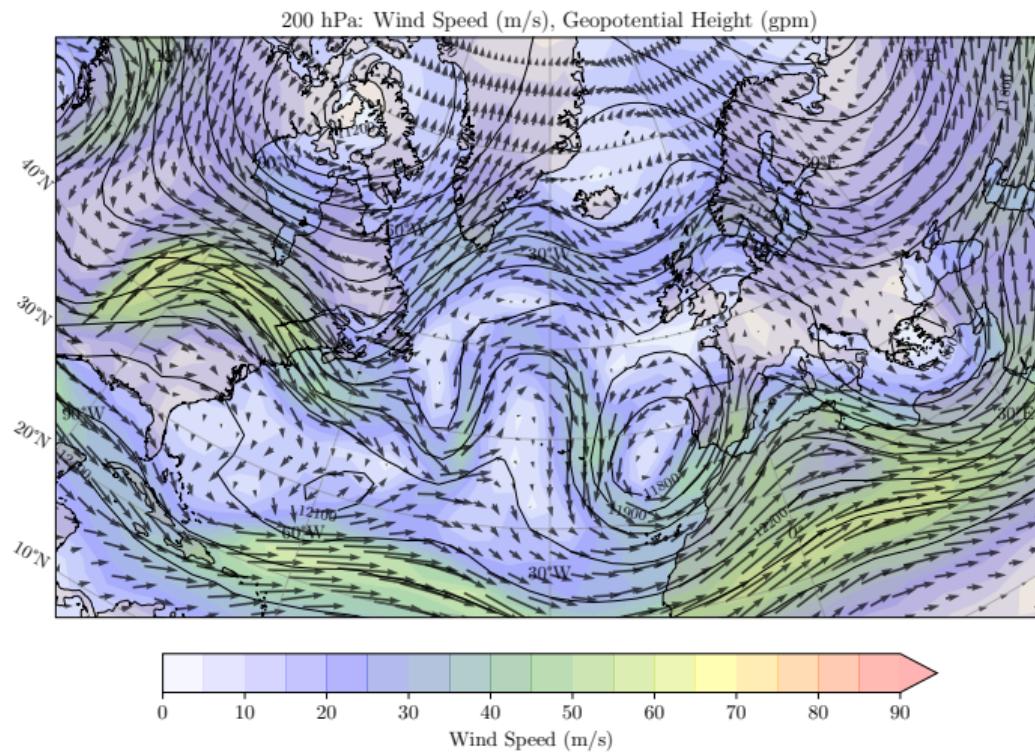
850 hPa: Wind Speed (m/s), Geopotential Height (gpm)



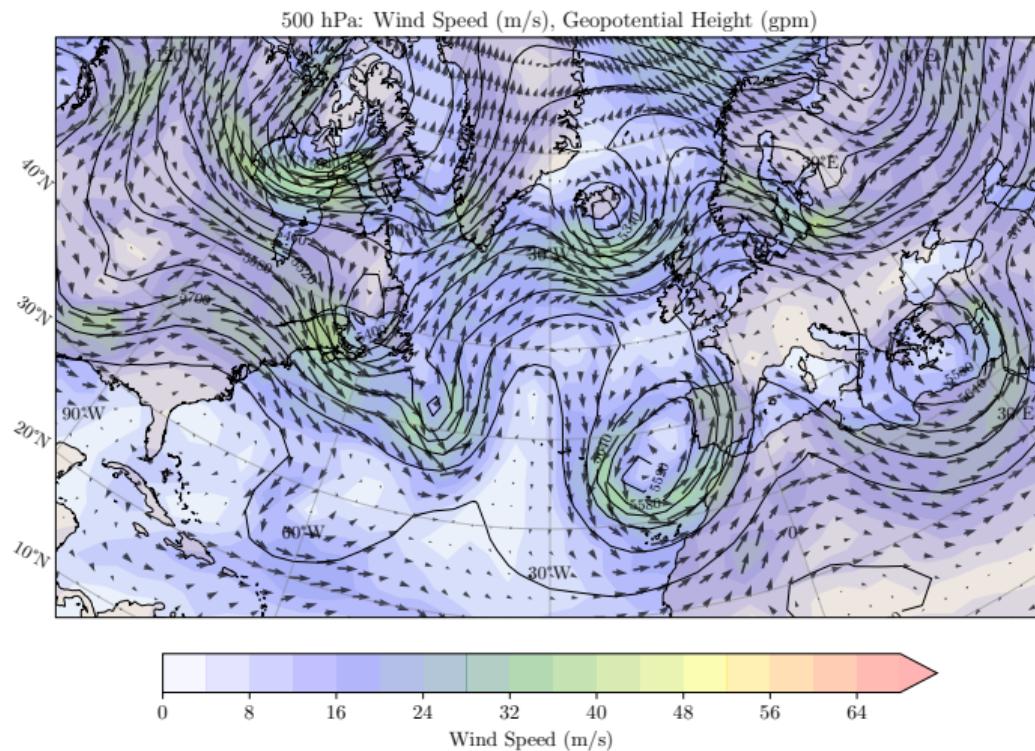
# Forecast: 01-May-2025 at 00:00, Level 500 hPa



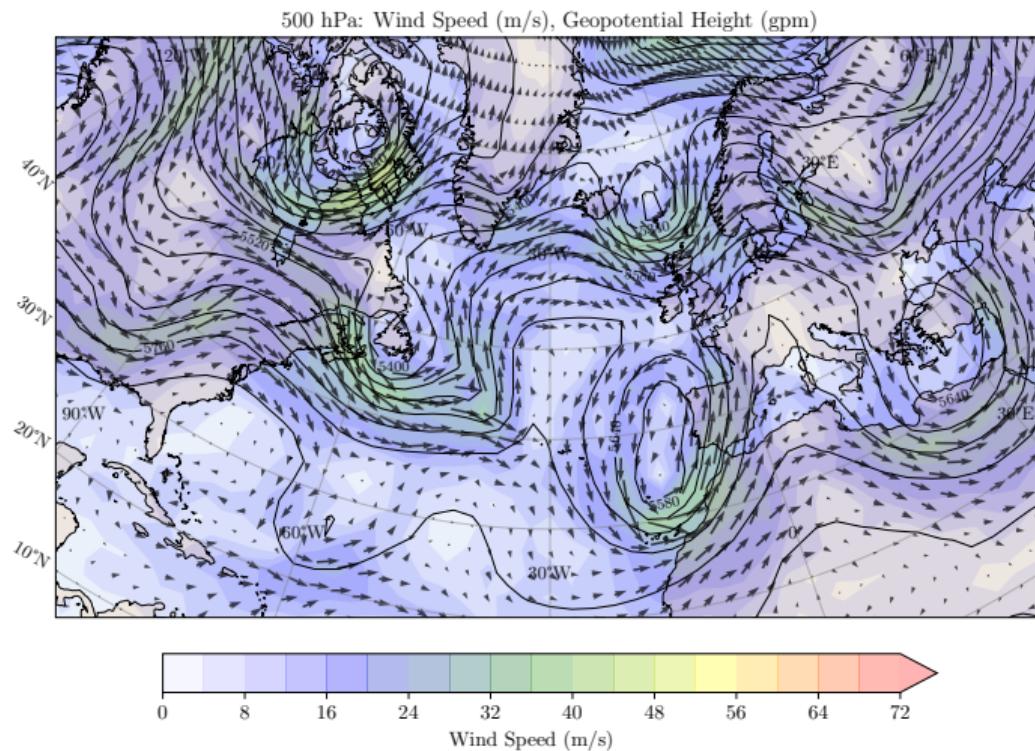
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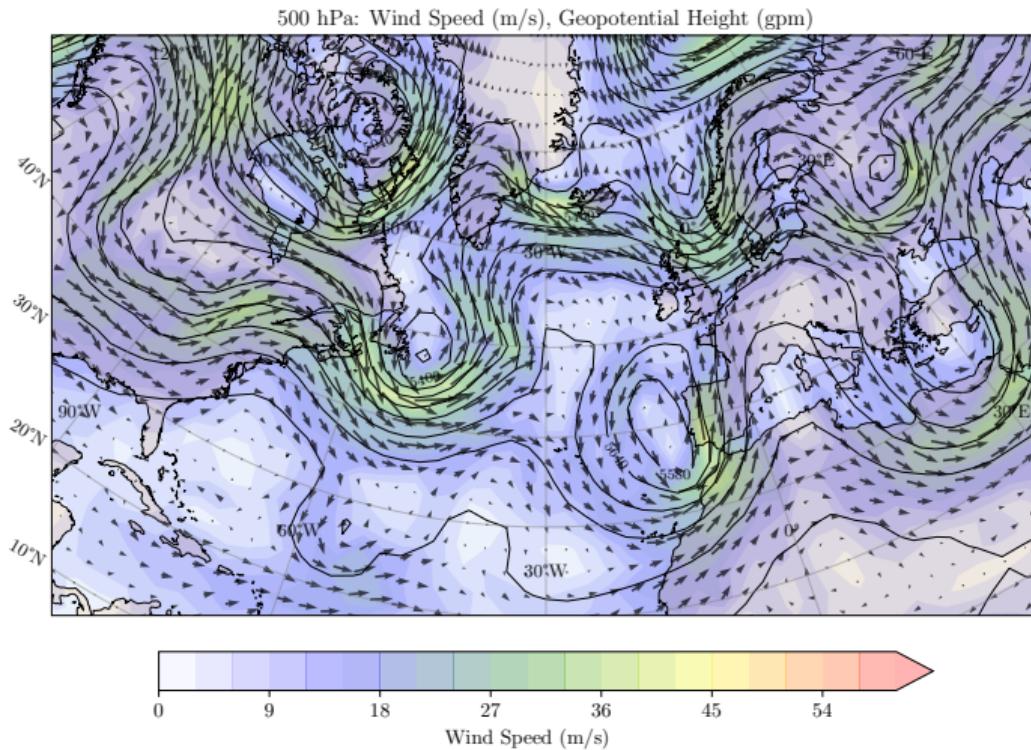
# Forecast: 1-May-2025 at 00:00, Level 500hPa



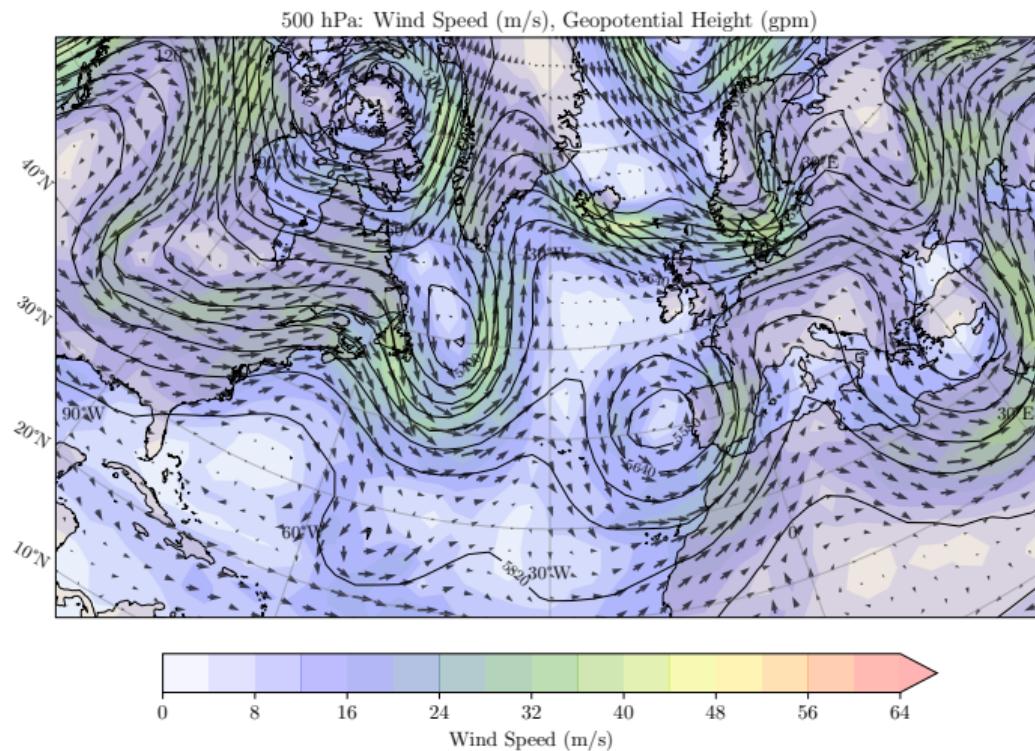
# Forecast: 1-May-2025 at 12:00, Level 500hPa



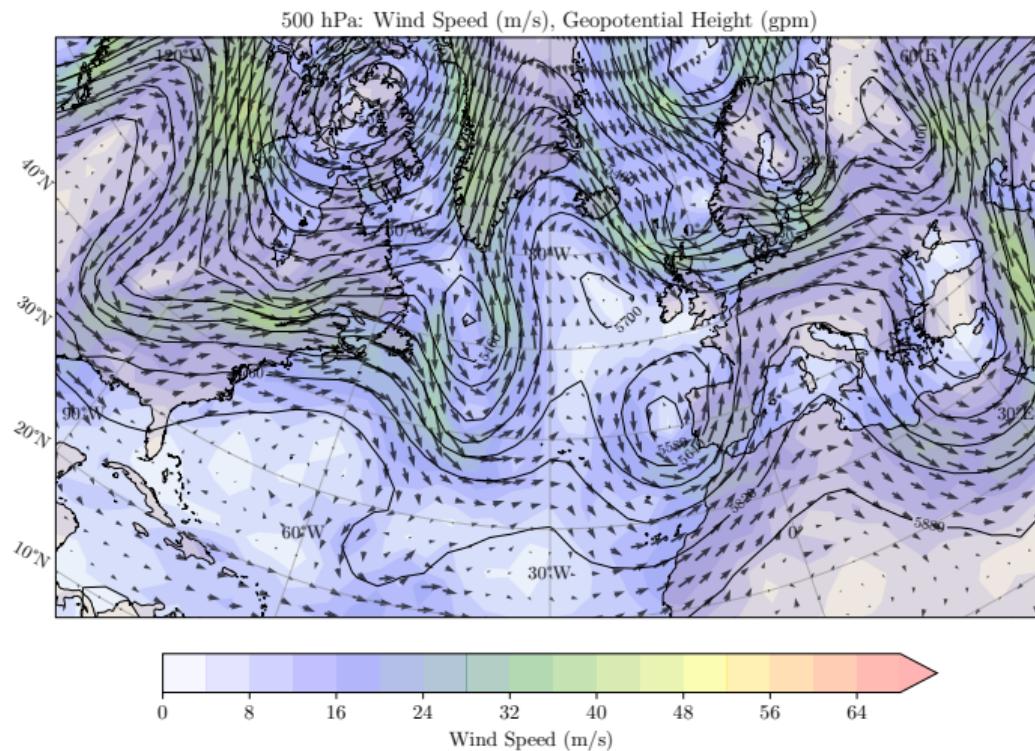
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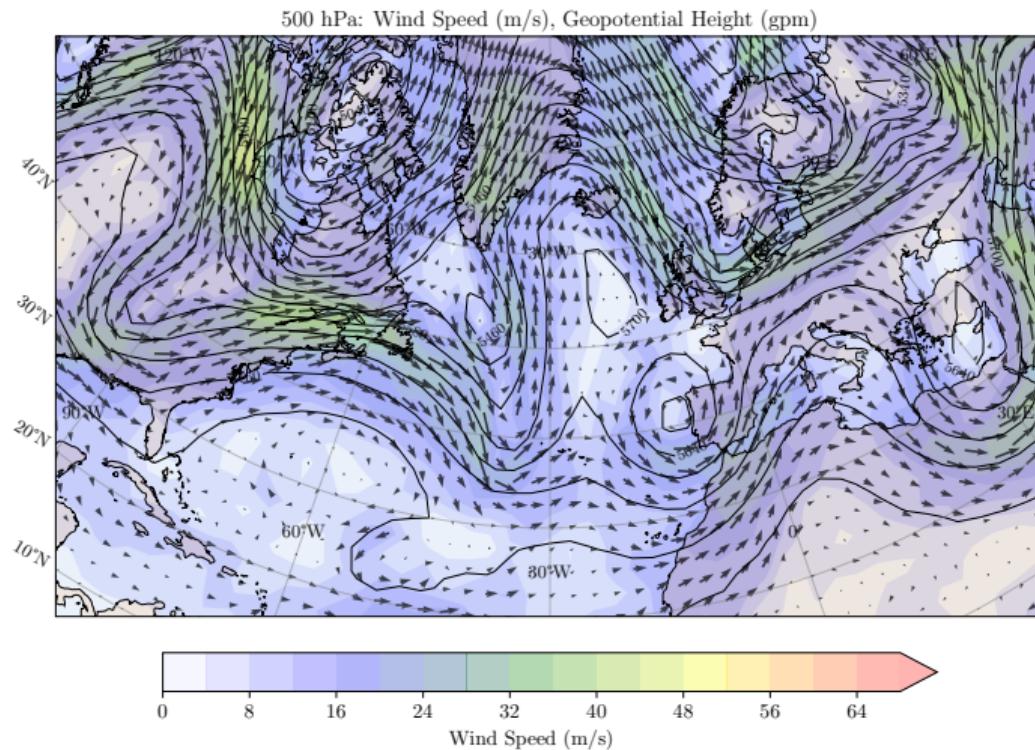
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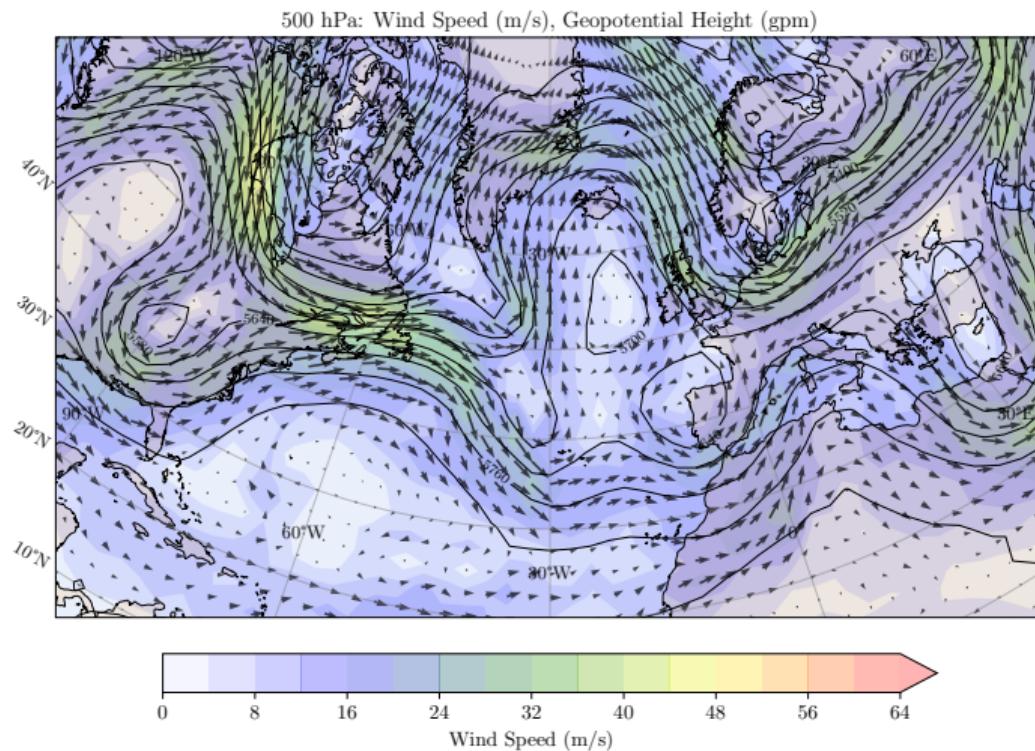
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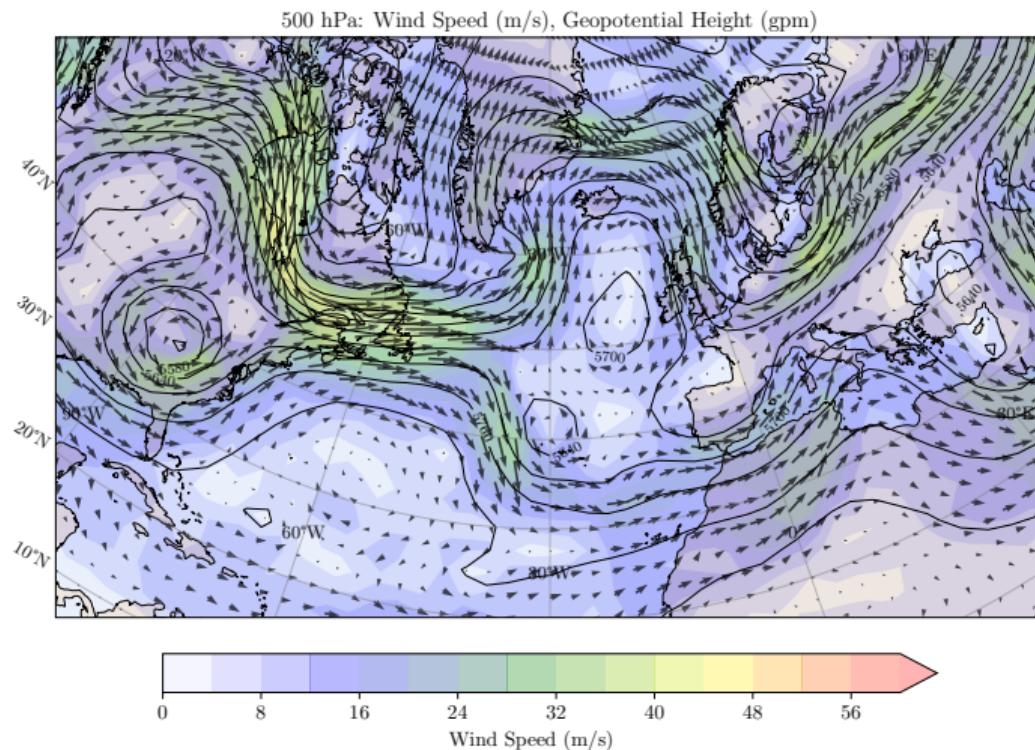
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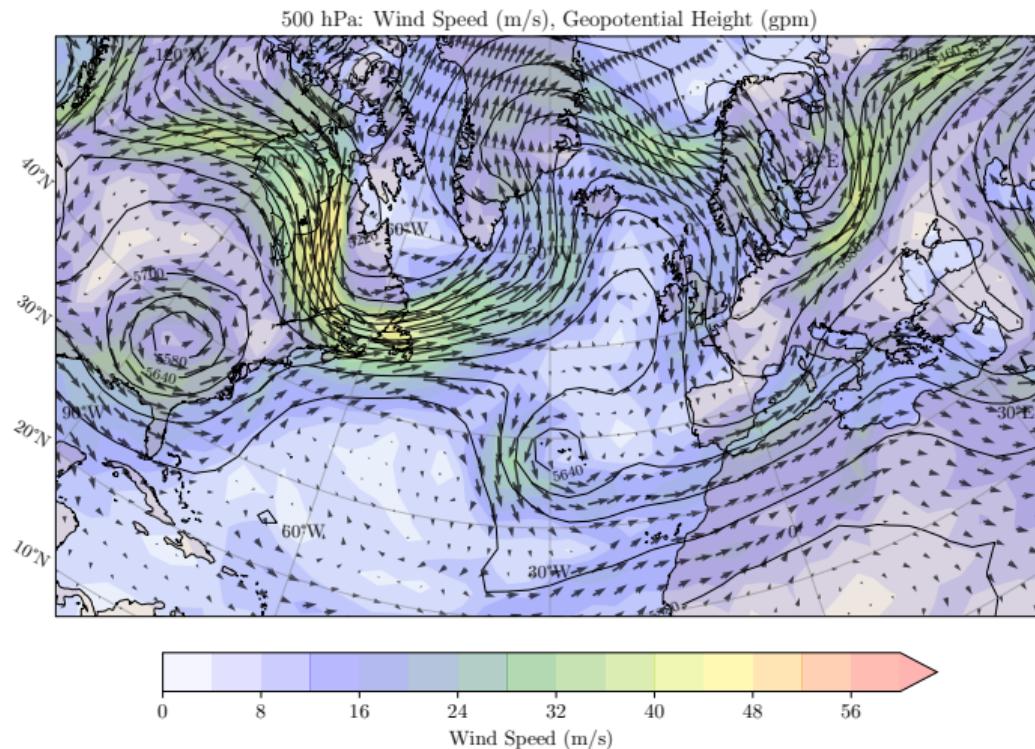
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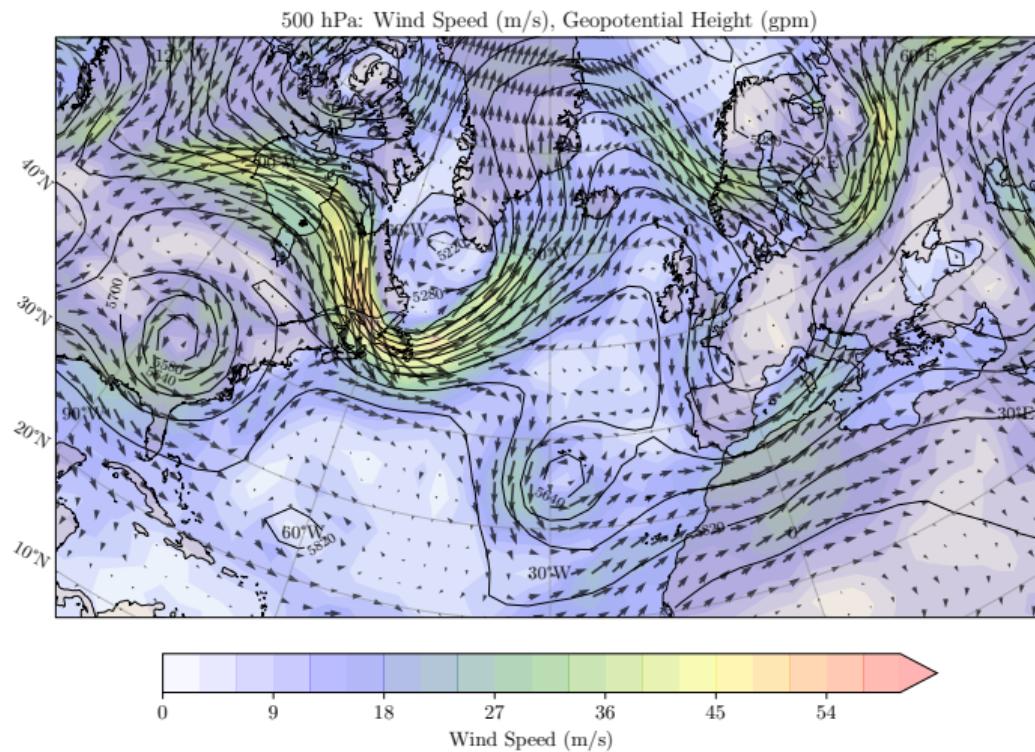
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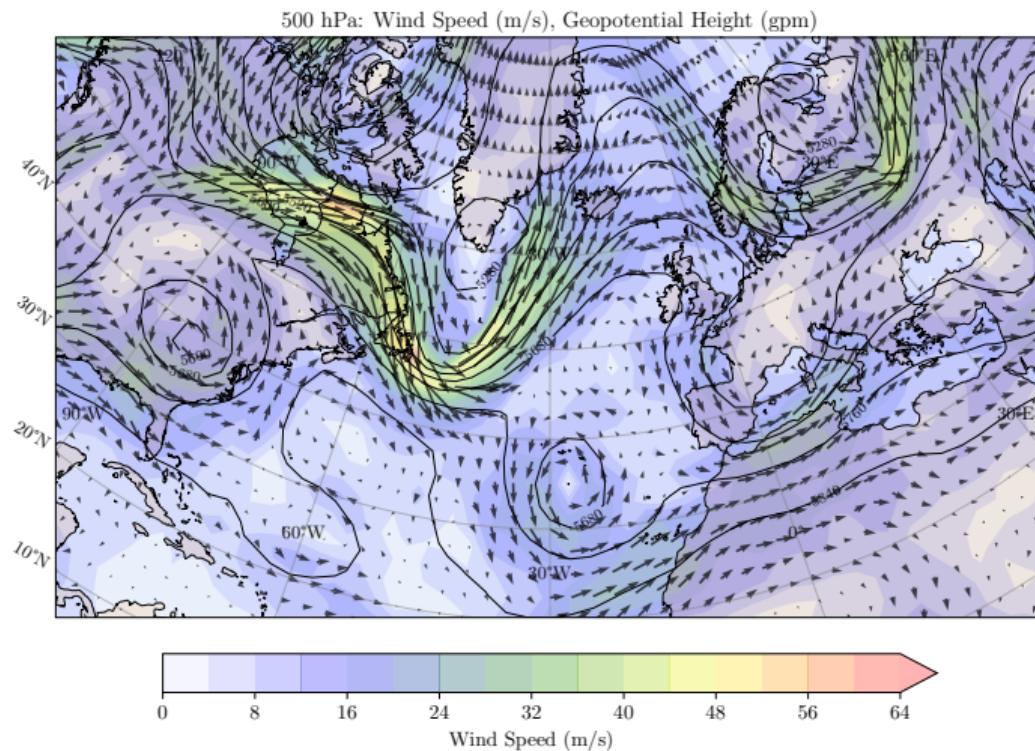
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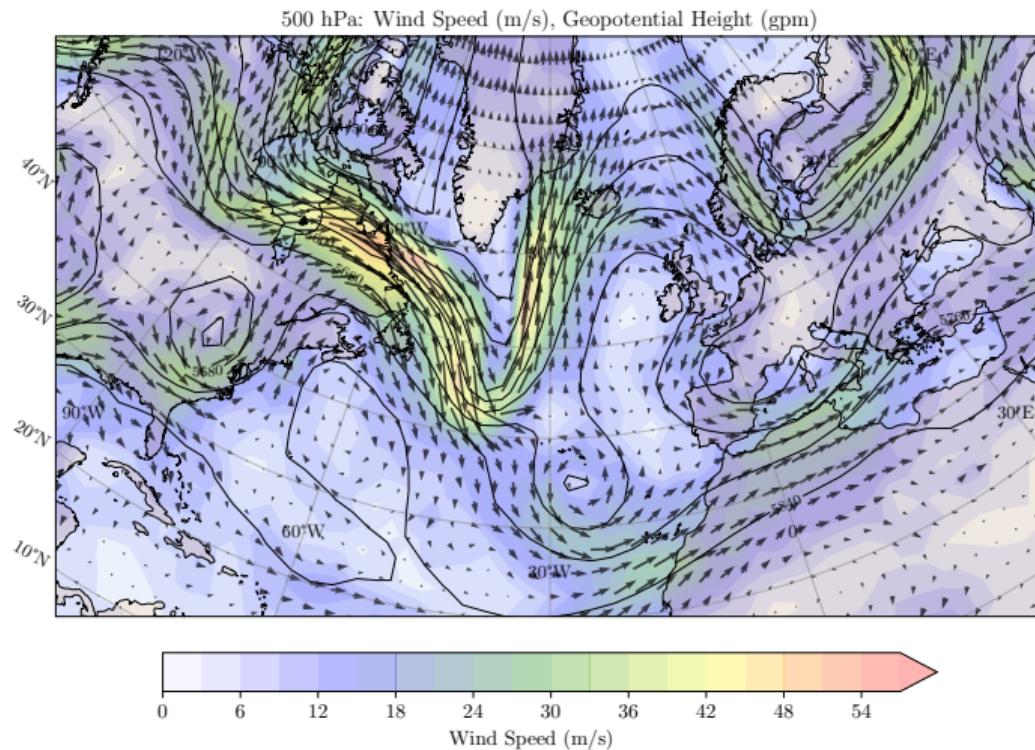
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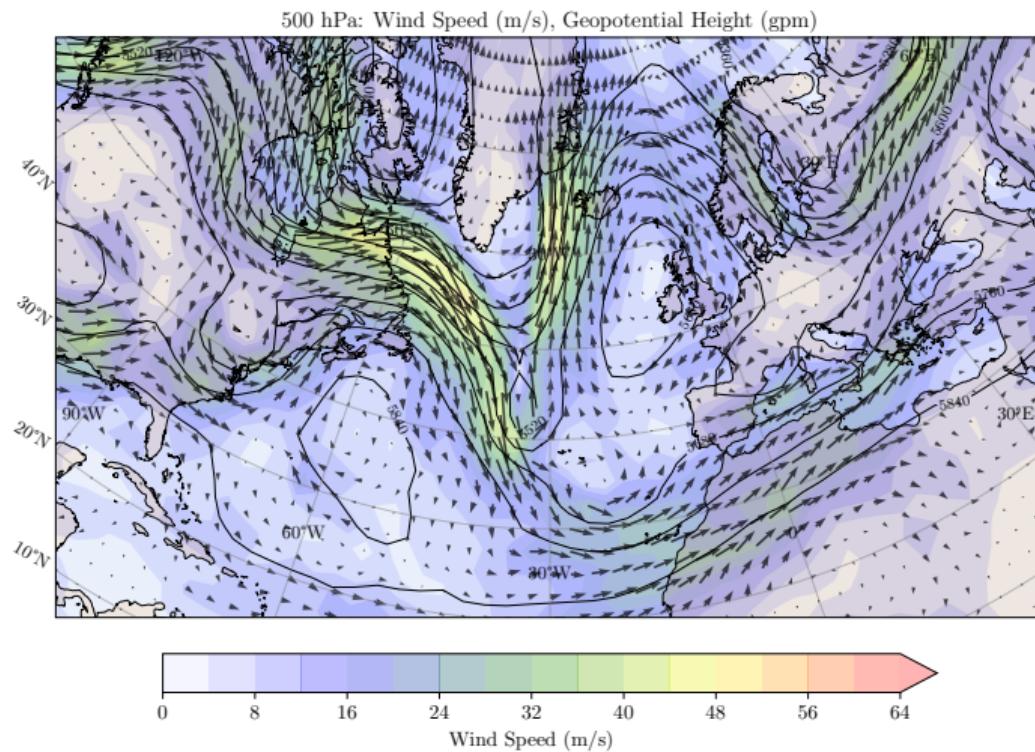
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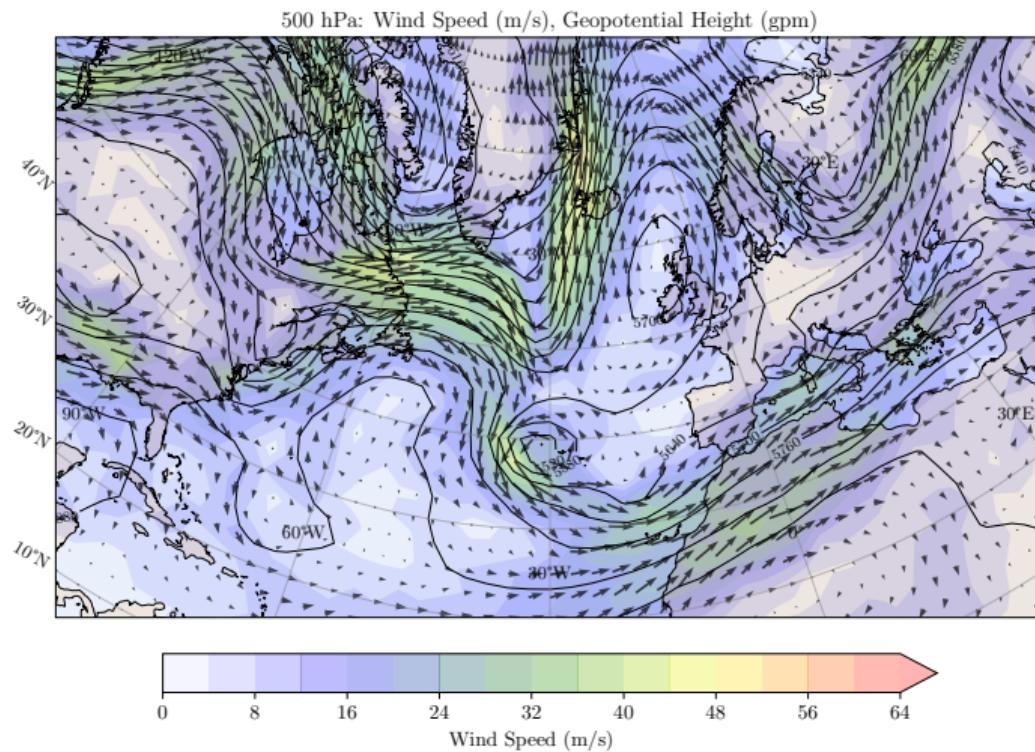
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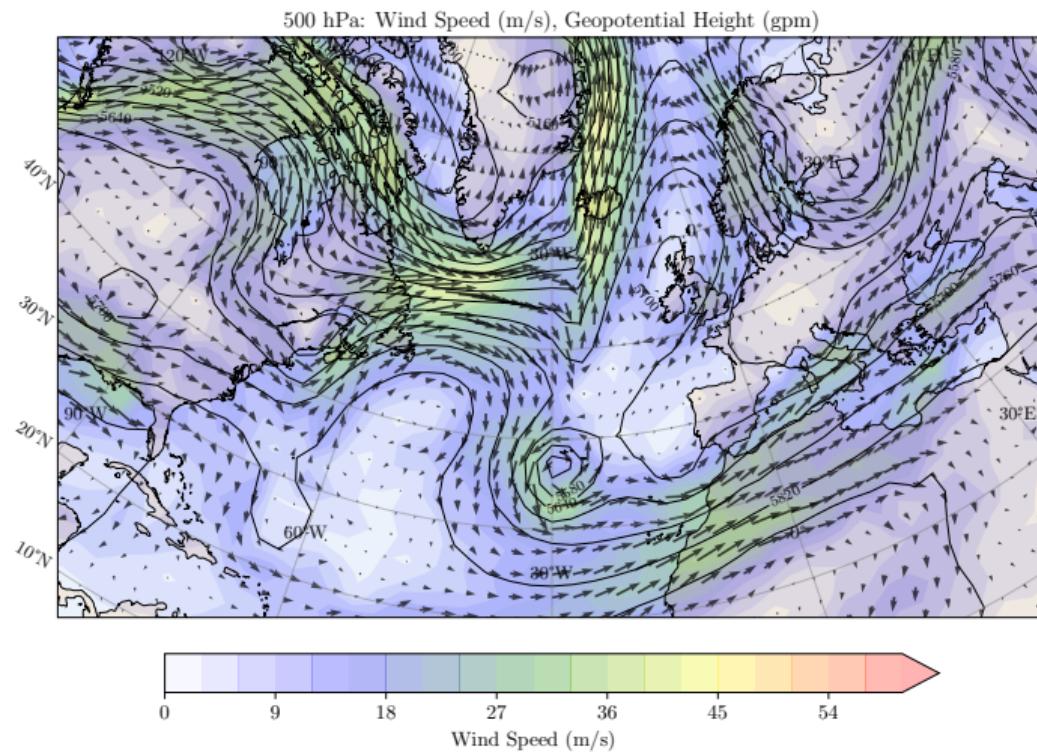
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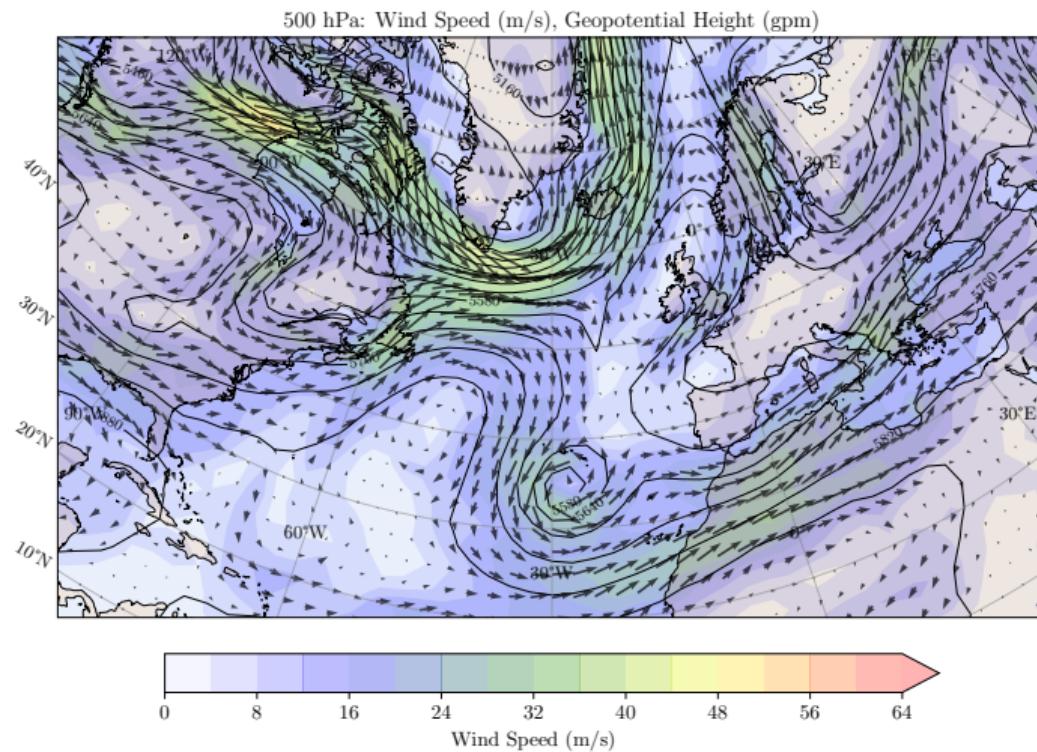
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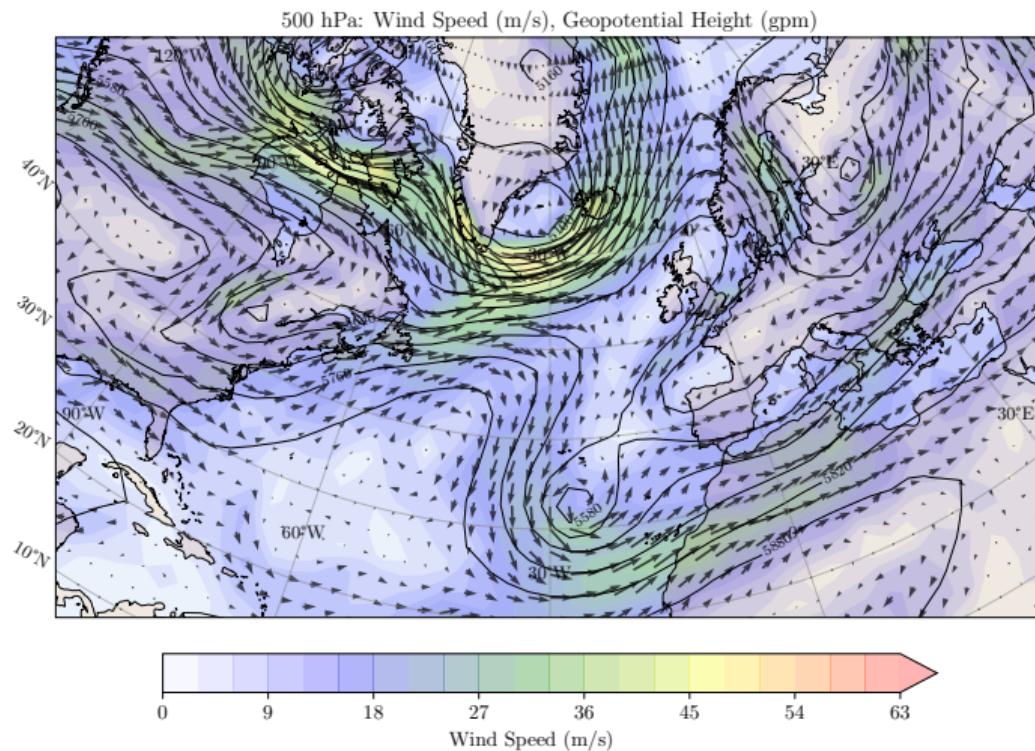
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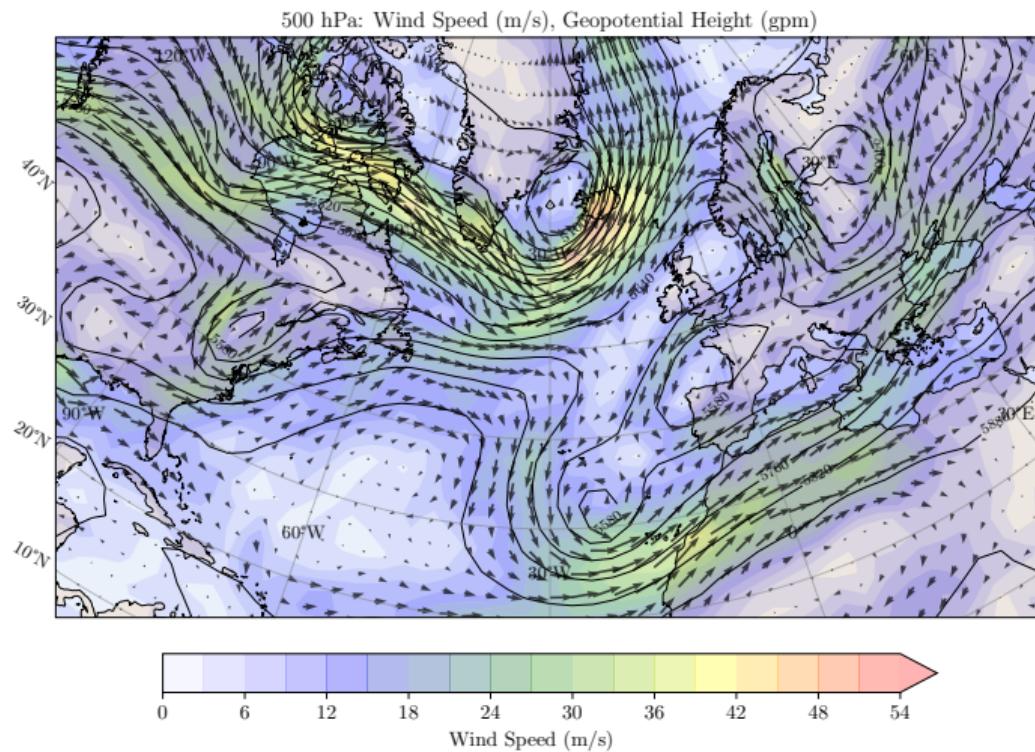
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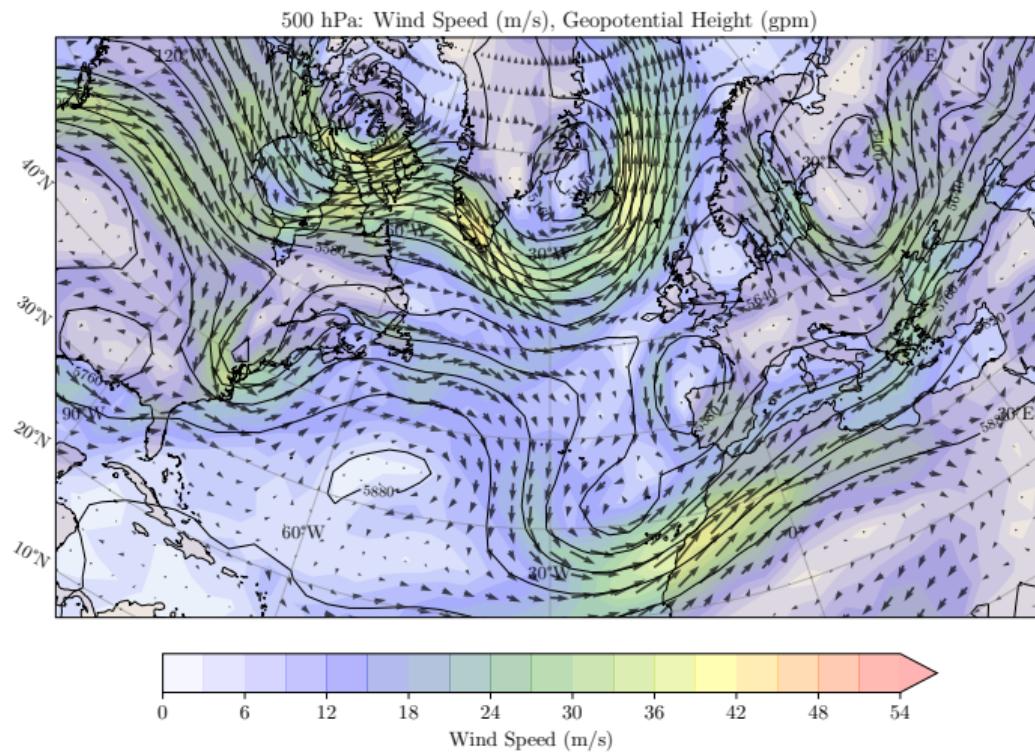
# Forecast: 9-May-2025 at 00:00, Level 500hPa



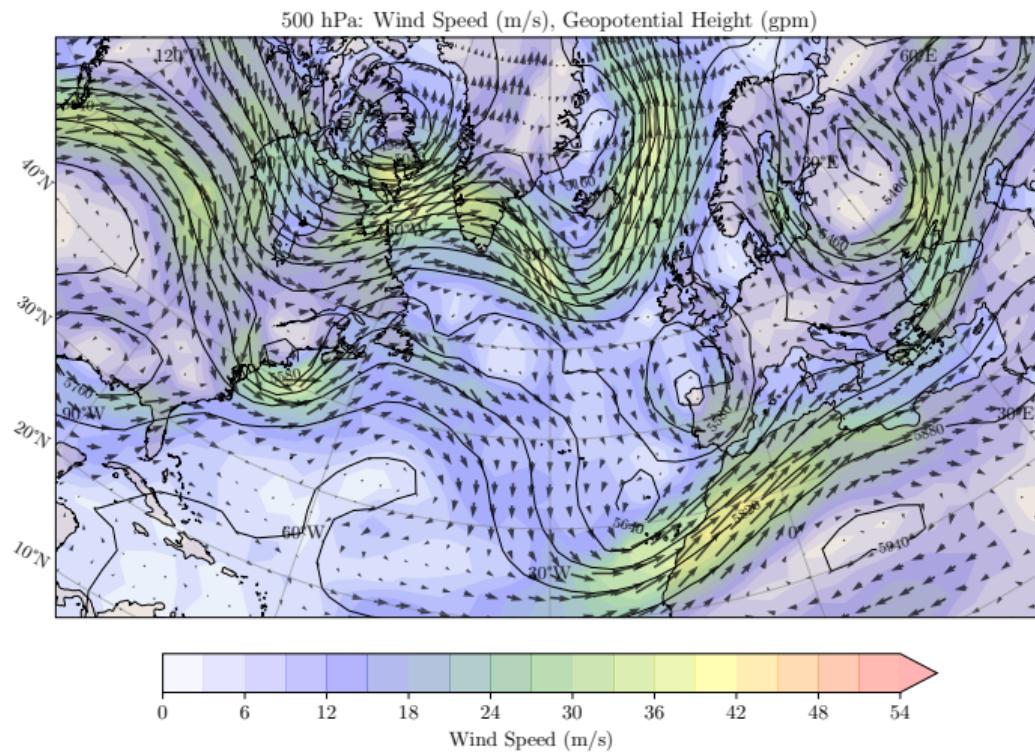
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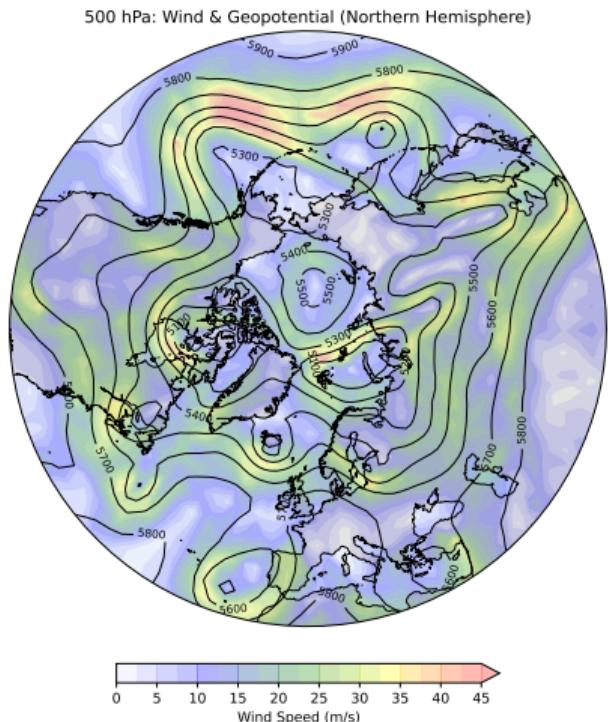
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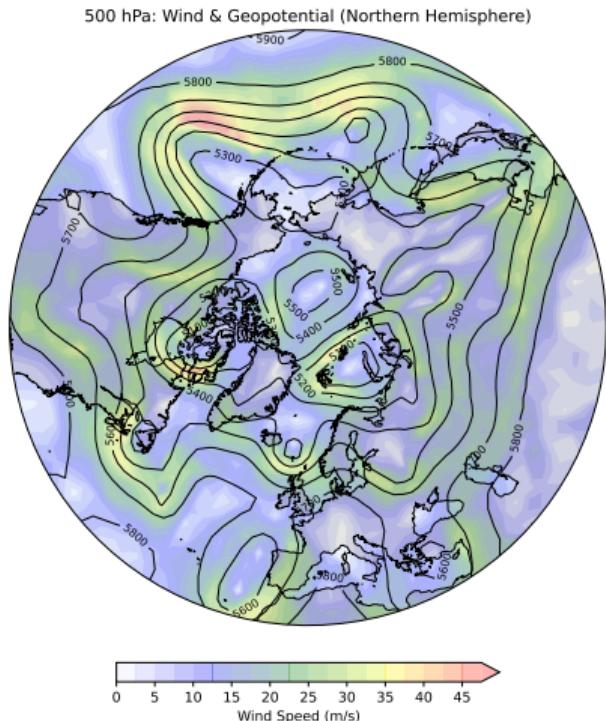
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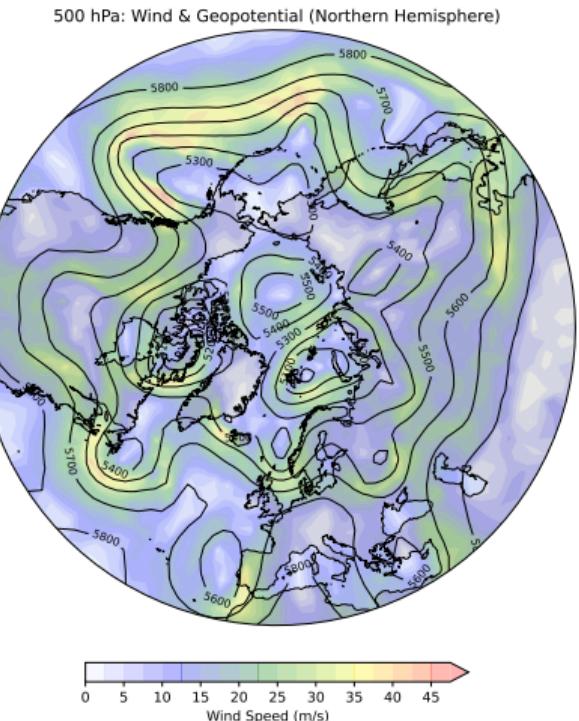
# Forecast: 1-May-2025 at 00:00, Level 500hPa



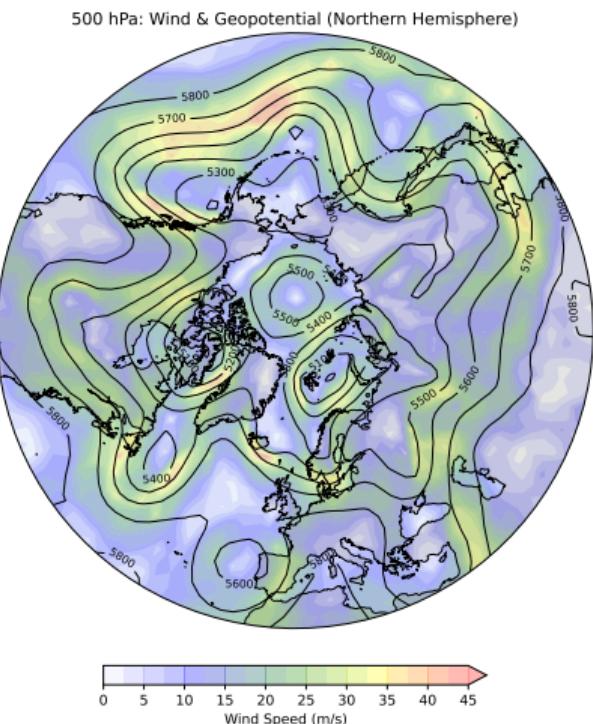
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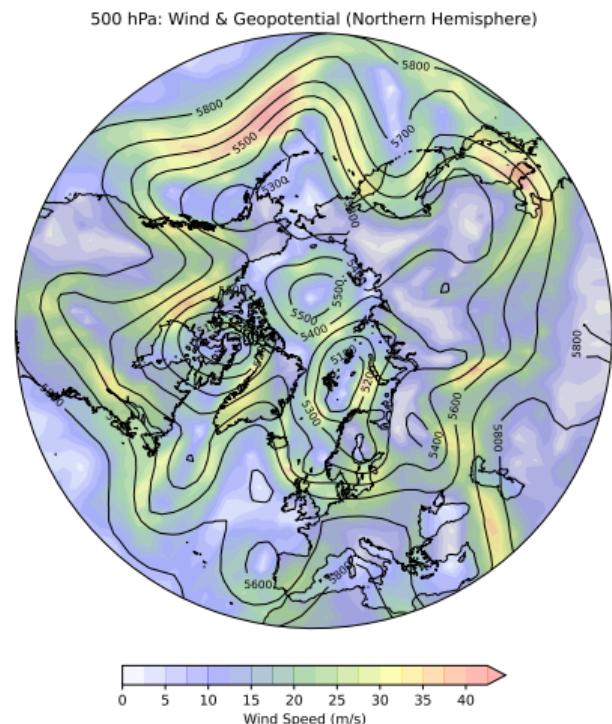
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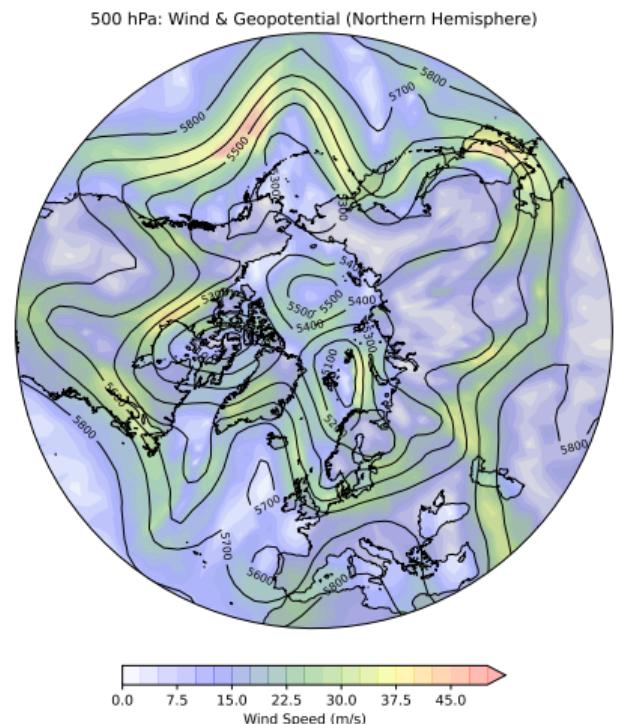
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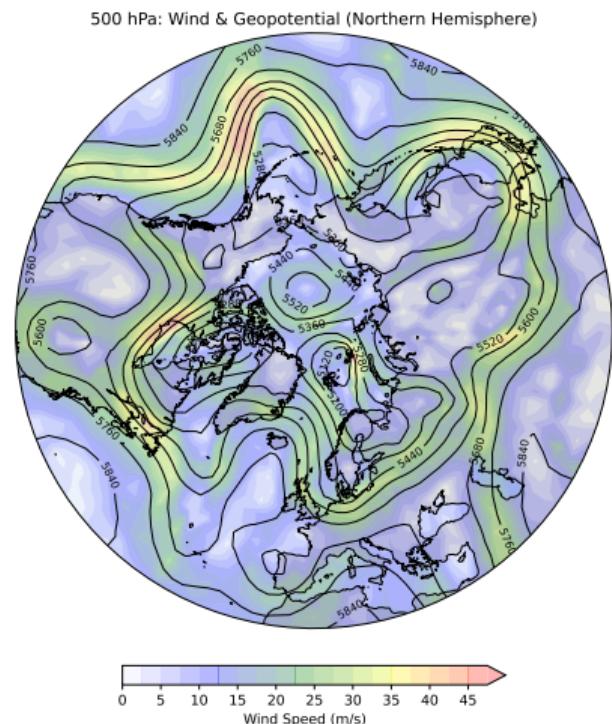
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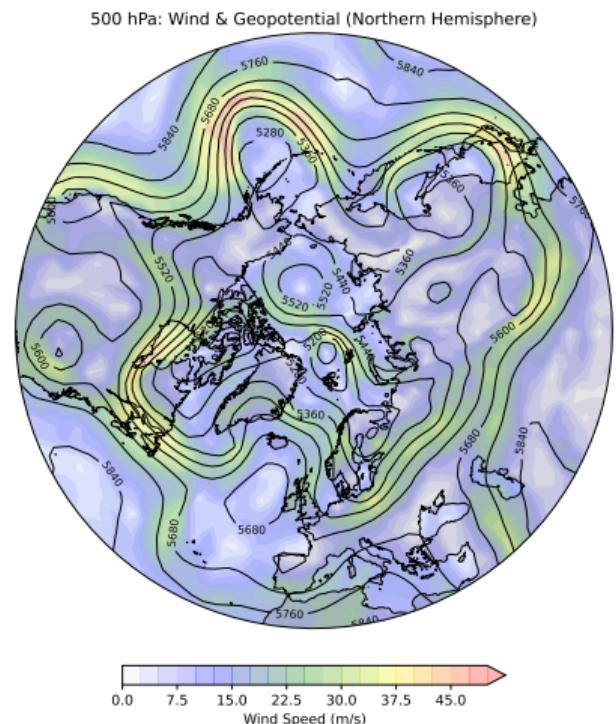
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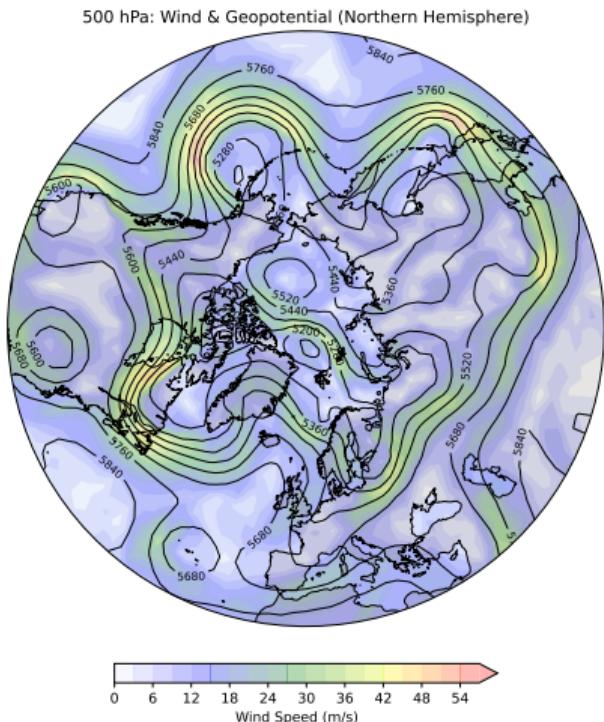
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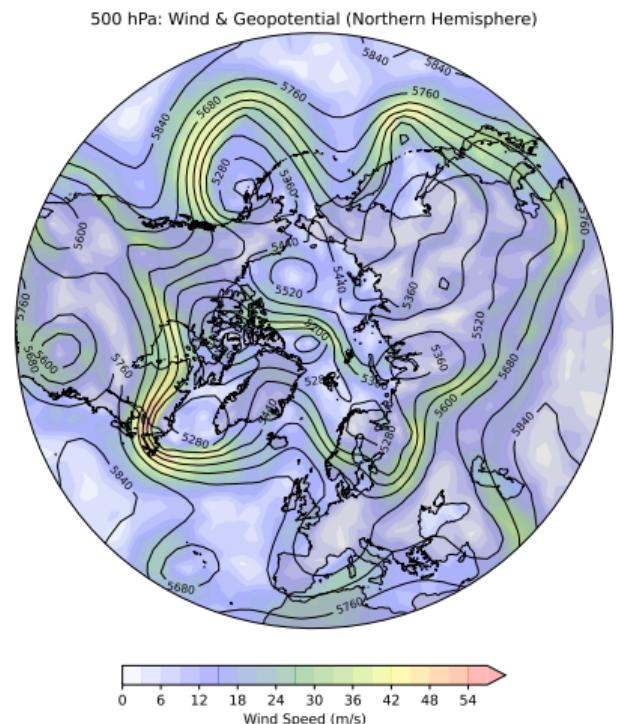
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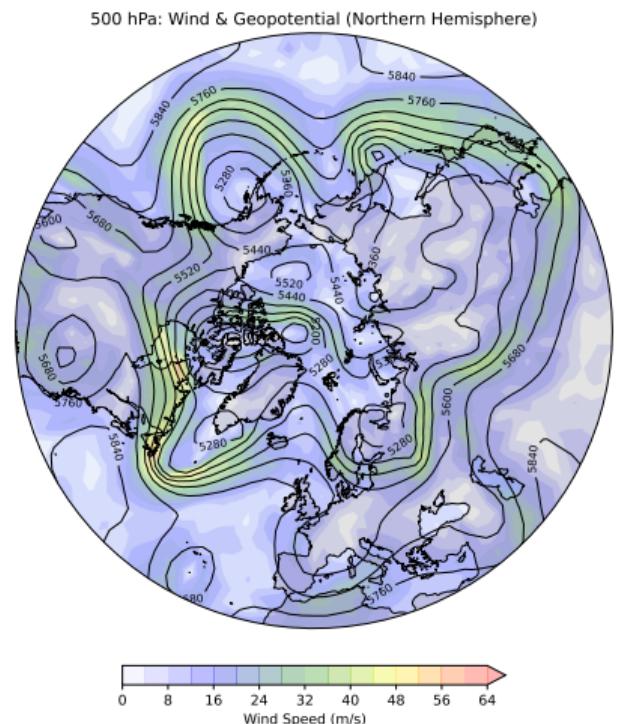
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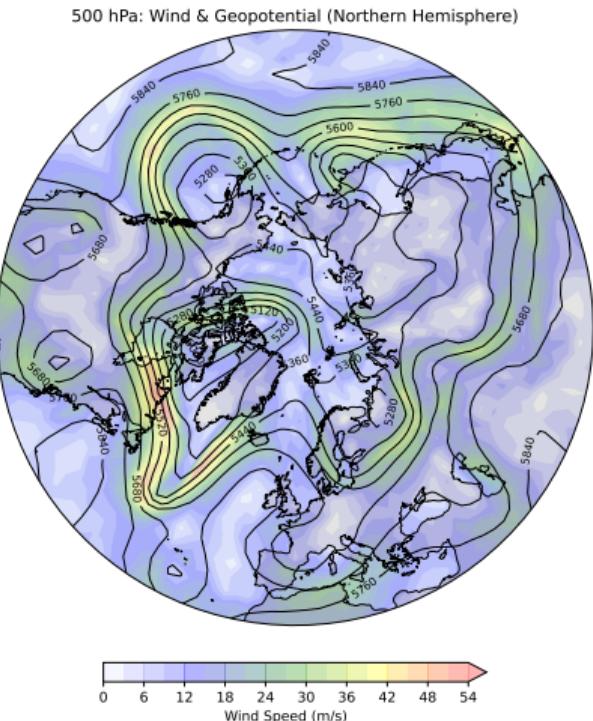
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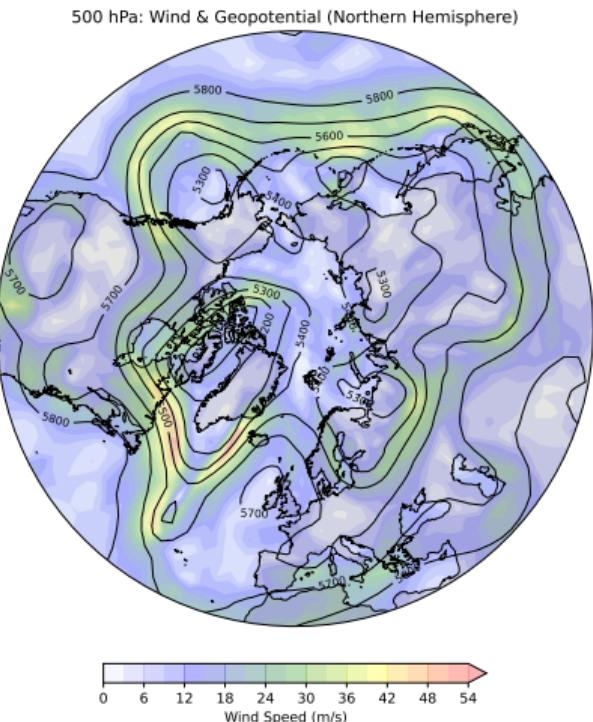
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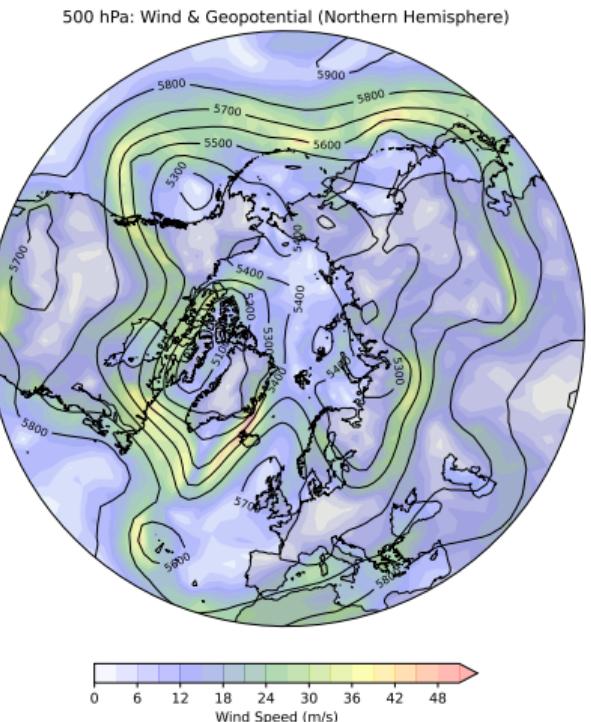
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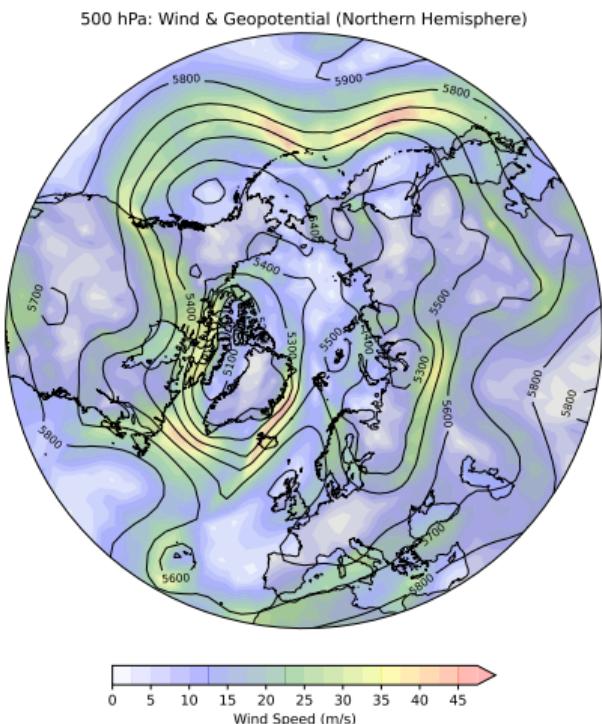
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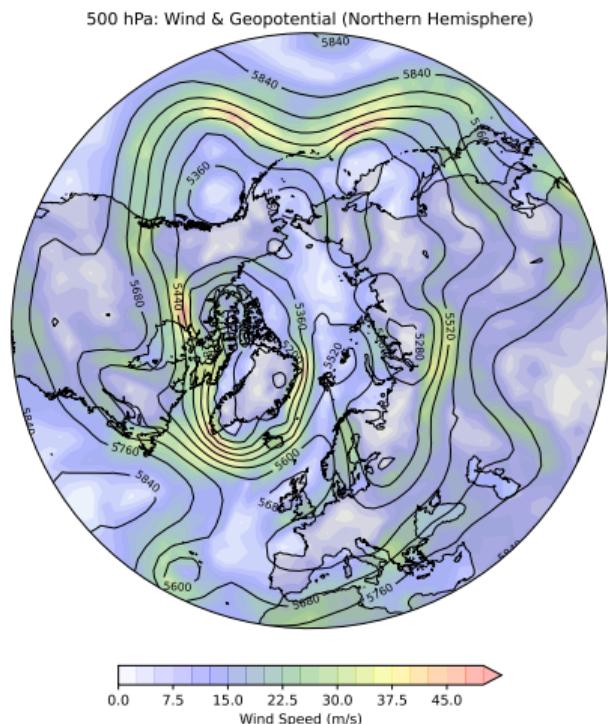
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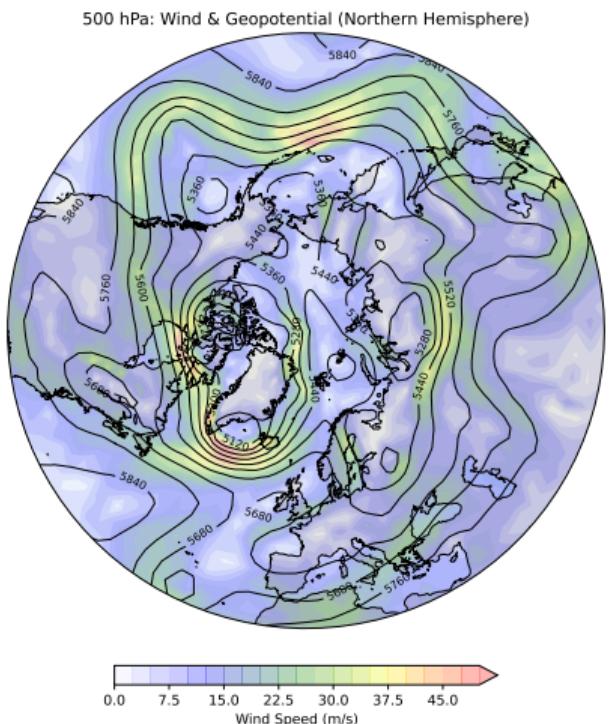
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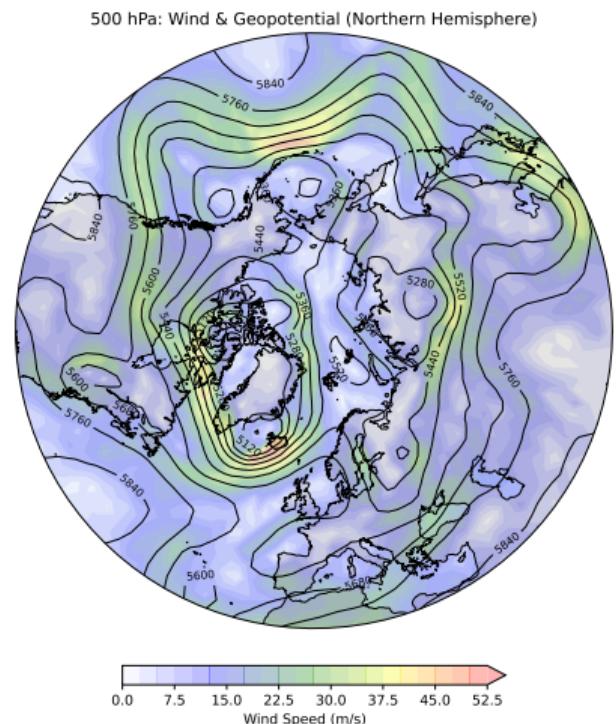
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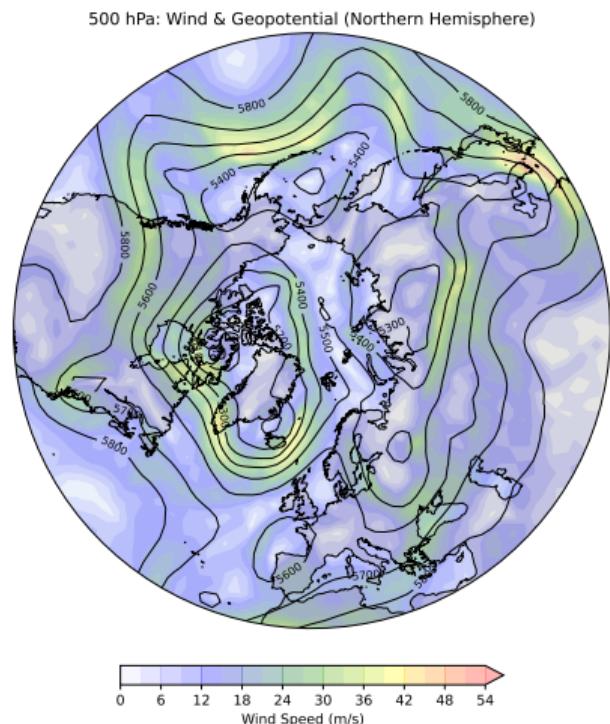
Forecast: 9-May-2025 at 00:00, Level 500hPa



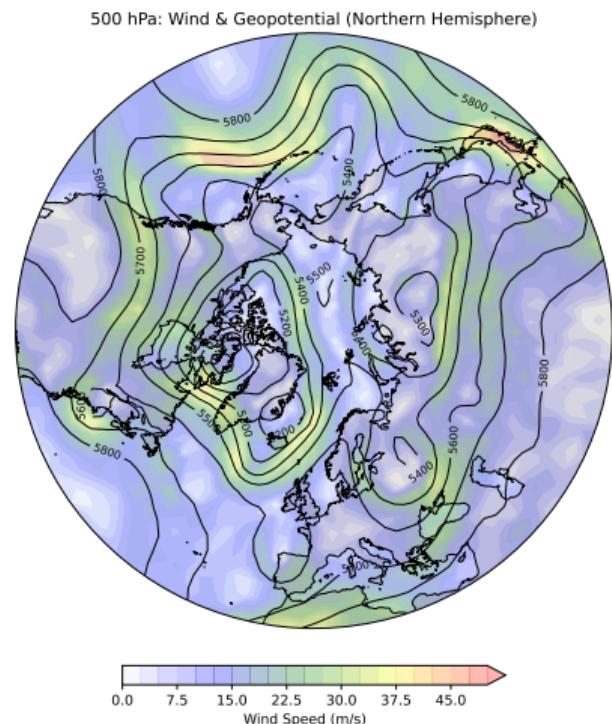
Forecast: 9-May-2025 at 12:00, Level 500hPa



Forecast: 10-May-2025 at 00:00, Level 500hPa

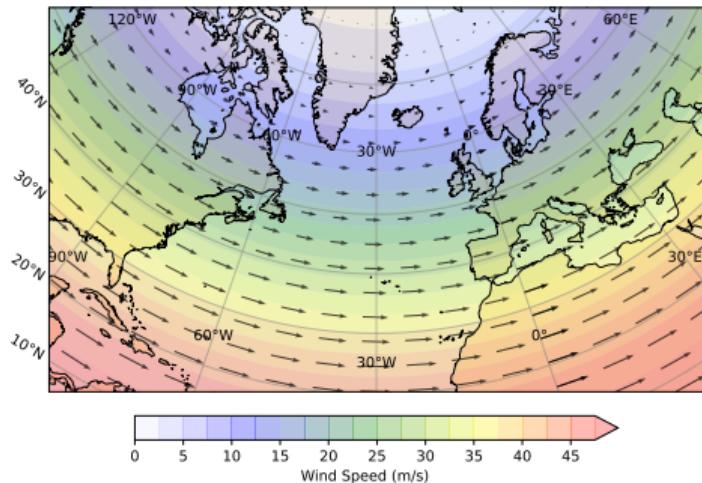


Forecast: 10-May-2025 at 12:00, Level 500hPa



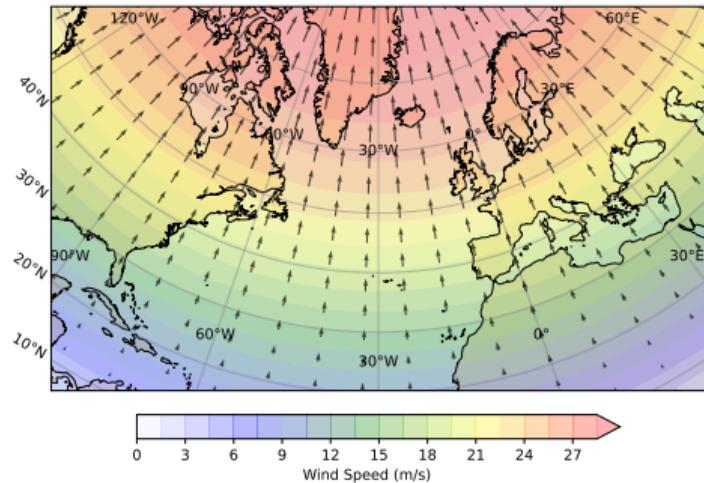
# Idealisiertes zonales Windfeld

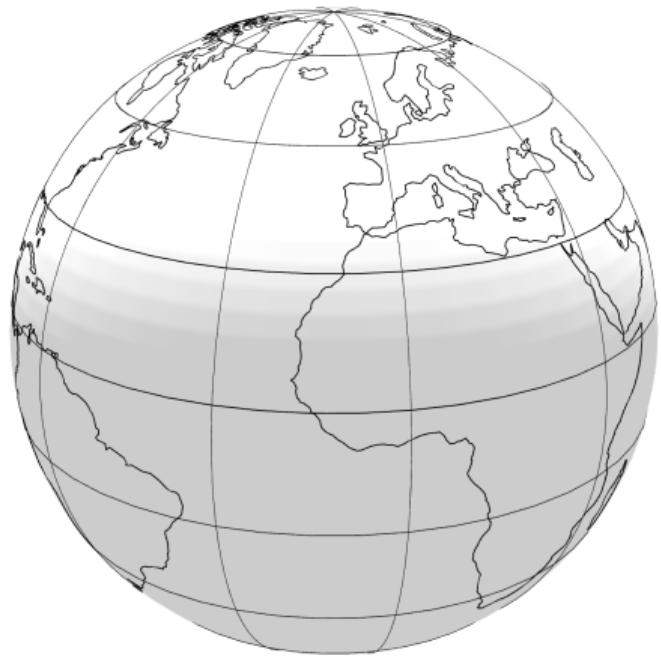
- Gezeigt ist ein simuliertes **zonales Windfeld**:
  - Reine Ost-West-Strömung ( $v = 0$ )
  - Geschwindigkeit abhängig von der Breite:  
 $u = U_0 \cdot \sin^2(\theta)$
  - Keine Druckgradienten oder vertikale Struktur
- Dieses idealisierte Feld dient als Basis zur Untersuchung großskaliger atmosphärischer Prozesse.
- Zonale Winde treten z. B. in der Realität als **Jetstreams** auf.



# Idealisiertes meridionales Windfeld

- Gezeigt ist ein simuliertes **meridionales Windfeld**:
  - Reine Nord-Süd-Strömung ( $u = 0$ )
  - Geschwindigkeit abhängig von der Breite:
$$v = V_0 \cdot \cos(\theta)$$
- Die Darstellung ist idealisiert, ohne Rückkopplung durch Coriolis oder Druckgradienten.















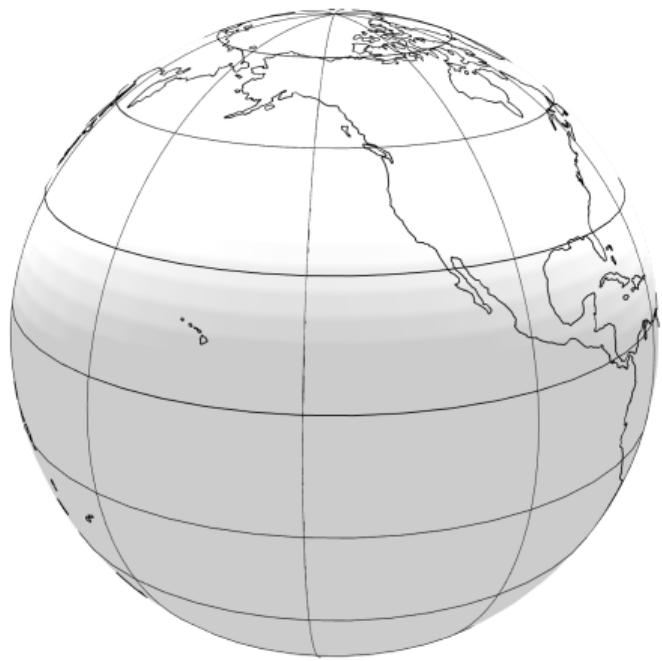


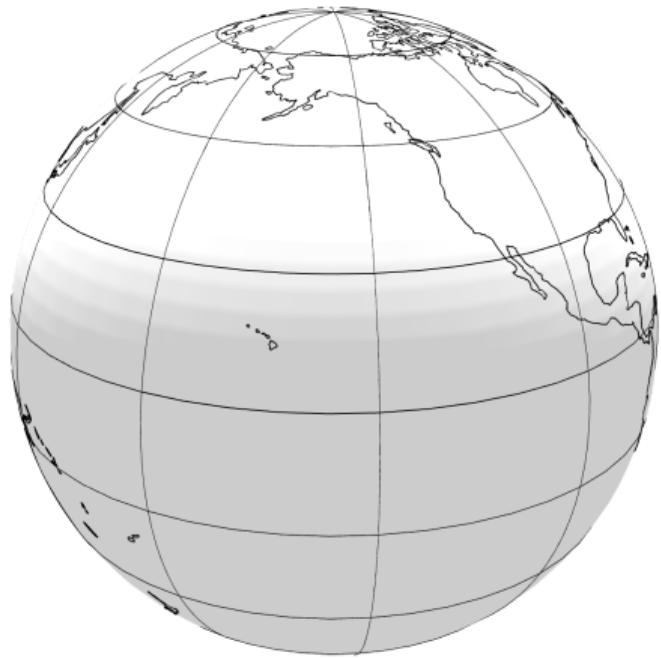


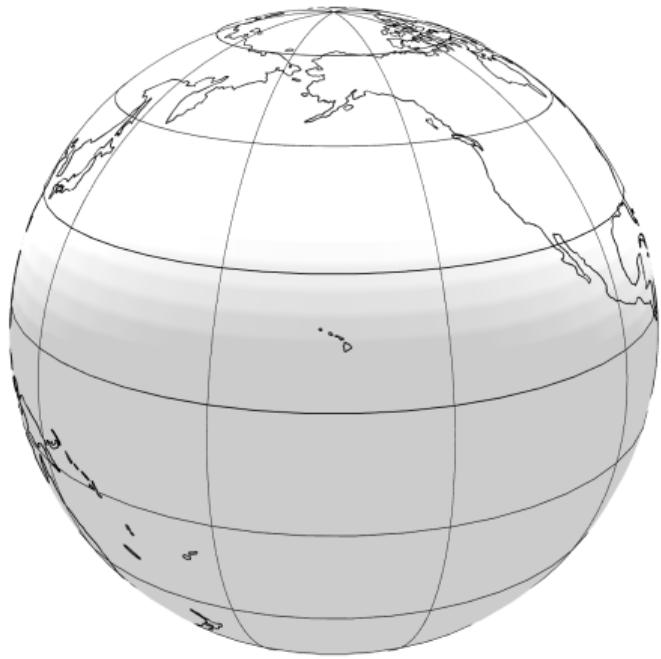


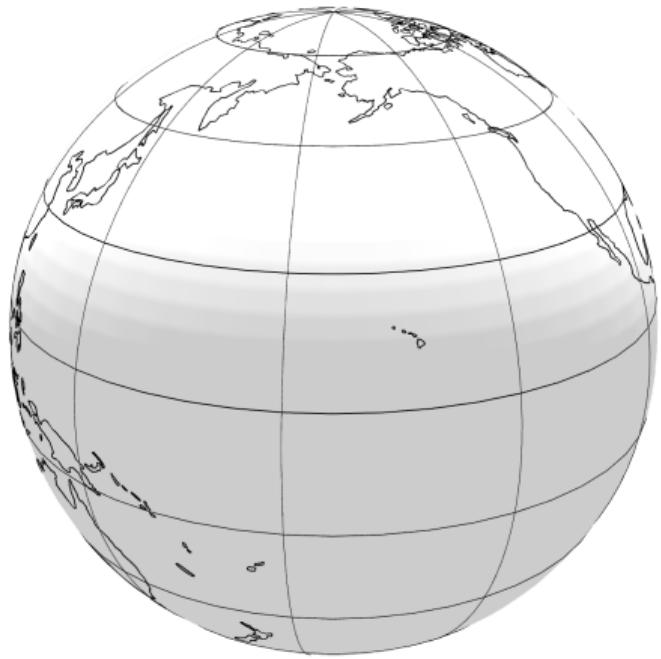


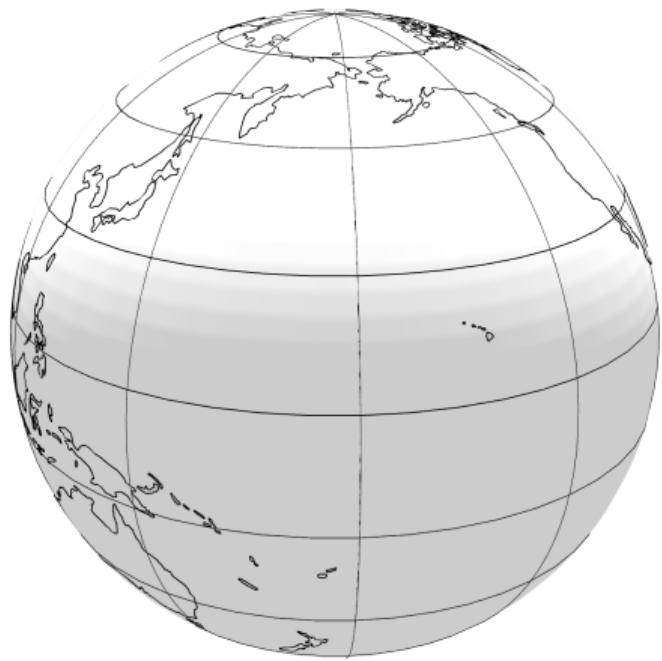










































# Corioliskraft: Grundprinzip

- Die **Corioliskraft** ist eine Scheinkraft, die in rotierenden Bezugssystemen wie der Erde wirkt.
- Sie verursacht eine Ablenkung von bewegten Luft- und Wassermassen:
  - **Nordhalbkugel:** Ablenkung nach rechts
  - **Südhalbkugel:** Ablenkung nach links
- Maximale Wirkung an den Polen, null am Äquator.

# Mathematische Formulierung

$$\vec{F}_C = -2m(\vec{\Omega} \times \vec{v})$$

- $m$ : Masse des Körpers
- $\vec{\Omega}$ : Rotationsvektor der Erde
- $\vec{v}$ : Geschwindigkeit relativ zur Erdoberfläche

# Breitenabhängigkeit und Coriolis-Parameter

- Der **Coriolis-Parameter** beschreibt die Breitenabhängigkeit:

$$f = 2\Omega \sin(\phi)$$

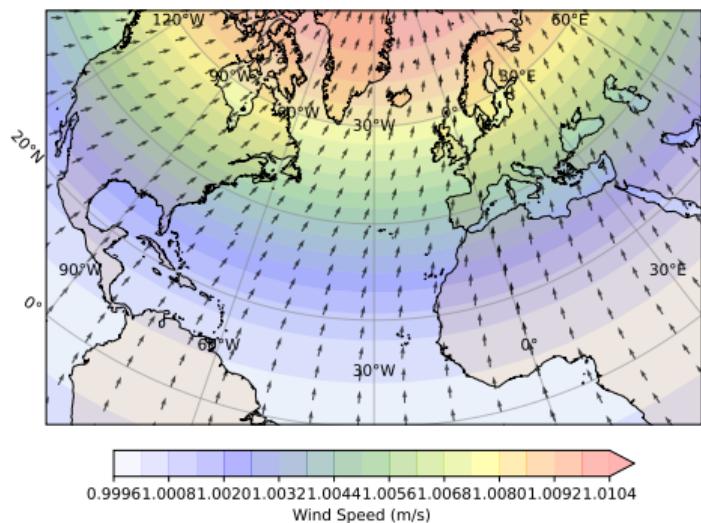
- Seine Änderung mit der Breite ergibt den  **$\beta$ -Parameter**:

$$\beta = \frac{\partial f}{\partial y} = \frac{2\Omega \cos(\phi)}{a}$$

- $a$ : Erdradius,     $y$ : Nord-Süd-Koordinate

# Simulation der Corioliskraft auf Nordströmung

- Darstellung: Ablenkung von Luftpaketen bei rein meridionaler Startgeschwindigkeit ( $v = 1, u = 0$ )
- Nach `steps = 100` Zeitschritten:
  - Auf Nordhalbkugel: Ablenkung nach Osten
  - Auf Südhalbkugel: Ablenkung nach Westen
- Breitenabhängigkeit durch  $f = 2\Omega \sin(\phi)$



# Numerische Simulation der Corioliskraft

- **Rotation der Erde:**

$$\Omega = 7.2921 \cdot 10^{-5} \text{ rad/s}, \quad f = 2\Omega \sin(\phi)$$

Der Coriolisparameter  $f$  steigt mit der geographischen Breite  $\phi$ , ist bei  $\phi = 0^\circ$  null.

- **Initialisierung:**

$$v = 1 \text{ (Nord)}, \quad u = 0 \text{ (Ost)}, \quad dx = dy = 0$$

Alle Luftpakete starten mit rein meridionaler Geschwindigkeit.

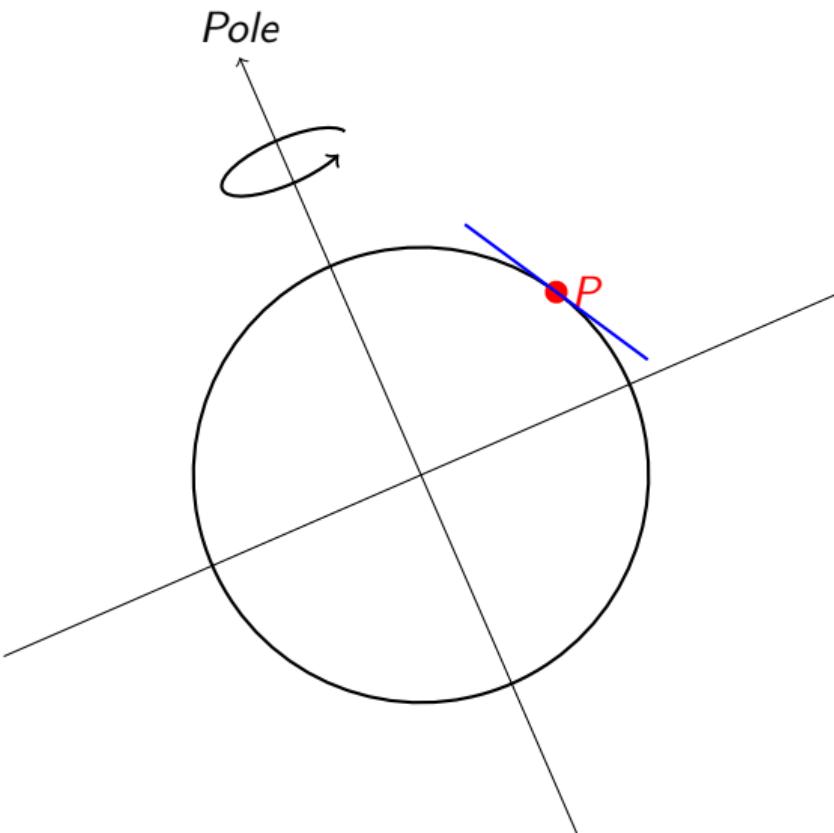
- **Zeitschritte:** (Euler-Verfahren)

$$u_{t+1} = u_t + f \cdot v_t \cdot \Delta t, \quad v_{t+1} = v_t - f \cdot u_t \cdot \Delta t$$

$$dx+ = u_{t+1} \cdot \Delta t, \quad dy+ = v_{t+1} \cdot \Delta t$$

Die Bewegung wird durch die Corioliskraft kontinuierlich abgelenkt.

# Tilted Earth with 3D Spin



# Was ist die $\beta$ -Ebene?

- Die  **$\beta$ -Ebene** ist eine lokale Approximation der Erdkugel nahe einer bestimmten Breite  $\phi_0$ .
- Ziel: Vereinfachung der Corioliskraft für mathematische Modelle großskaliger Strömungen.
- Der Coriolisparameter  $f$  wird linearisiert:

$$f(y) = f_0 + \beta y$$

mit:

- $f_0 = 2\Omega \sin(\phi_0)$ : Coriolisparameter an der Referenzbreite
- $\beta = \left. \frac{\partial f}{\partial y} \right|_{\phi_0} = \frac{2\Omega \cos(\phi_0)}{a}$
- $y$ : meridionale Entfernung vom Referenzbreitenkreis

# Was ist Vortizität?

- **Vortizität** beschreibt die lokale Rotation in einer Strömung.
- Definiert als das **Rotationsfeld** des Geschwindigkeitsvektors:

$$\vec{\zeta} = \nabla \times \vec{u}$$

- Für eine zweidimensionale Strömung  $\vec{u} = (u(x, y), v(x, y))$  ist nur die z-Komponente relevant:

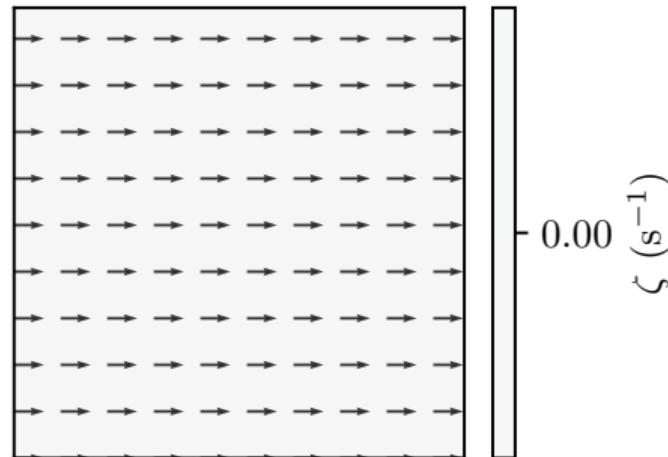
$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

- $\zeta > 0$ : Zyklonale Rotation (gegen den Uhrzeigersinn)
- $\zeta < 0$ : Antizyklonale Rotation (im Uhrzeigersinn)

## Zero Vorticity (Uniform Flow)

- $\vec{u} = (2, 0)$
- Uniform horizontal flow
- No shear or curvature
- $\zeta = 0$

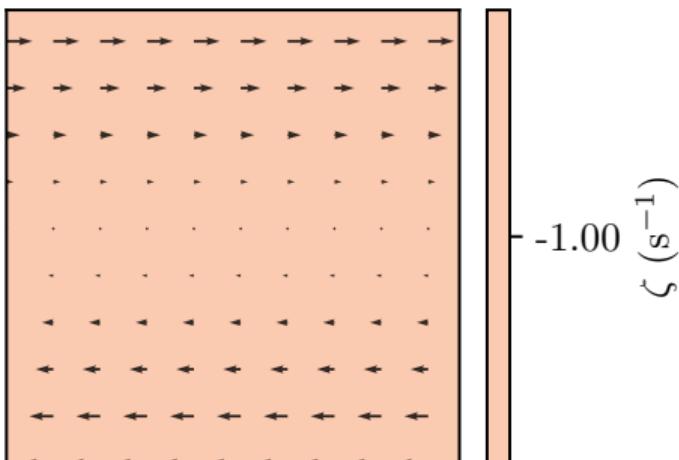
## Zero Vorticity (Uniform Flow)



# Shear Vorticity

- $\vec{u} = (y, 0)$
- Horizontal shear:  $\frac{\partial u}{\partial y} = 1$
- $\zeta = -1$

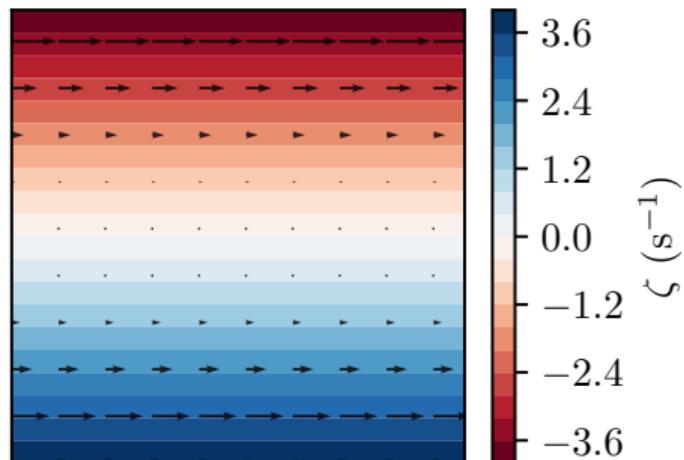
Shear Vorticity



# Nonlinear Shear Vorticity

- $\vec{u} = (y^2, 0)$
- $\zeta = -2y$
- Antisymmetric vorticity field
- Stronger at larger  $|y|$

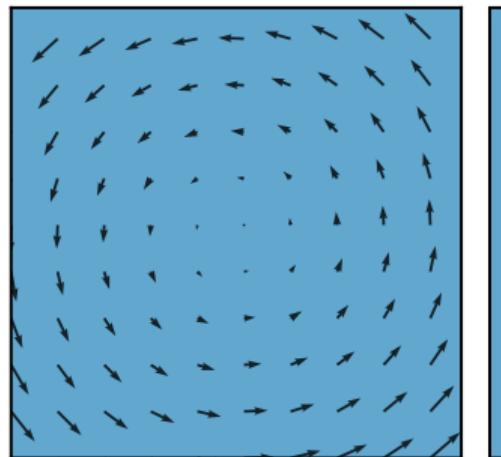
Nonlinear Shear Vorticity



# Positive Vorticity (Cyclonic)

- $\vec{u} = (-y, x)$
- Pure rotation, counter-clockwise
- $\zeta = 2$

Positive Vorticity (Cyclonic)

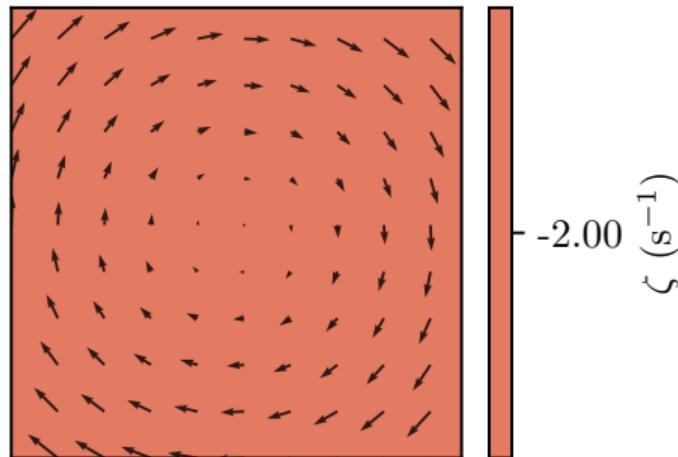


- 2.00  $\zeta \text{ (s}^{-1}\text{)}$

## Negative Vorticity (Anticyclonic)

- $\vec{u} = (y, -x)$
- Clockwise rotation
- $\zeta = -2$

Negative Vorticity (Anticyclonic)



# Absolute Vortizität

- In einem rotierenden Bezugssystem (wie der Erde) ergibt sich die **absolute Vortizität** zu:

$$\eta = f + \zeta$$

- $\zeta$ : relative Vortizität (durch Scherung und Krümmung der Strömung)
- $f = 2\Omega \sin \phi$ : Coriolis-Parameter, abhängig von der geografischen Breite
- Bedeutend für großräumige geophysikalische Strömungen (z. B. Rossby-Wellen, Erhaltung der potenziellen Vortizität)

# Konservierung der potentiellen Vortizität

- Die **potenzielle Vortizität (PV)** ist definiert als:

$$q = \frac{\eta}{H} = \frac{f + \zeta}{H}$$

- $\eta$ : absolute Vortizität, bestehend aus  $f + \zeta$
- $H$ : effektive Schichtdicke (z. B. Troposphärenhöhe oder isentrope Dicke)
- In einer reibungsfreien, adiabatischen Atmosphäre gilt:

$$\frac{Dq}{Dt} = 0$$

- Folge:** PV ist entlang von Teilchenbahnen erhalten  $\rightarrow$  zentrale Größe in der großskaligen Dynamik

# Mittlere Strömung und Anomalien

- In Äquatornähe dominiert eine mittlere Ost-West-Strömung  $U$
- Wir betrachten kleine Abweichungen davon:

$$u' = U + u, \quad v' = v \quad \text{mit } u, v \ll U$$

- Die Strömung ist quellenfrei  $\rightarrow$  Stromfunktion  $\psi$  existiert:

$$u = -\frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x}$$

# Zirkulation und Drehimpuls

- Relative Vorticity (Zirkulation):

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \Delta\psi$$

- Absolute Vorticity:

$$\zeta + f \quad (\text{mit Coriolisparameter } f = f(y))$$

- Annahme: Erhaltung der absoluten Vorticity:

$$\frac{d}{dt}(\zeta + f) = 0$$

# Bewegungsgleichung – Herleitung

- Kettenregel für totale Ableitung:

$$\frac{d}{dt}(\zeta + f) = \frac{\partial \zeta}{\partial t} + (U + u)\frac{\partial \zeta}{\partial x} + v \left( \frac{\partial \zeta}{\partial y} + \frac{\partial f}{\partial y} \right)$$

- Näherungen:

- $u \ll U \rightarrow$  vernachlässigbar
- $\partial \zeta / \partial y \ll \partial f / \partial y$
- $\partial f / \partial y = \beta$
- $v = \frac{\partial \psi}{\partial x}$

- Daraus ergibt sich:

$$\frac{\partial \zeta}{\partial t} + U \frac{\partial \zeta}{\partial x} + \beta \frac{\partial \psi}{\partial x} = 0$$

- Mit  $\zeta = \Delta \psi$ :

$$\frac{\partial \Delta \psi}{\partial t} + U \frac{\partial \Delta \psi}{\partial x} + \beta \frac{\partial \psi}{\partial x} = 0$$

# Wellenlösung der Gleichung

- Ansatz: ebene Wellen

$$\psi(x, y, t) = \cos(kx + ly - \omega t)$$

- Einsetzen in Gleichung ergibt Dispersionsrelation:

$$\omega = U k - \frac{\beta k}{k^2 + l^2}$$

- Phasengeschwindigkeit:

$$c = \frac{\omega}{k} = U - \frac{\beta}{k^2 + l^2}$$

- Interpretation: westwärts laufende Wellen mit geringer Geschwindigkeit als  $U$

# Physikalisches Feld am Beispiel der Vorticity

- Die **Vorticity**  $\zeta(x, y, t)$  beschreibt die Rotation eines Luftpakets.
- Sie ist ein **physikalisches Skalarfeld**, das jedem Punkt eine Wirbelstärke zuordnet:

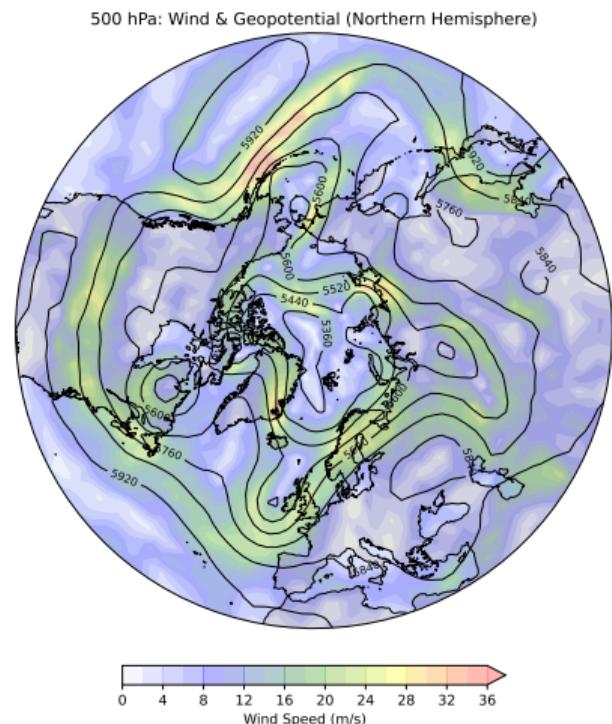
$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

- In der Atmosphäre entsteht Vorticity durch:
  - Wind-Scherung (Änderung der Windrichtung oder -geschwindigkeit)
  - Bewegung entlang der Breitenkreise ( $\beta$ -Effekt)
- Die **Vorticity-Gleichung** beschreibt, wie sich dieses Feld entwickelt:

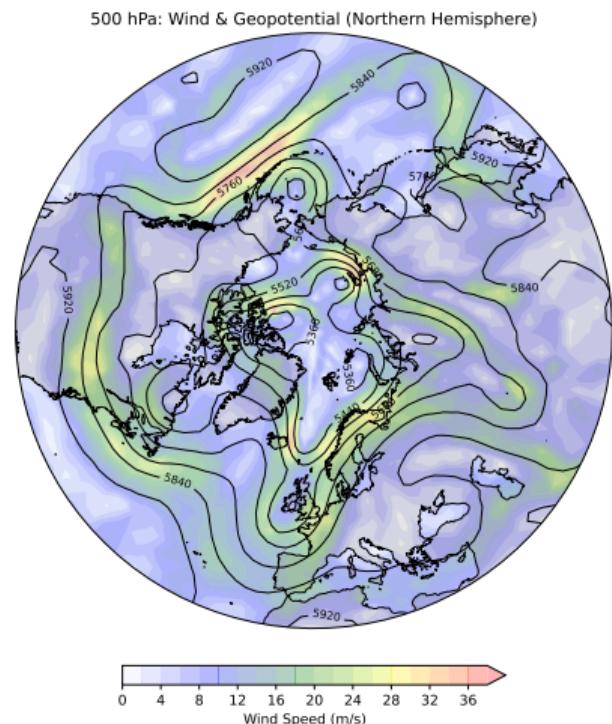
$$\frac{\partial \zeta}{\partial t} + \vec{v} \cdot \nabla \zeta + \beta v = 0$$

- → Diese Gleichung ist eine typische **Feldgleichung**, weil sie die Änderung eines Feldes durch lokale und advektive Prozesse beschreibt.

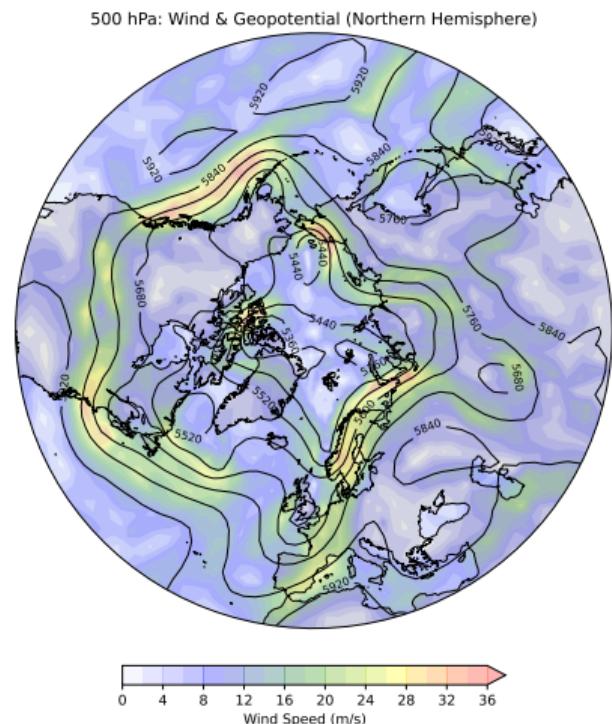
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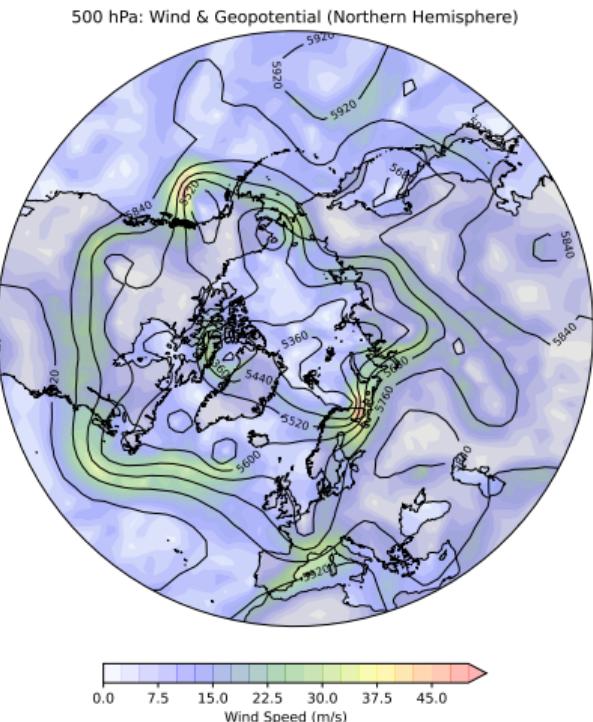
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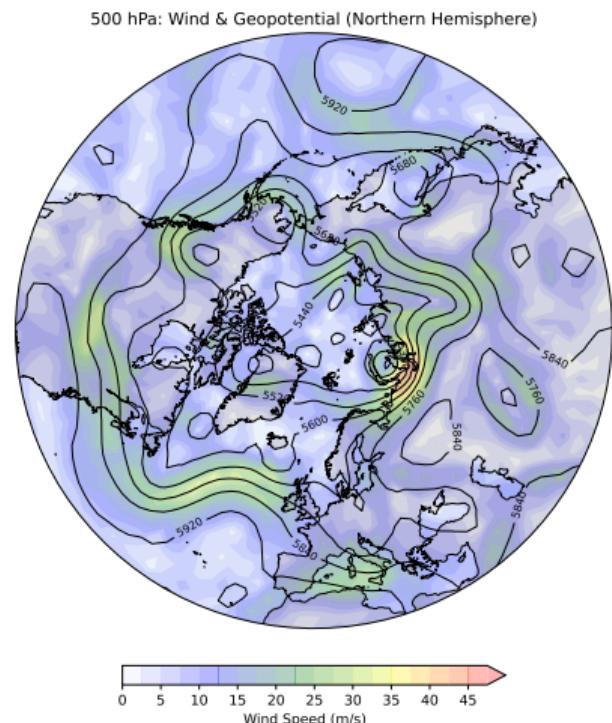
# Forecast: 22-July-2010 at 12:00, Level 500hPa



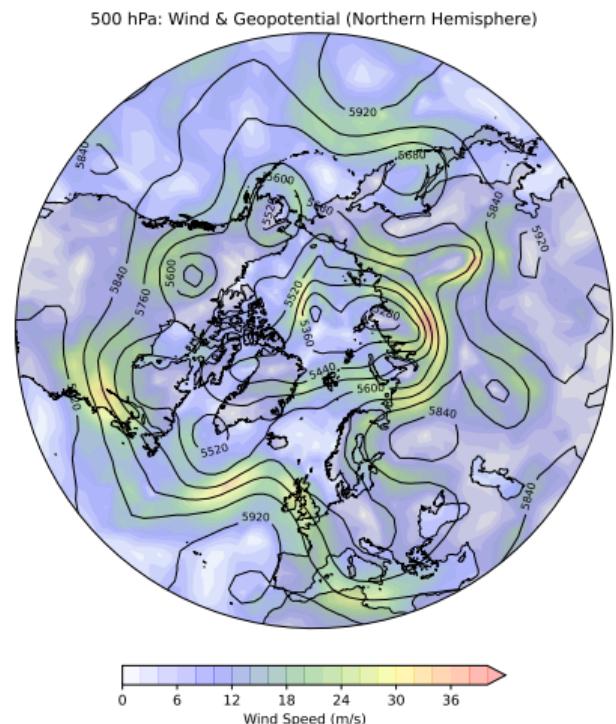
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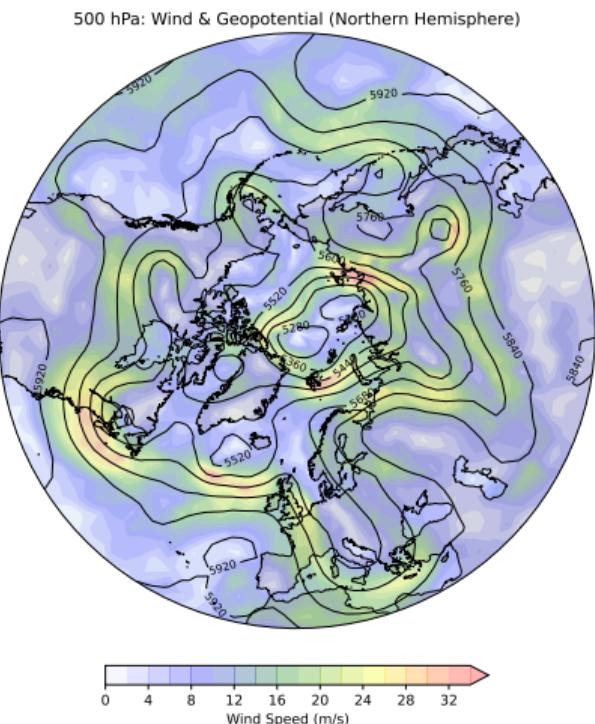
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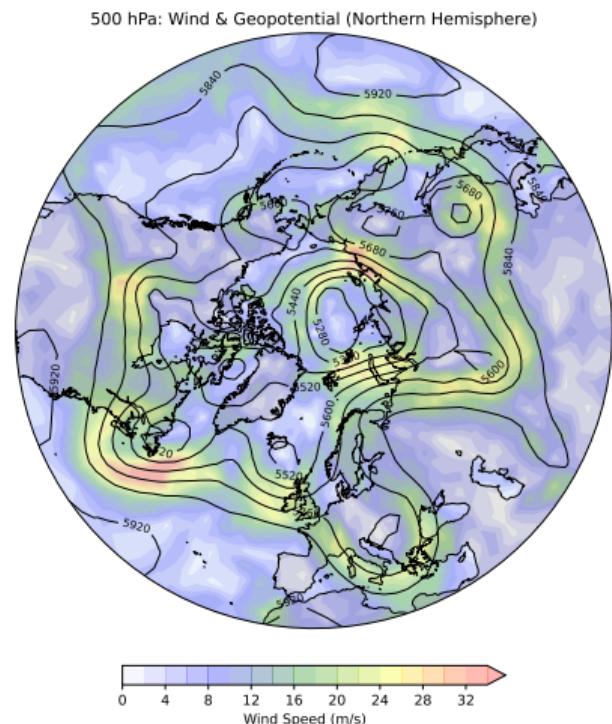
# Forecast: 25-July-2010 at 12:00, Level 500hPa



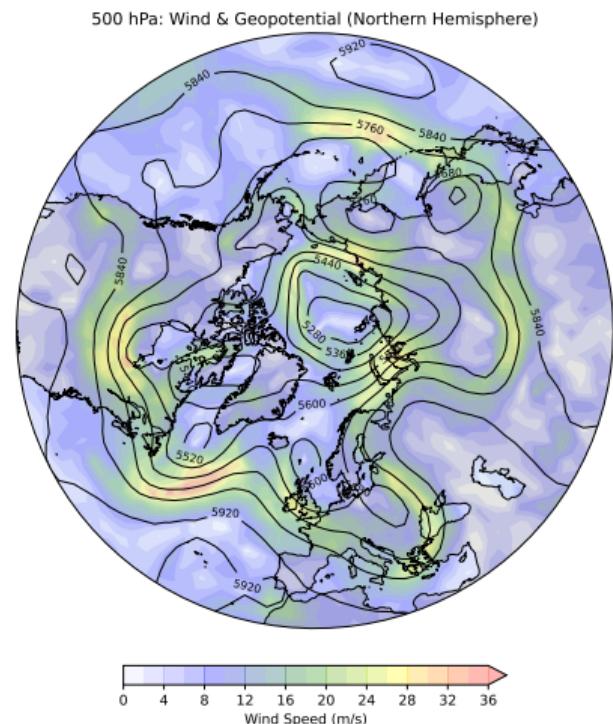
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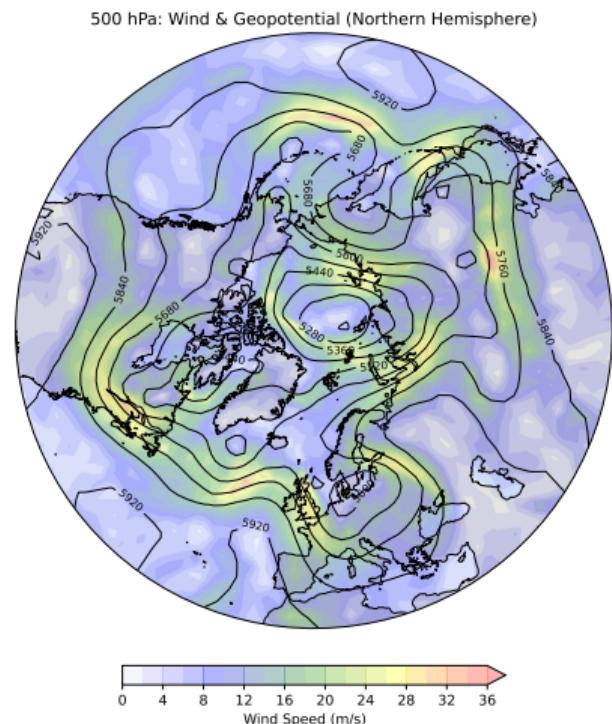
Forecast: 27-July-2010 at 12:00, Level 500hPa



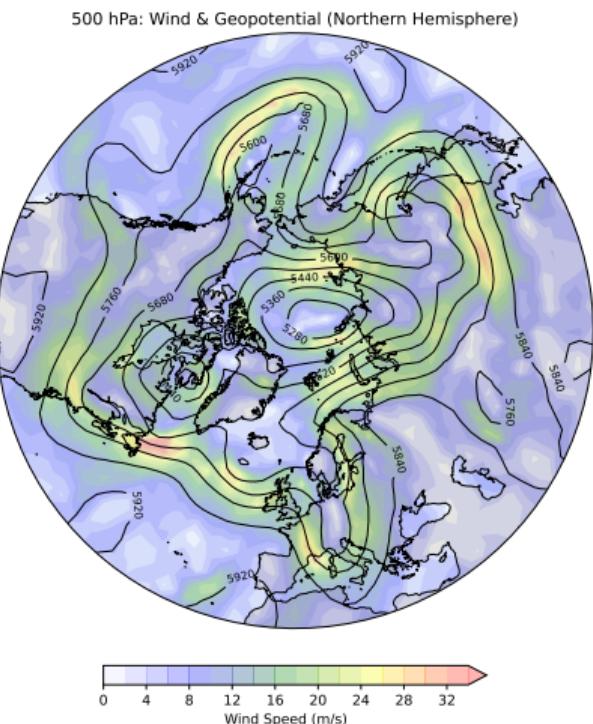
Forecast: 28-July-2010 at 12:00, Level 500hPa



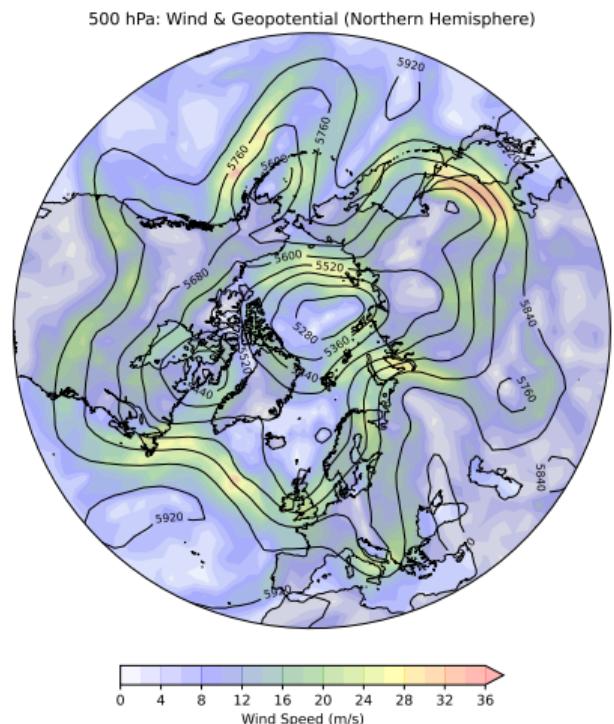
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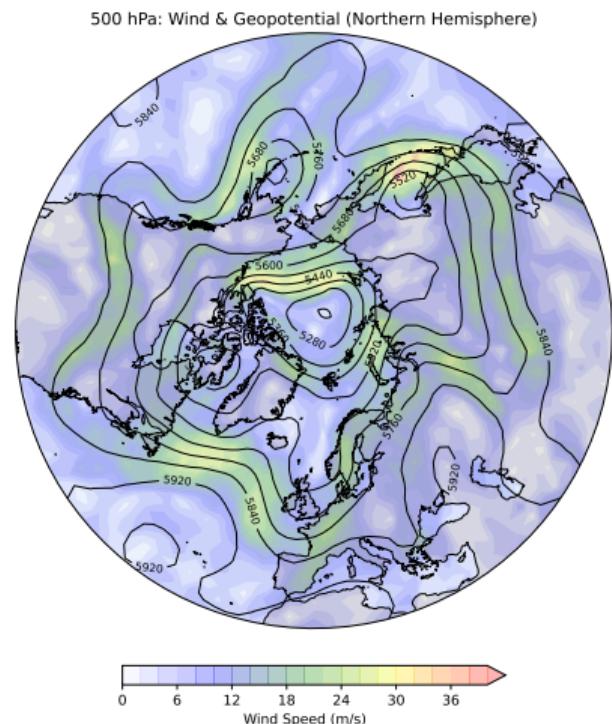
# Forecast: 30-July-2010 at 12:00, Level 500hPa



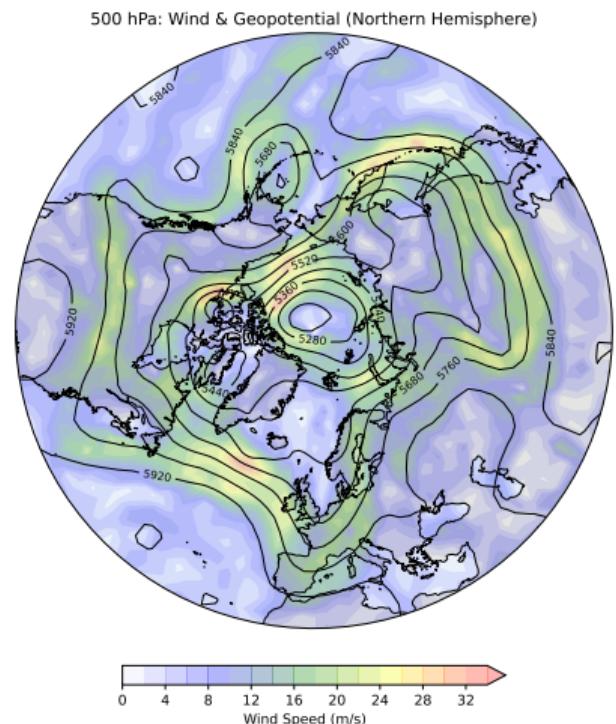
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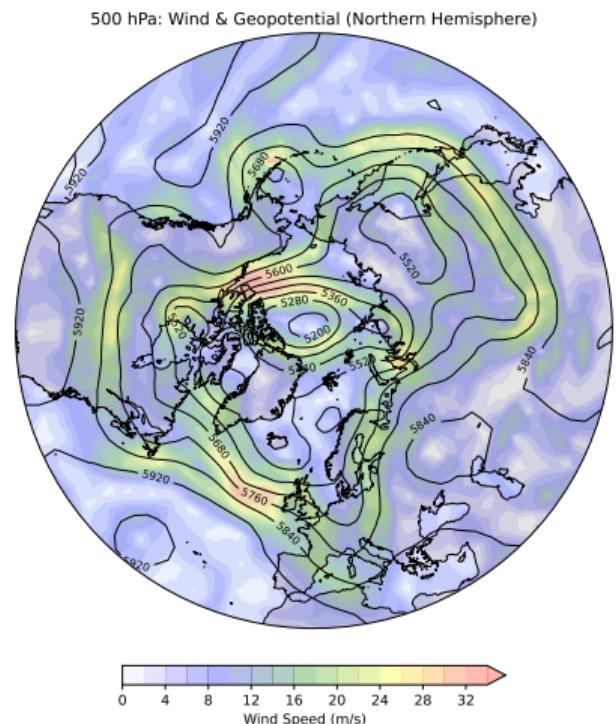
# Forecast: 1-August-2010 at 12:00, Level 500hPa



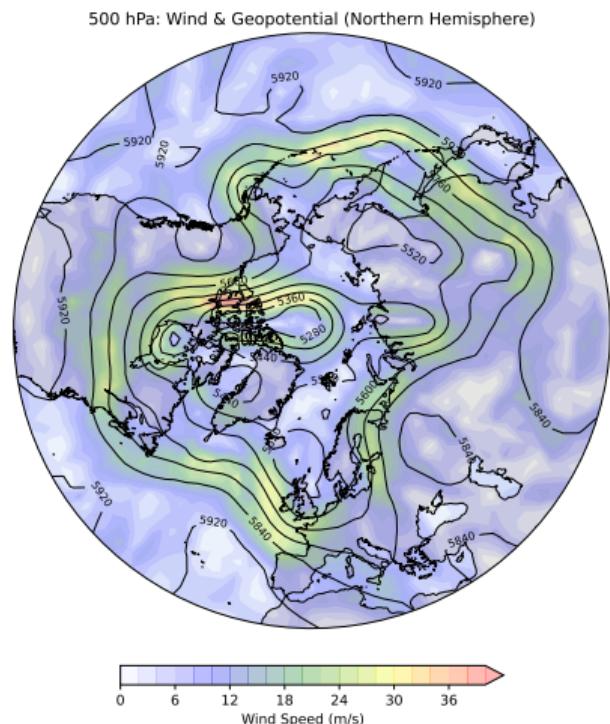
# Forecast: 2-August-2010 at 12:00, Level 500hPa



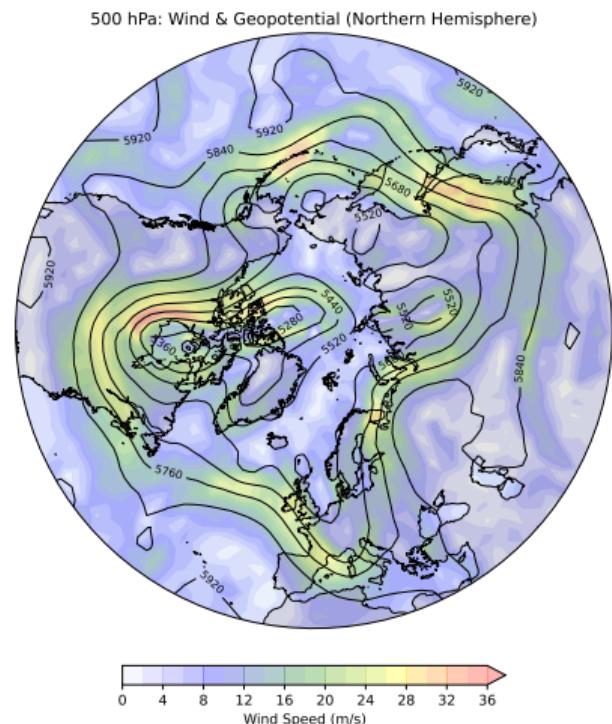
# Forecast: 3-August-2010 at 12:00, Level 500hPa



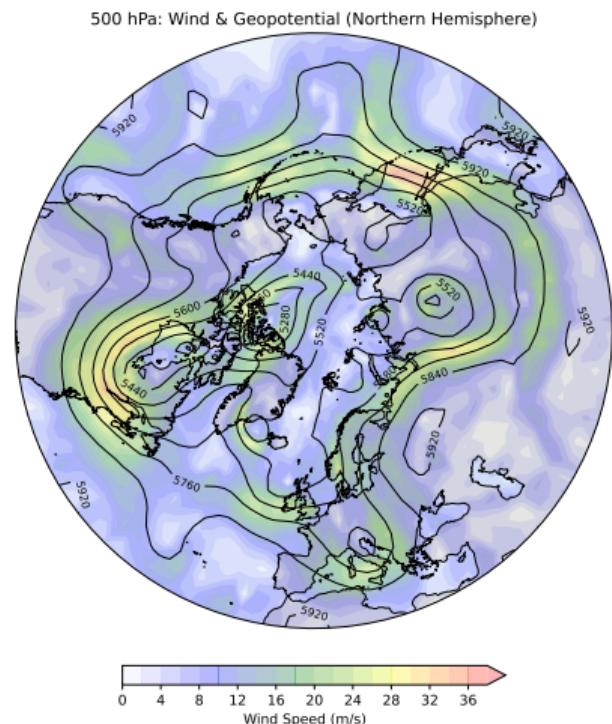
# Forecast: 4-August-2010 at 12:00, Level 500hPa



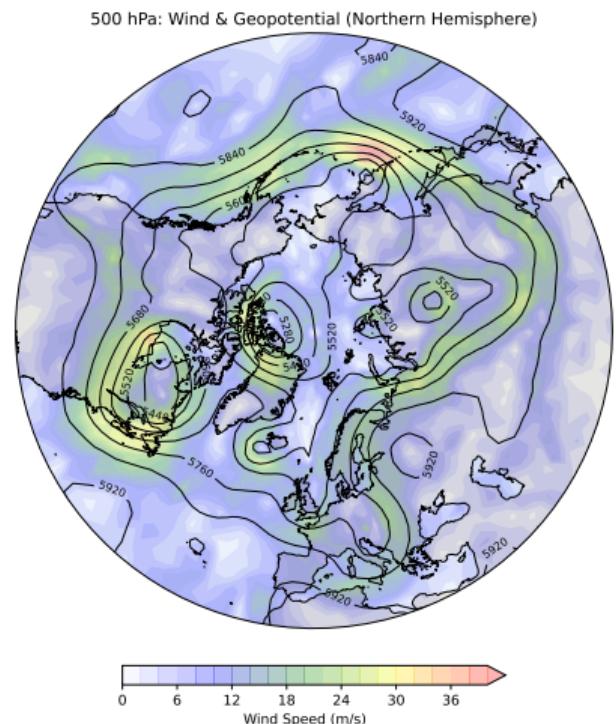
# Forecast: 5-August-2010 at 12:00, Level 500hPa



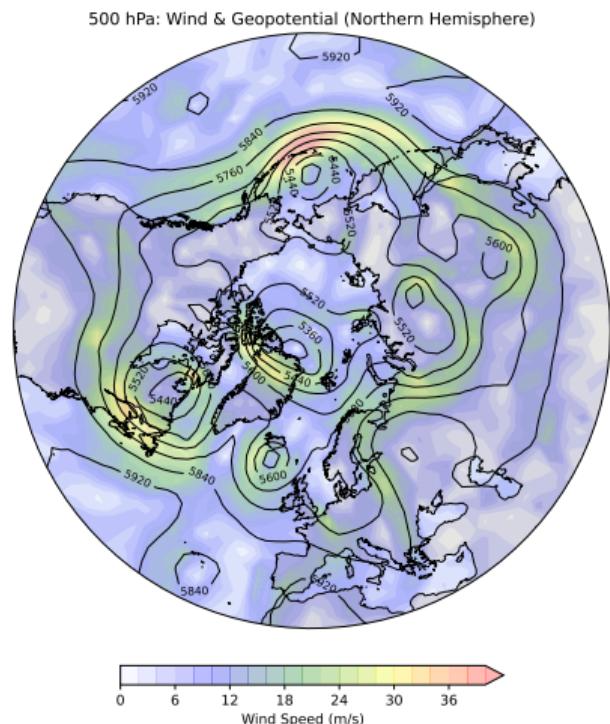
# Forecast: 6-August-2010 at 12:00, Level 500hPa



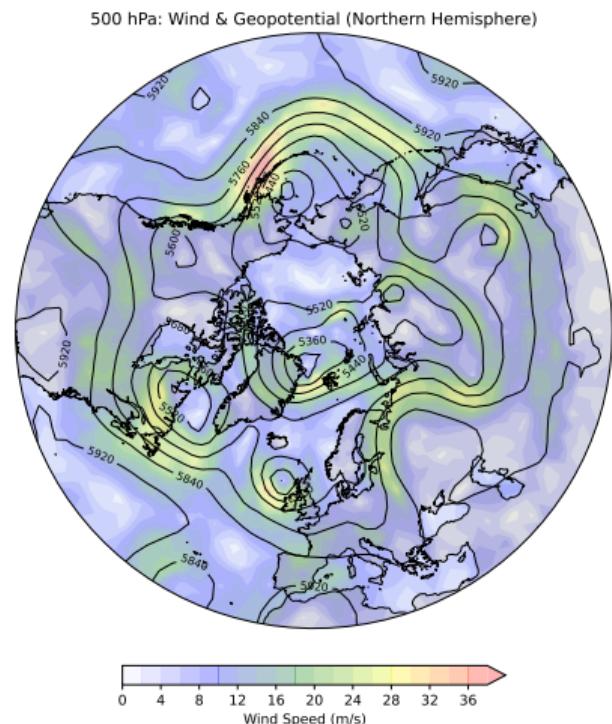
# Forecast: 7-August-2010 at 12:00, Level 500hPa



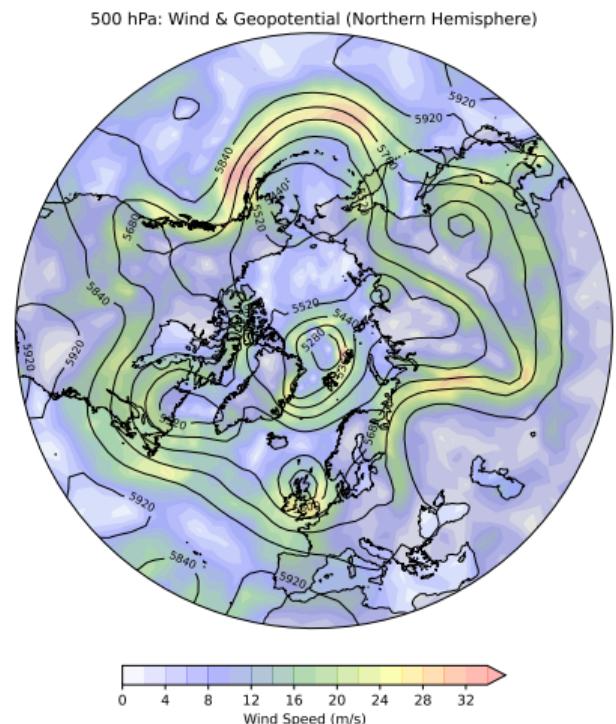
# Forecast: 8-August-2010 at 12:00, Level 500hPa



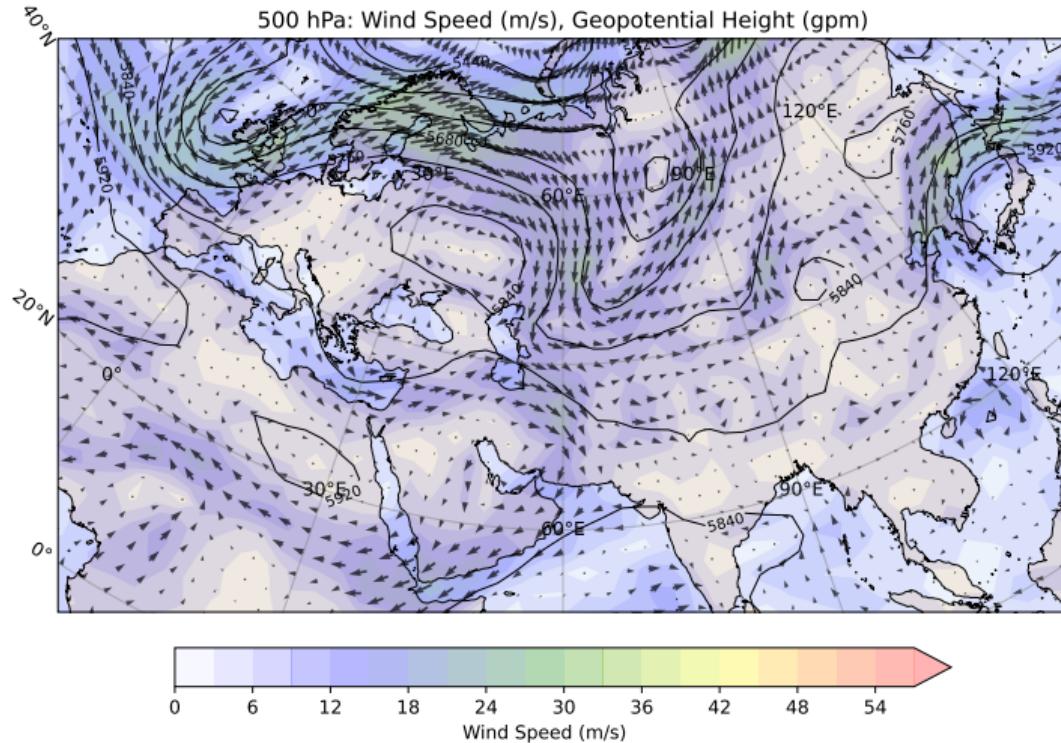
# Forecast: 9-August-2010 at 12:00, Level 500hPa



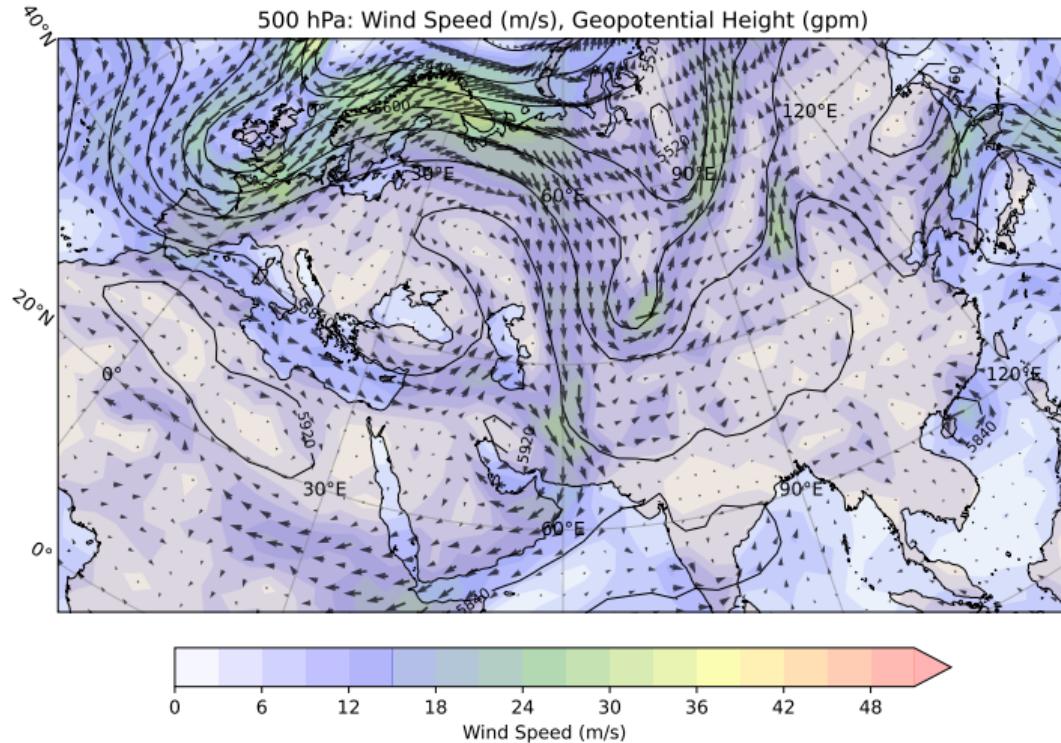
# Forecast: 10-August-2010 at 12:00, Level 500hPa



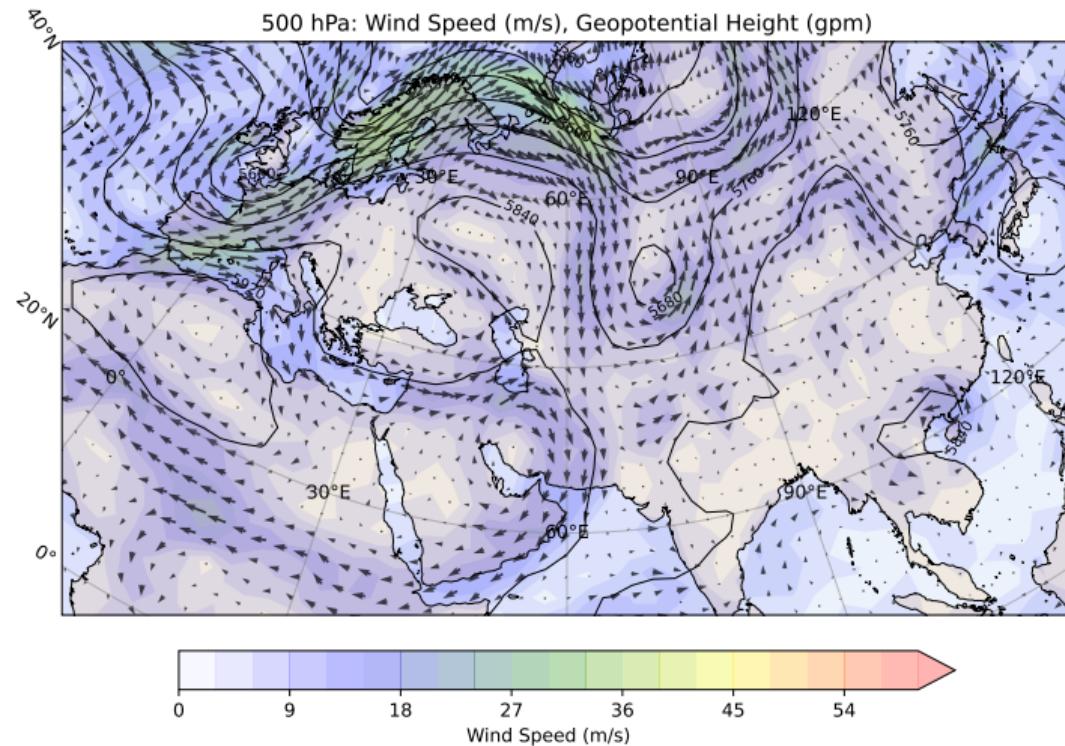
# Forecast: 20-July-2010 at 12:00, Level 500hPa



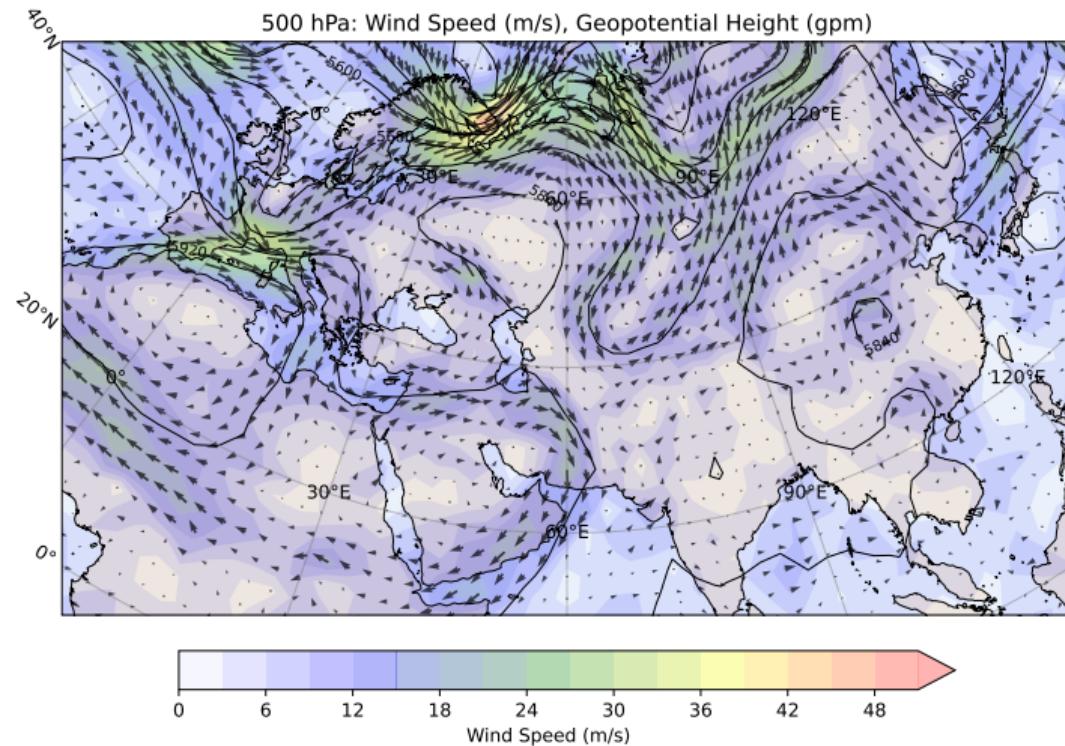
# Forecast: 21-July-2010 at 12:00, Level 500hPa



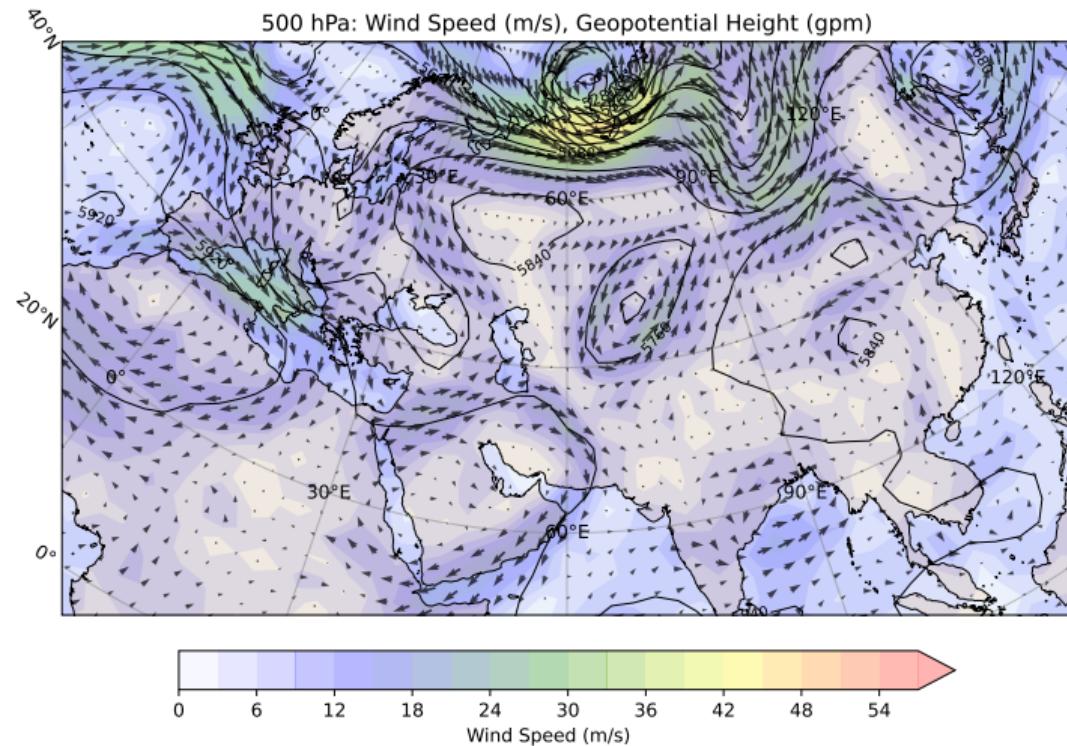
Forecast: 22-July-2010 at 12:00, Level 500hPa



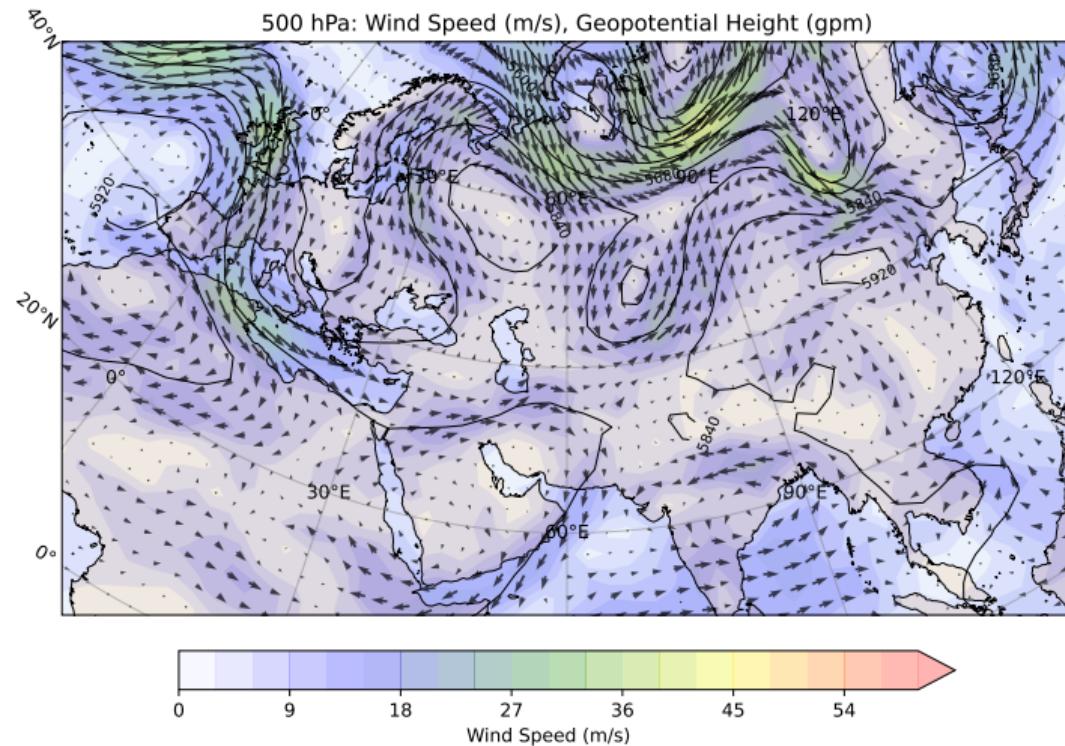
Forecast: 23-July-2010 at 12:00, Level 500hPa



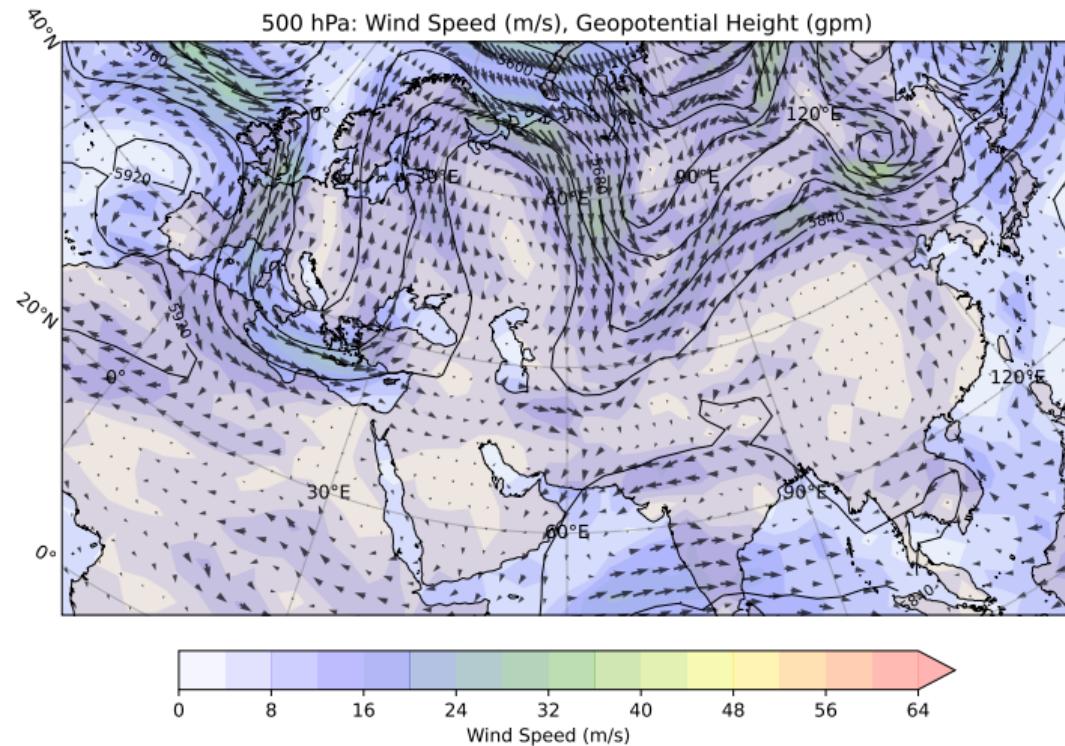
# Forecast: 24-July-2010 at 12:00, Level 500hPa



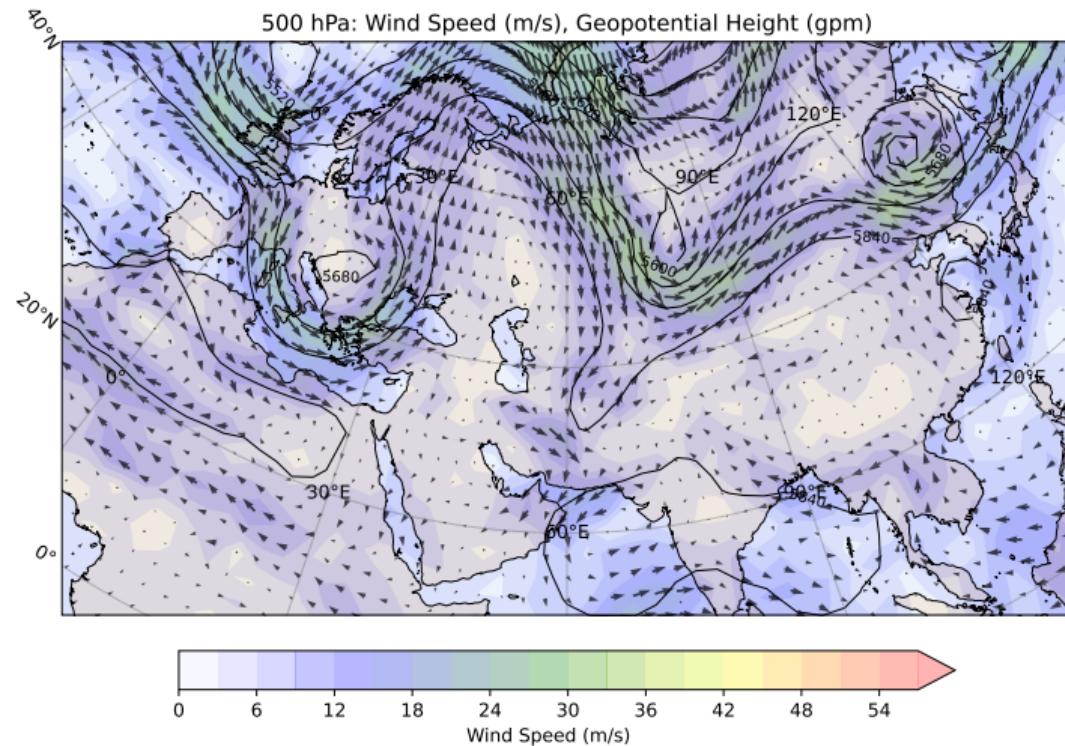
Forecast: 25-July-2010 at 12:00, Level 500hPa



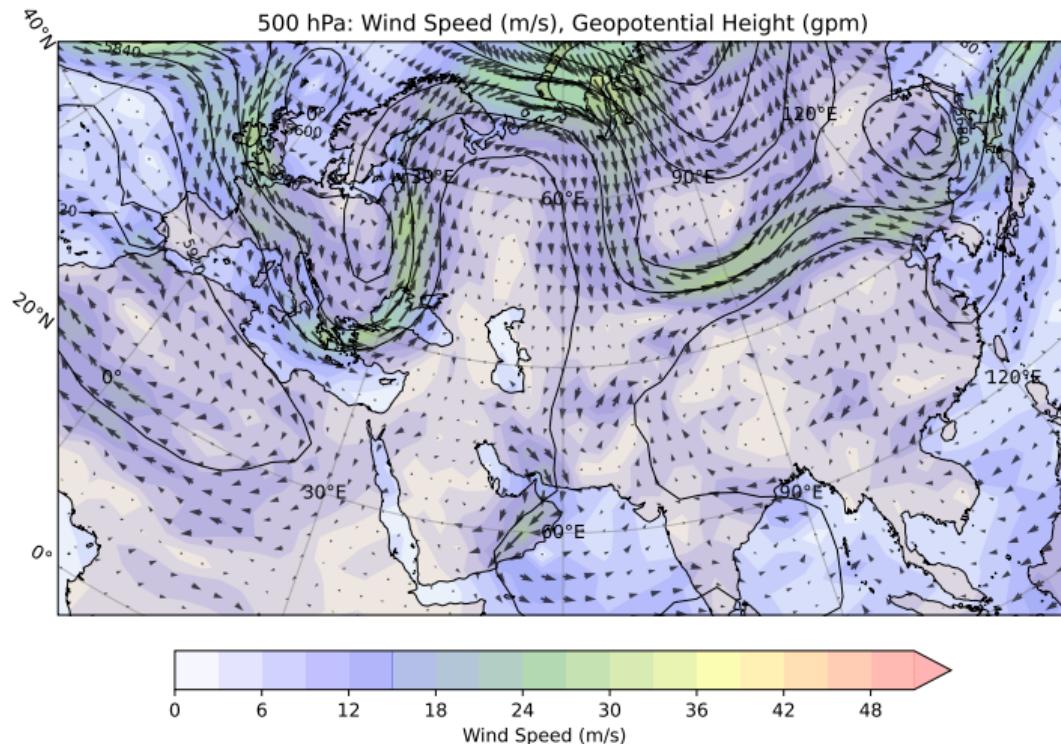
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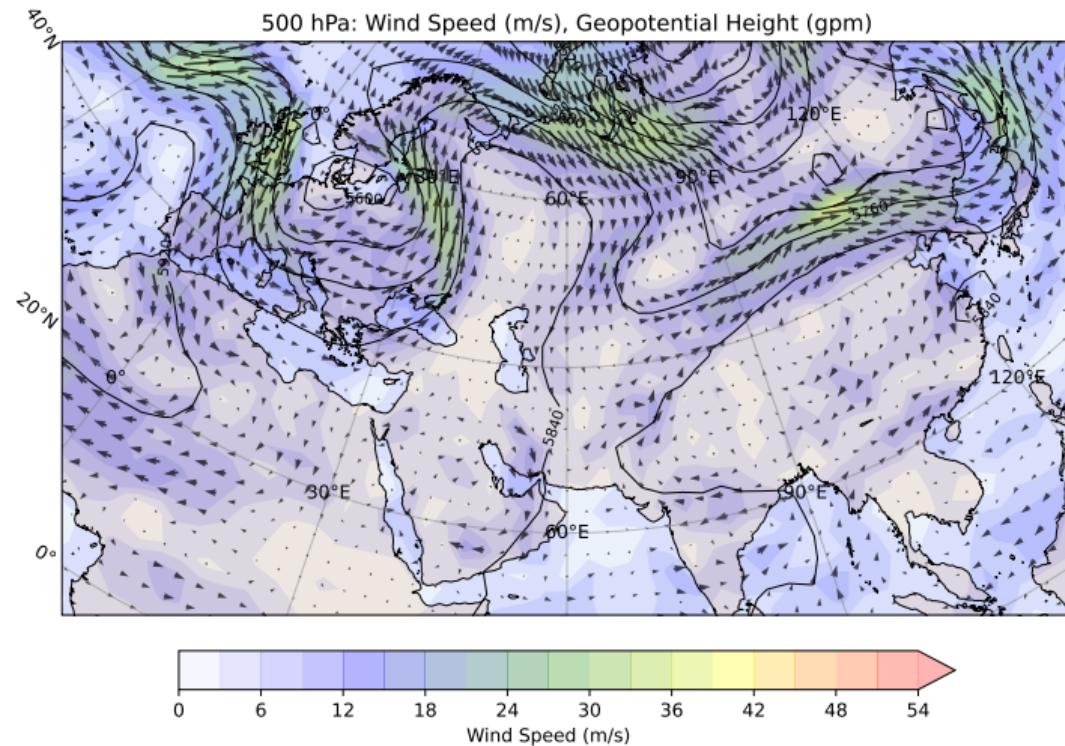
Forecast: 27-July-2010 at 12:00, Level 500hPa



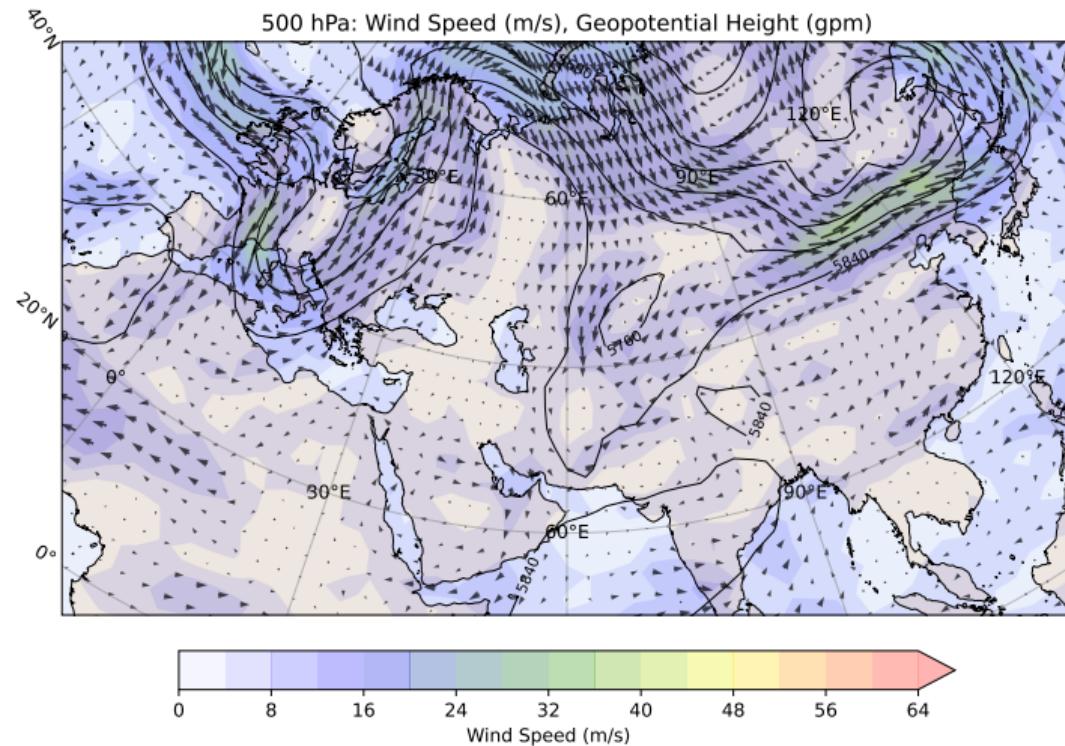
Forecast: 28-July-2010 at 12:00, Level 500hPa



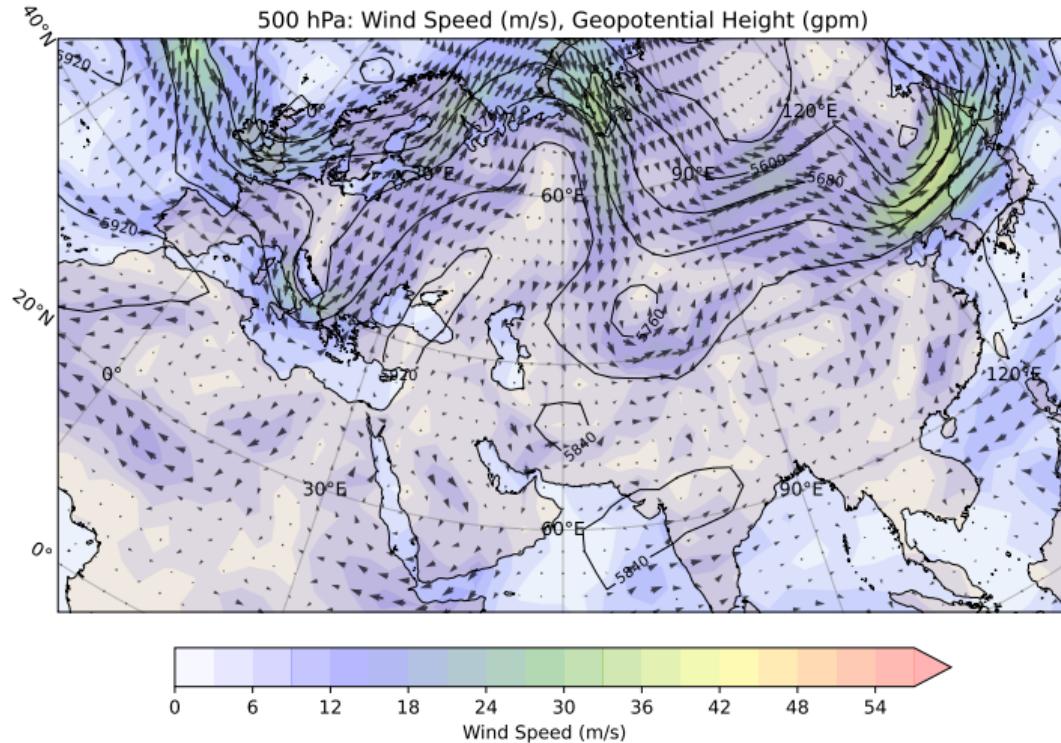
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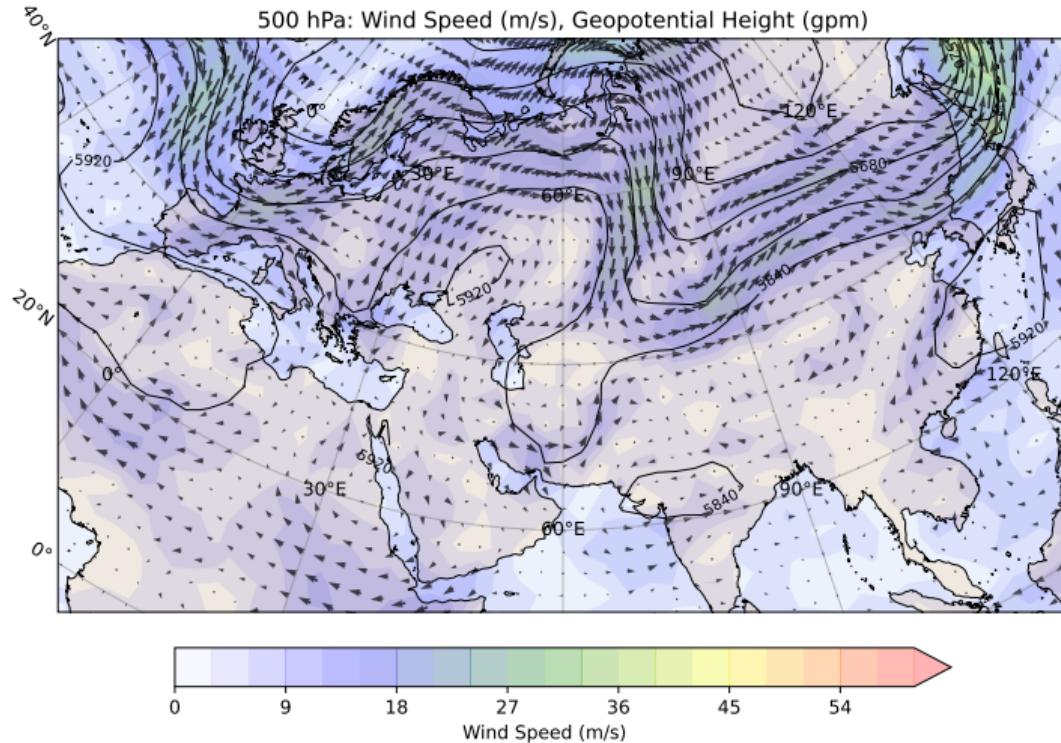
Forecast: 30-July-2010 at 12:00, Level 500hPa



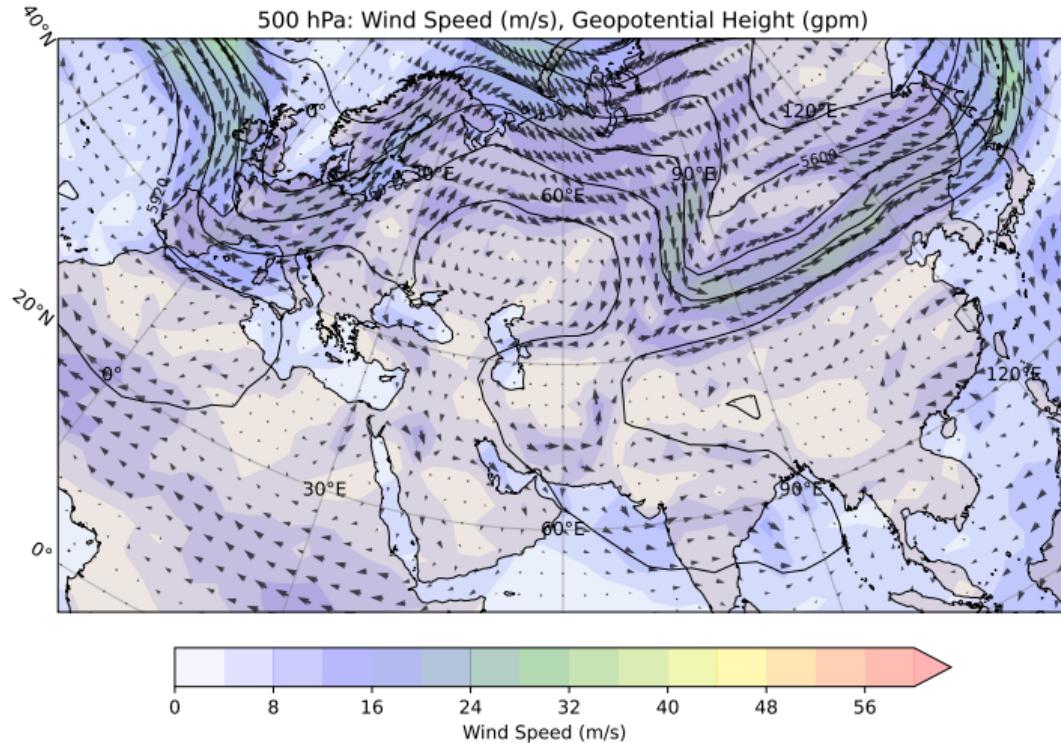
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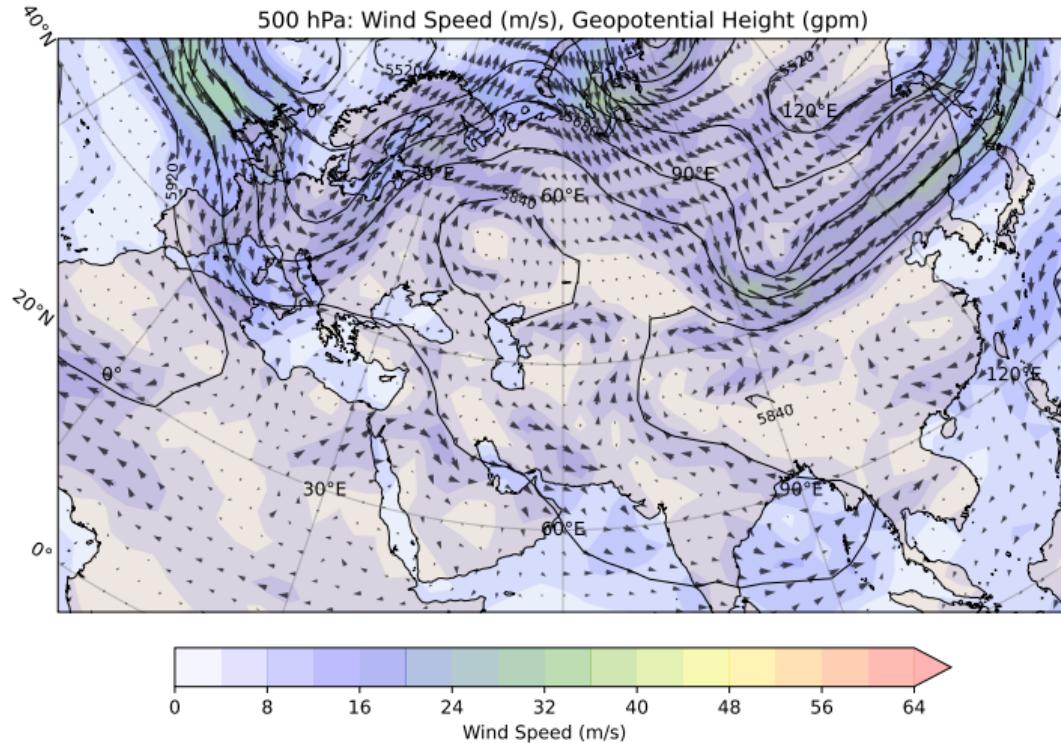
# Forecast: 1-August-2010 at 12:00, Level 500hPa



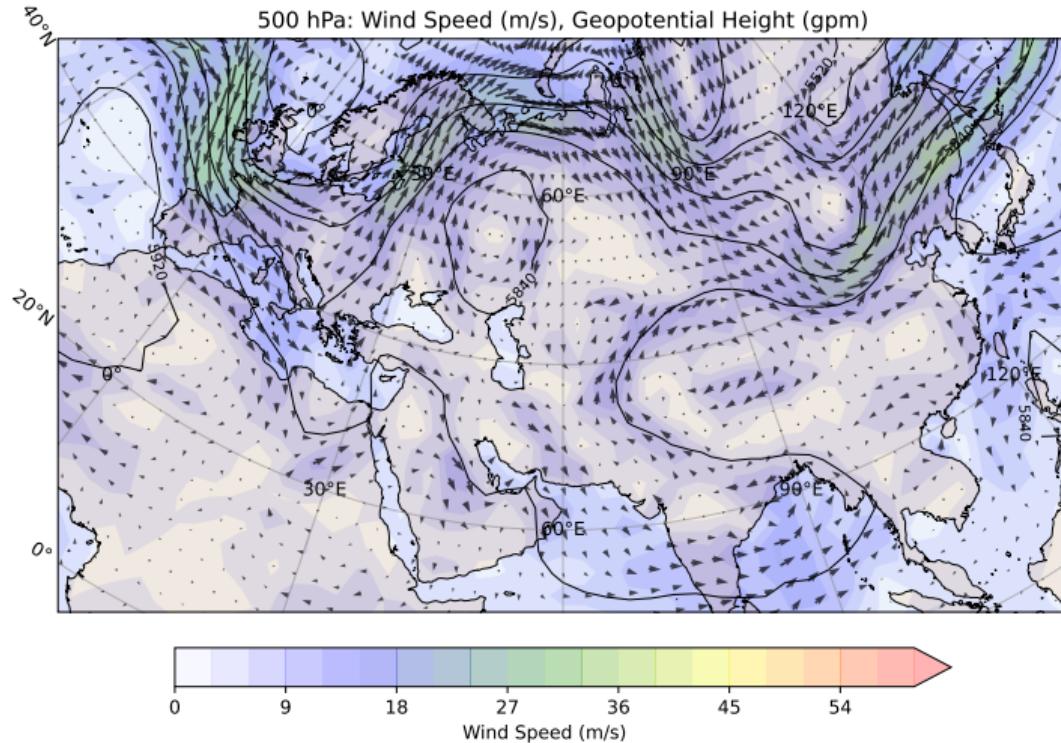
# Forecast: 2-August-2010 at 12:00, Level 500hPa



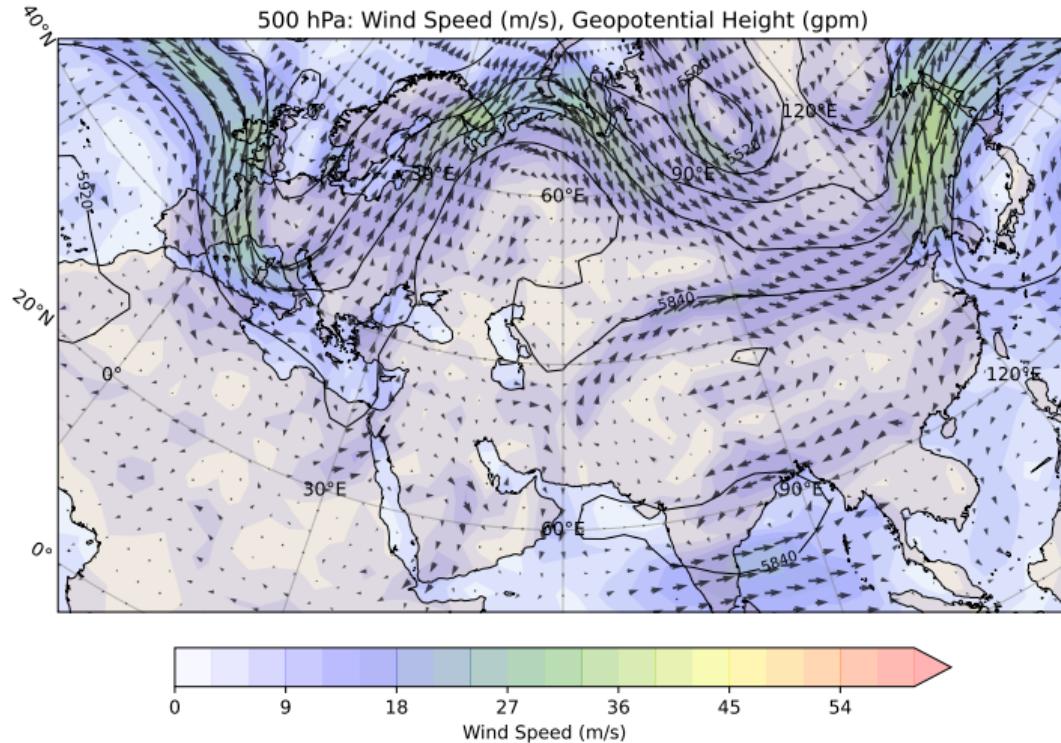
Forecast: 3-August-2010 at 12:00, Level 500hPa



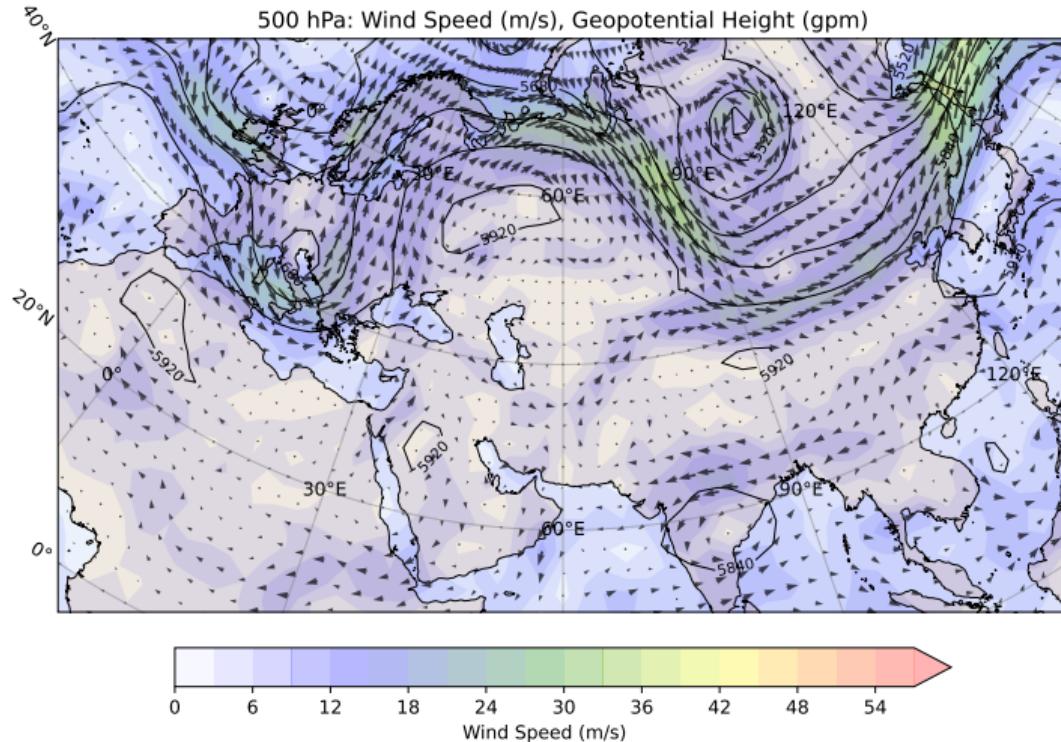
# Forecast: 4-August-2010 at 12:00, Level 500hPa



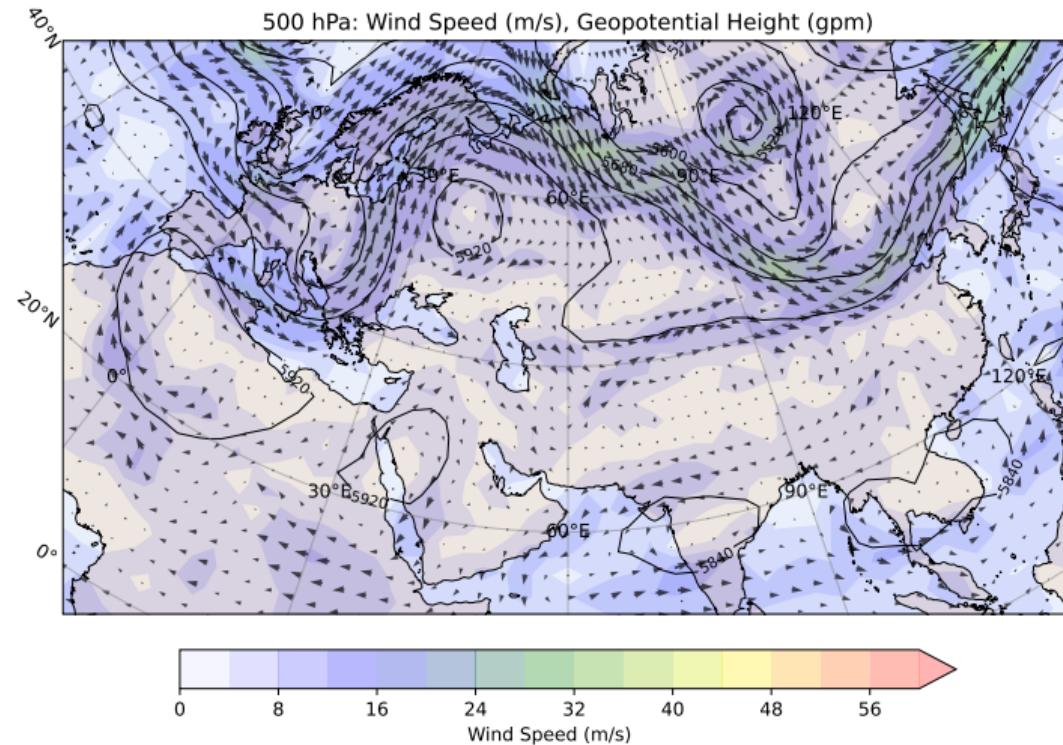
# Forecast: 5-August-2010 at 12:00, Level 500hPa



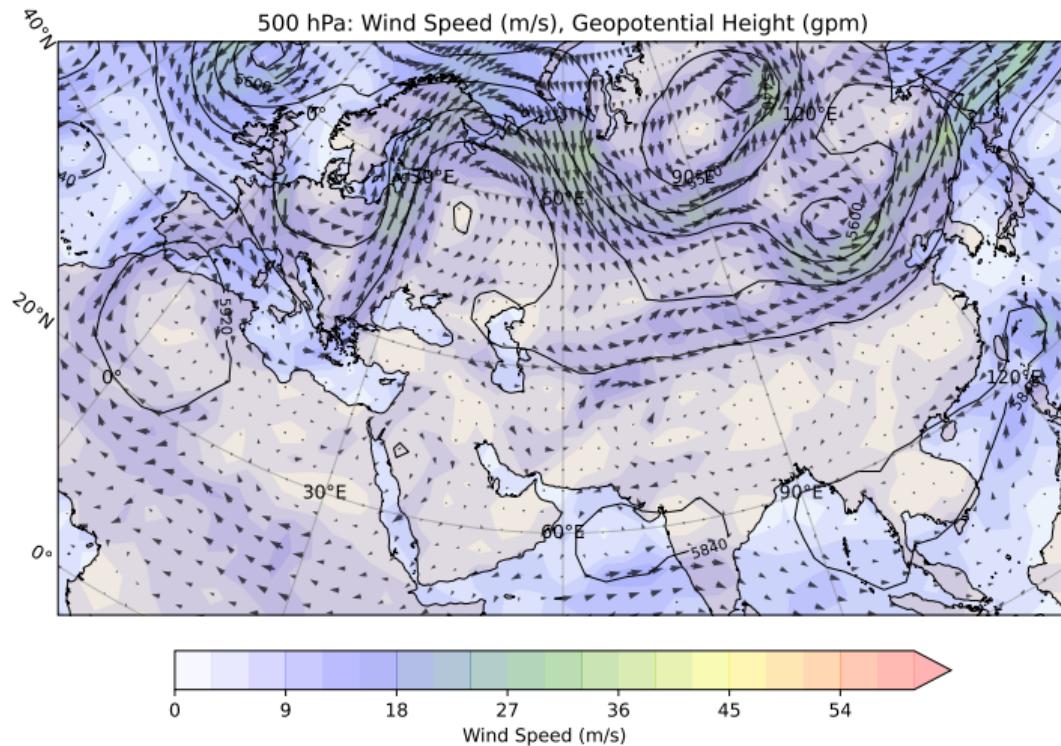
# Forecast: 6-August-2010 at 12:00, Level 500hPa



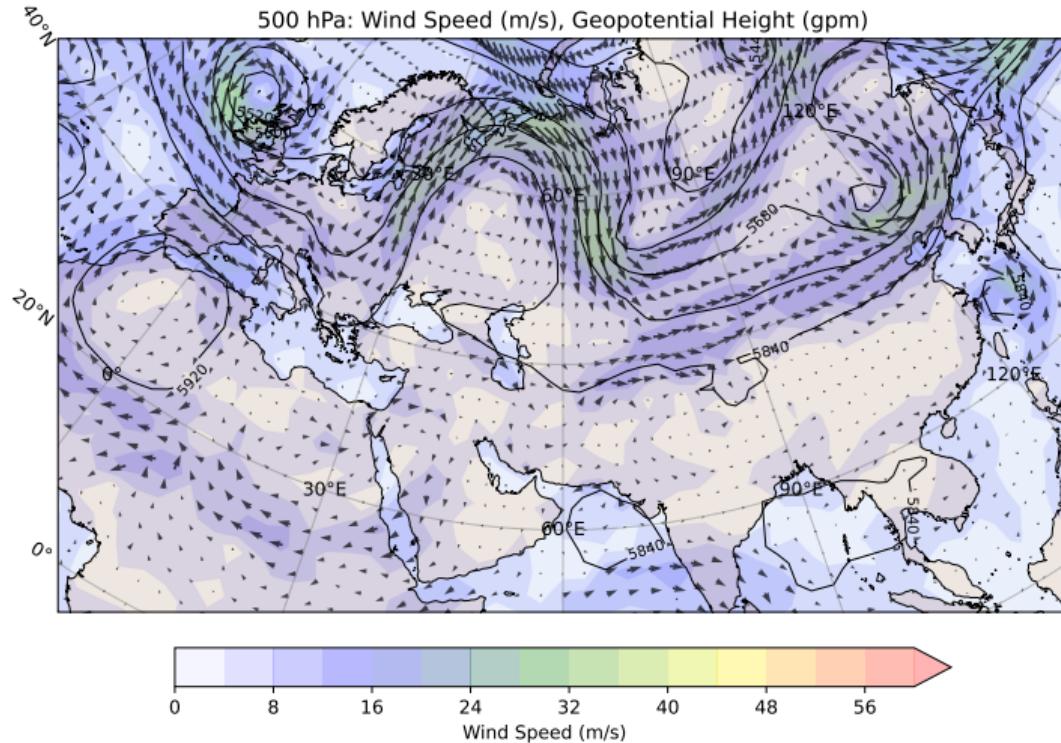
# Forecast: 7-August-2010 at 12:00, Level 500hPa



# Forecast: 8-August-2010 at 12:00, Level 500hPa



# Forecast: 9-August-2010 at 12:00, Level 500hPa



# Forecast: 10-August-2010 at 12:00, Level 500hPa

