

# Rossby Wellen

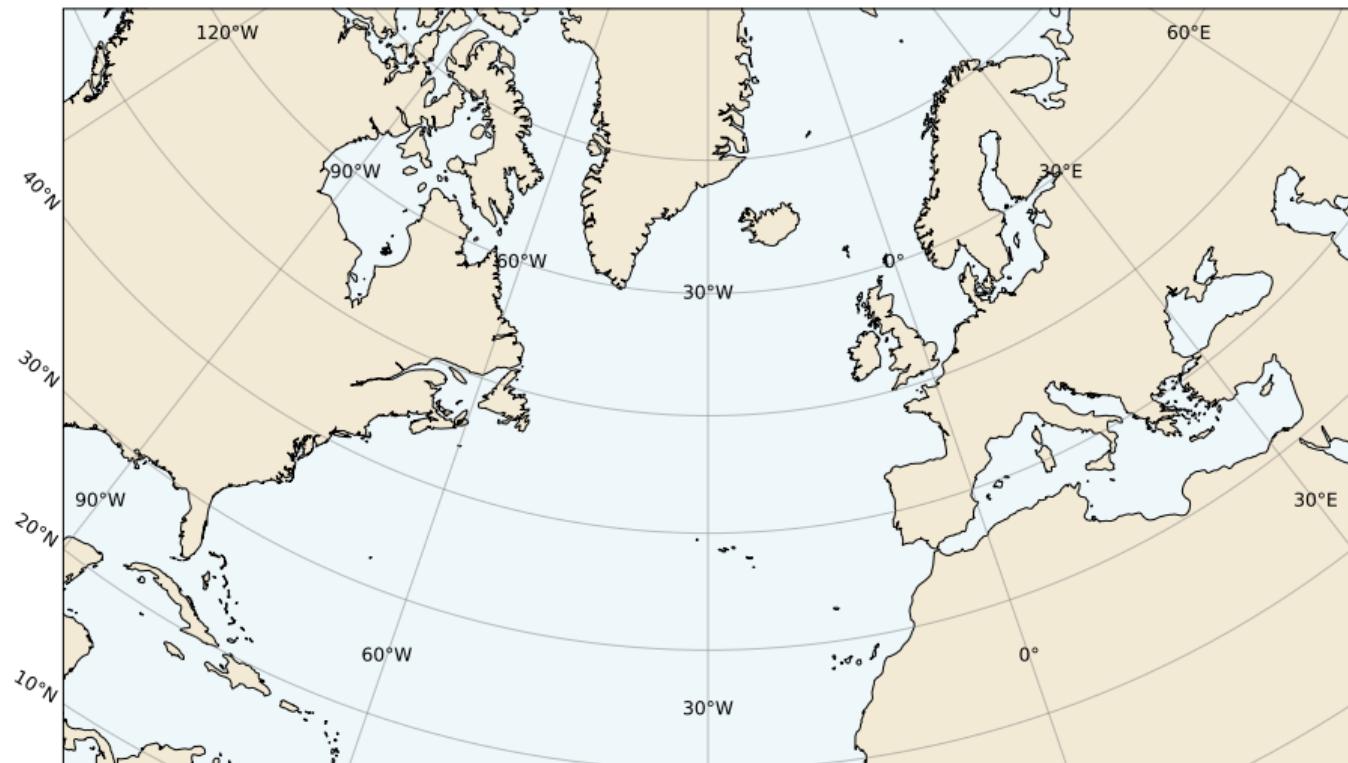


Michael Schmid

ICAI Interdisciplinary Center for Artificial Intelligence

June 5, 2025

# Karte



# ERA5-Analyse: Jetstream am Mai 2025

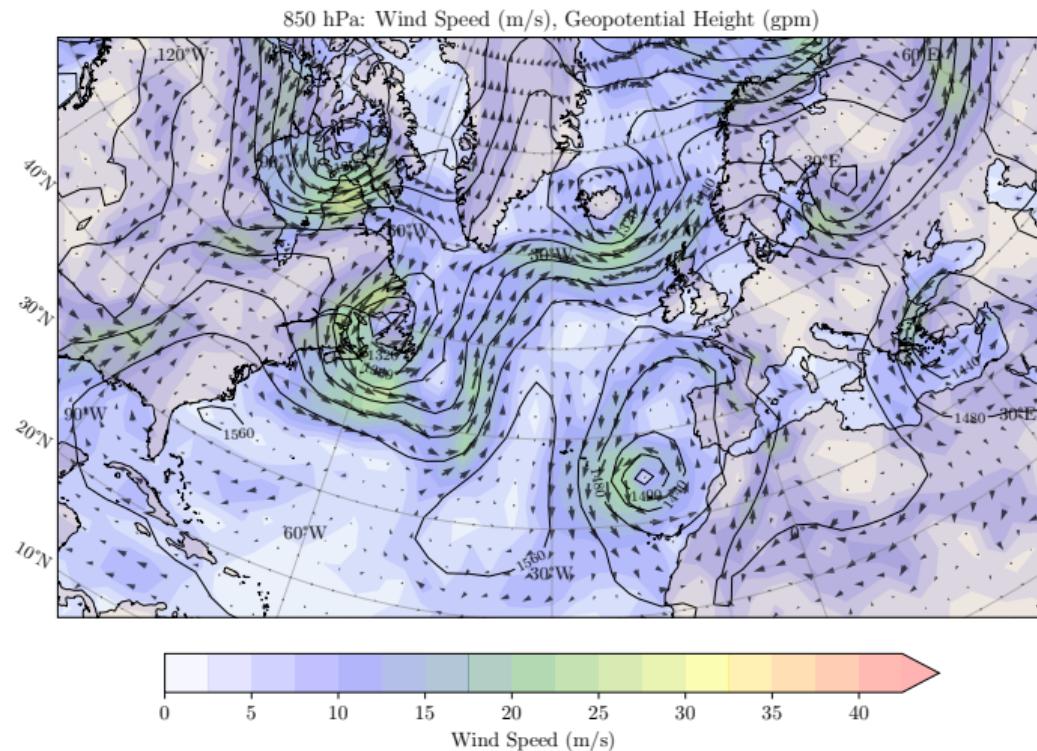
- Datenquelle: **ERA5 Reanalysis** (ECMWF) via CDS API
- Abfrageparameter:
  - Druckniveau: **500 hPa** (obere Troposphäre, Jetstream-Niveau)
  - Variablen: Geopotential,  $u$ - und  $v$ -Windkomponente
- Visualisierung:
  - Farbkarte: Windgeschwindigkeit (m/s)
  - Linien: Geopotentielle Höhe (gpm)
  - Pfeile: Windvektoren

# Warum 850, 500 und 200 hPa?

- Standard-Druckniveaus in der Meteorologie
- Repräsentieren typische Höhenbereiche der Troposphäre mit jeweils spezifischer dynamischer Bedeutung

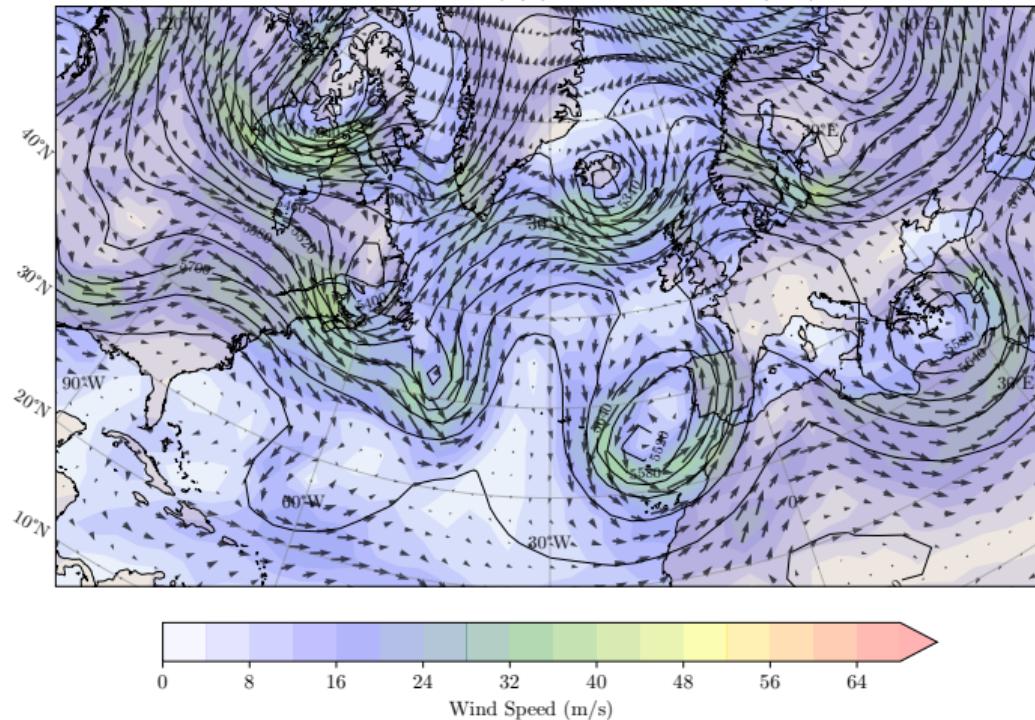
Niveau	Höhe (ca.)	Bedeutung
850 hPa	1.5 km	Untere Troposphäre: Temperatur, Feuchtigkeit, Hebung, Konvektion, Bodeninversion
500 hPa	5.5 km	Mittlere Troposphäre: Vorticity, Tröge/Rücken, synoptische Dynamik, Luftmassengrenzen
200 hPa	12 km	Obere Troposphäre: Jetstream, Divergenzfelder, Rossby-Wellen, barokline Kopplung

# Forecast: 01-May-2025 at 00:00, Level 850 hPa

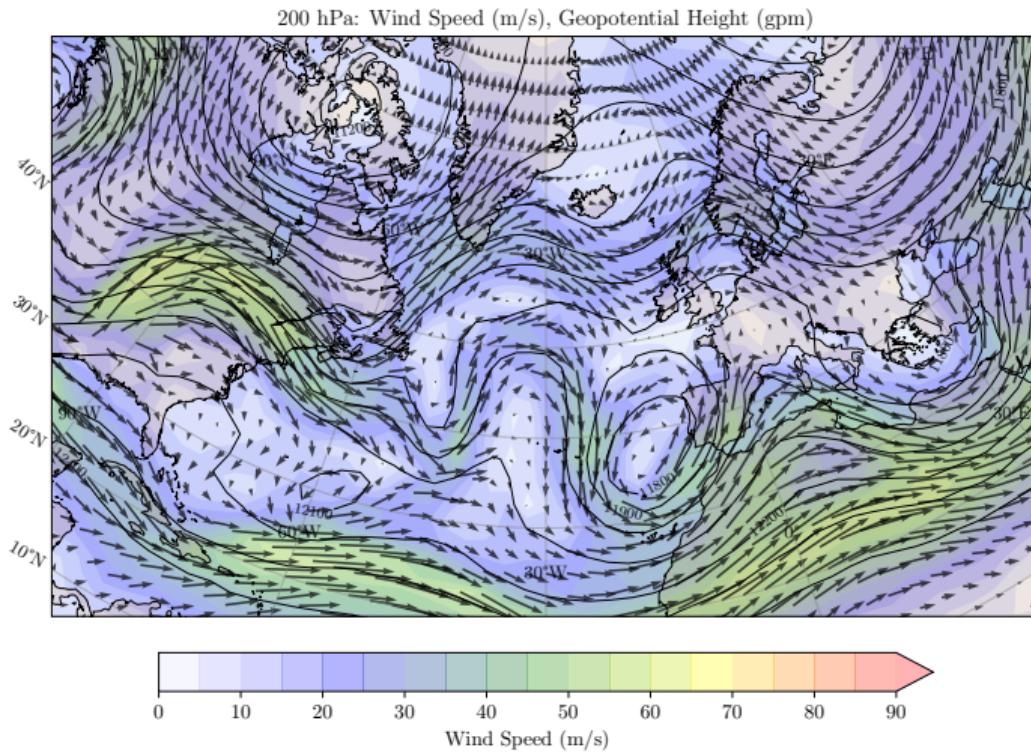


# Forecast: 01-May-2025 at 00:00, Level 500 hPa

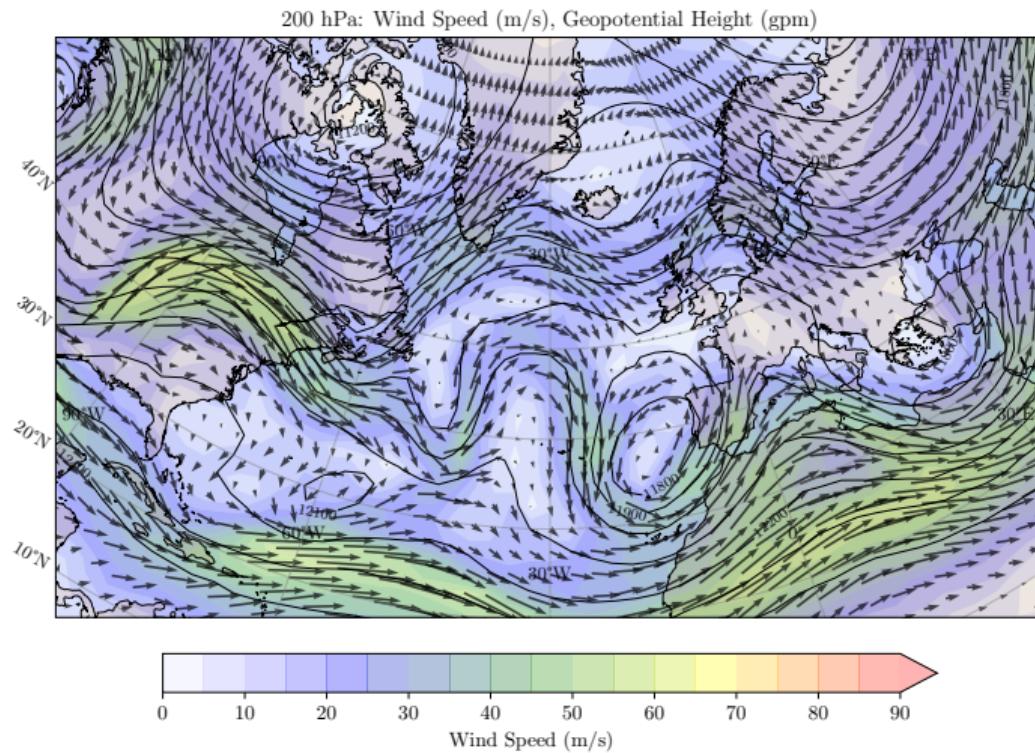
500 hPa: Wind Speed (m/s), Geopotential Height (gpm)



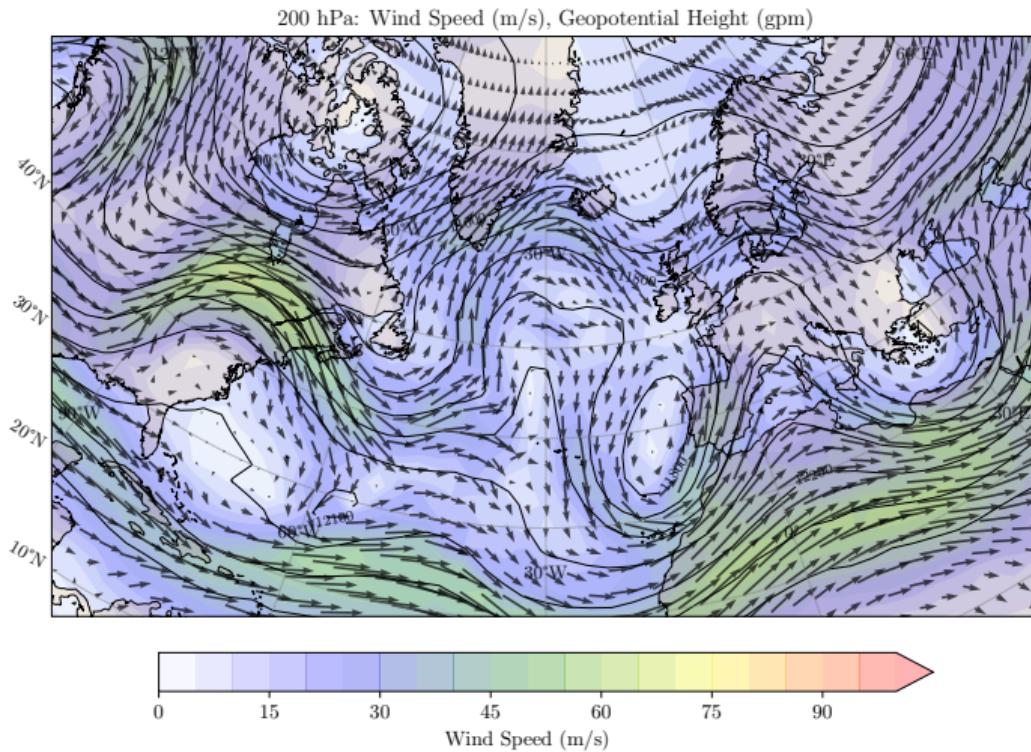
# Forecast: 01-May-2025 at 00:00, Level 200 hPa



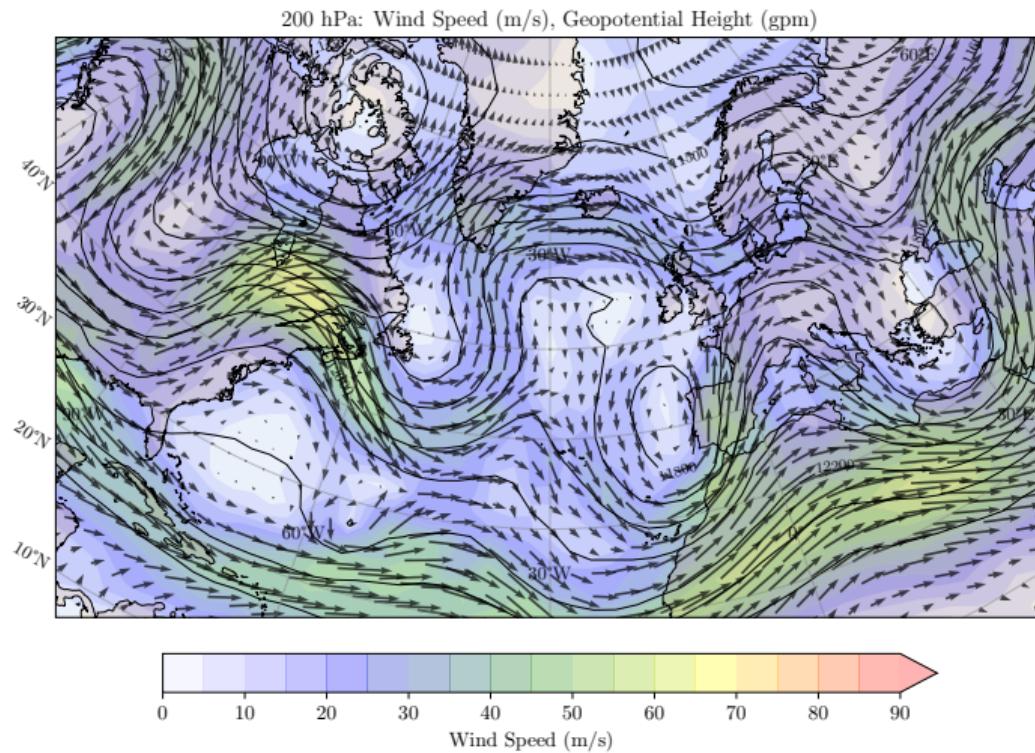
# Forecast: 1-May-2025 at 00:00, Level 200hPa



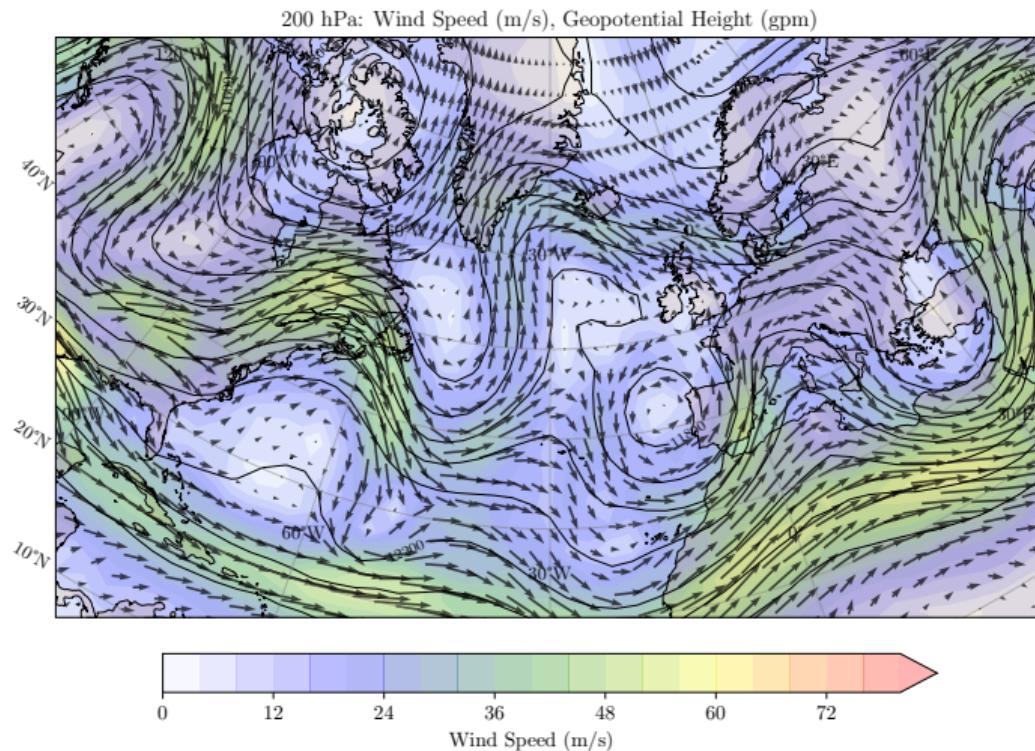
# Forecast: 1-May-2025 at 12:00, Level 200hPa



# Forecast: 2-May-2025 at 00:00, Level 200hPa

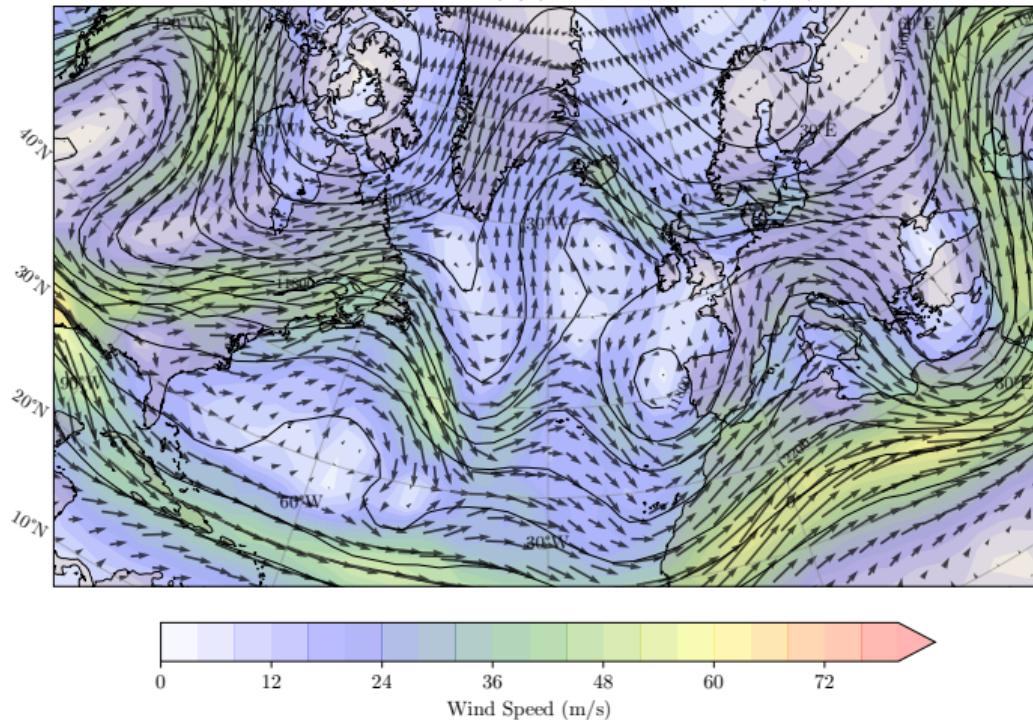


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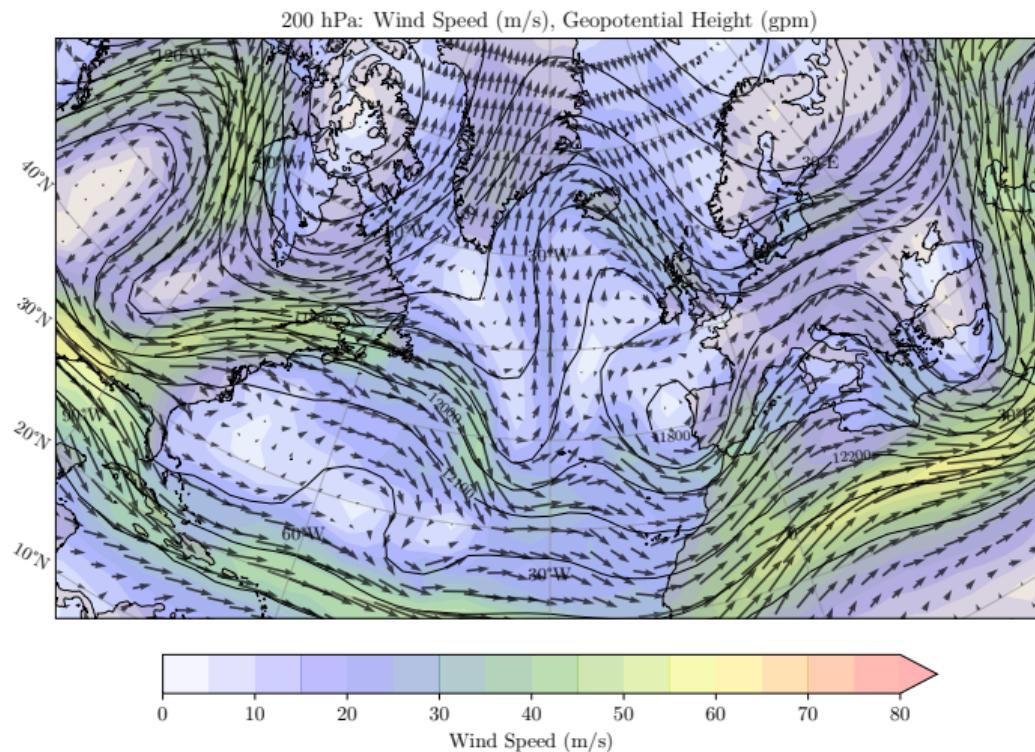


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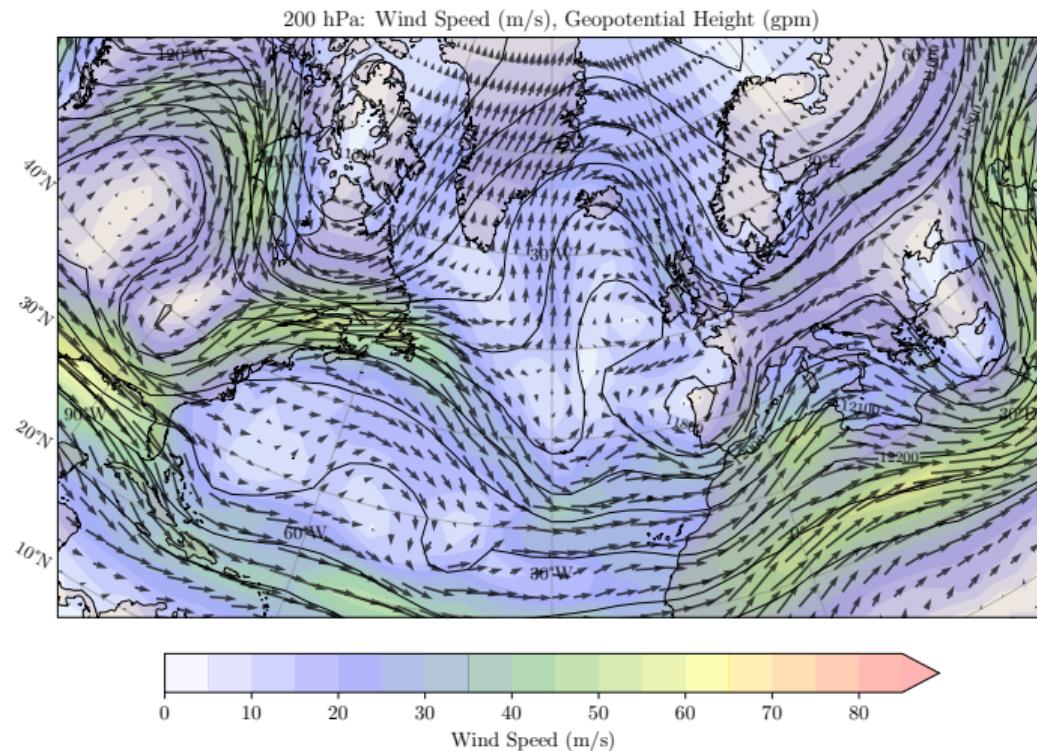
200 hPa: Wind Speed (m/s), Geopotential Height (gpm)



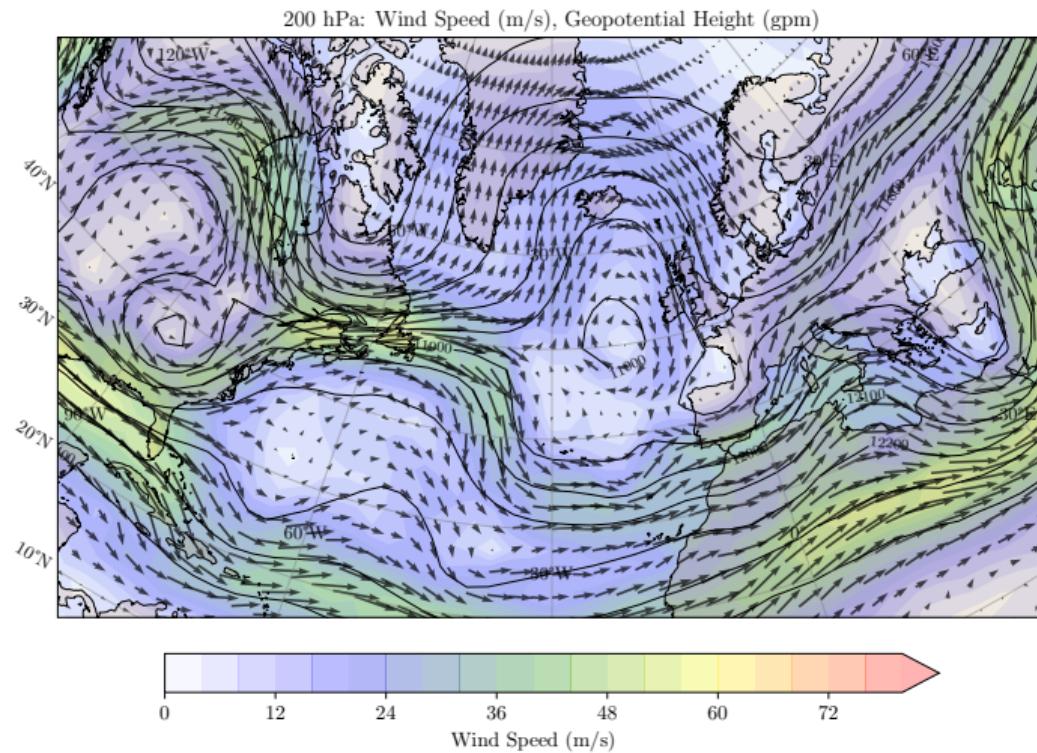
# Forecast: 3-May-2025 at 12:00, Level 200hPa



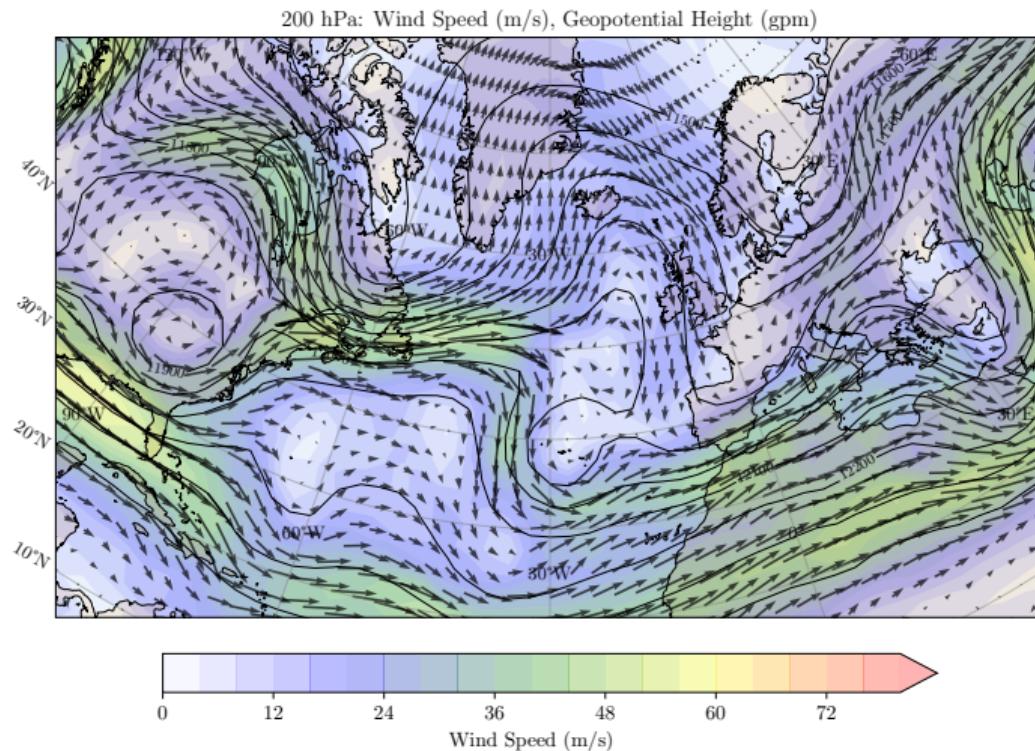
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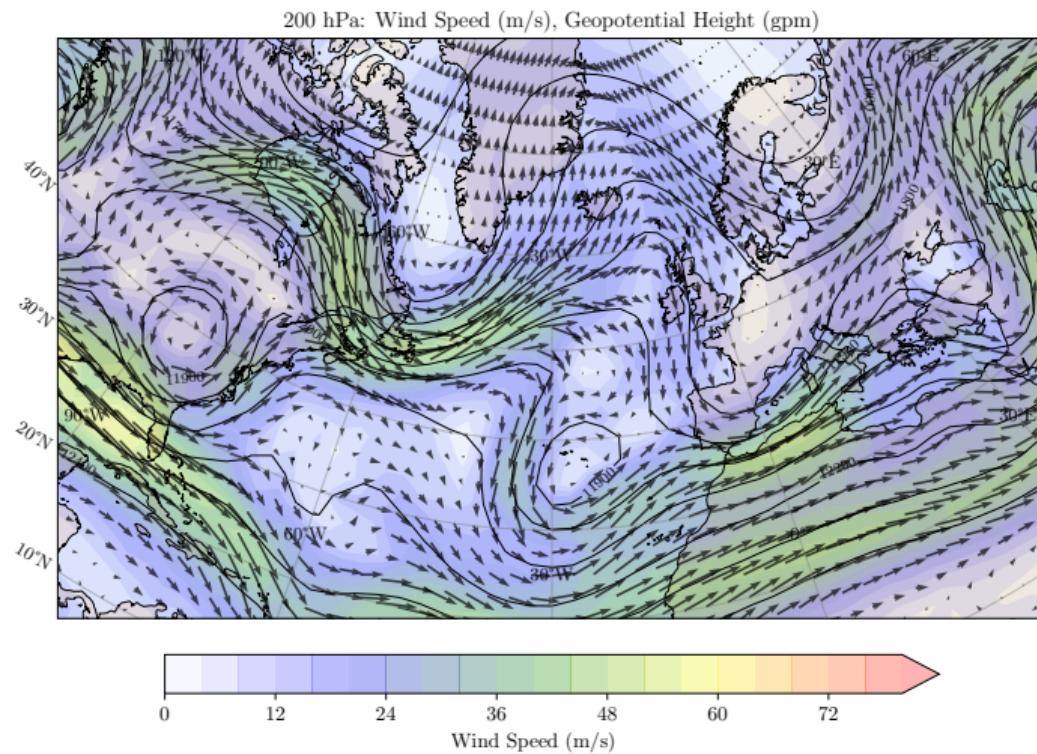
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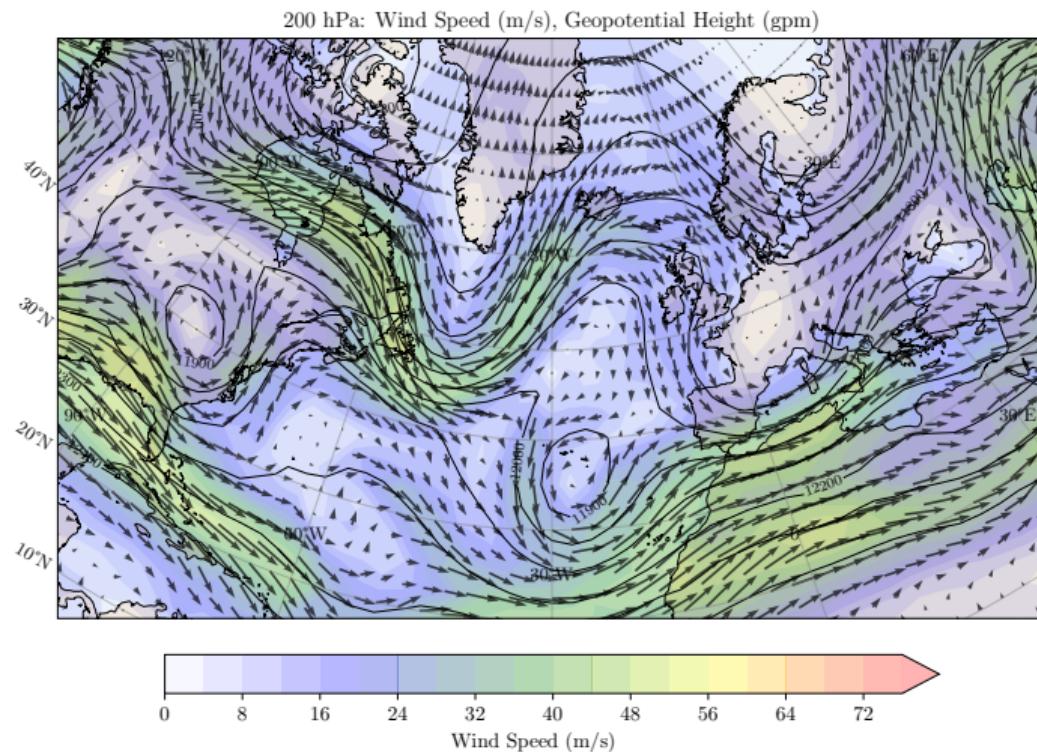
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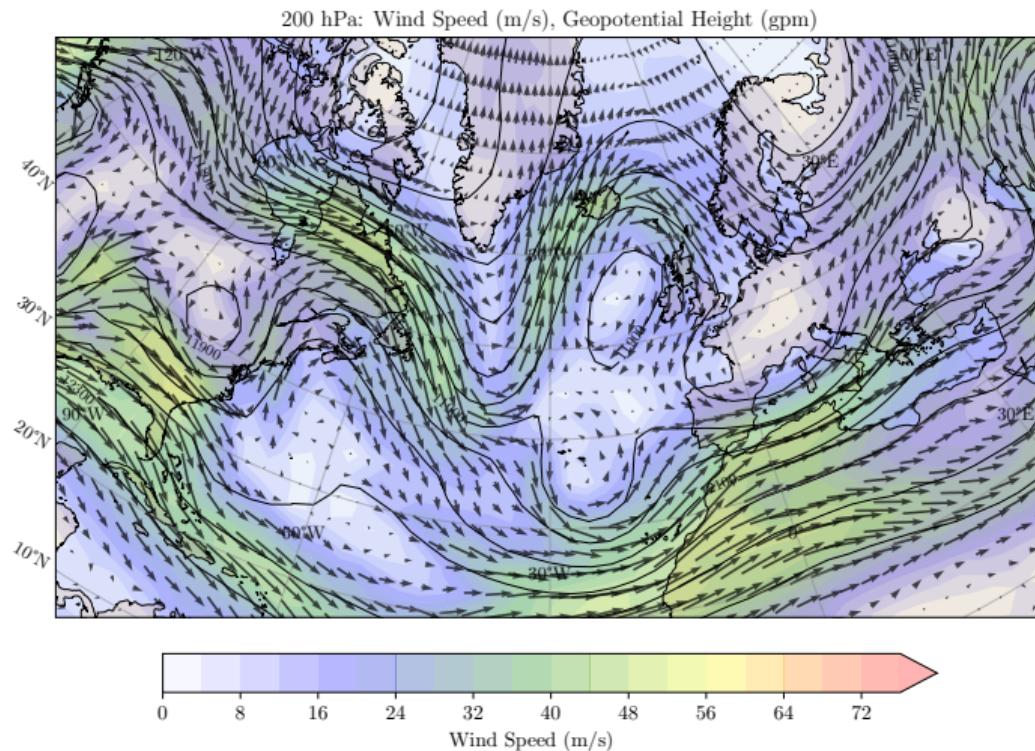
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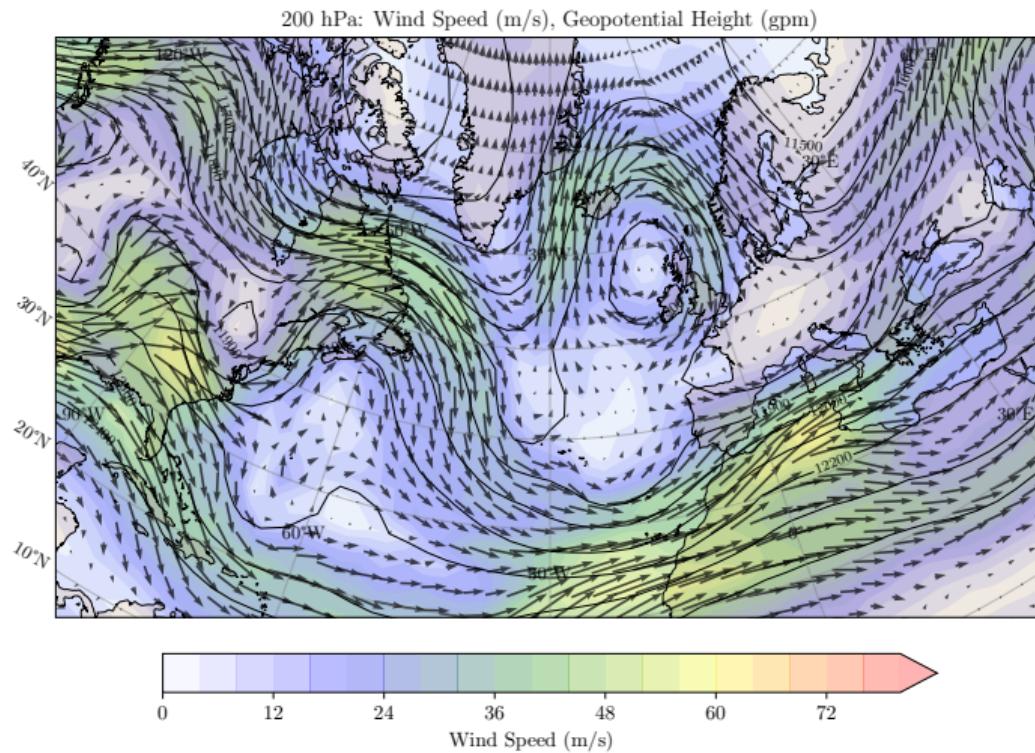
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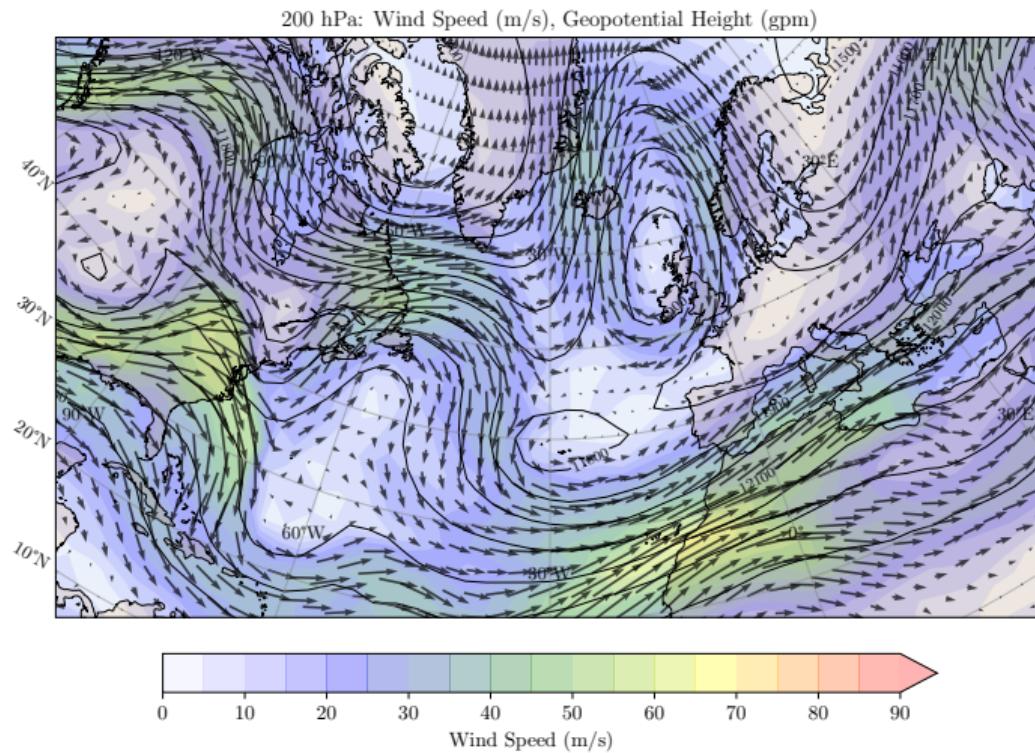
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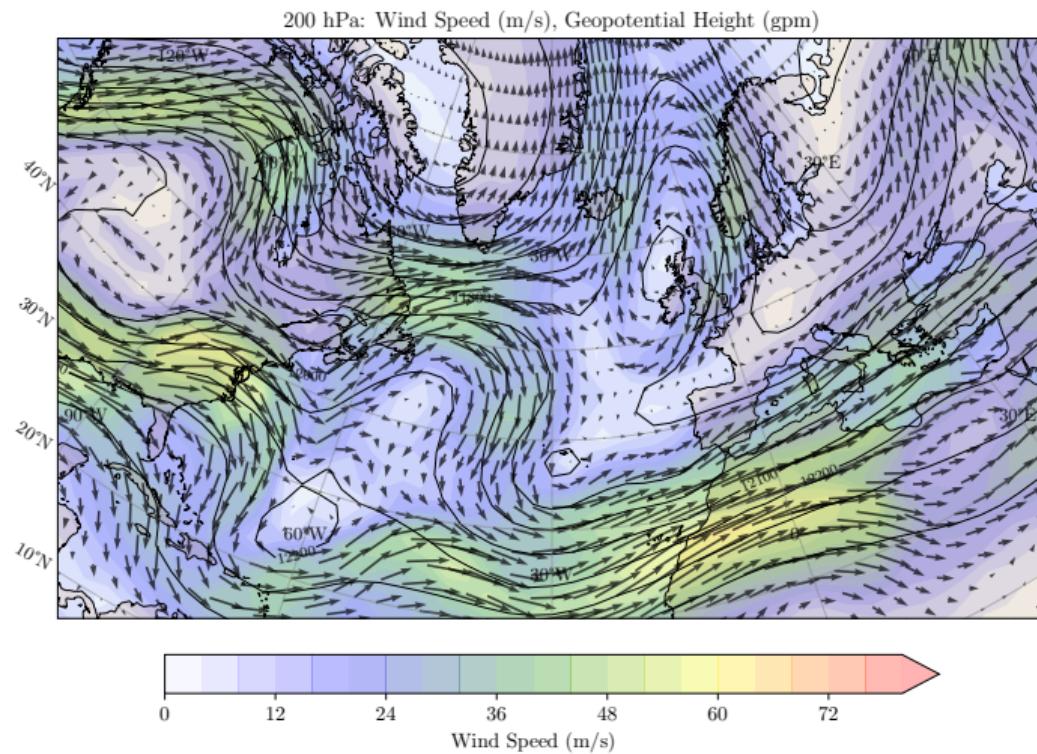
# Forecast: 7-May-2025 at 00:00, Level 200hPa



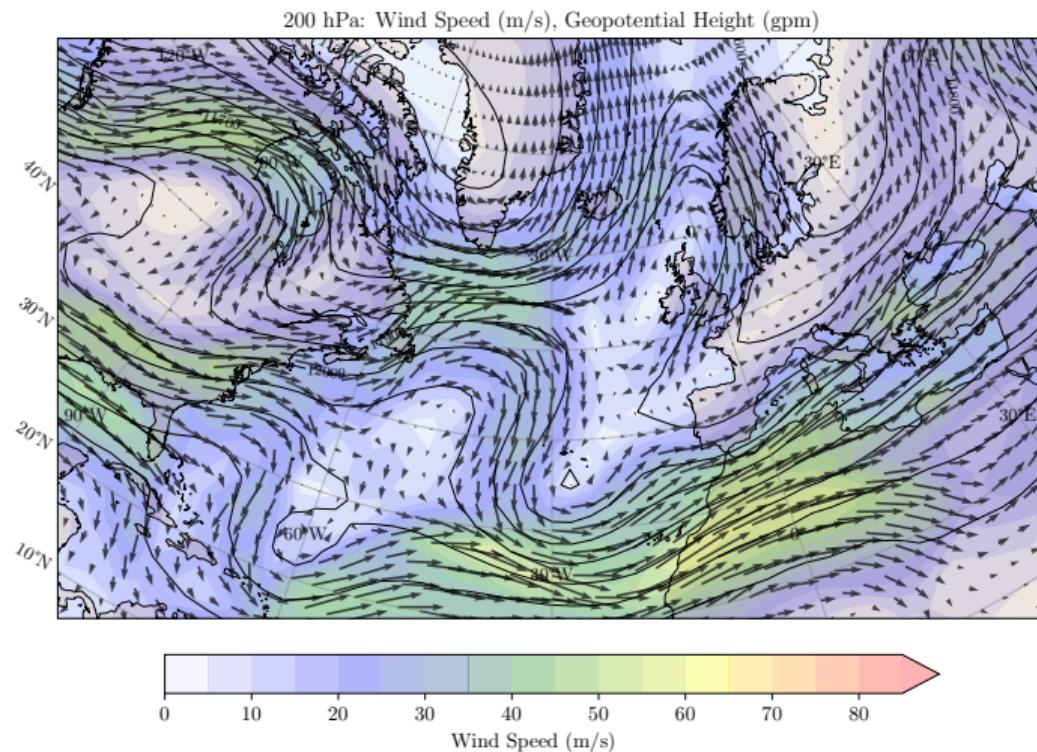
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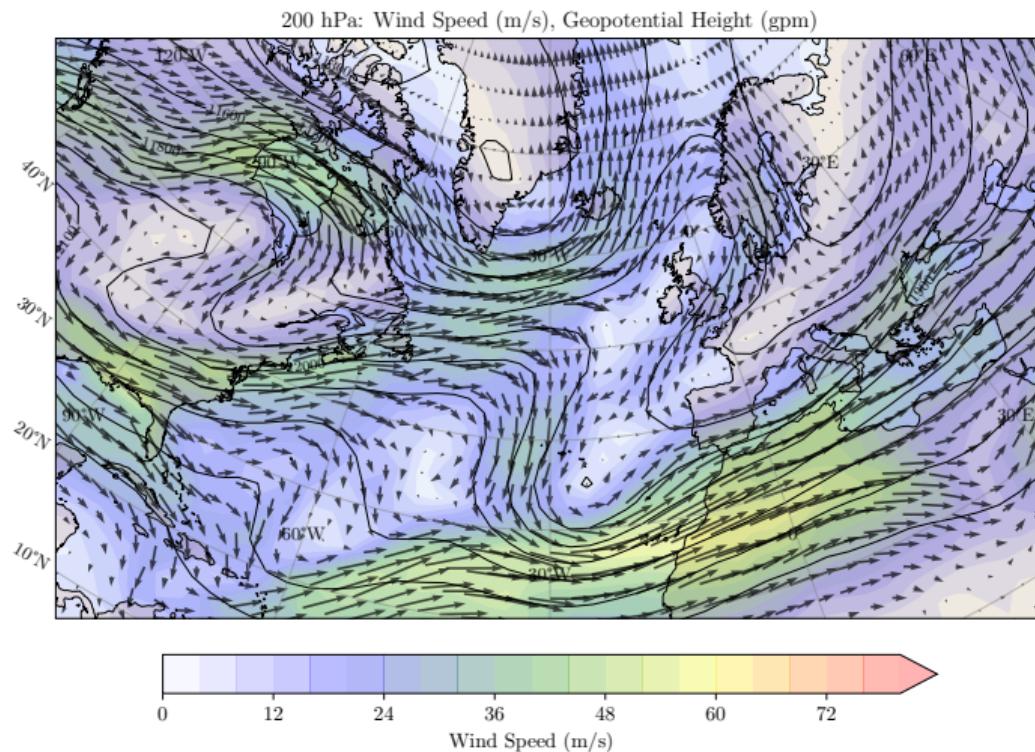
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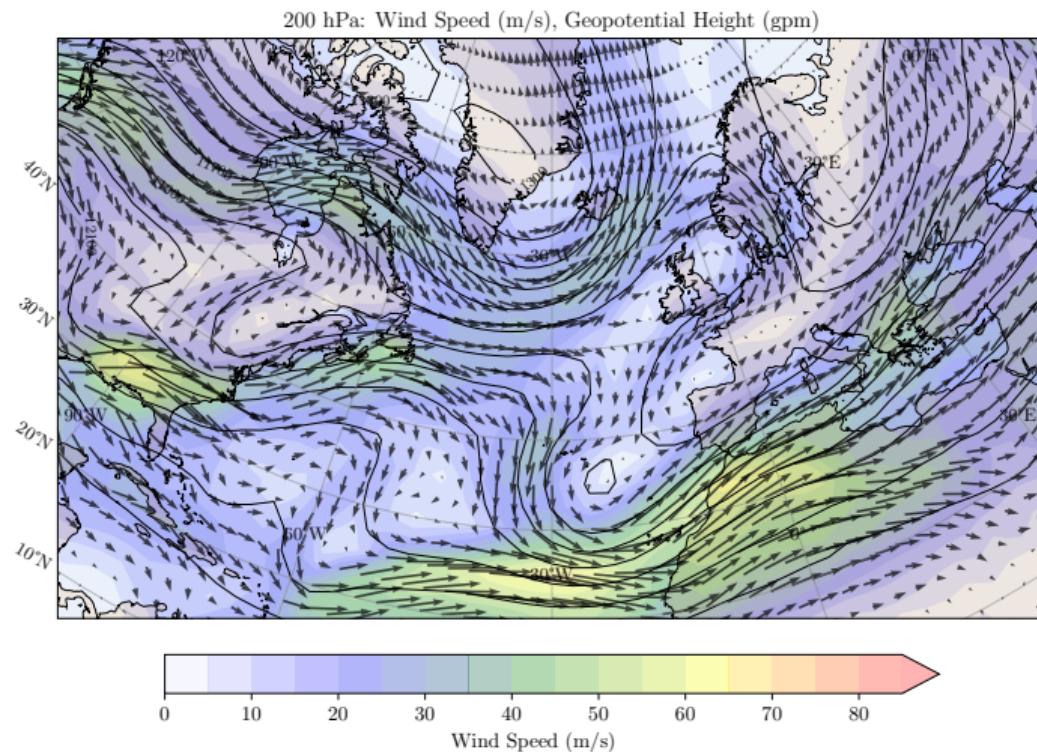
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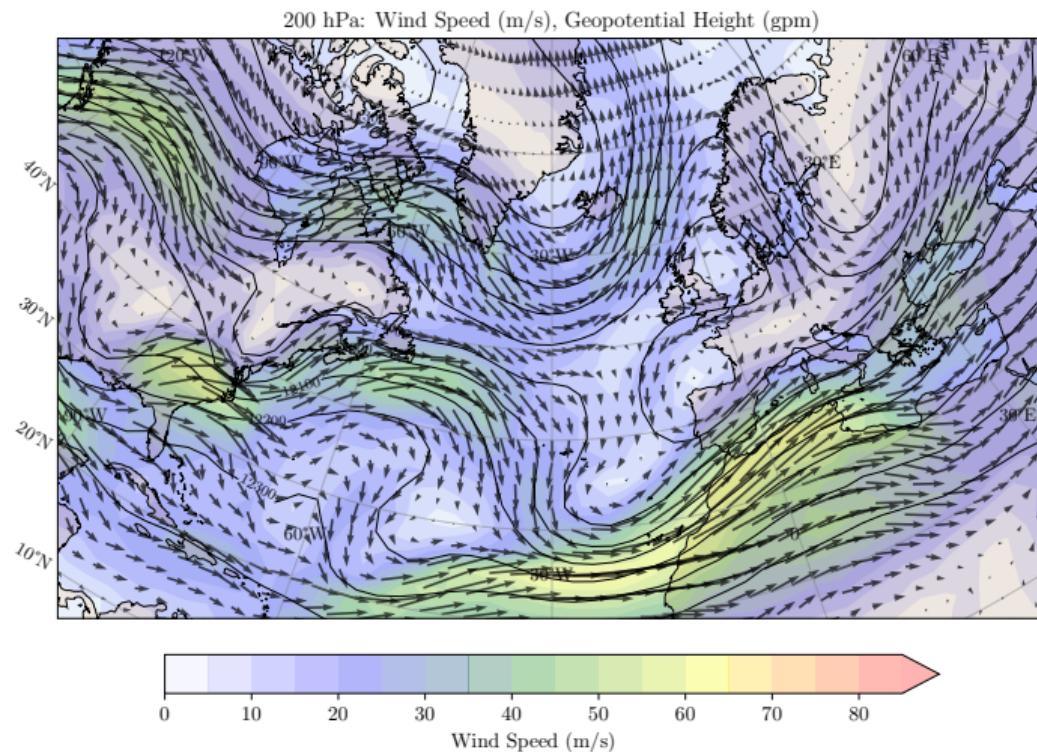
# Forecast: 9-May-2025 at 00:00, Level 200hPa



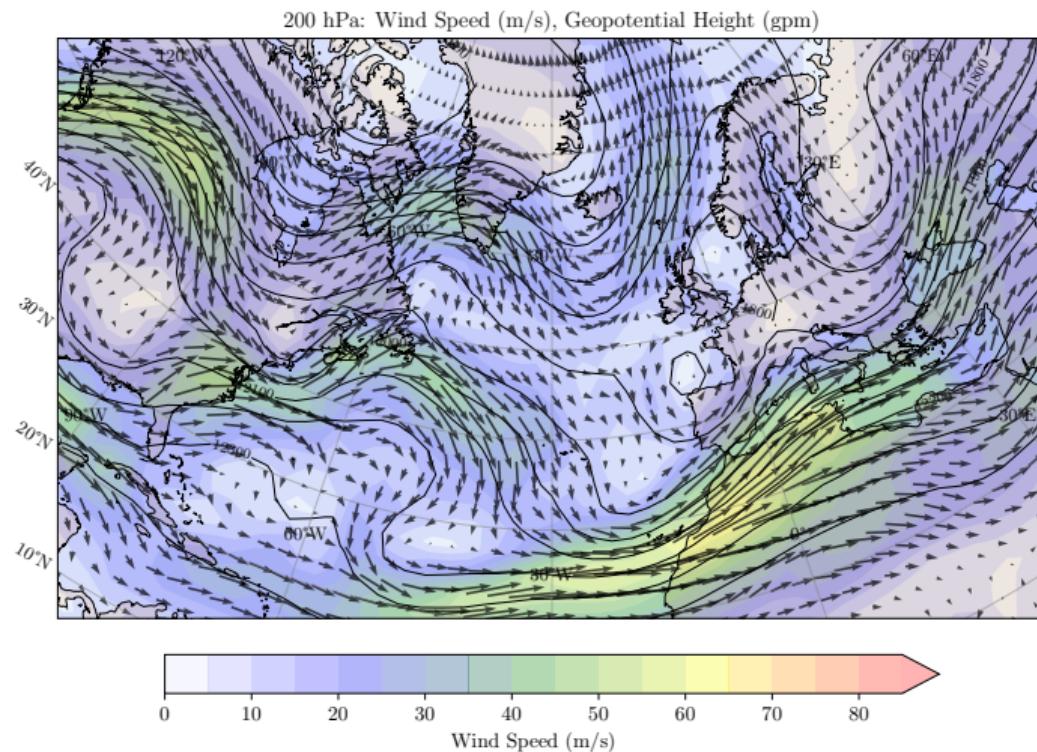
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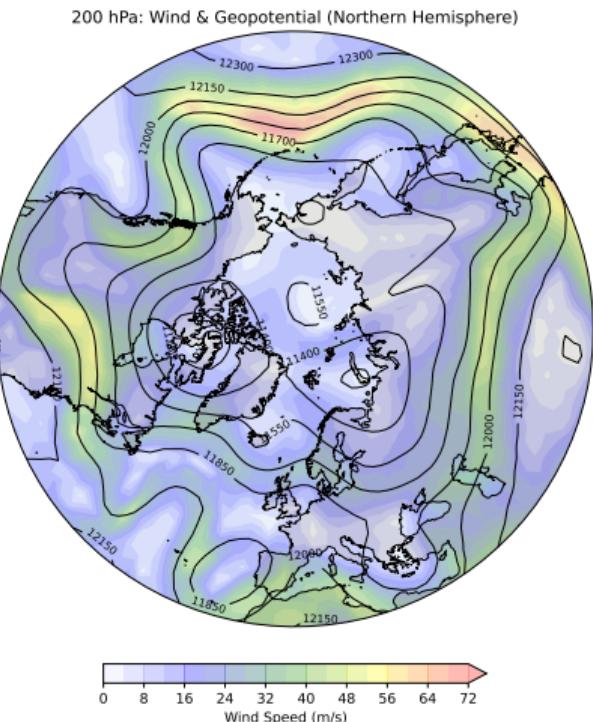
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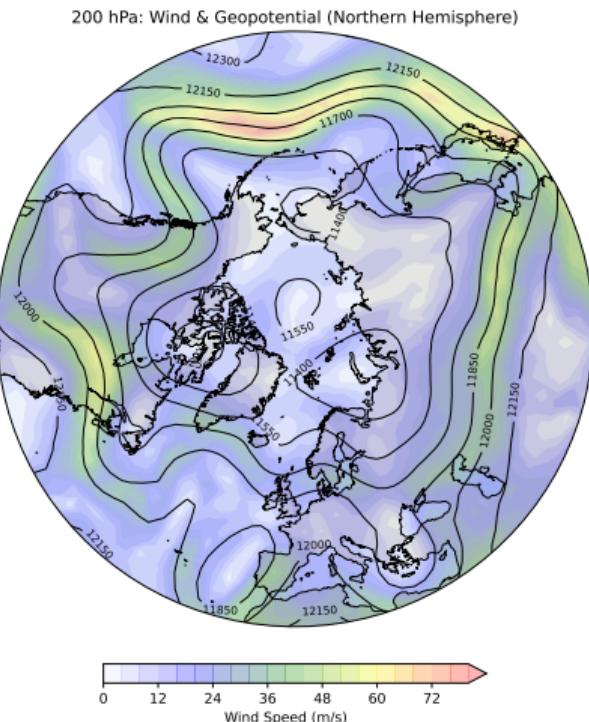
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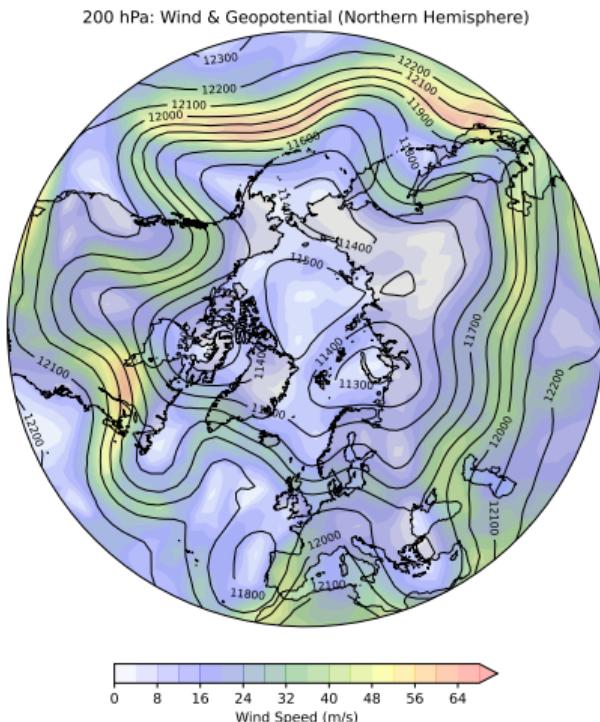
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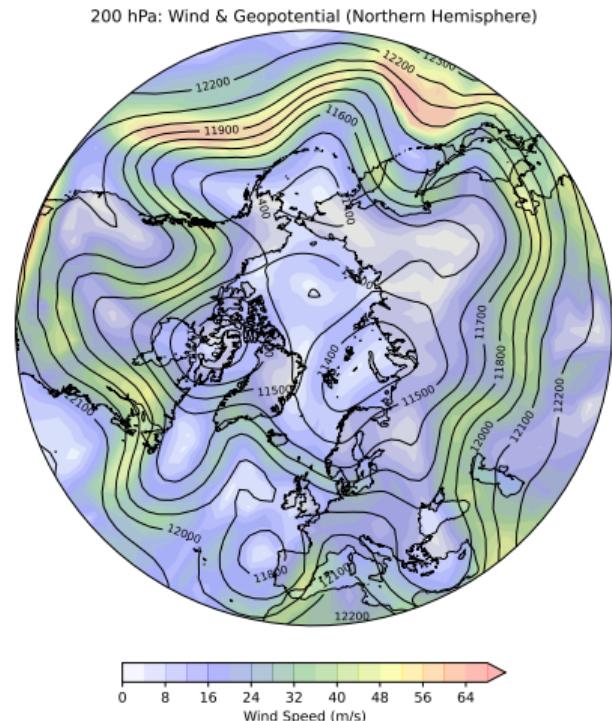
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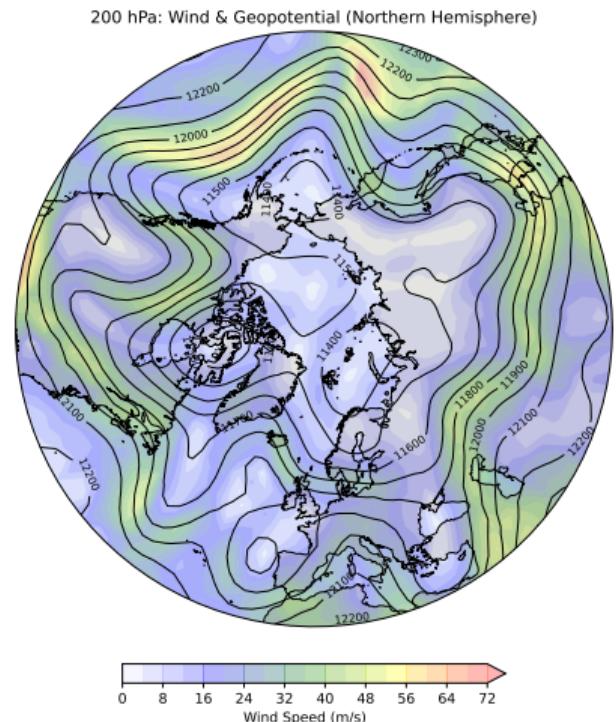
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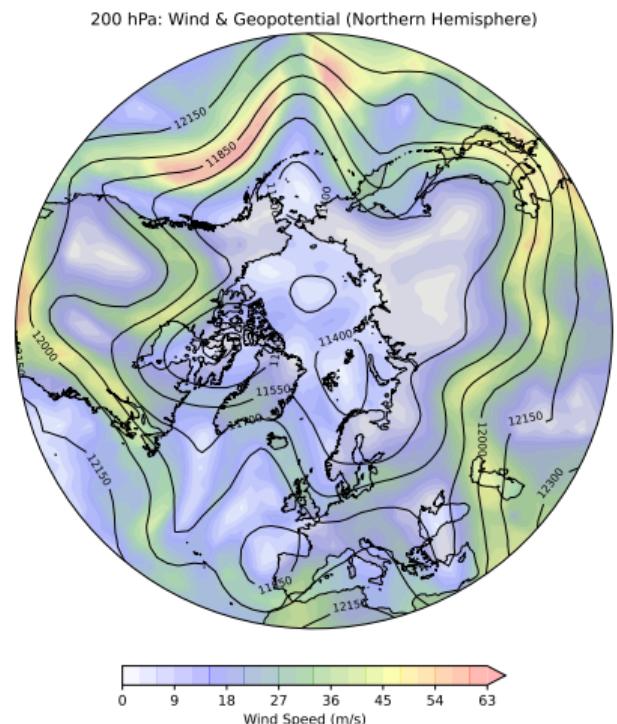
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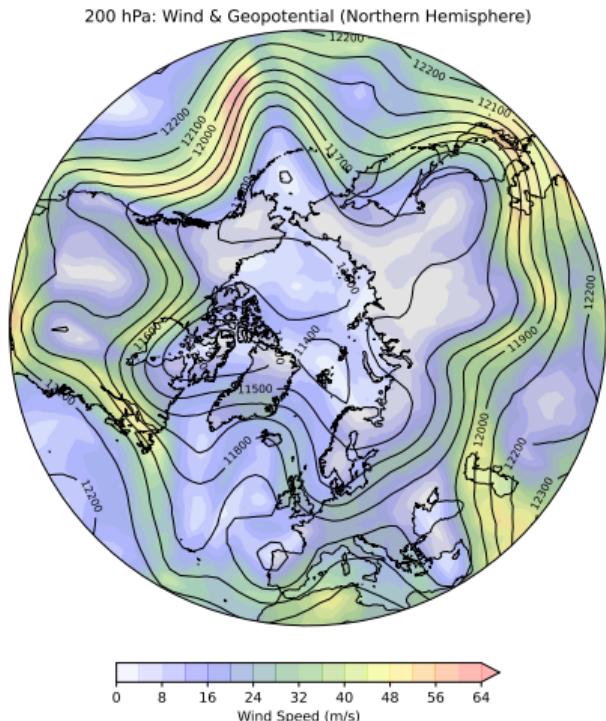
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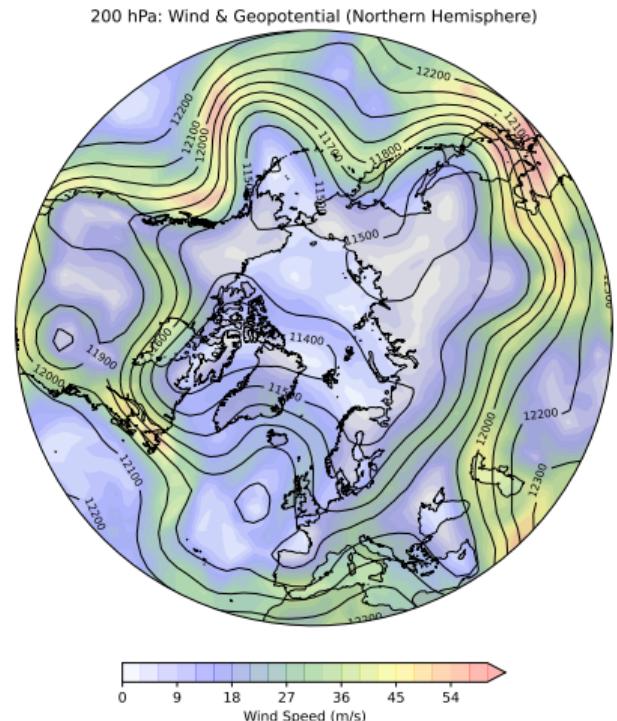
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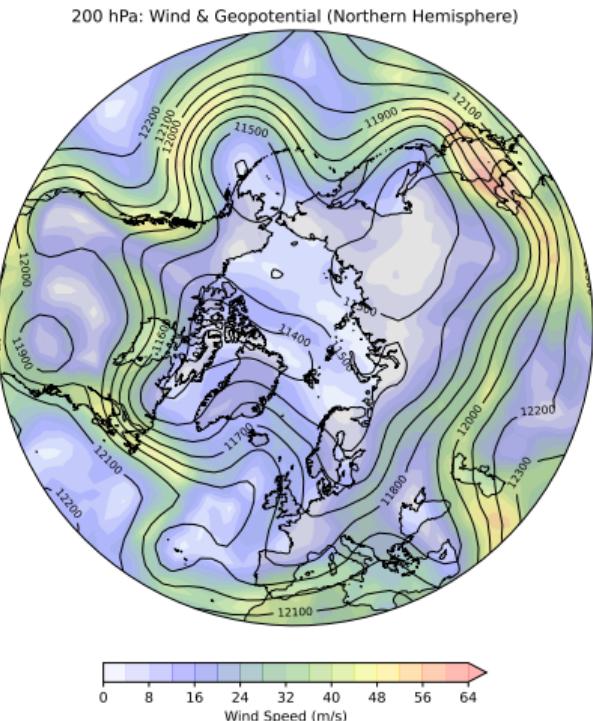
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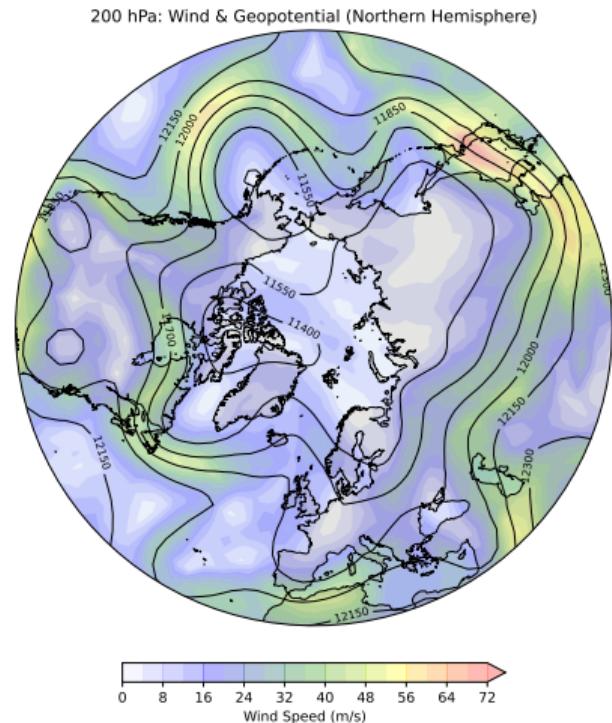
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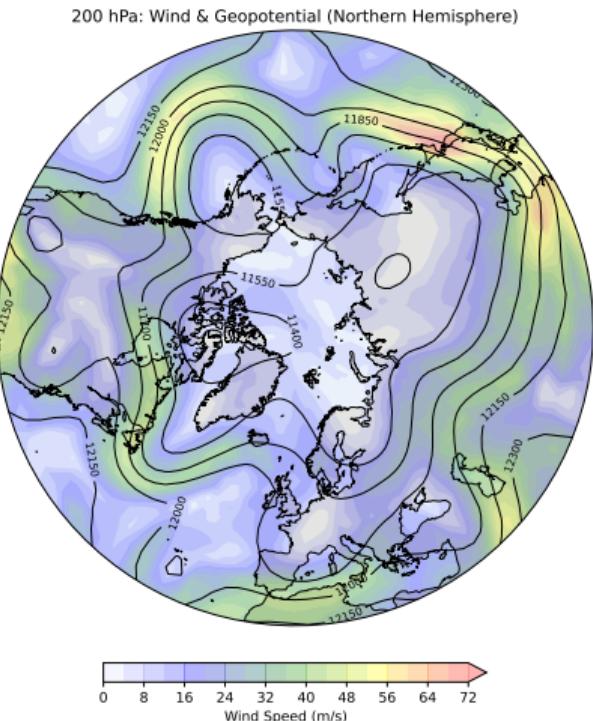
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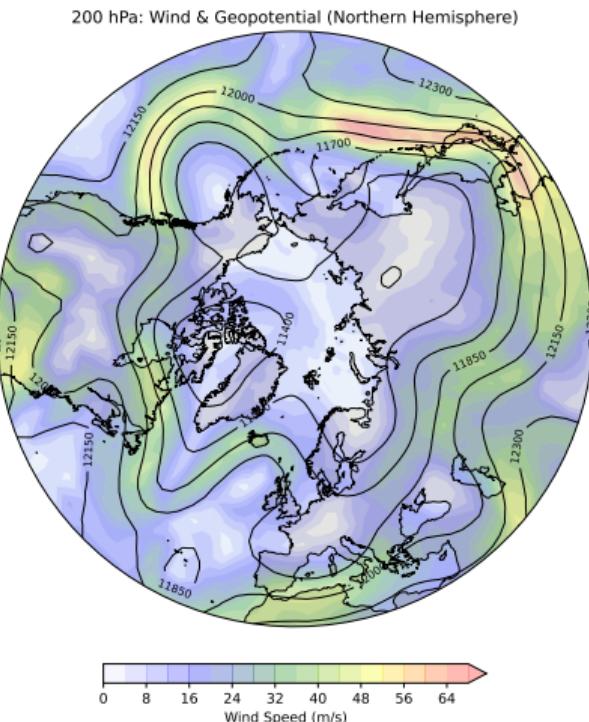
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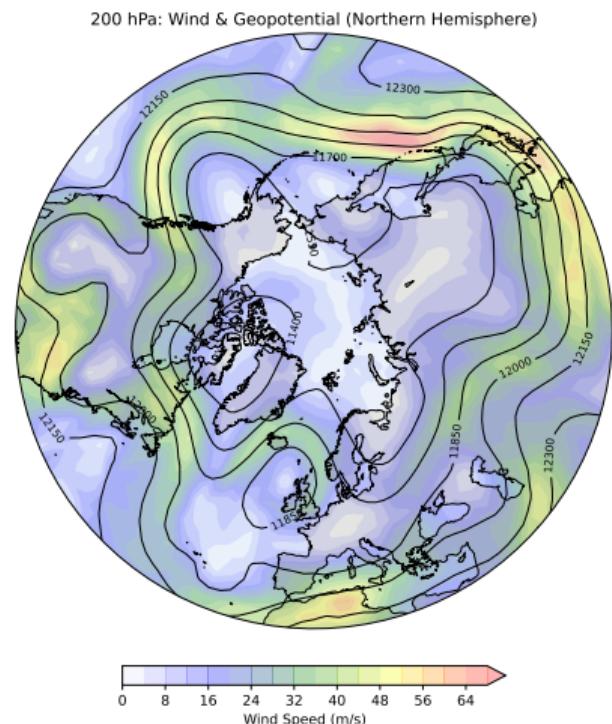
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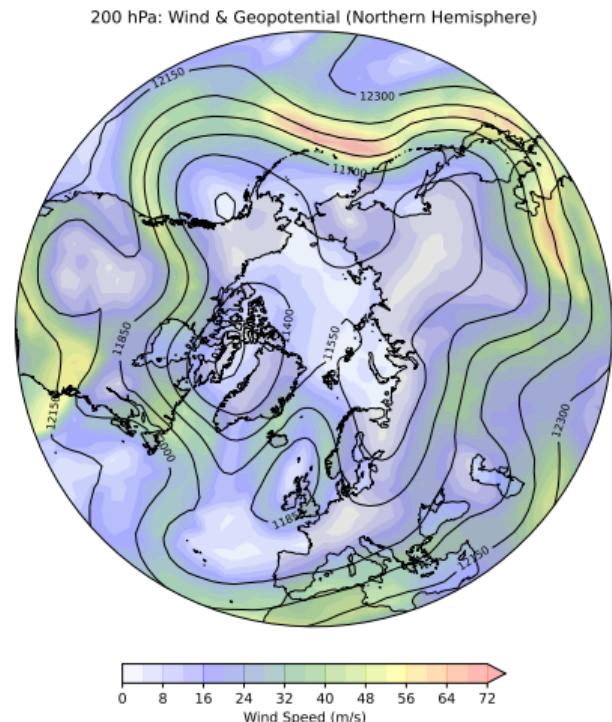
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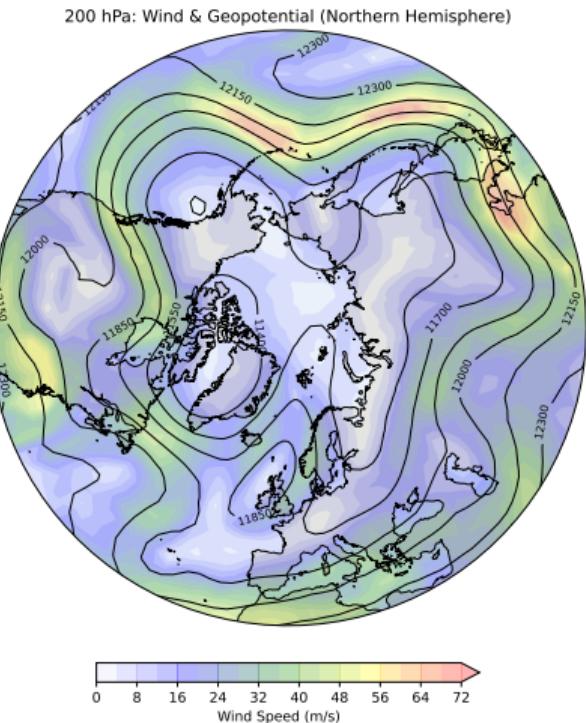
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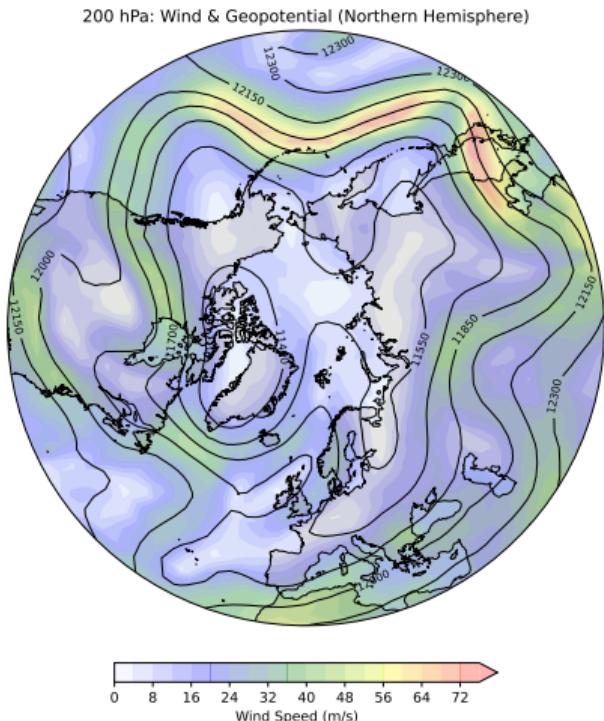
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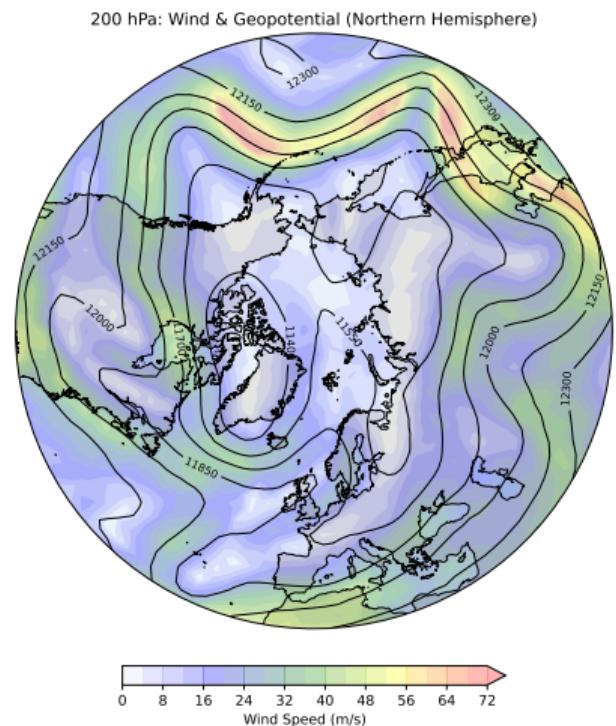
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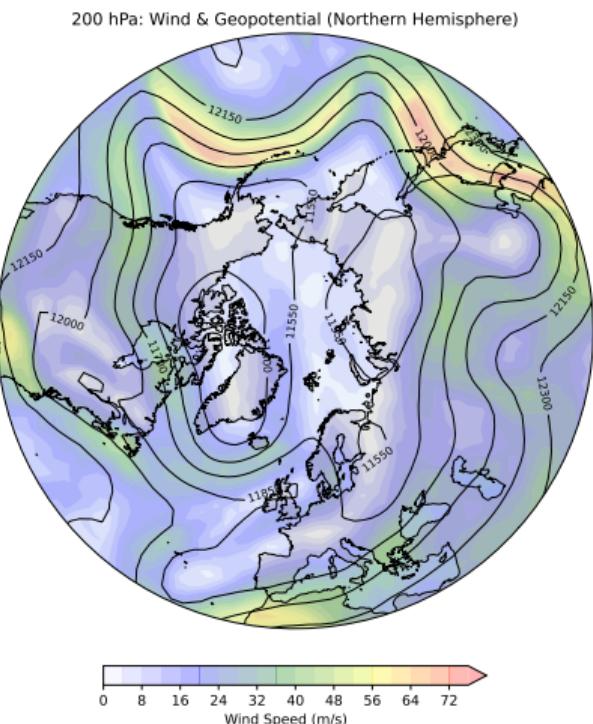
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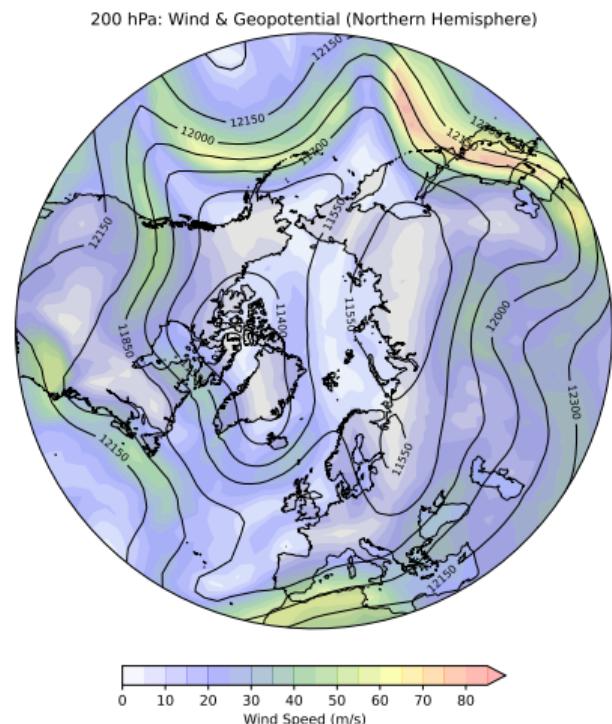
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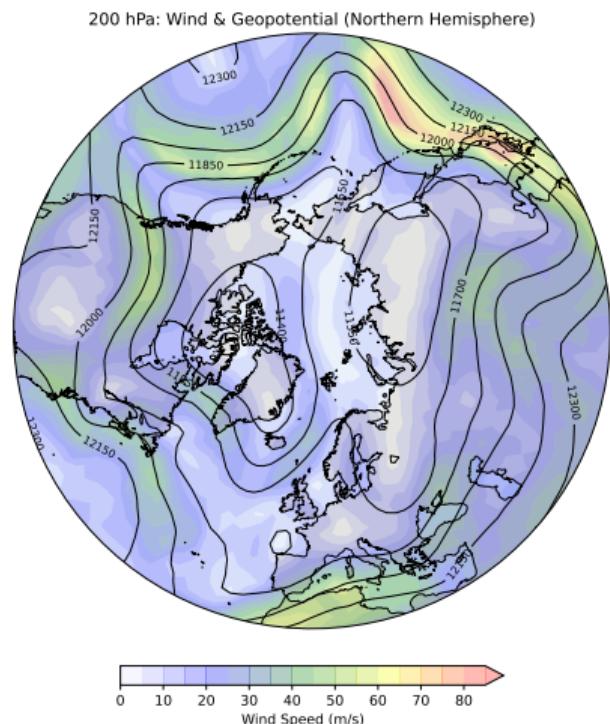
Forecast: 9-May-2025 at 12:00, Level 200hPa



Forecast: 10-May-2025 at 00:00, Level 200hPa

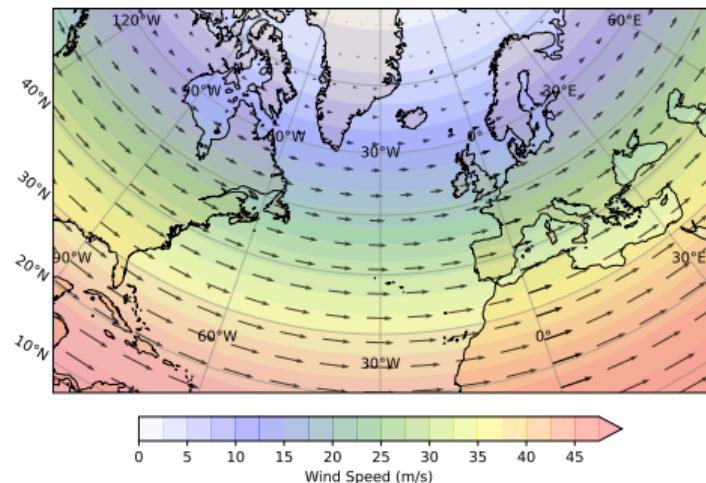


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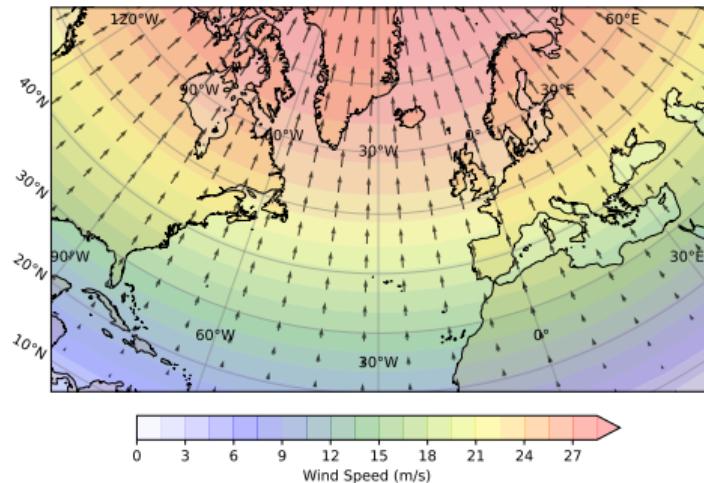
# Idealisiertes zonales Windfeld

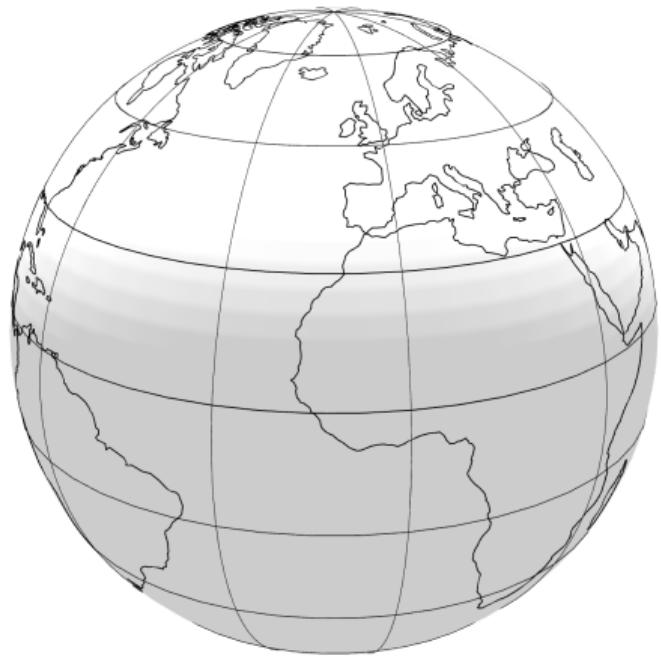
- Gezeigt ist ein simuliertes **zonales Windfeld**:
  - Reine Ost-West-Strömung ( $v = 0$ )
  - Geschwindigkeit abhängig von der Breite:  
 $u = U_0 \cdot \sin^2(\theta)$
  - Keine Druckgradienten oder vertikale Struktur
- Dieses idealisierte Feld dient als Basis zur Untersuchung grosskaliger atmosphärischer Prozesse.
- Zonale Winde treten z. B. in der Realität als **Jetstreams** auf.



# Idealisiertes meridionales Windfeld

- Gezeigt ist ein simuliertes **meridionales Windfeld**:
  - Reine Nord-Süd-Strömung ( $u = 0$ )
  - Geschwindigkeit abhängig von der Breite:
$$v = V_0 \cdot \cos(\theta)$$
- Die Darstellung ist idealisiert, ohne Rückkopplung durch Coriolis oder Druckgradienten.















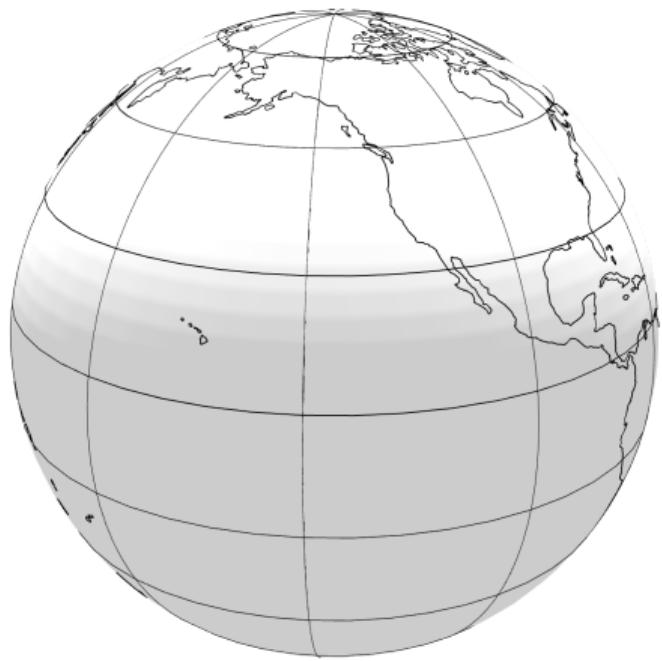


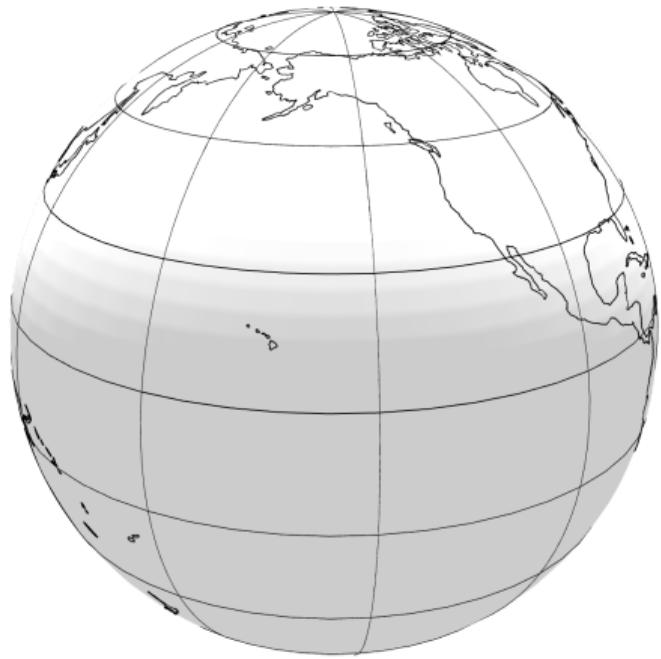


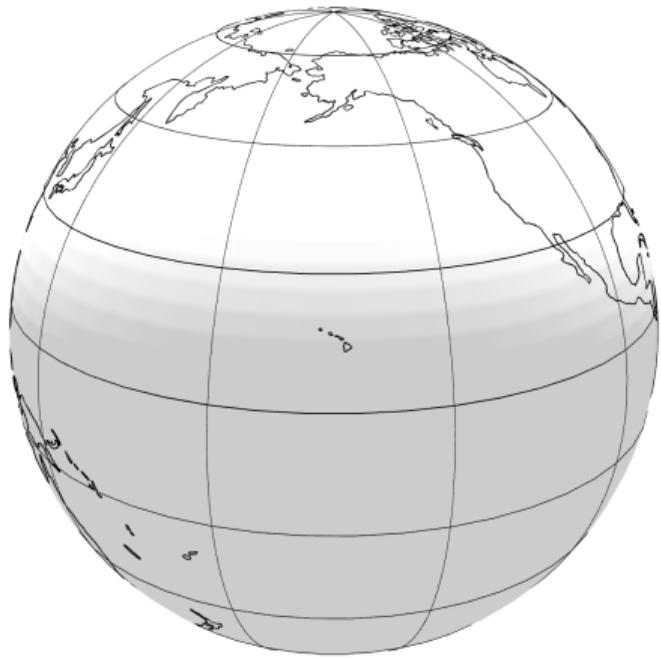


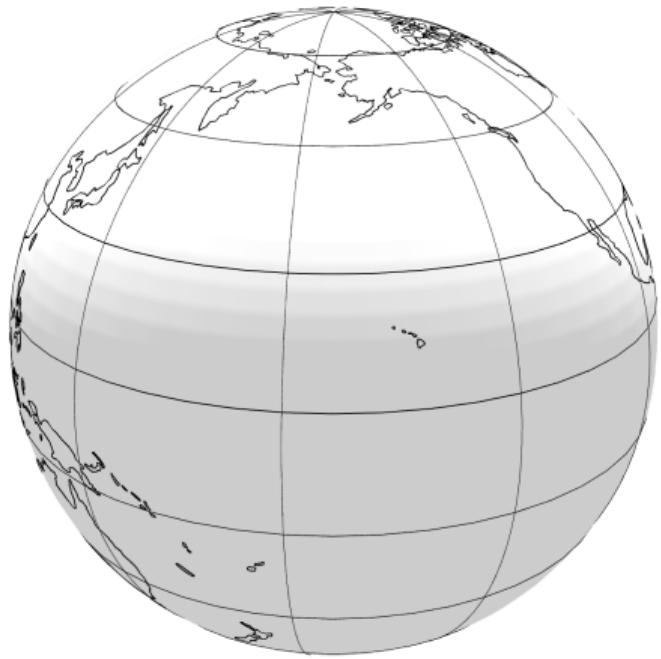


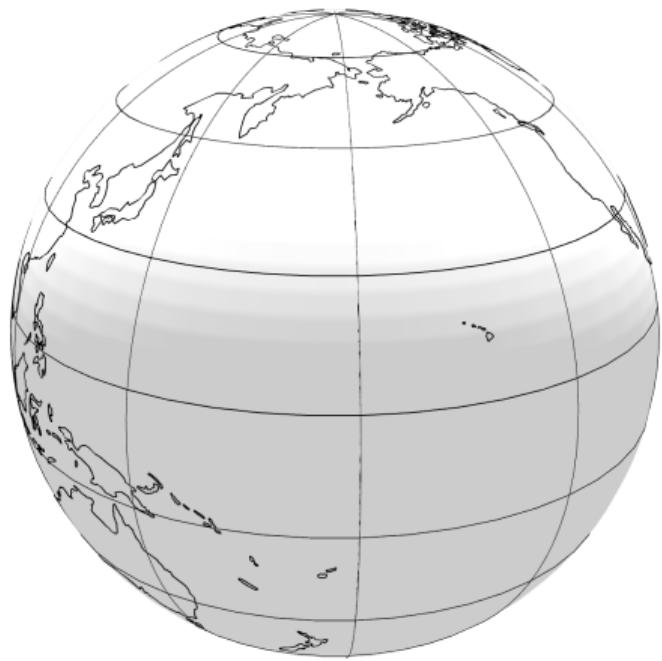










































# Corioliskraft: Grundprinzip

- Die **Corioliskraft** ist eine Scheinkraft, die in rotierenden Bezugssystemen wie der Erde wirkt.
- Sie verursacht eine Ablenkung von bewegten Luft- und Wassermassen:
  - **Nordhalbkugel:** Ablenkung nach rechts
  - **Südhalbkugel:** Ablenkung nach links
- Maximale Wirkung an den Polen, null am Äquator.

# Mathematische Formulierung

$$\vec{F}_C = -2m(\vec{\Omega} \times \vec{v})$$

- $m$ : Masse des Körpers
- $\vec{\Omega}$ : Rotationsvektor der Erde
- $\vec{v}$ : Geschwindigkeit relativ zur Erdoberfläche

# Beispiel: Corioliskraft beim Velofahren

## Gegeben:

- Geschwindigkeit:  $\vec{v} = 8.33 \text{ m/s}$  (30 km/h)
- Masse:  $m = 80 \text{ kg}$
- Breite:  $\varphi = 47^\circ$  (Zürich)
- Erdrotation:

$$23 \text{ h } 56 \text{ min } 4 \text{ s} \quad \Rightarrow \quad \vec{\Omega} = \frac{2\pi}{86164 \text{ s}} \approx 7.292 \times 10^{-5} \text{ rad/s}$$

## Formel:

$$F_C = 2m\vec{v}\vec{\Omega} \sin(\varphi)$$

## Einsetzen:

$$F_C \approx 2 \cdot 80 \cdot 8.33 \cdot 7.292 \times 10^{-5} \cdot \sin(47^\circ) \approx 0.070 \text{ N}$$

# Breitenabhängigkeit und Coriolis-Parameter

- Der **Coriolis-Parameter** beschreibt die Breitenabhängigkeit:

$$f = 2\Omega \sin(\phi)$$

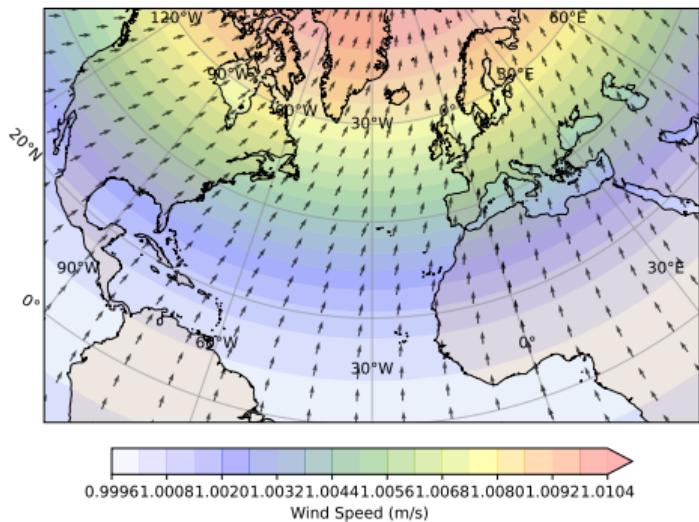
- Seine Änderung mit der Breite ergibt den  **$\beta$ -Parameter**:

$$\beta = \frac{\partial f}{\partial y} = \frac{2\Omega \cos(\phi)}{a}$$

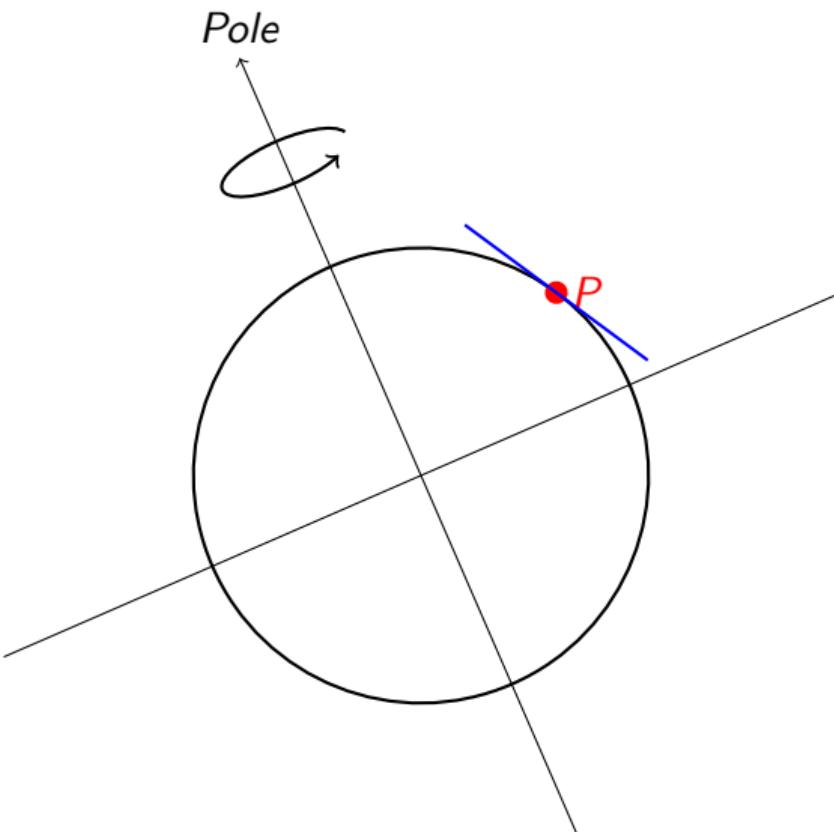
- $a$ : Erdradius,     $y$ : Nord-Süd-Koordinate

# Simulation der Corioliskraft auf Nordströmung

- Darstellung: Ablenkung von Luftpaketen bei rein meridionaler Startgeschwindigkeit ( $v = 1, u = 0$ )
- Nach `steps = 100` Zeitschritten:
  - Auf Nordhalbkugel: Ablenkung nach Osten
  - Auf Südhalbkugel: Ablenkung nach Westen
- Breitenabhängigkeit durch  $f = 2\Omega \sin(\phi)$



## $\beta$ -Ebene Approximation



# Was ist die $\beta$ -Ebene?

- Die  **$\beta$ -Ebene** ist eine lokale Approximation der Erdkugel nahe einer bestimmten Breite  $\phi_0$ .
- Ziel: Vereinfachung der Corioliskraft für mathematische Modelle grosskaliger Strömungen.
- Der Coriolisparameter  $f$  wird linearisiert:

$$f(y) = f_0 + \beta y$$

mit:

- $f_0 = 2\Omega \sin(\phi_0)$ : Coriolisparameter an der Referenzbreite
- $\beta = \left. \frac{\partial f}{\partial y} \right|_{\phi_0} = \frac{2\Omega \cos(\phi_0)}{a}$
- $y$ : meridionale Entfernung vom Referenzbreitenkreis

# Was ist Vortizität?

- **Vortizität** beschreibt die lokale Rotation in einer Strömung.
- Definiert als das **Rotationsfeld** des Geschwindigkeitsvektors:

$$\vec{\zeta} = \nabla \times \vec{u}$$

- Für eine zweidimensionale Strömung  $\vec{u} = (u(x, y), v(x, y))$  ist nur die z-Komponente relevant:

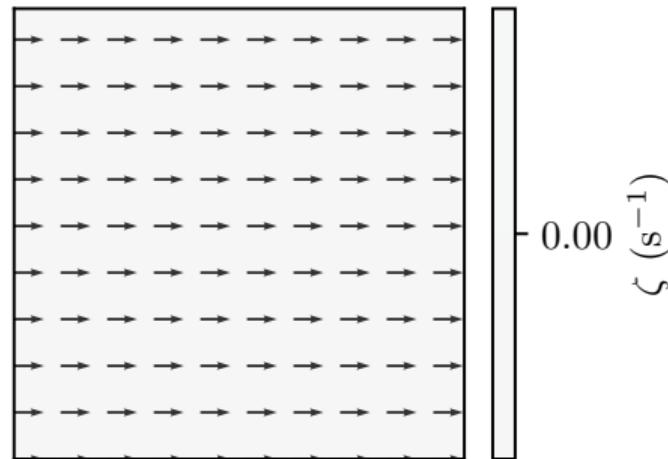
$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

- $\zeta > 0$ : Zyklonale Rotation (gegen den Uhrzeigersinn)
- $\zeta < 0$ : Antizyklonale Rotation (im Uhrzeigersinn)

## Zero Vorticity (Uniform Flow)

- $\vec{u} = (2, 0)$
- Uniform horizontal flow
- No shear or curvature
- $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 - 0 = 0$

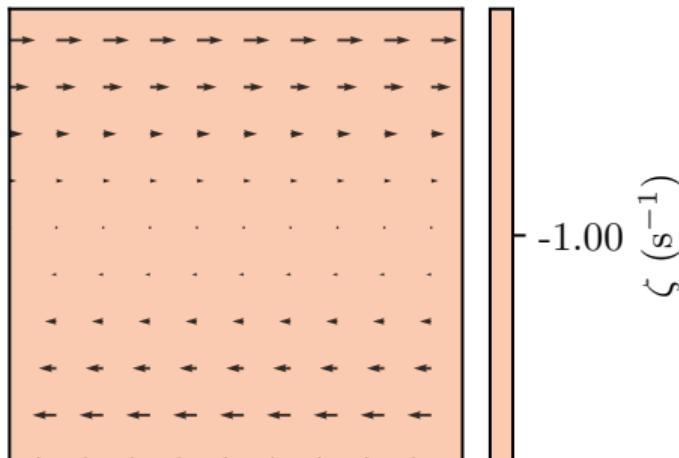
## Zero Vorticity (Uniform Flow)



# Shear Vorticity

- $\vec{u} = (y, 0)$
- Horizontal shear:  $\frac{\partial u}{\partial y} = 1$
- $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 - 1 = -1$

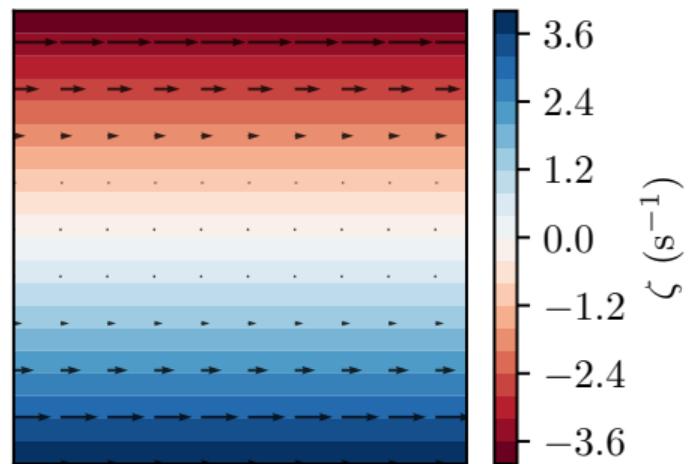
Shear Vorticity



# Nonlinear Shear Vorticity

- $\vec{u} = (y^2, 0)$
- Antisymmetric vorticity field
- Stronger at larger  $|y|$
- $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 - 2y = -2y$

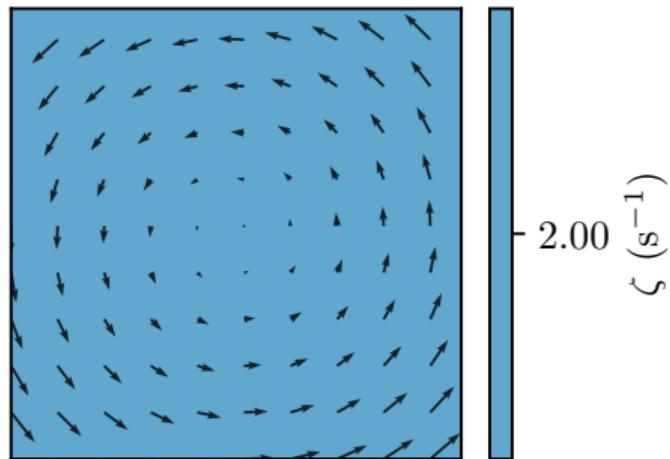
Nonlinear Shear Vorticity



# Positive Vorticity (Cyclonic)

- $\vec{u} = (-y, x)$
- Pure rotation, counter-clockwise
- $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 1 - (-1) = 2$

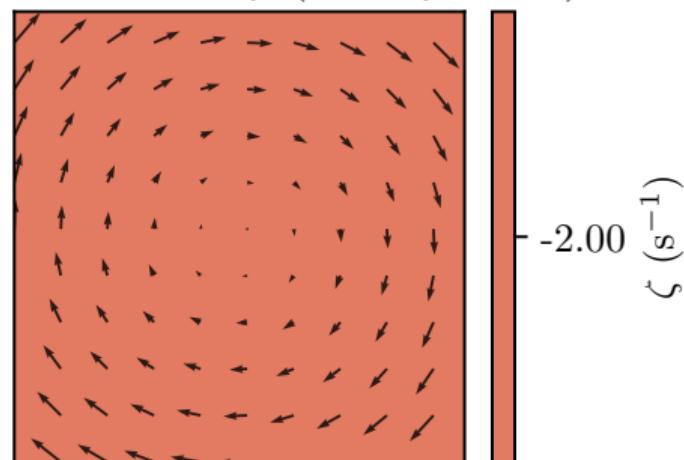
Positive Vorticity (Cyclonic)



# Negative Vorticity (Anticyclonic)

- $\vec{u} = (y, -x)$
- Clockwise rotation
- $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = -1 - 1 = -2$

Negative Vorticity (Anticyclonic)



# Absolute Vortizität

- In einem rotierenden Bezugssystem (wie der Erde) ergibt sich die **absolute Vortizität** zu:

$$\eta = f + \zeta$$

- $\zeta$ : relative Vortizität (durch Scherung und Krümmung der Strömung)
- $f = 2\Omega \sin \phi$ : Coriolis-Parameter, abhängig von der geografischen Breite
- Bedeutend für grossräumige geophysikalische Strömungen (z. B. Rossby-Wellen, Erhaltung der potenziellen Vortizität)

# Konservierung der potentiellen Vortizität

- Die **potenzielle Vortizität (PV)** ist definiert als:

$$q = \frac{\eta}{H} = \frac{f + \zeta}{H}$$

- $\eta$ : absolute Vortizität, bestehend aus  $f + \zeta$
- $H$ : effektive Schichtdicke (z. B. Troposphärenhöhe oder isentrope Dicke)
- In einer reibungsfreien, adiabatischen Atmosphäre gilt:

$$\frac{Dq}{Dt} = 0$$

- Folge:** PV ist entlang von Teilchenbahnen erhalten  $\rightarrow$  zentrale Grösse in der grosskaligen Dynamik

# Warum ist PV-Erhaltung wichtig für Rossby-Wellen?

- Rossby-Wellen entstehen durch meridionale Bewegung von Luftpaketen (nach Norden oder Süden)
- Dabei ändert sich der Coriolis-Parameter:  $f = 2\Omega \sin \phi$
- **PV-Erhaltung:**

$$q = \frac{f + \zeta}{H} = \text{konstant}$$

- Wenn ein Luftpaket nach Norden wandert:  $f \uparrow \Rightarrow \zeta \downarrow$
- Folge: Luftpaket rotiert weniger → Rückstellkraft → beginnt zu oszillieren
- **Diese Oszillation ist die Rossby-Welle**
- Ohne PV-Erhaltung gäbe es keine Rückstellmechanismus → keine Welle

# Mittlere Strömung und Anomalien

- In Äquatornähe dominiert eine mittlere Ost-West-Strömung  $U$
- Wir betrachten kleine Abweichungen davon:

$$u' = U + u, \quad v' = v \quad \text{mit } u, v \ll U$$

- Die Strömung ist quellenfrei  $\rightarrow$  Stromfunktion  $\psi$  existiert:

$$u = -\frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x}$$

# Zirkulation und Drehimpuls

- Relative Vorticity (Zirkulation):

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \Delta\psi$$

- Absolute Vorticity:

$$\zeta + f \quad (\text{mit Coriolisparameter } f = f(y))$$

- Annahme: Erhaltung der absoluten Vorticity:

$$\frac{d}{dt}(\zeta + f) = 0$$

# Bewegungsgleichung - Herleitung

- Kettenregel für totale Ableitung:

$$\frac{d}{dt}(\zeta + f) = \frac{\partial \zeta}{\partial t} + (U + u)\frac{\partial \zeta}{\partial x} + v \left( \frac{\partial \zeta}{\partial y} + \frac{\partial f}{\partial y} \right)$$

- Näherungen:

- $u \ll U \rightarrow$  vernachlässigbar
- $\partial \zeta / \partial y \ll \partial f / \partial y$
- $\partial f / \partial y = \beta$
- $v = \frac{\partial \psi}{\partial x}$

- Daraus ergibt sich:

$$\frac{\partial \zeta}{\partial t} + U \frac{\partial \zeta}{\partial x} + \beta \frac{\partial \psi}{\partial x} = 0$$

- Mit  $\zeta = \Delta \psi$ :

$$\frac{\partial \Delta \psi}{\partial t} + U \frac{\partial \Delta \psi}{\partial x} + \beta \frac{\partial \psi}{\partial x} = 0$$

# Wellenlösung der Gleichung

- Ansatz: ebene Wellen

$$\psi(x, y, t) = \cos(kx + ly - \omega t)$$

- Einsetzen in Gleichung ergibt Dispersionsrelation:

$$\omega = U k - \frac{\beta k}{k^2 + l^2}$$

- Phasengeschwindigkeit:

$$c = \frac{\omega}{k} = U - \frac{\beta}{k^2 + l^2}$$

- Interpretation: westwärts laufende Wellen mit geringer Geschwindigkeit als  $U$

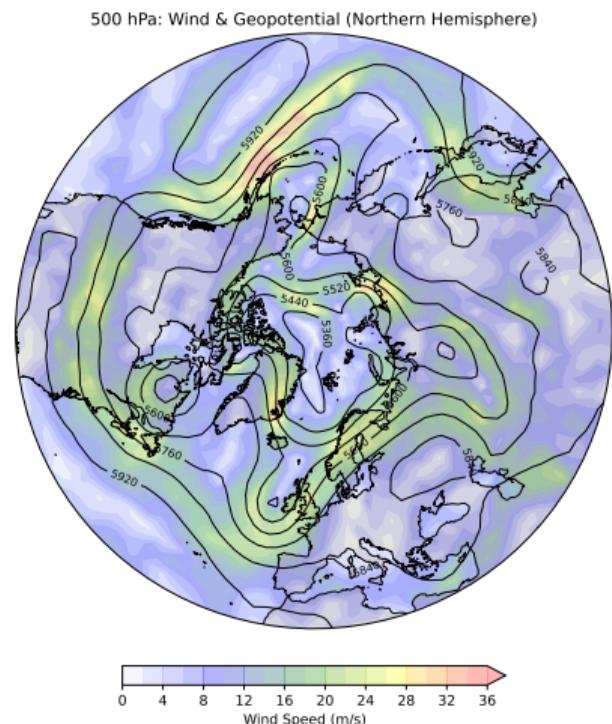
# Extremereignisse 2010: Russland und Pakistan

- Sommer 2010: Zwei extreme Wetterereignisse gleichzeitig
  - **Russland:** Hitzewelle,  $>40$  °C, Waldbrände, Smog, tausende Todesfälle
  - **Pakistan:** Jahrhundertflut, Monsunregen, 20 Mio. Menschen betroffen
- Ursache: **Quasi-stationäre Rossby-Welle mit  $k = 7$** 
  - Hoch über Russland → blockierte Hitze
  - Tief über Pakistan → anhaltender Monsunregen
- Atmosphärische Wellenstruktur führte zu **gleichzeitigen Extremen**
- Beispiel für Wirkung grosskaliger Dynamik auf regionale Wetterkatastrophen <sup>1</sup>

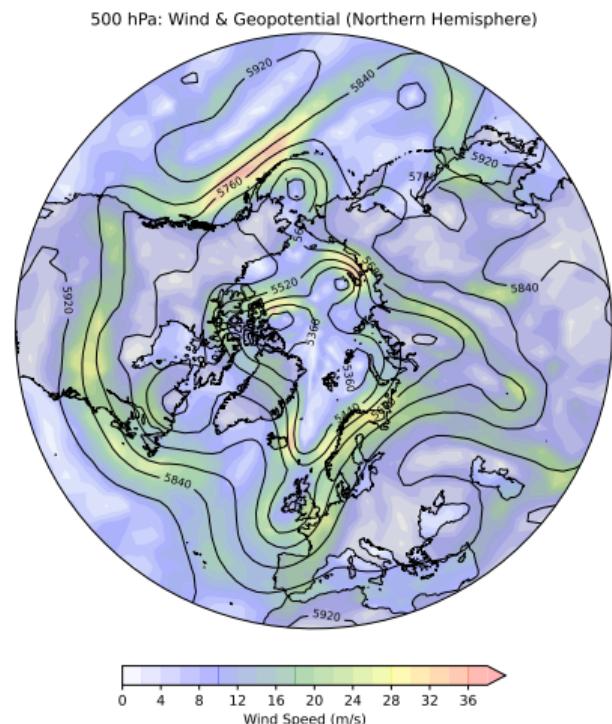
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<sup>1</sup>Petoukhov et al. (2013), PNAS, DOI: [10.1073/pnas.1222000110](https://doi.org/10.1073/pnas.1222000110)

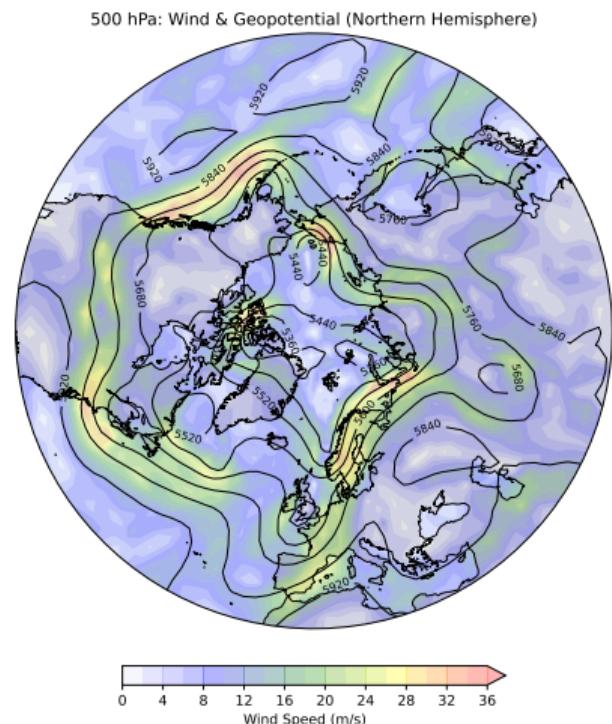
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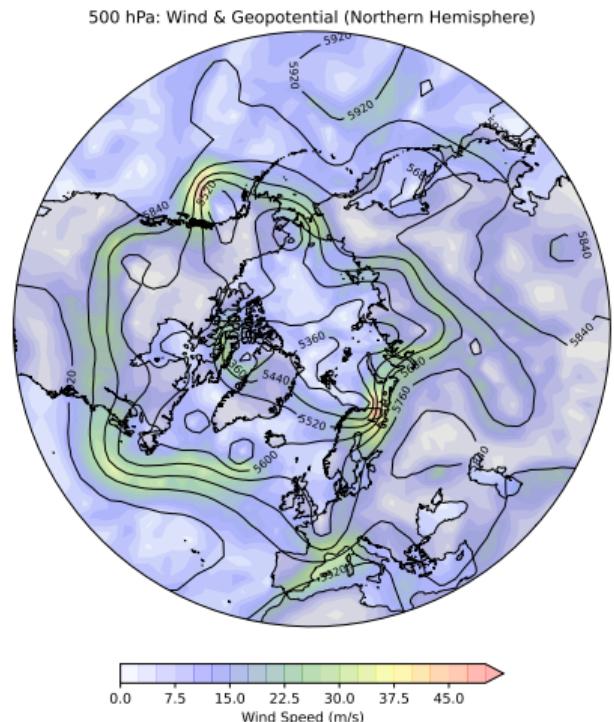
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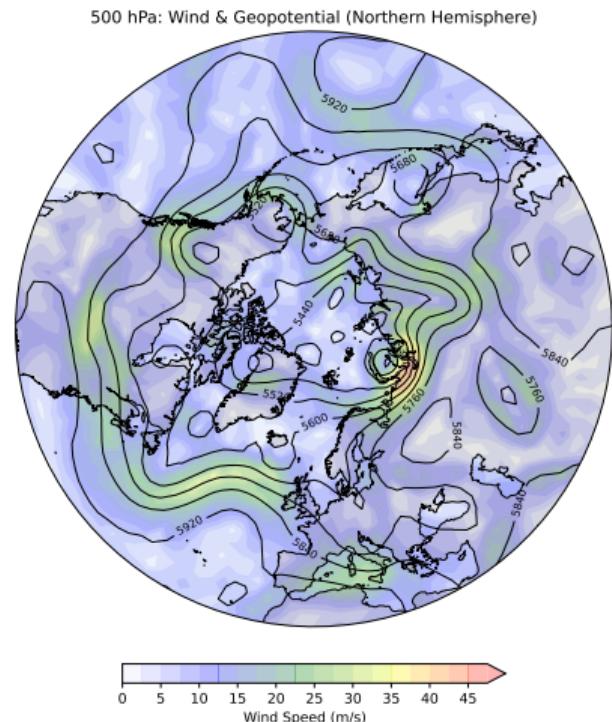
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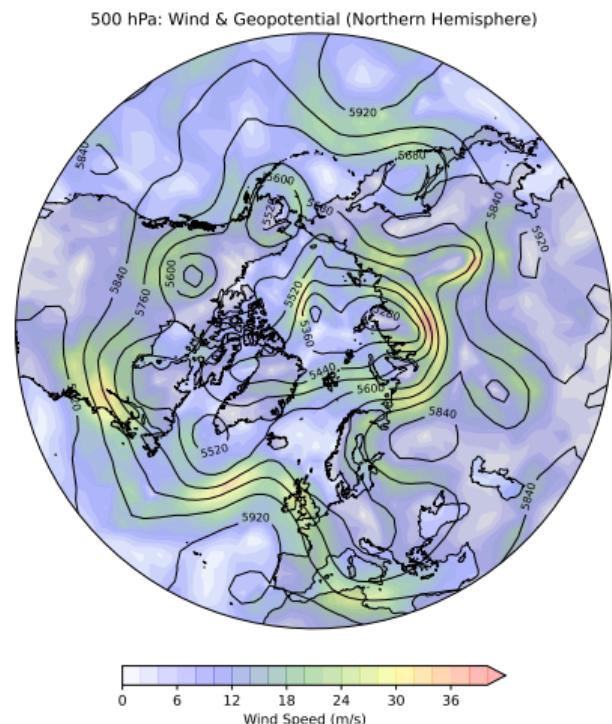
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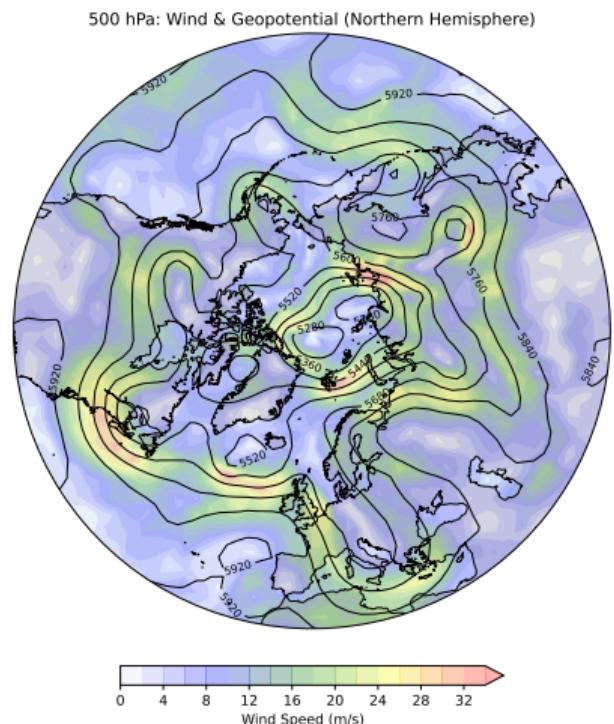
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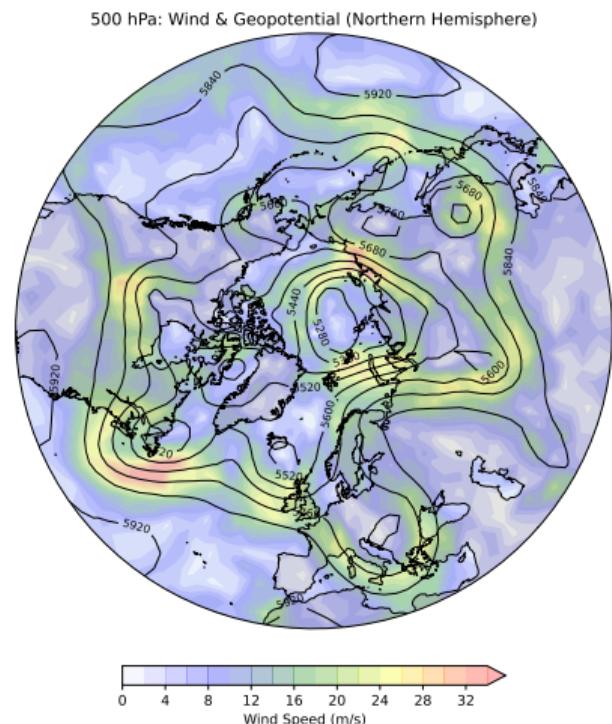
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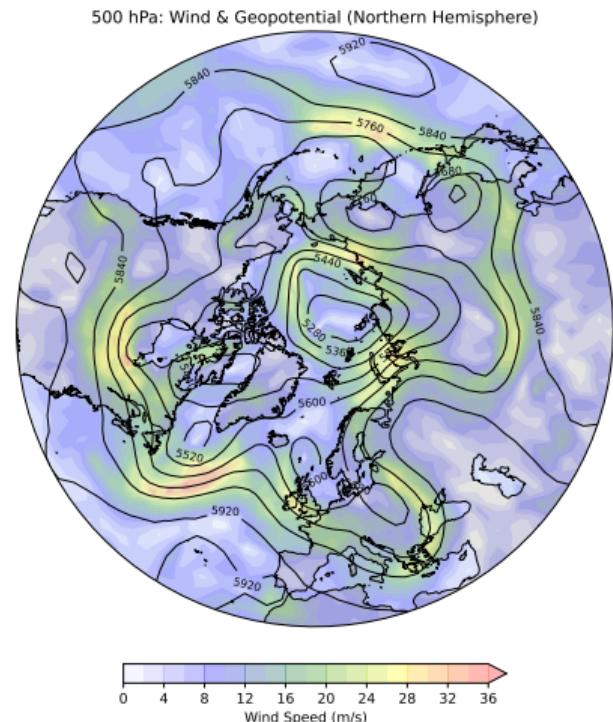
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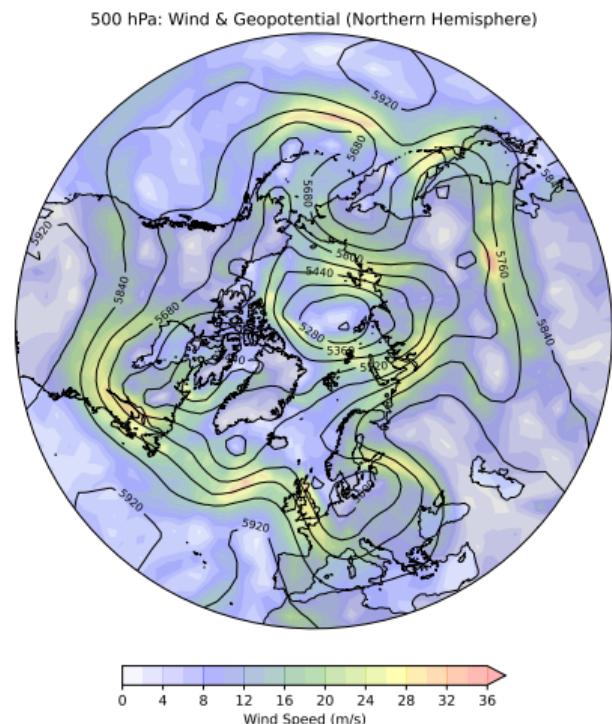
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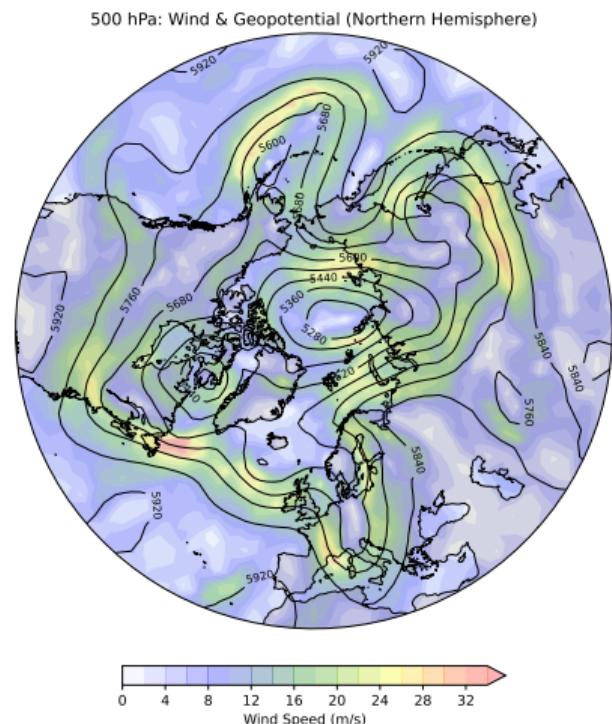
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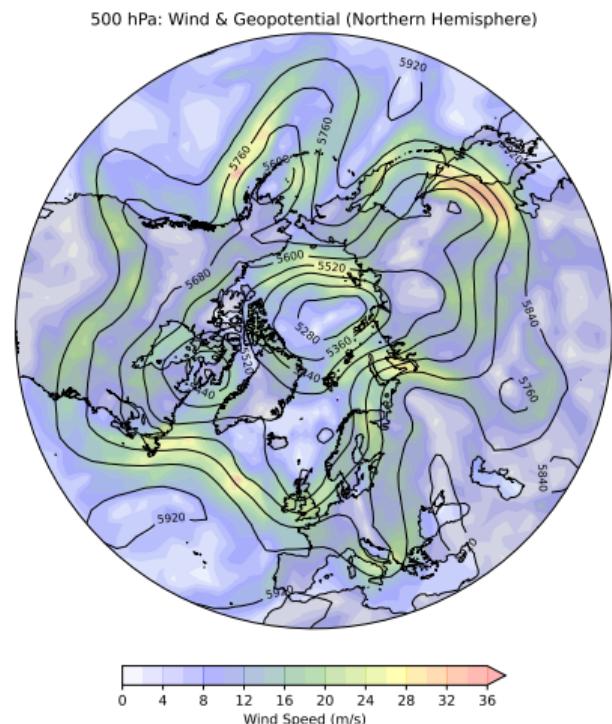
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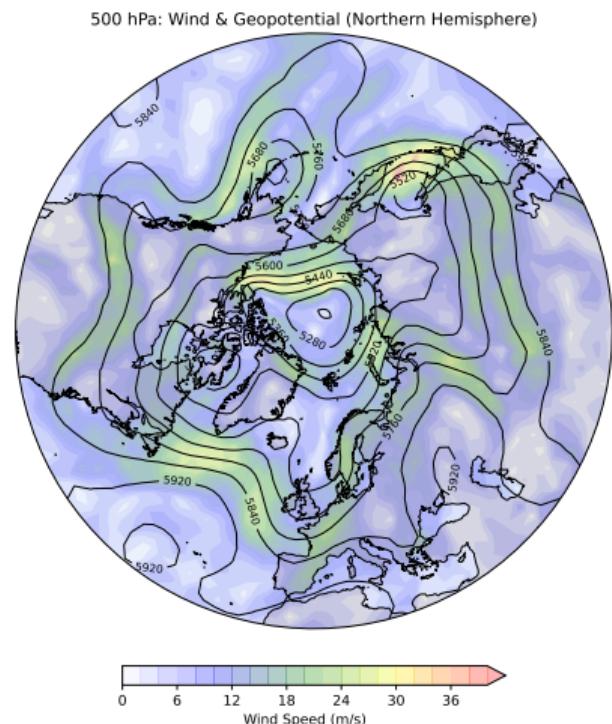
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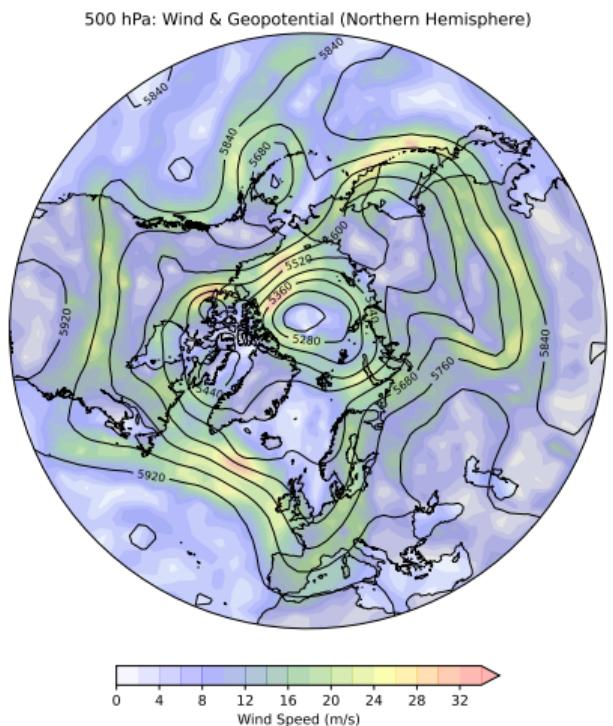
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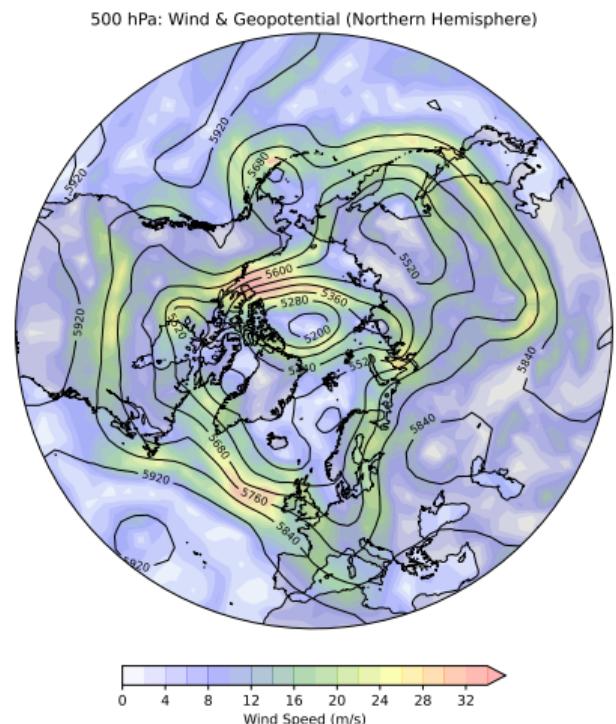
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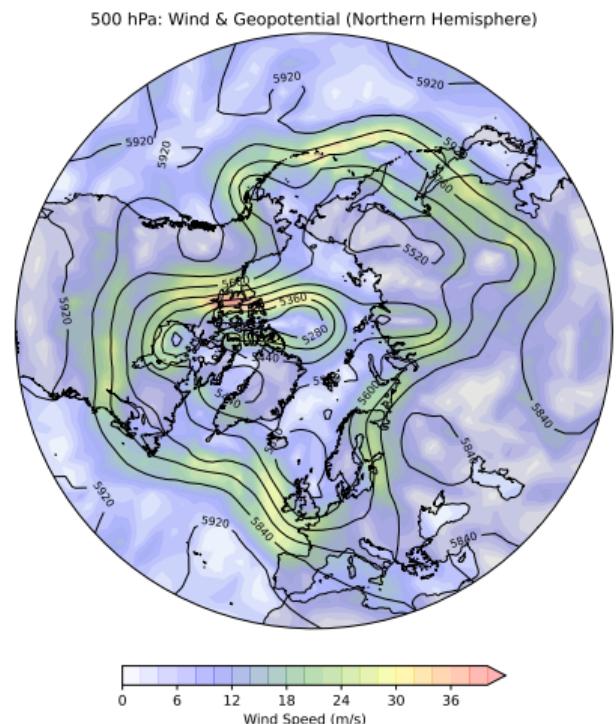
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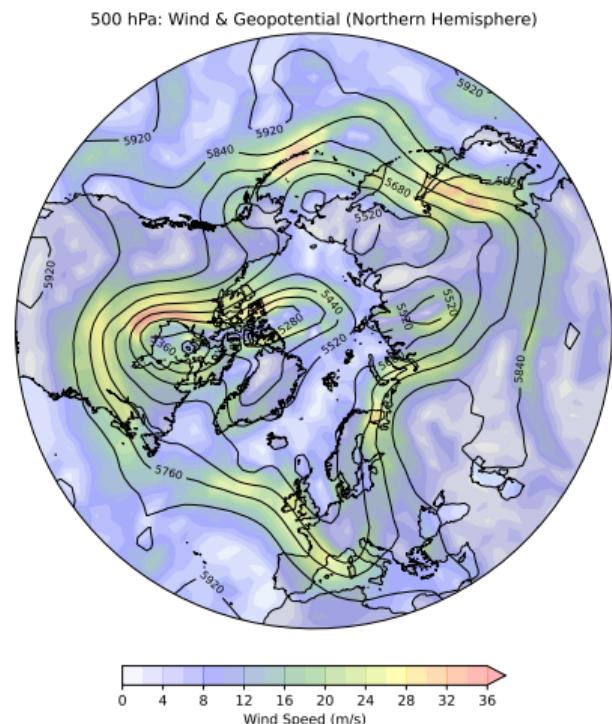
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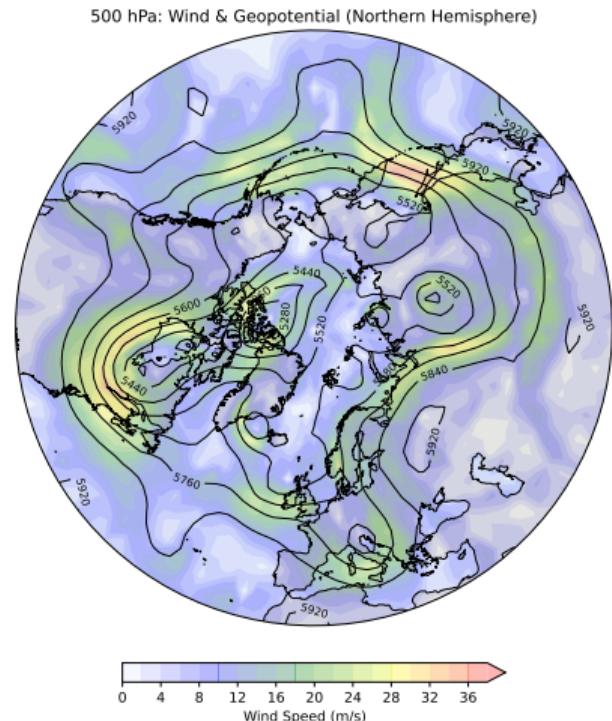
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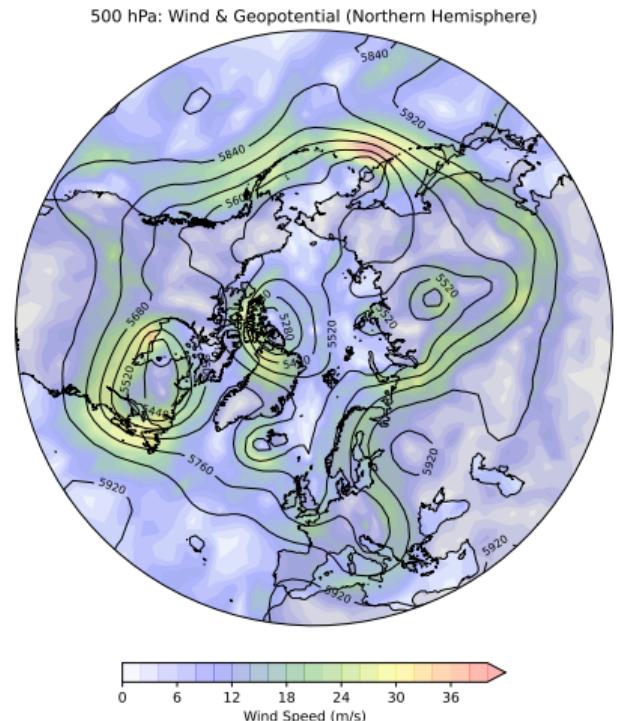
# Forecast: 5-August-2010 at 12:00, Level 500hPa



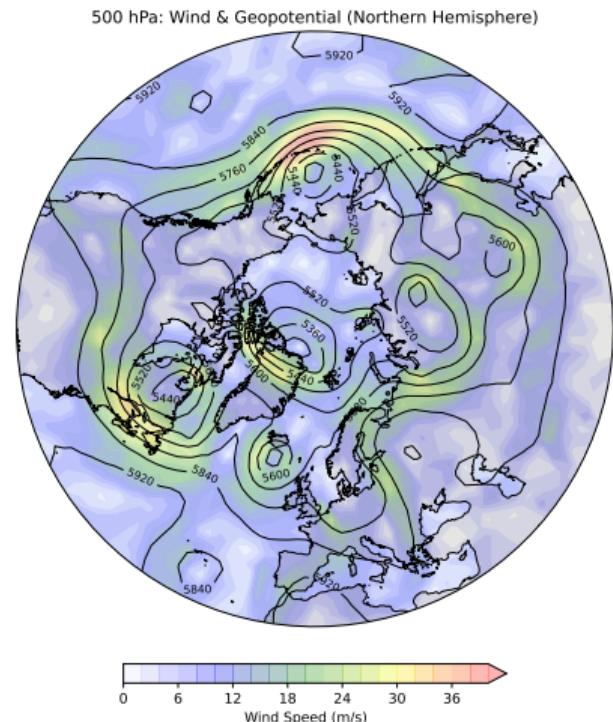
# Forecast: 6-August-2010 at 12:00, Level 500hPa



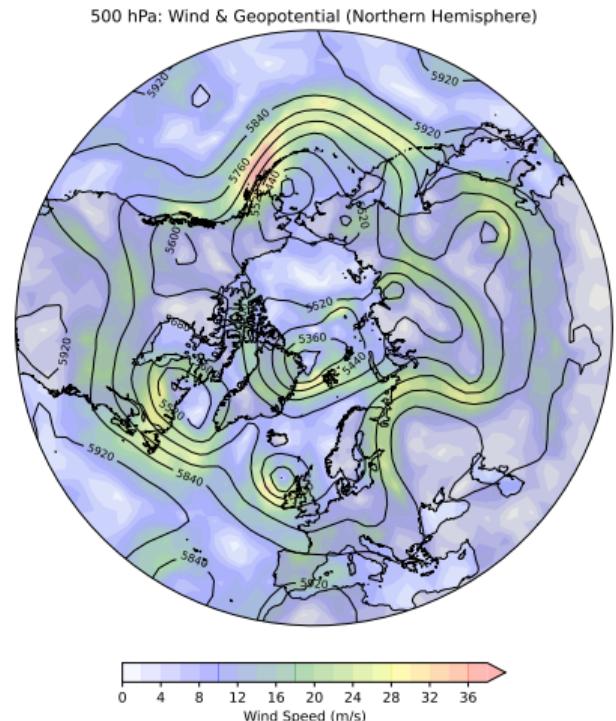
# Forecast: 7-August-2010 at 12:00, Level 500hPa



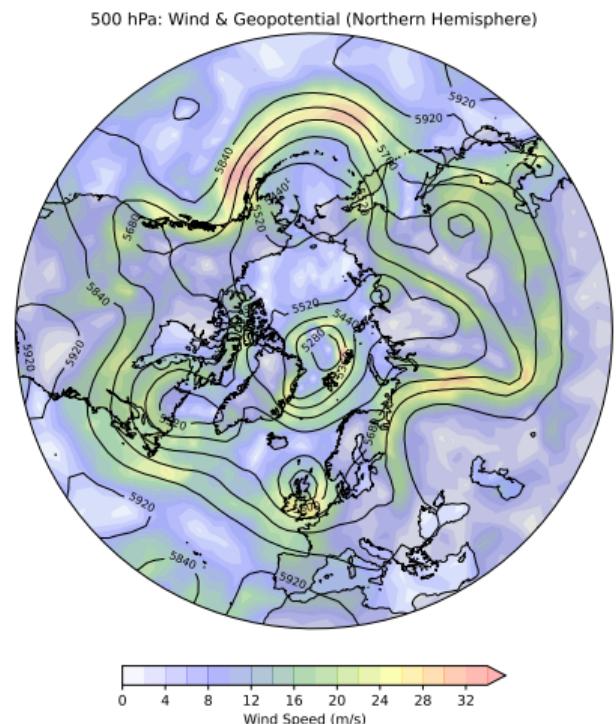
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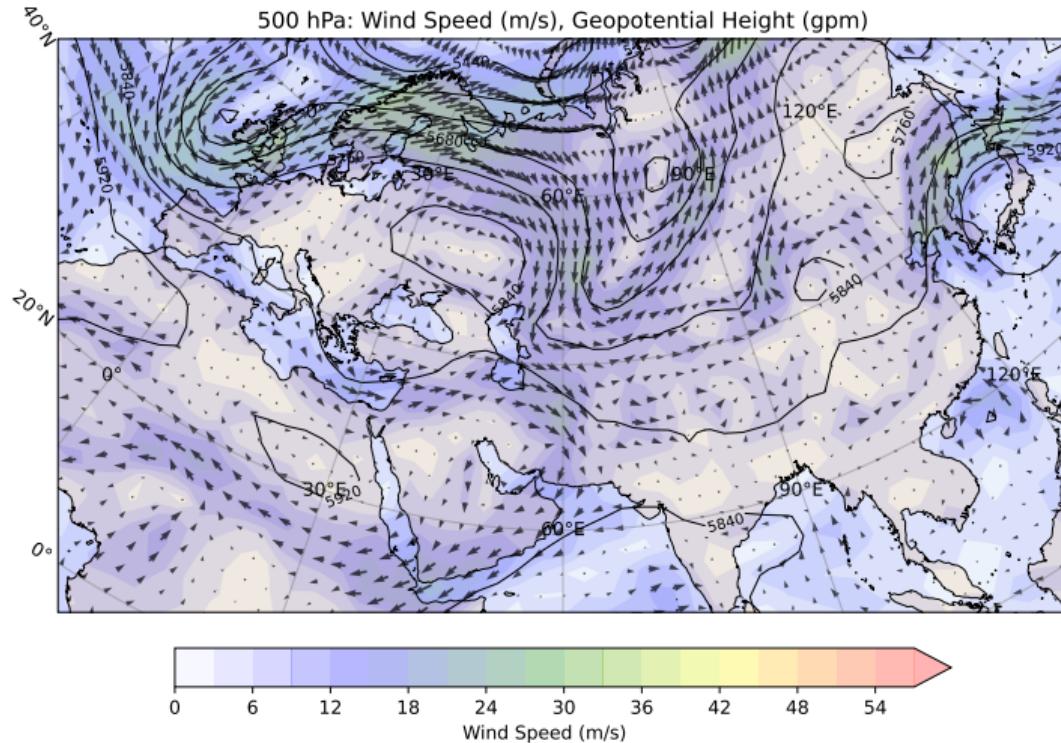
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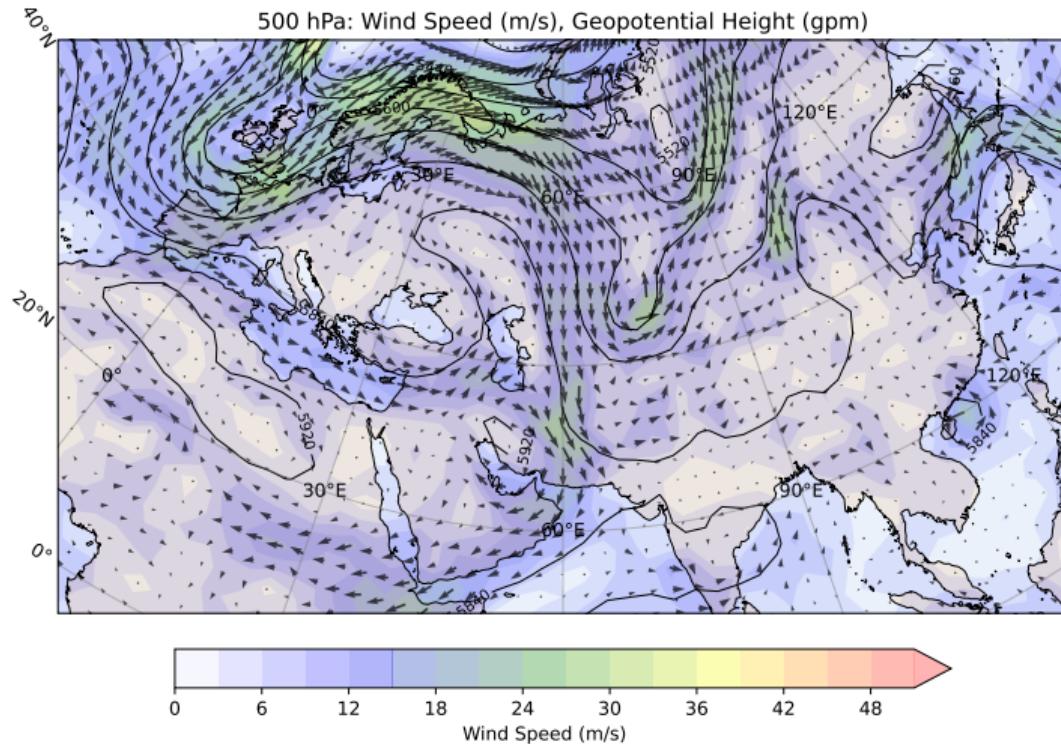
# Forecast: 10-August-2010 at 12:00, Level 500hPa



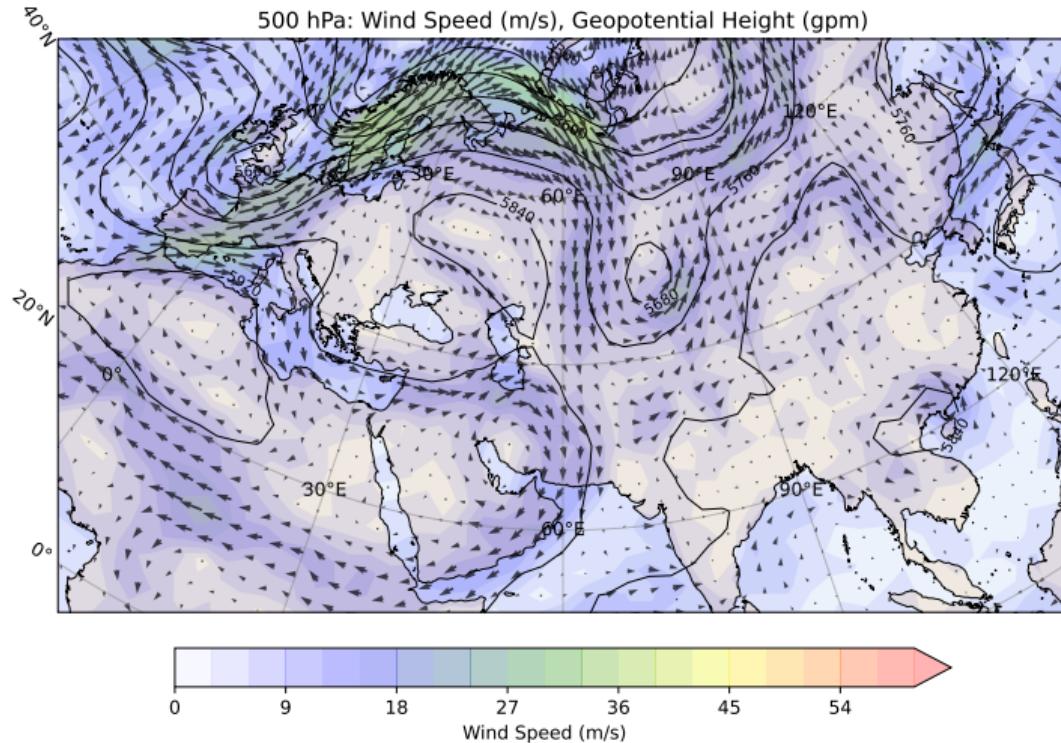
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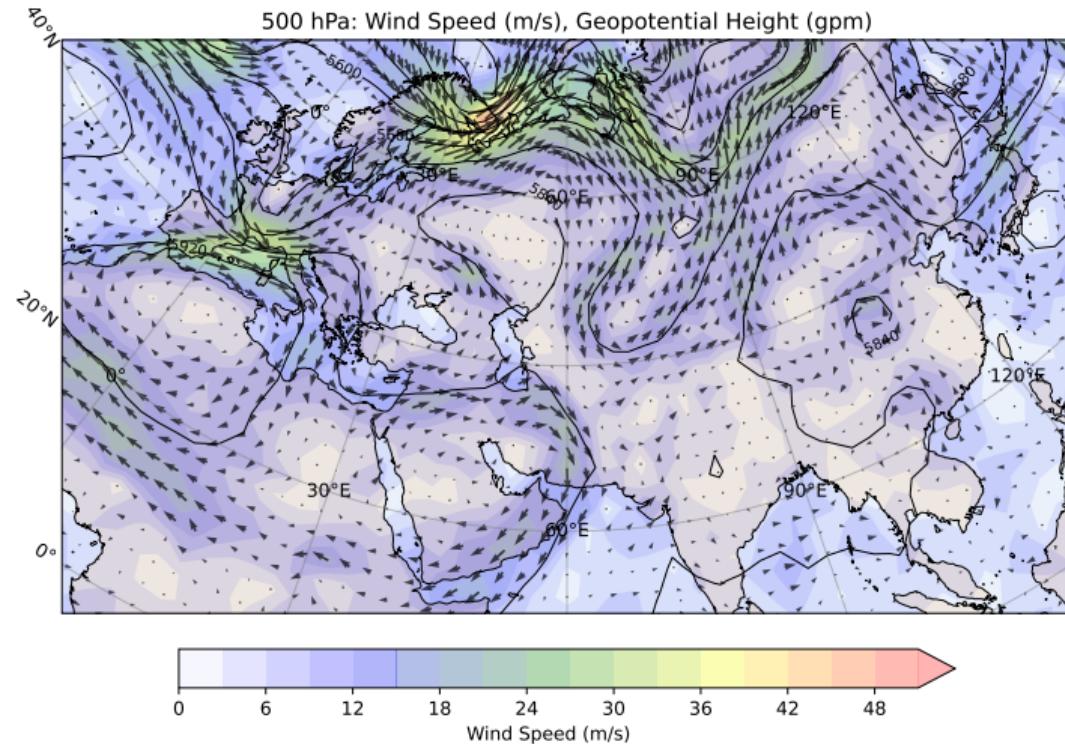
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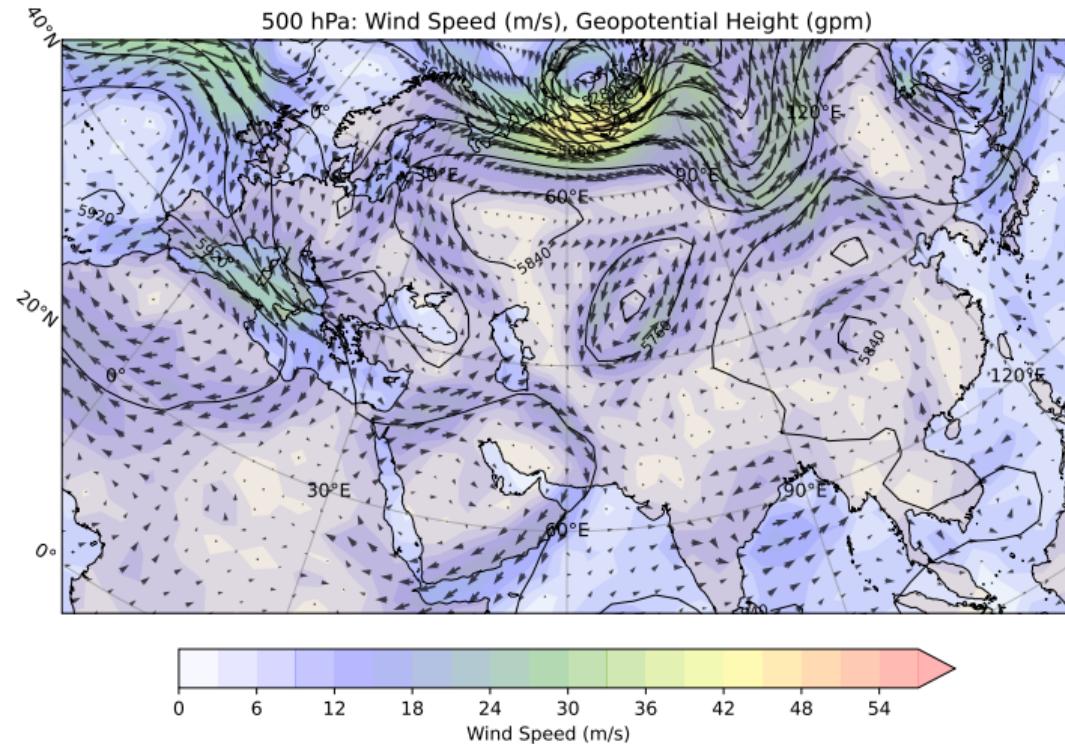
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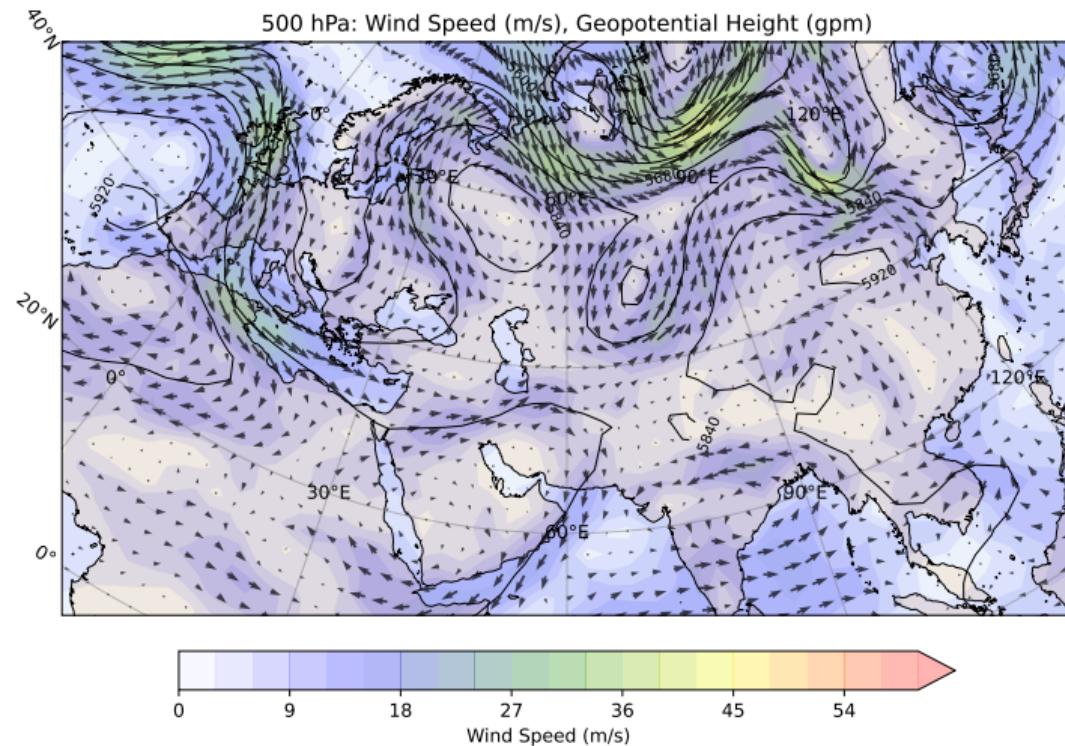
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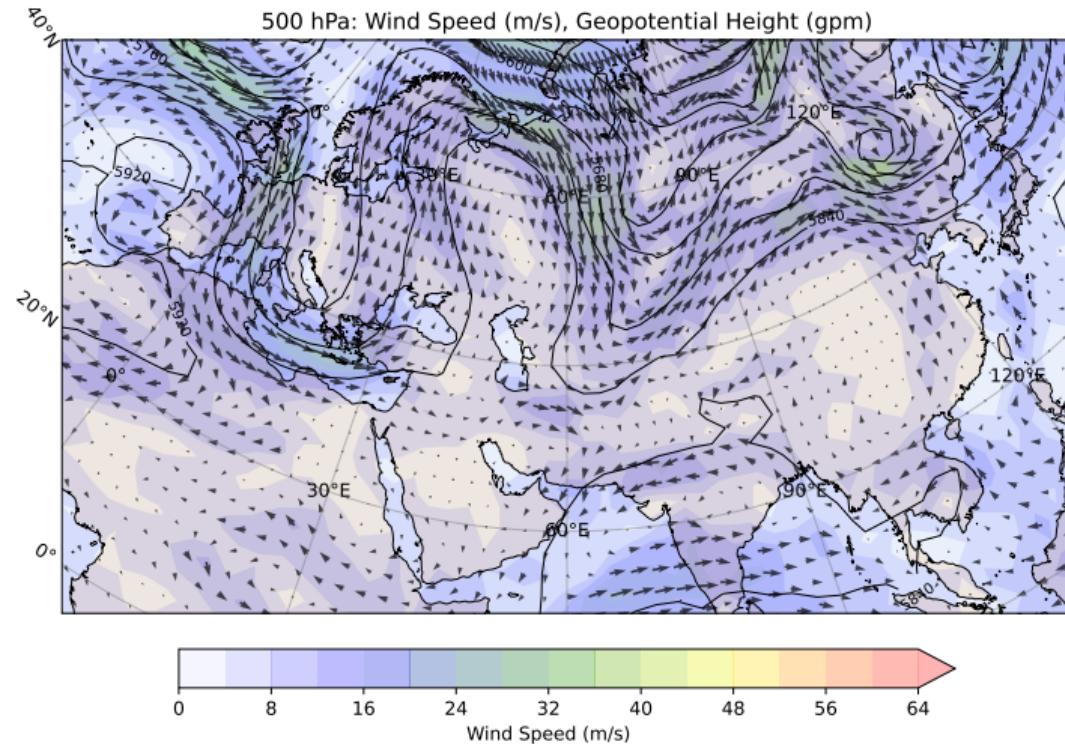
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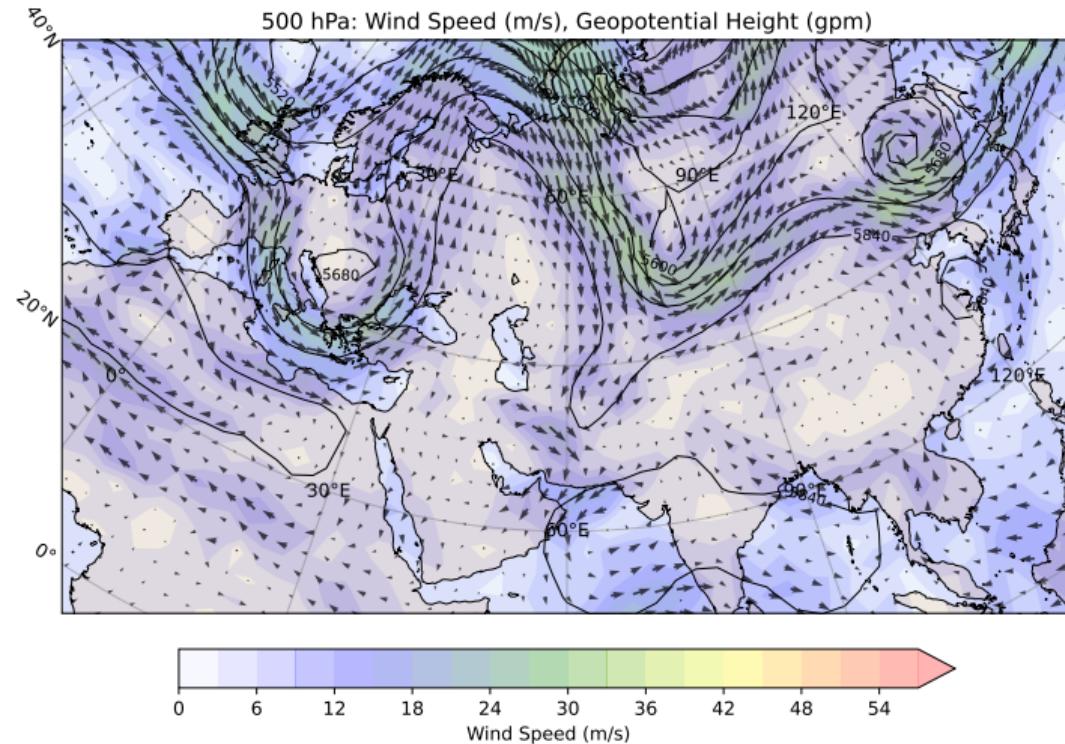
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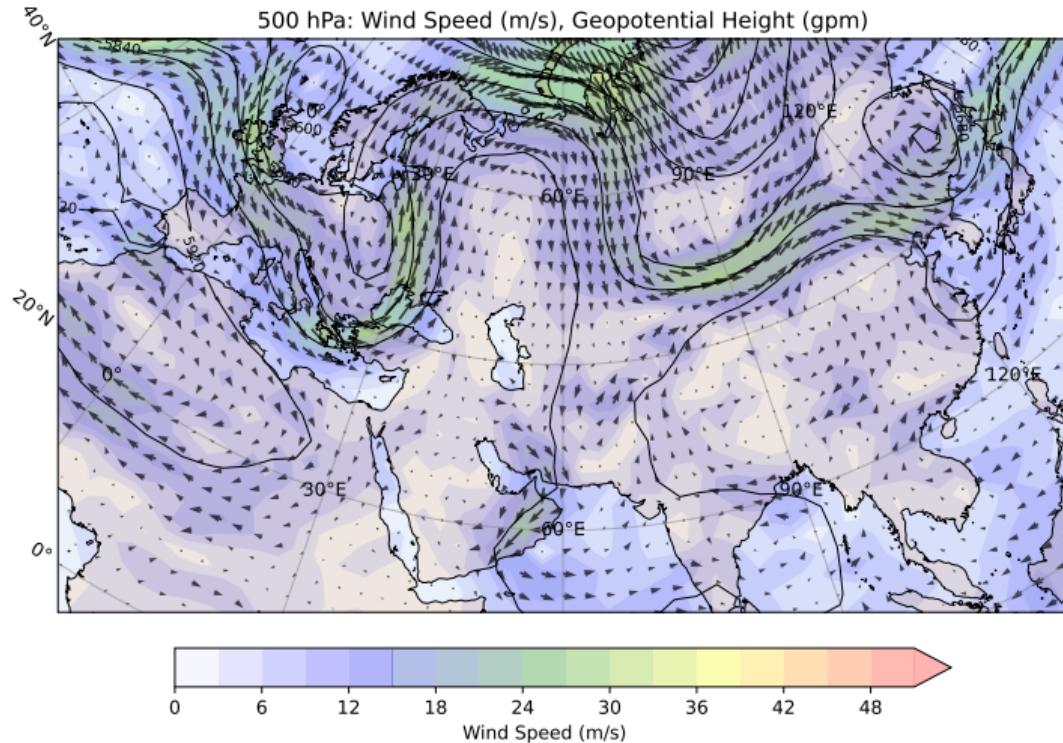
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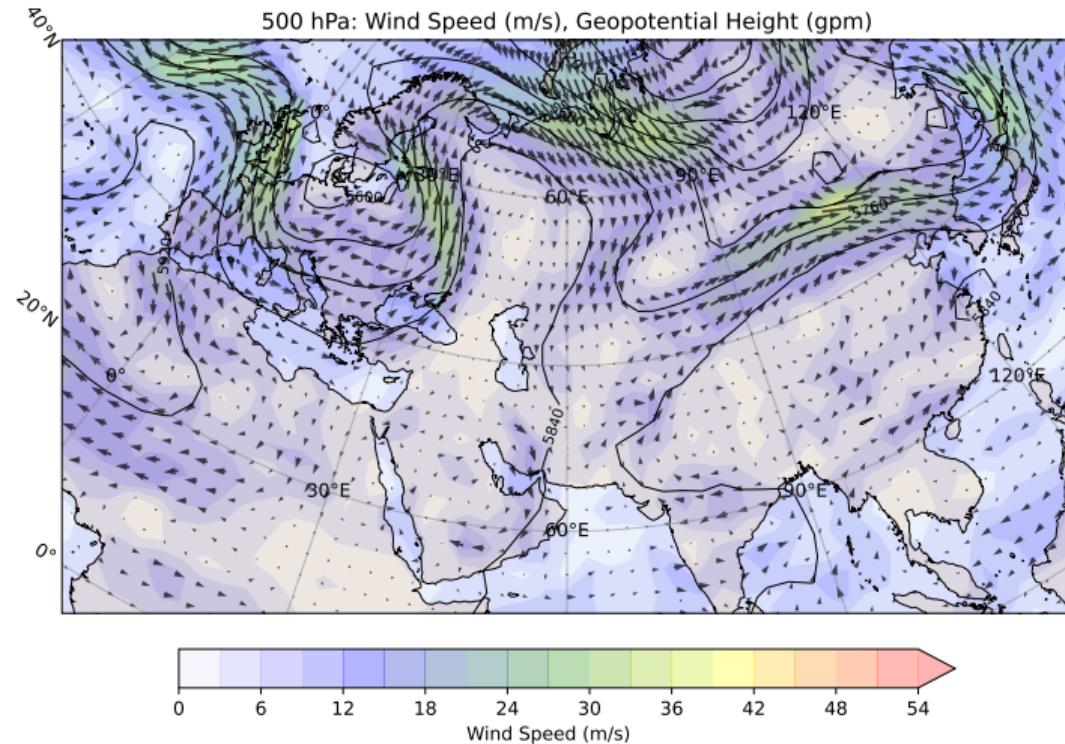
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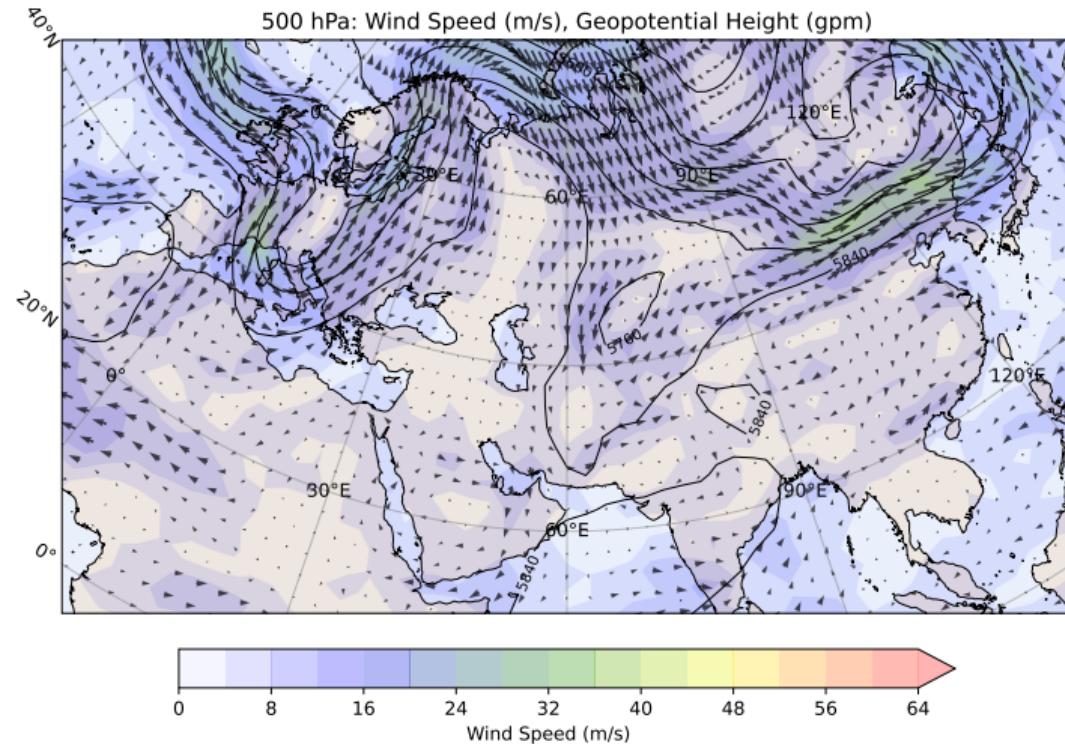
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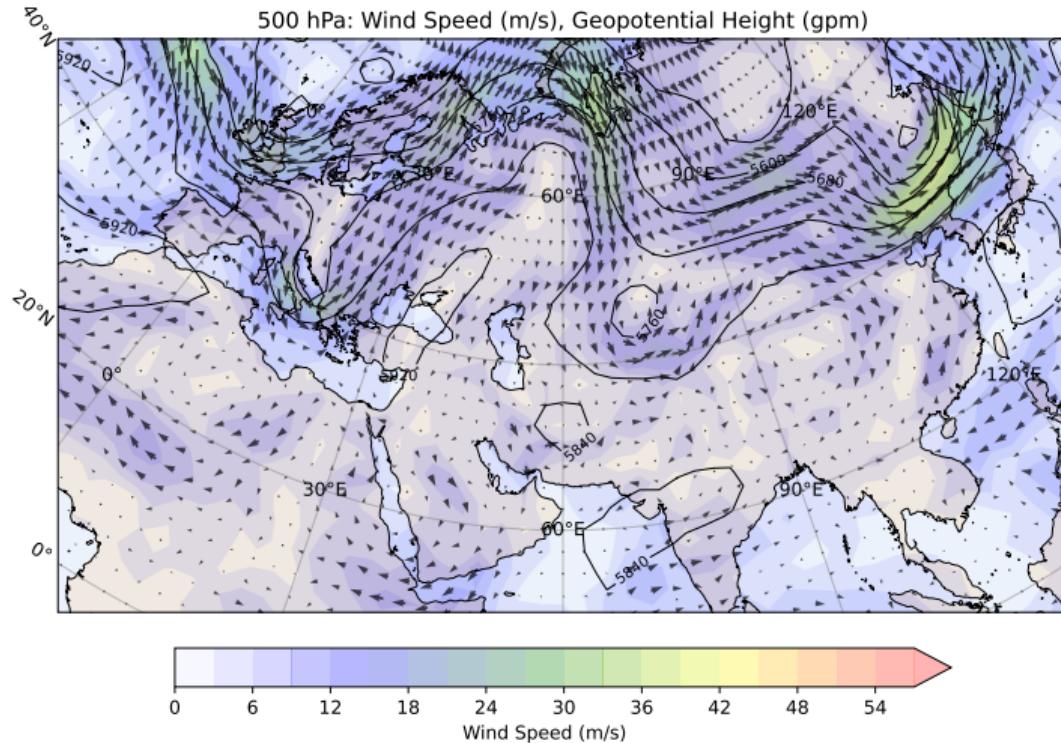
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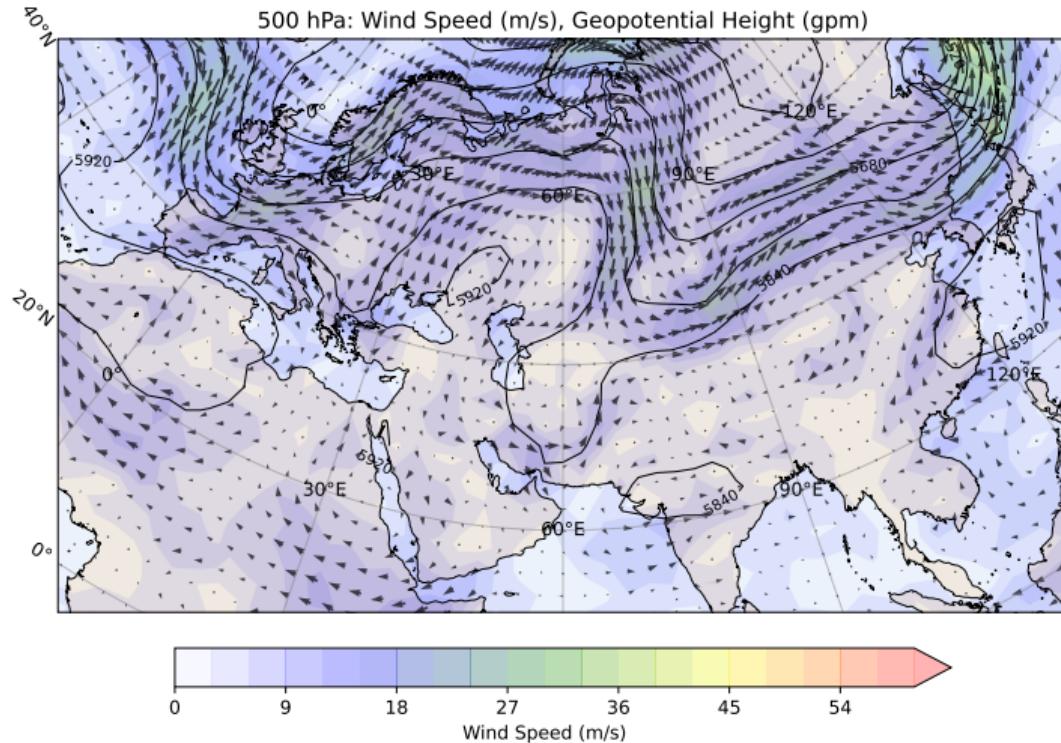
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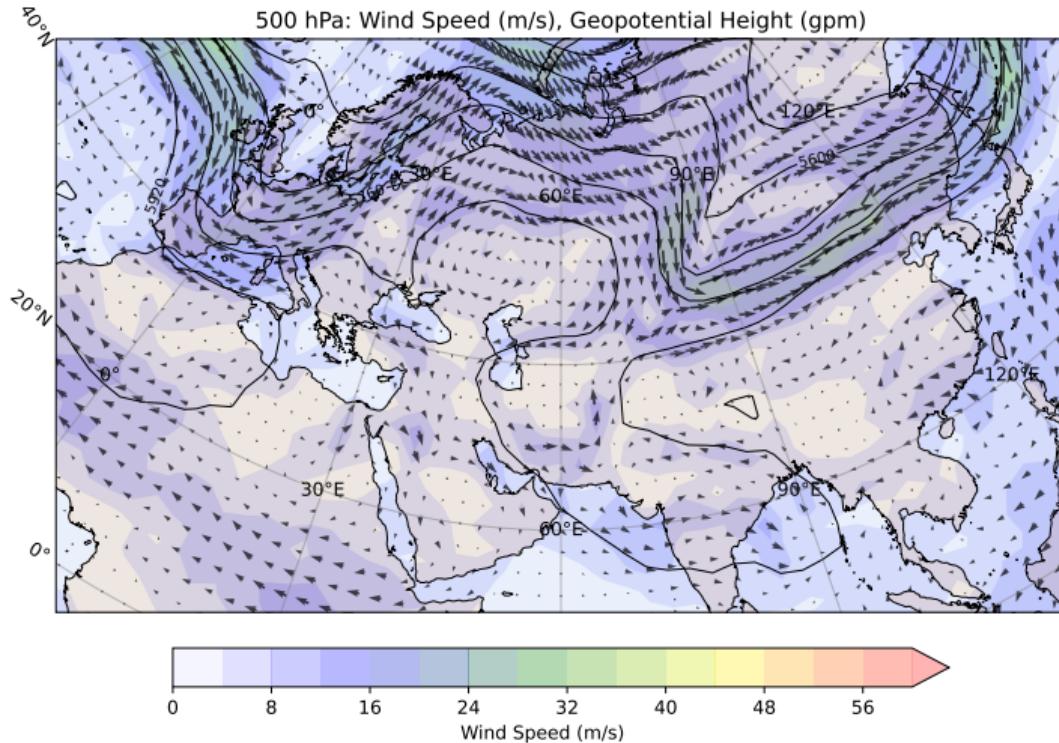
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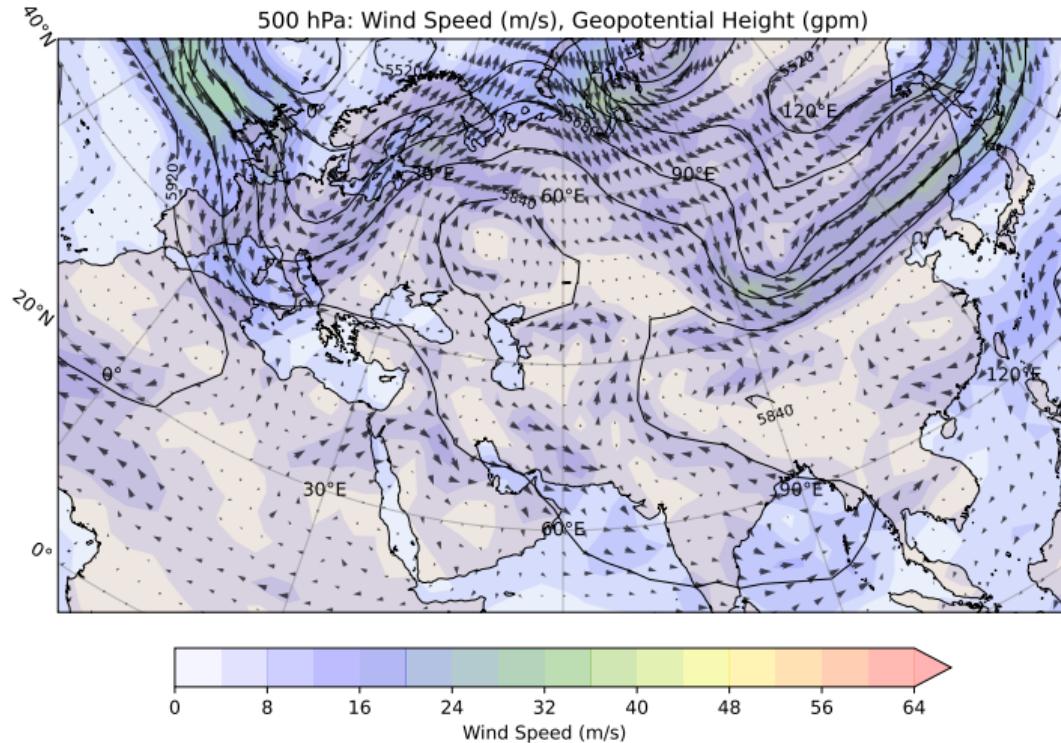
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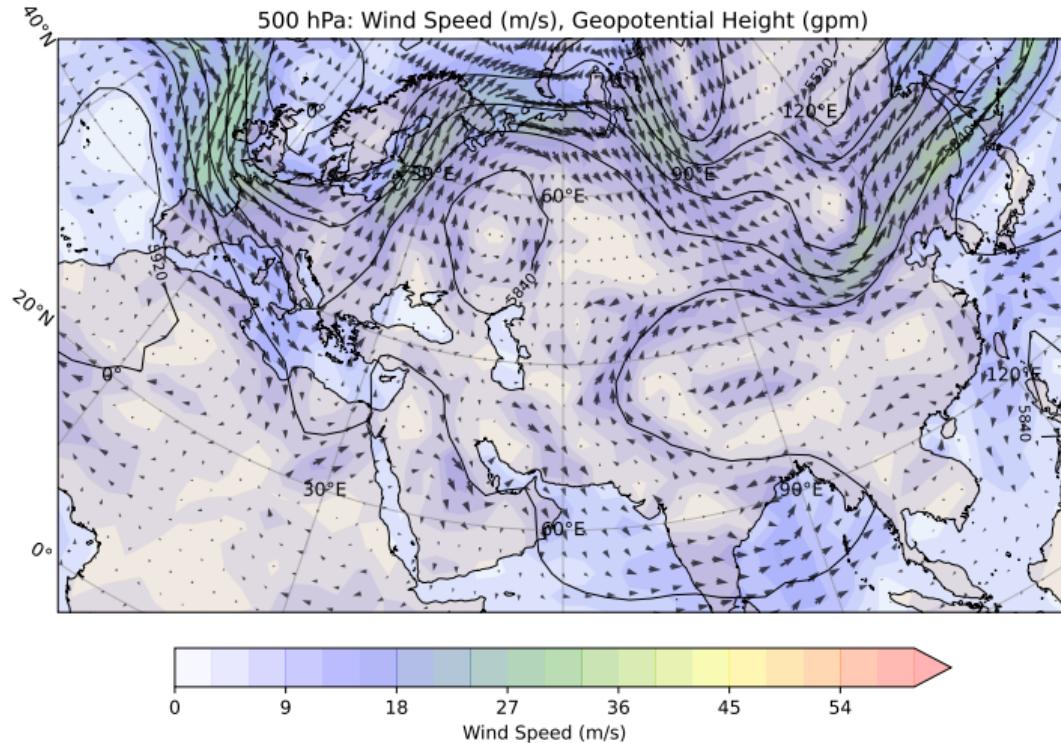
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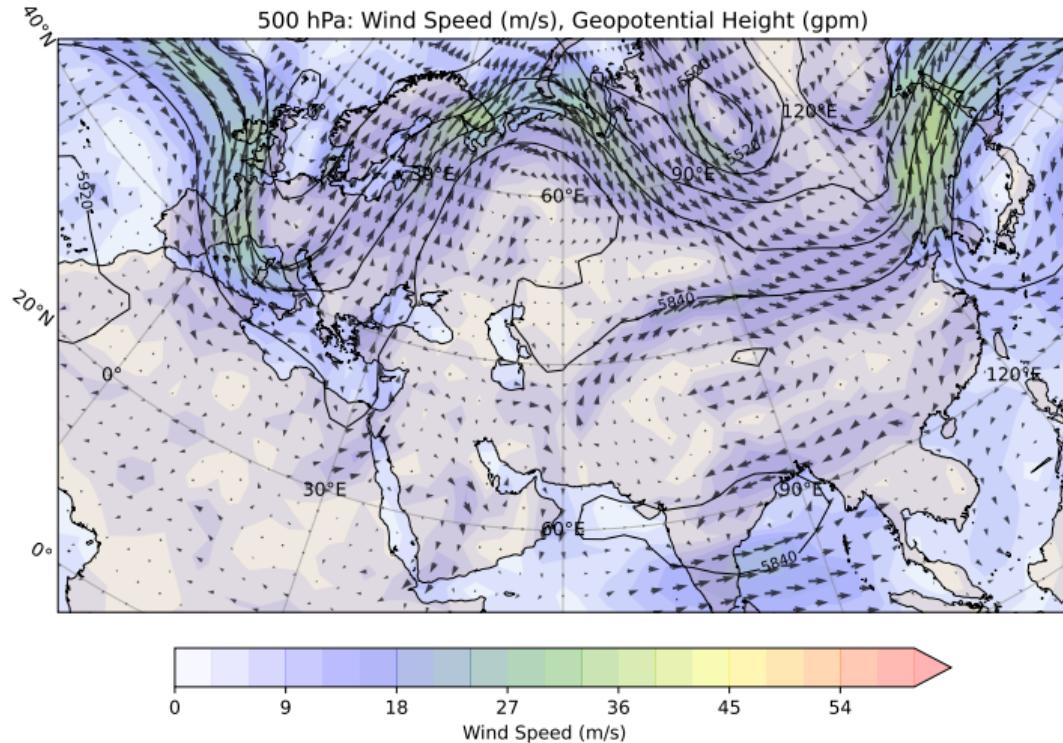
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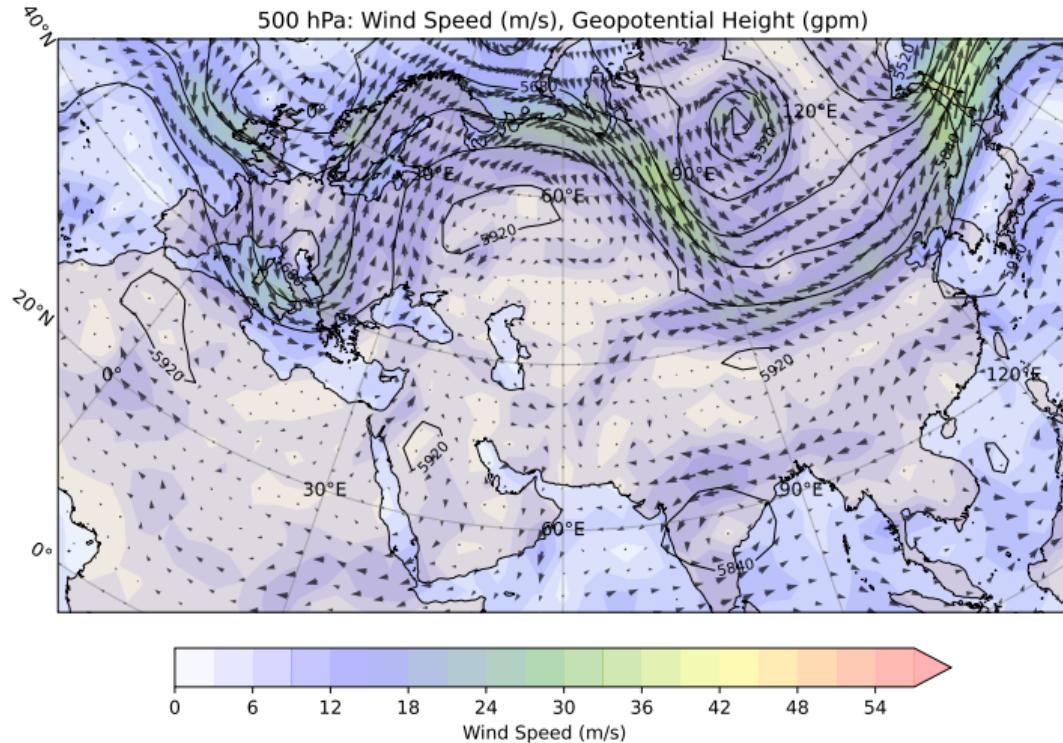
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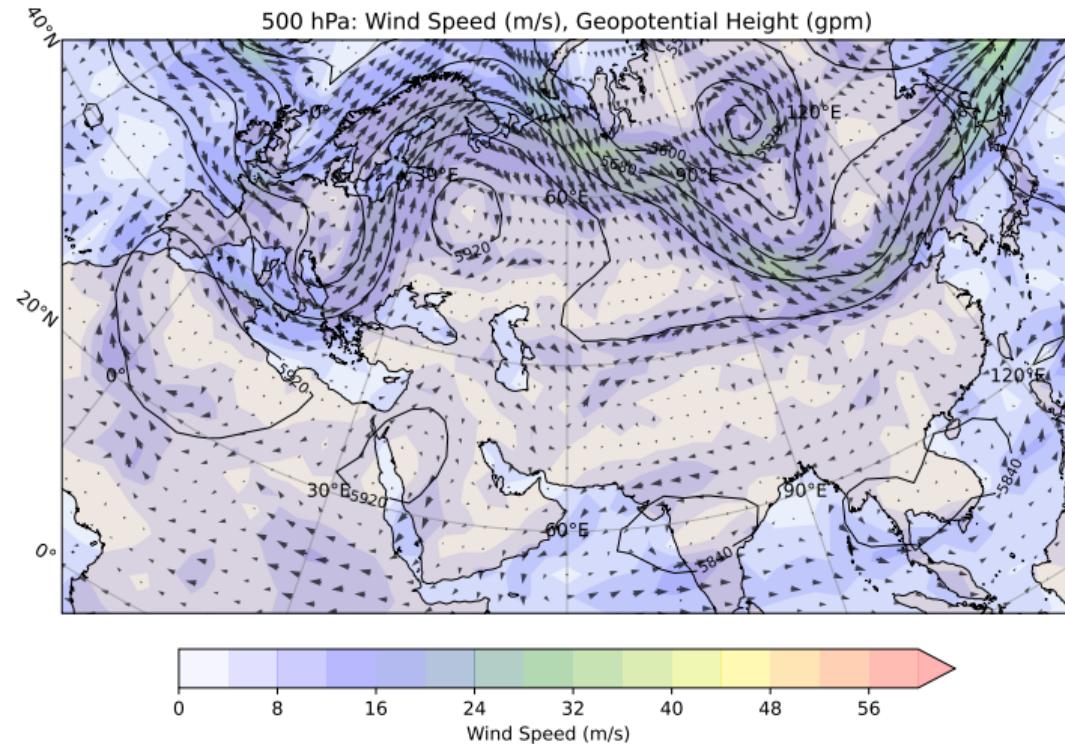
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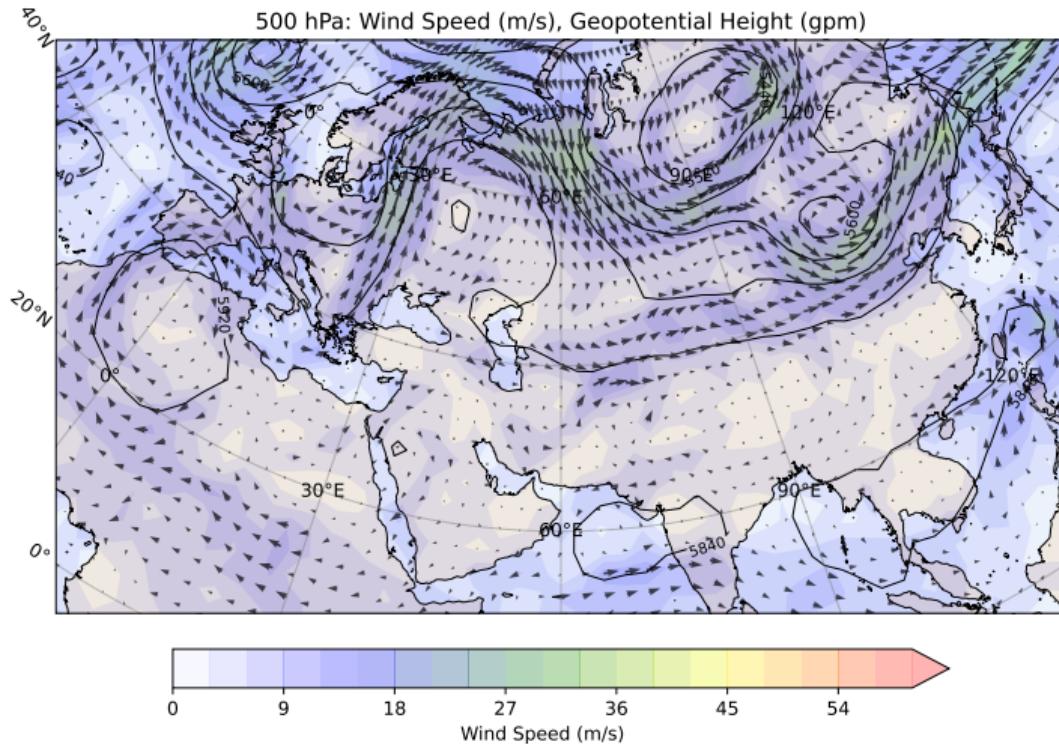
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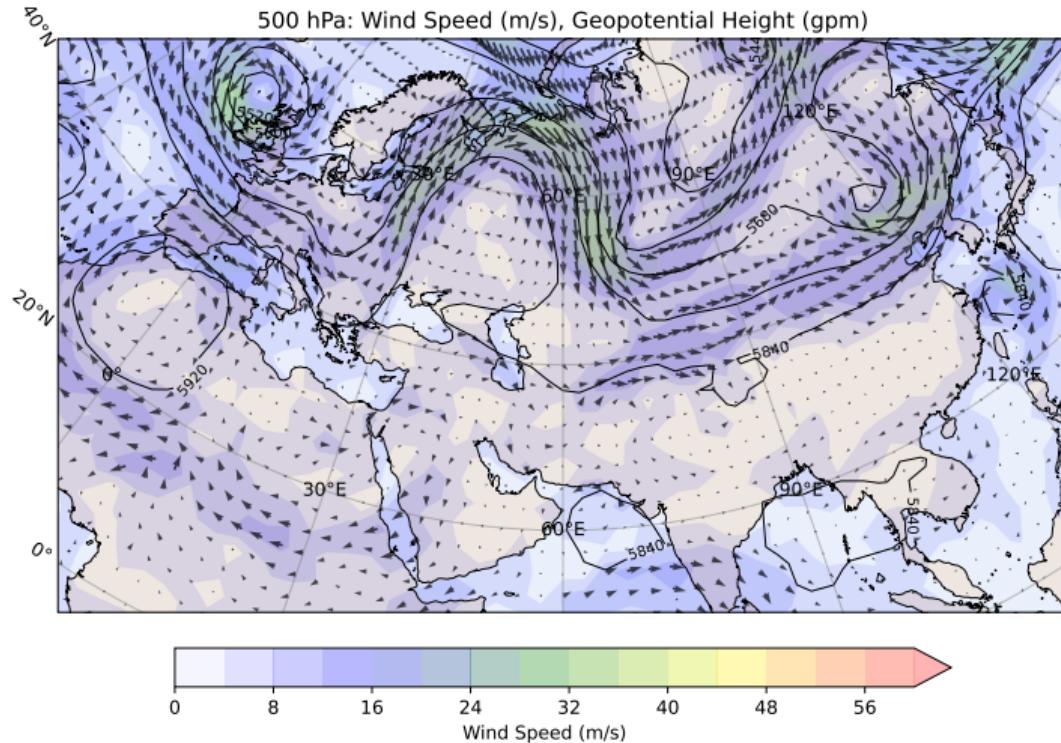
# Forecast: 7-August-2010 at 12:00, Level 500hPa



# Forecast: 8-August-2010 at 12:00, Level 500hPa



Forecast: 9-August-2010 at 12:00, Level 500hPa



# Forecast: 10-August-2010 at 12:00, Level 500hPa

