Курсова 1.1.

Задача 2

а) $y' \cos x + y(1+y)\sin x = 0$

$$y(x) = -\frac{e^{c_1}\cos(x)}{e^{c_1}\cos(x) - 1}$$

$$\cos(x)\frac{dy(x)}{dx} + (y(x)+1)\sin(x)\,y(x) = 0$$

$$\frac{dy(x)}{dx} = -(y(x)+1)\tan(x)\,y(x)$$

$$\int \frac{\frac{dy(x)}{dx}}{(y(x)+1)y(x)}\,dx = \int -\tan(x)\,dx$$

$$-\log(y(x)+1) + \log(y(x)) = \log(\cos(x)) + c_1$$

$$y(x) = \frac{e^{c_1}\cos(x)}{-e^{c_1}\cos(x)+1}$$

$$y(x) = \frac{c_1\cos(x)}{-c_1\cos(x)+1}$$

б) $(x^2+1)y' - (2x+1)y = 0$

$$x^2 y'(x) + y'(x) - 2x\,y(x) = 0$$

$$y'(x)(x^2+1) - y(x)(2x+1) = 0$$

$$y'(x) + x^2 y'(x) - y(x)(2x+1) = 0$$

$$y'(x) + x^2 y'(x) + (-y(x) - 2x\,y(x)) = 0$$

$$ye^{x}\,dy + xe^{y^2}\,dx = 0$$

$$e^{y(x)^2}x + e^{x}\frac{dy(x)}{dx}y(x) = 0$$

$$\frac{dy(x)}{dx} = -\frac{e^{-x+y(x)^2}x}{y(x)}$$

$$e^{-y(x)^2}\frac{dy(x)}{dx}y(x) = -e^{-x}x$$

$$\int e^{-y(x)^2}\frac{dy(x)}{dx}y(x)\,dx = \int -e^{-x}x\,dx$$

$$-\frac{1}{2}e^{-y(x)^2} = -e^{-x}(-x-1) + C_1$$

$$y(x) = -i\sqrt{\log(-2e^{-x}(x+C_1e^{x}+1))}$$

$$(1+y^2)\sin x\,dx - (1+\cos x)y\,dy = 0$$

$$\cancel{y(x) = \sqrt{C_1 S}}$$

$$(y(x)^2+1)\sin(x) - \frac{dy(x)}{dx}(\cos(x)+1)y(x) = 0$$

$$\frac{dy(x)}{dx}(-\cos(x)-1)y(x) - (-\cos(x)-1)\tan\left(\frac{x}{2}\right)y(x)^2 =$$

$$(-\cos x-1)\tan\left(\frac{x}{2}\right)$$

$$2\frac{dy(x)}{dx}y(x) - 2\tan\left(\frac{x}{2}\right)y(x)^2 = 2\tan\left(\frac{x}{2}\right)$$

$$\frac{dv(x)}{dx} - 2\tan\left(\frac{x}{2}\right)v(x) = 2\tan\left(\frac{x}{2}\right)$$

$$\cos^4\left(\frac{x}{2}\right)\frac{dv(x)}{dx}-\left(2\cos^3\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)\right)v(x)=2\cos^3\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)$$ d)

$$\cos^4\left(\frac{x}{2}\right)\frac{dv(x)}{dx}+\frac{d}{dx}\left(\cos^4\left(\frac{x}{2}\right)\right)v(x)=2\cos^3\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)$$

$$\frac{d}{dx}\left(\cos^4\left(\frac{x}{2}\right)v(x)\right)=2\cos^3\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)$$

$$\int\frac{d}{dx}\left(\cos^4\left(\frac{x}{2}\right)v(x)\right)dx=\int 2\cos^3\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)dx$$

$$\cos^4\left(\frac{x}{2}\right)v(x)=-\cos^4\left(\frac{x}{2}\right)+C1$$

$$v(x)=C1\sec^4\left(\frac{x}{2}\right)-1$$

$$y(x)=-\sqrt{C1\sec^4\left(\frac{x}{2}\right)-1}$$

3 задача

a)

$$y'=(x+y-1)^2$$

$$y'=y(x^2)+(-2+2x)y(x)+1-2x+x^2$$

$$y'(\cancel{x})=y(x)^2+(-2+2x)y(x)+1-2x+x^2$$

$$y'(\cancel{x})=(x+y-1)^2$$

$$((x-1)+y(x))((x-1)+y(x))=$$

$$=(x-1)(x-1)+(x-1)(y(x))+(y(x))(\cancel{x}-1)+(y(x))(y(x))$$

$$=(\cancel{x}-1)^2+((x-1)y(x))+(y(x)(x-1))+y(x)^2$$

$$=(x-1)^2+2((x-1)y(x))+y(x)^2$$

$$y(x)=((x-1)^2+2y(x)(x-1)+y(x^2)^2)$$

[redacted]

$$y'(x)=(x^2-2x+1)+2y(x)(x-1)+y(x^2)$$

$$2y(x)(x-1)=2xy(x)-2y(x)$$

$$y'(x)=1-2x+x^2+(2xy(x)-2y(x))+y(x)^2$$

$$y' = \cos(x - y + 1)$$

$$y(x) = -2\cot^{-1}\left(\frac{1}{2}(c_1 + 2x + 2)\right) + x + 1$$

$$\frac{dy(x)}{dx} = \cos(x - y(x) + 1)$$

$$= \frac{dv(x)}{dx} + 1 = \cos(v(x) + 1)$$

$$\frac{dv(x)}{dx} = -\cos(v(x) + 1) + 1$$

$$\frac{\frac{dv(x)}{dx}}{-\cos(v(x)+1)+1} = 1$$

$$\int \frac{\frac{dv(x)}{dx}}{-\cos(v(x)+1)} \, dx = \int 1 \, dx$$

$$-\cot\left(\frac{1}{2}(v(x)+1)\right) = x + c_1$$

$$v(x) = -2\cot^{-1}(x + c_1) - 1$$

$$\Rightarrow$$

$$y(x) = x + 2\cot^{-1}(x + c_1) + 1$$

в) $(y-3x+2)dx+(3x-y-1)dy=0$

$y(x)=\frac{1}{2}(W(-e^{-4x+c_1-1})+1)+3x-1$

$-3x+\frac{dy(x)}{dx}(3x-y(x)-1)+y(x)+2=0$

$\left(\frac{dv(x)}{dx}+3\right)(-v(x)-1)+v(x)+2=0$

$-\left(\frac{dv(x)}{dx}+3\right)(v(x)+1)+v(x)+2=0$

$\frac{dv(x)}{dx}=\frac{-2v(x)-1}{v(x)+1}$

$\frac{\frac{dv(x)}{dx}(v(x)+1)}{-2v(x)-1}=1$

$\int\frac{\frac{dv(x)}{dx}(v(x)+1)}{-2v(x)-1}dx=\int 1\,dx$

$\frac{1}{4}(-\log(2v(x)+1)-2v(x)-1)=x+c_1$

$v(x)=\frac{1}{2}(W(e^{-4(x+c_1)})-1)$

$y(x)=\frac{1}{2}(6x+W(e^{-4(x+c_1)})-1$

2) $(2y-x+1)dx+(4y-2x+6)dy=0$

$M(x,y)=2y-x+1$ $N(x,y)=4y-2x+6$

$P(x)=\frac{\partial M}{\partial y}=2$

$Q(y)=\frac{\partial N}{\partial x}=-2$

$I = e^{\int 2dx} = e^{2x}$

$e^{2x} \cdot (2y - x + 1)dx + e^{2x} \cdot (4y - 2x + 6)dy = 0$

$dF = \frac{\partial F}{\partial x} dx + \frac{\partial F}{\partial y} dy = 0$

$\frac{\partial F}{\partial x} = e^{2x} (2y - x + 1)$

$F = \int e^{2x} \cdot (2y - x + 1) dx$

$\frac{\partial F}{\partial y} e^{2x} \cdot (4y - 2x + 6)$

$F = \int e^{2x} \cdot (4y - 2y + 6) dy$

4 Ejercicio

2) $xy' = y(1 + \ln \frac{y}{x})$

$xy' = y + y \ln \left(\frac{y}{x}\right)$

$y' = \frac{y}{x} + \frac{y}{x} \ln \left(\frac{y}{x}\right)$

$v = \frac{y}{x} \quad y = vx \quad y' = v + xv'$

$v + xv' = v + v \ln(v)$

$xv' = v \ln(v)$

$\frac{dv}{v \ln(v)} = \frac{dx}{x}$

$\int \frac{1}{\ln(x)} dv = \int \frac{1}{x} dx$

$\mathrm{li}(x) = \ln(\ln(v)) + C_1$

$\ln(v)\ e^{\ln(|x|)+C}$

$v = e^{C}|x|$

$v = \frac{y}{x}$

$\frac{y}{x} = e^{C}|x|$

$y = e^{C}x^{2}$

$xy' = y(1+\ln(\frac{y}{x}))$

б) $xy' = \frac{x^2+y^2}{x+y}$ $\qquad u = \frac{y}{x} \quad y = ux \quad y' = u'x+u$

$x(u'x+u) = \frac{x^2+(ux)^2}{x+ux}$

$u'x^2+u.x = \frac{x^2+u^2x^2}{x(1+u)}$

$u'x^2+u.x = \frac{x^2(1+u^2)}{x(1+u)}$

$u'x+u = \frac{x(1+u^2)}{1+u}$

$\frac{du}{1+u} = \frac{dx}{x}$

$\int\frac{1}{1+u}du = \int\frac{1}{x}dx$

$\ln|1+u| = \ln|x|+C$

$1+u = e^{\ln|x|+C}$

$1+u = e^{C}|x|$

$u = e^{C}|x|-1$

$\frac{y}{x} = e^{C}|x|-1$

$y = (e^{C}|x|-1)x$

2)

$$xy\,dx = (x^2 - y^2)\,dy$$

$$\frac{dx}{dy} = \frac{x^2 - y^2}{xy}$$

$$\frac{dx}{dy} = \frac{x^2}{xy} - \frac{y^2}{xy}$$

$$\frac{dx}{dy} = \frac{x}{y} - \frac{y}{x} \qquad u = \frac{x}{y} \quad \frac{dx}{dy} = y\frac{du}{dy} + u$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{\frac{x}{y} - \frac{y}{x}}$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{\frac{x}{y} - \frac{y}{x}}$$

$$\frac{dy}{dx} = \frac{1}{u - \frac{1}{u}} = \frac{1}{\frac{u^2 - 1}{u}} = \frac{u}{u^2 - 1}$$

$$u(u^2 - 1)\,du = dx$$

$$\int u(u^2 - 1)\,du = \int dx$$

$$\int (u^3 - u)\,du = x + C \qquad u = \frac{x}{y}$$

$$\frac{u^4}{4} - \frac{u^2}{2} = x + C$$

$$\frac{(x/y)^4}{4} - \frac{(x/y)^2}{2} = x + C$$

5 задача

а) $(2x+y)y' = x+2y$ $\qquad y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$(2x+y) \quad \frac{dy}{dx} = \frac{x^2+2y}{2x+y}$

$v + x\frac{dv}{dx} = \frac{x+2(vx)}{2x+(vx)}$

$v + x\frac{dv}{dx} = \frac{x+2v}{2+v}$

$v(2+v) + x(2+v)\frac{dv}{dx} = x+2v$

$2v + v^2 + 2x\frac{dv}{dx} + xv\frac{dv}{dx} = x+2v$

$v^2 + 2x\frac{dv}{dx} + xv\frac{dv}{dx} - v = x$

$v^2 + (2x+x^2)\frac{dv}{dx} - v = x$

$v^2 - v = x - (2x+x^2)\frac{dv}{dx}$

$v(v-1) = x(1-2+x)\frac{dv}{dx})$

$\frac{v}{v-1}dv = \frac{x}{x(1+x)}dx$

$\int\frac{v}{v-1}dv = \int\frac{1}{1+x}dx$

б)

6 задача

а)

$\frac{dy}{dx}$

$(y-x)y' = x+y$

$(y-x)\frac{dy}{dx} = x+y$

$(y-x)\frac{dy}{dx} = x+y$

$\frac{dy}{dx} = \frac{x+y}{y-x}$

$\frac{dy}{dx} = \frac{x}{y-x} + \frac{y}{y-x}$ $\qquad u = \frac{y}{x} \quad \frac{dy}{dx} = u + x\frac{du}{dx}$

$\frac{u}{x} + x\frac{du}{dx} = \frac{1}{1-u} + u$

$\frac{u}{1-u} - u = \frac{1}{2}\frac{dx}{du}$

$\frac{du}{\frac{u}{1-u} - u} = \frac{dx}{x}$

6 група a)

$2x^3y' = 2x^2y - 3$

$y' = \frac{2x^2y - 3}{2x^3}$

$y' = \frac{2y}{x} - \frac{3}{2x^3}$

$\frac{dy}{dx} = \frac{2y}{x} - \frac{3}{2x^3}$

$dy = \left(\frac{2y}{x} - \frac{3}{2x^3}\right)dx$

$$\int \frac{1}{y} dy = 2\int \frac{1}{x} dx - \frac{3}{2}\int \frac{1}{x^3} dx$$

$$\ln|y| = 2\ln|x| + \frac{3}{2x^2} + C$$

$$\ln|y| = \ln|x^2| + \frac{3}{2x^2} + C$$

$$\ln|y| = \ln|x^2| + \ln\left(e^{\frac{3}{2x^2}}\right) + C$$

$$\ln|y| = \ln\left(x^2 e^{\frac{3}{2x^2}}\right) + C$$

$$y = \pm x^2 e^{\frac{3}{2x^2}} e^{C}$$

$$2x^3y' = 2x^2y - 3$$

$$2x^3y' = 2x^2 e^{\frac{3}{2x^2}} C$$

7 задача $4xy' + (4x+1)y^2 - 4y = 0$

a)

$$4x\frac{dy(x)}{dx} - 4y(x) = (-4x-1)y(x)^2$$

$$\frac{4x\frac{dy(x)}{dx} - 4y(x) = (-4x-1)y(x)^2}{-\frac{dx}{y(x)^2}}$$

$v(x) \frac{1}{y(x)}$ $\qquad \frac{dv(x)}{dx} = \frac{-\frac{dy(x)}{dx}}{y(x)^2}$

$$\frac{dv(x)}{dx} + \frac{v(x)}{x} = \frac{1}{4x} + 1$$

$$x\frac{dv(x)}{dx} + v(x) = -x\left(-\frac{1}{4x} - 1\right)$$

$$x\frac{dv(x)}{dx} + \frac{d}{dx}(x)\,v(x) = -x\left(-\frac{1}{4x} - 1\right)$$

$$\frac{d}{dx}(xv(x)) = -x\left(-\frac{1}{4x} - 1\right)$$

$$\int \frac{d}{dx}(xv(x))\,dx = \int -x\left(-\frac{1}{4x} - 1\right)dx$$

$$xv(x) = \frac{x^2}{2} + \frac{x}{4} + C_1$$

$$v(x) = \frac{x}{2} + \frac{C1}{x} + \frac{1}{4}$$

$$y(x) = \frac{1}{v(x)} = \frac{4x}{2x^2 + x + 4C1}$$

$$y(x) = \frac{4x}{2x^2 + x + C1}$$

1задача

а) $(1-3x^2-y)dx=(x-3y^2)dy$

$$y(x)=-\frac{\sqrt[3]{2}x}{\sqrt[3]{\sqrt{(27c_1-27x^3+27x)^2-108x^3}+27c_1-27x(x^2-1)}}$$

$$\frac{\sqrt[3]{\sqrt{(27c_1-27x^3+27x)^2-108x^3}+27c_1-27x(x^2-1)}}{3\sqrt[3]{2}}$$

$$y(x)=\frac{(1+i\sqrt{3})x}{2^{2/3}\sqrt[3]{\sqrt{(27c_1-27x^3+27x)^2-108x^3}+27c_1-27x(x^2-1)}}$$

$$\frac{(1-i\sqrt{3})\sqrt[3]{\sqrt{(27c_1-27x^3+27x)^2-108x^3}+27c_1-27x(x^2-1)}}{6\sqrt[3]{2}}$$

$$-3x^2-y(x)+1=(x-3y(x)^2)\frac{dy(x)}{dx}$$

$$1-3x^2-y(x)+(-x+3y(x)^2)\frac{dy(x)}{dx}=0$$

$P(x,y)=-3x^2-y+1 \qquad Q(x,y)=-x+3y^2$

$$f(x,y)=\int(-3x^2-y+1)dx=x-3-yx+g(y)$$

$$\frac{\partial f(x,y)}{\partial y}=\frac{\partial}{\partial y}(x-x^3-yx+g(y))=-x+\frac{dg(y)}{dy}$$

$$\frac{\partial f(x,y)}{\partial y}=Q(x,y)$$

$$-x+\frac{dg(y)}{dy}=-x+3y^2$$

$$\frac{dg(y)}{dy}=3y^2 \qquad \frac{dg(y)}{dy}$$

$g(y)=\int 3y^2dy=y^3$

$f(x,y)=-x^3+x+y^3-yx \qquad -x^3+x+y^3-yx=C_1$

2/09/2020

a) $2xy\,dx = (x^2 - 2y^3)\,dy$

$$y(x) = \frac{\sqrt[3]{\frac{2}{3}}\,C_1}{\sqrt[3]{\sqrt{3}\,\sqrt{4C_1^3+27x^4}+9x^2}} - \frac{\sqrt[3]{\sqrt{3}\,\sqrt{4C_1^3+27x^4}+9x^2}}{\sqrt[3]{2}\,3^{2/3}}$$

$$y(x) = \frac{(1-i\sqrt{3})\sqrt[3]{\sqrt{3}\,\sqrt{4C_1^3+27x^4}+9x^2}}{2\sqrt[3]{2}\,3^{2/3}} - \frac{(1+i\sqrt{3})C_1}{2^{2/3}\sqrt[3]{3}\sqrt[3]{\sqrt{4C_1^3+27x^4}+9x^2}}$$

$$y(x) = \frac{(1+i\sqrt{3})\sqrt[3]{\sqrt{3}\,\sqrt{4C_1^3+27x^4}+9x^2}}{2\sqrt[3]{2}\,3^{2/3}} - \frac{(1-i\sqrt{3})C_1}{2^{2/3}\sqrt[3]{3}\sqrt[3]{\sqrt{4C_1^3+27x^4}+9x^2}}$$

$$2x\,y(x) = (x^2 - 2y(x)^3)\frac{dy(x)}{dx} = 0$$

$R(x,y) = 2xy \quad S(x,y) = -x^2 + 2y^3$

$$2y\,\cancel{d\mu(y)\,R(x,y) = \frac{\partial}{\partial x}(\mu(y)S(x,y))} - 2\mu(y)x$$

$$\frac{\frac{\partial\mu(y)}{\partial y}}{u(y)} = -\frac{2}{y}$$

$\log(u(y)) = -2\log(y)$

$u(y) = \frac{1}{y^2}$

$P(x,y) = \frac{2x}{y}$

$Q(x,y) = 2y - \frac{x^2}{y^2}$

$$\frac{2x}{y(x)} + \left(2y(x) - \frac{x^2}{y(x)^2}\right)\frac{dy(x)}{dx} = 0$$

$$\frac{\partial f(x,y)}{\partial x}$$

$$f(x,y) = \int \frac{2x}{y}\,dx = \frac{x^2}{y} + g(y)$$

$$\frac{\partial f(x,y)}{\partial y} = \frac{\partial}{\partial y}\left(\frac{x^2}{y} + g(y)\right) = -\frac{x^2}{y^2} + \frac{dg(y)}{dy}$$

$$-\frac{x^2}{y^2} + \frac{dg(y)}{dy} = 2y - \frac{x^2}{y^2}$$

$$\frac{dg(y)}{dy} = 2y$$

$$g(y) = \int 2y\, dy = y^2$$

$$f(x,y) = y^2 + \frac{x^2}{y}$$

$$f(x,y) = c_1$$

$$y^2 + \frac{x^2}{y} = c_1$$

$$y^2 + \frac{x^2}{y} = C_1$$

~~Задача 3~~

3 задача

a)

$$y' = \ln \frac{y}{y'-1}$$

$$y'(y'-1)\ln\left(\frac{y}{y'-1}\right) \quad (y'-1)$$

$$y'(y'-1) = y\ln\left(\frac{y}{z-1}\right) \qquad z = y' \qquad y'' = \frac{dz}{dx}$$

$$z(z-1) = y\ln\left(\frac{y}{z-1}\right) \qquad y = z.w \qquad y' = z + z'w$$

$$y'' = z''w + 2z'w' + zw''$$

$$z(z-1) = z.w\ln\left(\frac{z-w}{z-1}\right)$$

$$y = z.w$$

$$y' = z \qquad w = \frac{y}{z}$$

$$\frac{dw}{dz} = \frac{d}{dz}\left(\frac{y}{z}\right)$$

$$\frac{dw}{dz} = \frac{z\left(\frac{y}{z}\right) - y}{z^2}$$

$\frac{dw}{dz} = \frac{y-y}{z^2}$ $\quad w = c \quad y = z.w$

$y = z.c$

$y = c.y' \quad y' = \frac{y}{c}$

$\frac{dw}{dz} = 0$

$z(z-1) = z.w \ln\left(\frac{z.w}{z-1}\right)$

$z(z-1) = z.c.\ln\left(\frac{z.c}{z-1}\right)$

$z(z-1) = cz \ln\left(\frac{cz}{z-1}\right)$

$y = xy' - e^{y'}$

$\frac{dy}{dx} = x\frac{dy'}{dx} \qquad u = x \quad v = y'$

$y = xy' - e^{y'} \quad y' = \frac{dy}{dx}$

$y = x\frac{dy}{dx} - e^{\frac{dy}{dx}}$

$y = x(y')^2 - \frac{1}{y'}$

$y = xy' - \frac{x}{y'}$

$y' = \left(\frac{y + \frac{x}{y'}}{x}\right)$