Курсова 1.1.

Задача 2

а) $y' \cos x + y(1+y)\sin x = 0$

$y(x) = -\frac{e^{c_1}\cos(x)}{e^{c_1}\cos(x)-1}$

$\cos(x)\frac{dy(x)}{dx} + (y(x)+1)\sin(x)\,y(x) = 0$

$\frac{dy(x)}{dx} = (y(x)+1)(\tan(x)\,y(x))$

$\int \frac{\frac{dy(x)}{dx}}{(y(x)+1)y(x)}\,dx = \int -\tan(x)\,dx$

$-\log(y(x)+1) + \log(y(x)) = \log(\cos(x)) + c_1$

$y(x) = \frac{e^{c_1}\cos(x)}{-e^{c_1}\cos(x)+1}$

$y(x) = \frac{c_1\cos(x)}{-c_1\cos(x)+1}$

б) $(x^2+1)y' - (2x+1)y = 0$

$x^2y'(x) + y'(x) - 2x\,y(x) = 0$

$y'(x)(x^2+1) - y(x)(2x+1) = 0$

$y'(x) + x^2y'(x) - y(x)(2x+1) = 0$

$y'(x) + x^2\,y'(x) + (-y(x) - 2x\,y(x)) = 0$

$$y e^x dy + x e^{y^2} dx = 0$$

$$e^{y(x)^2} x + e^x \frac{dy(x)}{dx} y(x) = 0$$

$$\frac{dy(x)}{dx} = -\frac{e^{-x+y(x)^2} x}{y(x)}$$

$$e^{-y(x)^2} \frac{dy(x)}{dx} y(x) = -e^{-x} x$$

$$\int e^{y(x)^2} \frac{dy(x)}{dx} y(x)\, dx = \int -e^{-x} x\, dx$$

$$-\frac{1}{2} e^{-y(x)^2} = -e^{-x}(-x-1) + C_1$$

$$y(x) = -i\sqrt{\log(-2e^{-x}(x + C_1 e^x + 1))}$$

$$(1+y^2)\sin x\, dx - (1+\cos x) y\, dy = 0$$

~~$y(x) = \sqrt{C_1 - 2}$~~

$$(y(x)^2+1)\sin(x) - \frac{dy(x)}{dx}(\cos(x)+1) y(x) = 0$$

$$\frac{dy(x)}{dx}(-\cos(x)-1) y(x) - (-\cos(x)-1)\tan\left(\frac{x}{2}\right) y(x)^2 =$$

$$(-\cos x - 1)\tan\left(\frac{x}{2}\right)$$

$$2\frac{dy(x)}{dx} y(x) - 2\tan\left(\frac{x}{2}\right) y(x)^2 = 2\tan\left(\frac{x}{2}\right)$$

$$\frac{dy(x)}{dx} - 2\tan\left(\frac{x}{2}\right) y(x) = 2\tan\left(\frac{x}{2}\right)$$

$$\cos^4\left(\frac{x}{2}\right)\frac{dv(x)}{dx} - \left(2\cos^3\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)\right)v(x) = 2\cos^3\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)$$

$$\cos^4\left(\frac{x}{2}\right)\frac{dv(x)}{dx} + \frac{d}{dx}\left(\cos^4\left(\frac{x}{2}\right)\right)v(x) = 2\cos^3\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)$$

$$\frac{d}{dx}\left(\cos^4\left(\frac{x}{2}\right)v(x)\right) = 2\cos^3\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)$$

$$\int \frac{d}{dx}\left(\cos^4\left(\frac{x}{2}\right)v(x)\right)dx = \int 2\cos^3\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)dx$$

$$\cos^4\left(\frac{x}{2}\right)v(x) = -\cos^4\left(\frac{x}{2}\right) + C_1$$

$$v(x) = C_1\sec^4\left(\frac{x}{2}\right) - 1$$

$$y(x) = -\sqrt{C_1\sec^4\left(\frac{x}{2}\right) - 1}$$

3 задача

а) $y' = (x + y - 1)^2$

$y' = y(x^2) + (-2 + 2x)y(x) + 1 - 2x + x^2$

$y'(x) = y(x^2) + (-2 + 2x)y(x) + 1 - 2x + x^2$

$y'(x) = (x + y - 1)^2$

$[(x-1) + y(x)][(x-1) + y(x)] =$

$= (x-1)(x-1) + (x-1)(y(x)) + (y(x))(x-1) + (y(x))(y(x))$

$= (x-1)^2 + [(x-1)y(x)] + (y(x)(x-1)) + y(x)^2$

$= (x-1)^2 + 2[(x-1)y(x)] + y(x)^2$

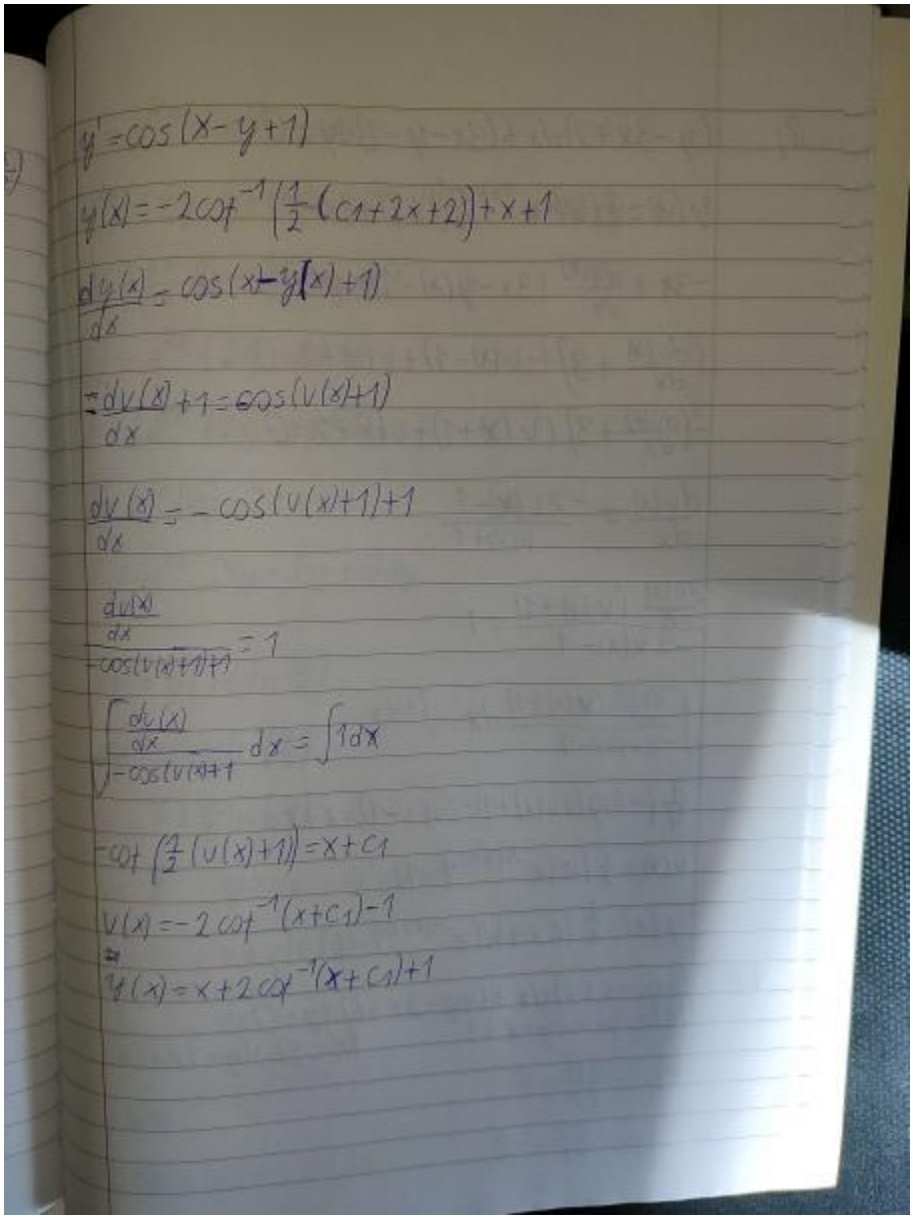
$y(x) = [(x-1)^2 + 2y(x)(x-1) + y(x^2)^2]$

[redacted]

$y'(x) = (x^2 - 2x + 1) + 2y(x)(x-1) + y(x^2)$

$2y(x)(x-1) = 2xy(x) - 2y(x)$

$y'(x) = 1 - 2x + x^2 + (2xy(x) - 2y(x)) + y(x)^2$



$y' = \cos(x - y + 1)$

$y(x) = -2\cot^{-1}\left(\frac{1}{2}(c_1 + 2x + 2)\right) + x + 1$

$\frac{dy(x)}{dx} = \cos(x - y(x) + 1)$

$\frac{dv(x)}{dx} + 1 = \cos(v(x) + 1)$

$\frac{dv(x)}{dx} = -\cos(v(x) + 1) + 1$

$\frac{\frac{dv(x)}{dx}}{-\cos(v(x)+1)+1} = 1$

$\int \frac{\frac{dv(x)}{dx}}{-\cos(v(x)+1)+1}\,dx = \int 1\,dx$

$-\cot\left(\frac{1}{2}(v(x)+1)\right) = x + c_1$

$v(x) = -2\cot^{-1}(x + c_1) - 1$

$y(x) = x + 2\cot^{-1}(x + c_1) + 1$

8) $(y-3x+2)dx+(3x-y-1)dy=0$

$y(x)=\frac{1}{2}(W(-e^{-4x+c-1})+1)+3x-1$

$-3x+\frac{dy(x)}{dx}(3x-y(x)-1)+y(x)+2=0$

$\left(\frac{dv(x)}{dx}+3\right)(-v(x)-1)+v(x)+2=0$

$-\left(\frac{dv(x)}{dx}+3\right)(v(x)+1)+v(x)+2=0$

$\frac{dv(x)}{dx}=\frac{-2v(x)-1}{v(x)+1}$

$\frac{\frac{dv(x)}{dx}(v(x)+1)}{-2v(x)-1}=1$

$\int\frac{\frac{dv(x)}{dx}(v(x)+1)}{-2v(x)-1}dx=\int 1\,dx$

$\frac{1}{4}(-\log(2v(x)+1)-2v(x)-1)=x+c_1$

$v(x)=\frac{1}{2}(W(e^{-4(x+c_1)})-1)$

$y(x)=\frac{1}{2}(6x+W(e^{-4(x+c_1)})-1$

2) $(2y-x+1)dx+(4y-2x+6)dy=0$

$M(x,y)=2y-x+1 \qquad N(x,y)=4y-2x+6$

$P(x)=\frac{\partial M}{\partial y}=2$

$Q(y)=\frac{\partial N}{\partial x}=-2$

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$I = e^{\int 2dx} = e^{2x}$

$e^{2x} \cdot (2y - x + 1)dx + e^{2x} \cdot (4y - 2x + 6)dy = 0$

$dF = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy = 0$

$\frac{\partial F}{\partial x} = e^{2x}(2y - x + 1)$

$F = \int e^{2x} \cdot (2y - x + 1)dx$

$\frac{\partial F}{\partial y} e^{2x} \cdot (4y - 2x + 6)$

$F = \int e^{2x} \cdot (4y - 2y + 6)dy$

4 Exercício

2) $xy' = y(1 + \ln\frac{y}{x})$

$xy' = y + y\ln(\frac{y}{x})$

$y' = \frac{y}{x} + \frac{y}{x}\ln(\frac{y}{x})$

$v = \frac{y}{x} \quad y = vx \quad y' = v + xv'$

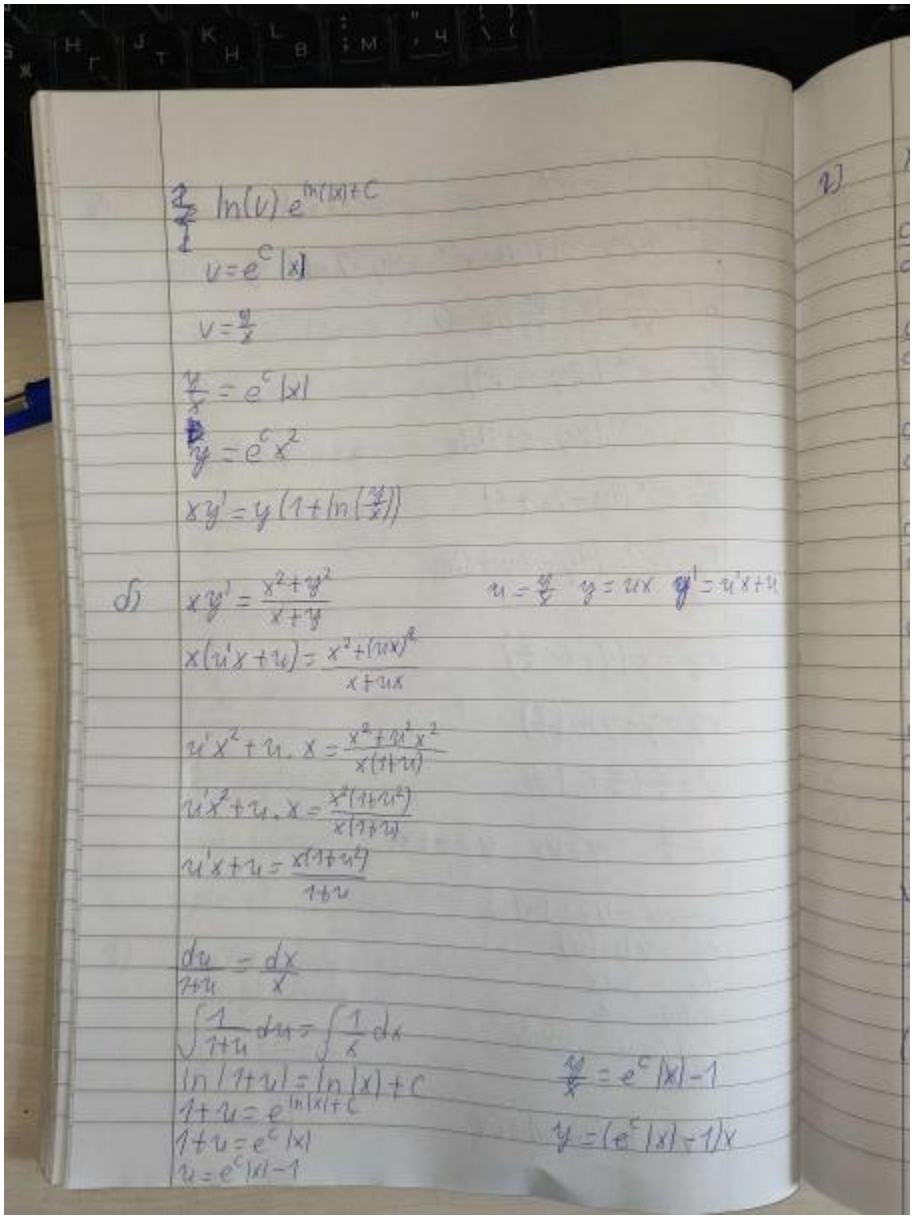
$v + xv' = v + v\ln(v)$

$xv' = v\ln(v)$

$\frac{dv}{v\ln(v)} = \frac{dx}{x}$

$\int \frac{1}{\ln(v)}dv = \int \frac{1}{x}dx$

$li(v) = \ln[\ln(v)] + C_1$



$\ln(v) = e^{\ln|x|+C}$

$v = e^C |x|$

$v = \frac{y}{x}$

$\frac{y}{x} = e^C |x|$

$y = e^C x^2$

$xy' = y(1 + \ln(\frac{y}{x}))$

б) $xy' = \frac{x^2+y^2}{x+y}$ $u = \frac{y}{x}$ $y = ux$ $y' = u'x+u$

$x(u'x+u) = \frac{x^2+(ux)^2}{x+ux}$

$u'x^2 + u \cdot x = \frac{x^2+u^2x^2}{x(1+u)}$

$u'x^2 + u \cdot x = \frac{x^2(1+u^2)}{x(1+u)}$

$u'x + u = \frac{x(1+u^2)}{1+u}$

$\frac{du}{1+u} = \frac{dx}{x}$

$\int \frac{1}{1+u} du = \int \frac{1}{x} dx$

$\ln|1+u| = \ln|x| + C$

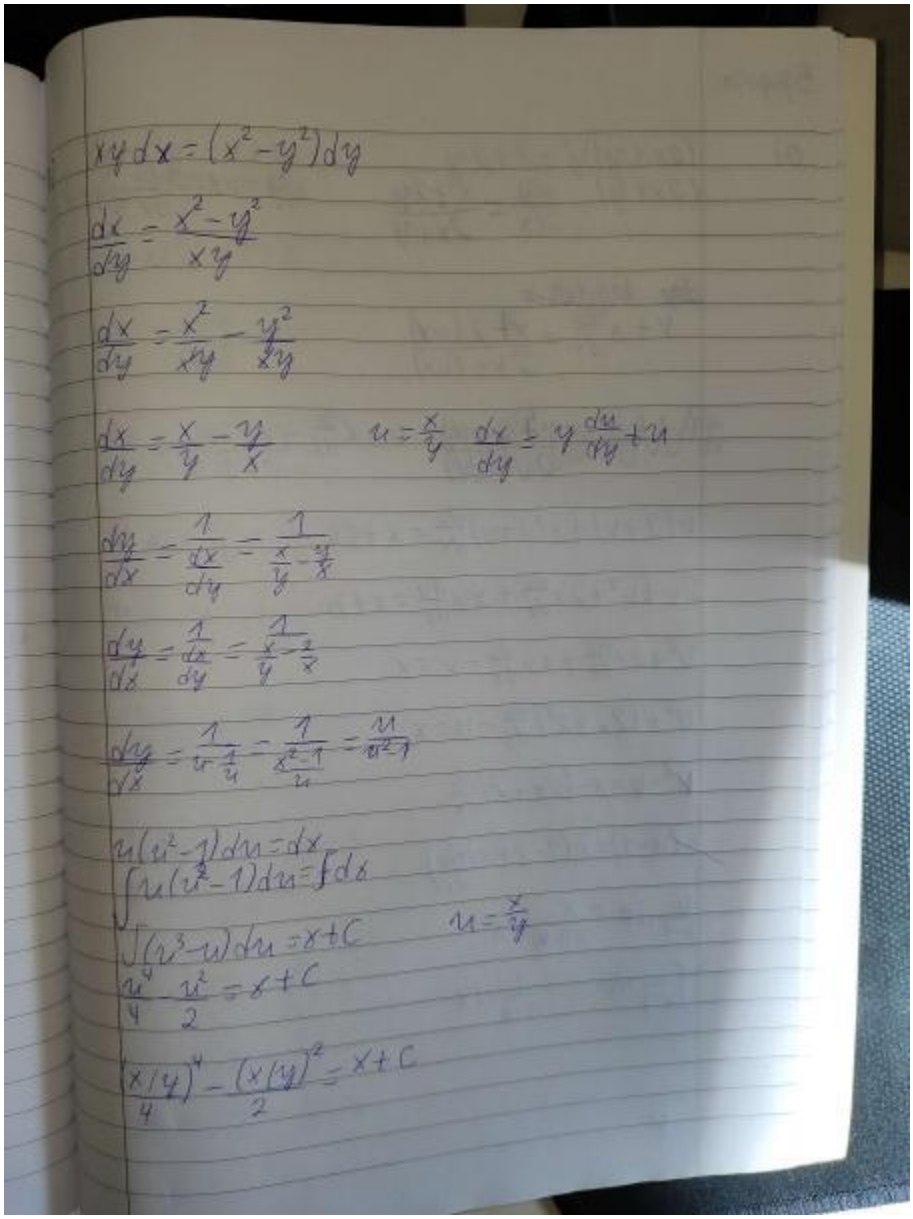
$1+u = e^{\ln|x|+C}$

$1+u = e^C |x|$

$u = e^C |x| - 1$

$\frac{y}{x} = e^C |x| - 1$

$y = (e^C |x| - 1)x$



$$xy\,dx = (x^2 - y^2)\,dy$$

$$\frac{dx}{dy} = \frac{x^2 - y^2}{xy}$$

$$\frac{dx}{dy} = \frac{x^2}{xy} - \frac{y^2}{xy}$$

$$\frac{dx}{dy} = \frac{x}{y} - \frac{y}{x} \qquad u = \frac{x}{y} \quad \frac{dx}{dy} = y\frac{du}{dy} + u$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{\frac{x}{y} - \frac{y}{x}}$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{\frac{x}{y} - \frac{y}{x}}$$

$$\frac{dy}{dx} = \frac{1}{u - \frac{1}{u}} = \frac{1}{\frac{u^2-1}{u}} = \frac{u}{u^2-1}$$

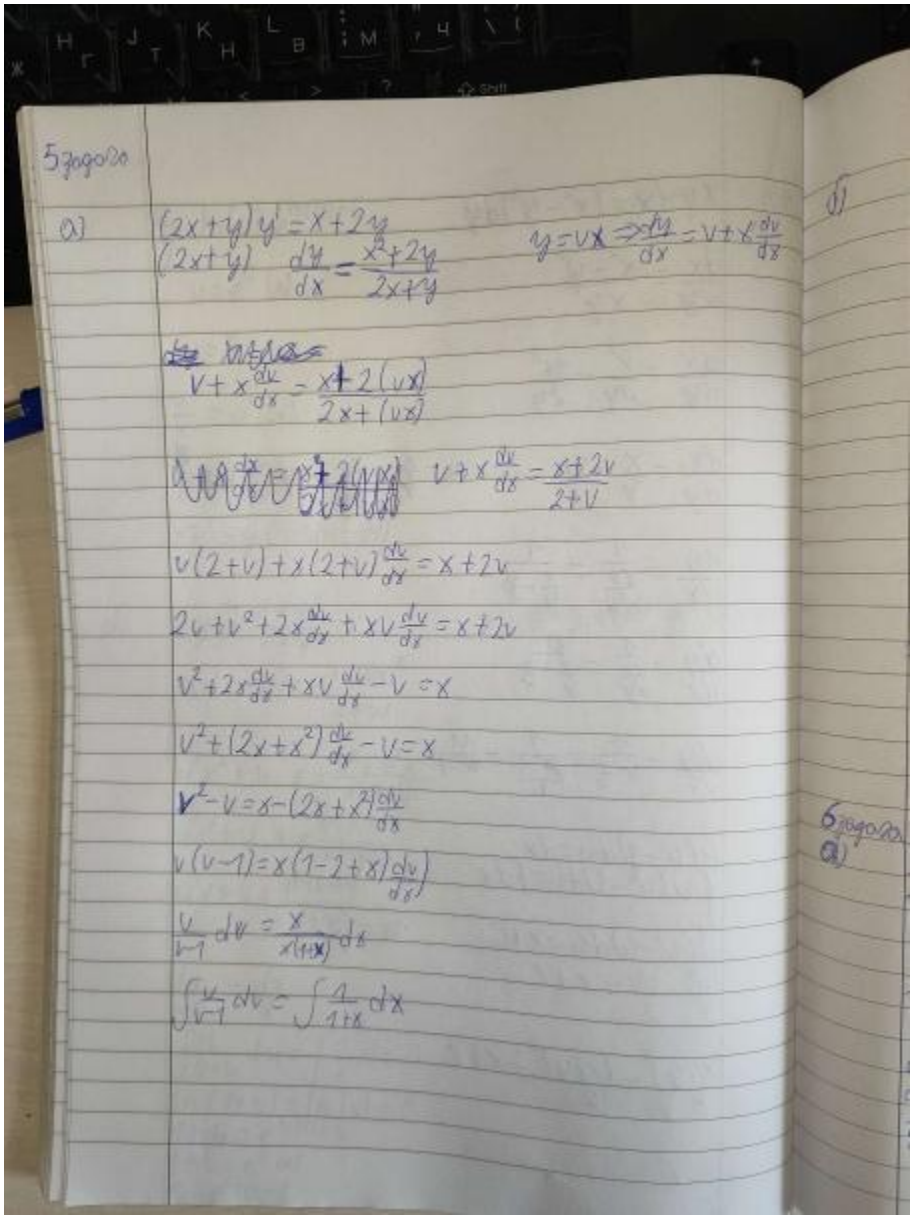
$$u(u^2-1)\,du = dx$$

$$\int u(u^2-1)\,du = \int dx$$

$$\int (u^3 - u)\,du = x + C \qquad u = \frac{x}{y}$$

$$\frac{u^4}{4} - \frac{u^2}{2} = x + C$$

$$\frac{(x/y)^4}{4} - \frac{(x/y)^2}{2} = x + C$$



5 задача

а) $(2x+y)y' = x+2y$

$(2x+y)\frac{dy}{dx} = \frac{x+2y}{2x+y}$

$y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{x+2(vx)}{2x+(vx)}$

$v + x\frac{dv}{dx} = \frac{x+2v}{2+v}$

$v(2+v) + x(2+v)\frac{dv}{dx} = x+2v$

$2v + v^2 + 2x\frac{dv}{dx} + xv\frac{dv}{dx} = x+2v$

$v^2 + 2x\frac{dv}{dx} + xv\frac{dv}{dx} - v = x$

$v^2 + (2x+x^2)\frac{dv}{dx} - v = x$

$v^2 - v = x - (2x+x^2)\frac{dv}{dx}$

$v(v-1) = x(1-2+x)\frac{dv}{dx}$

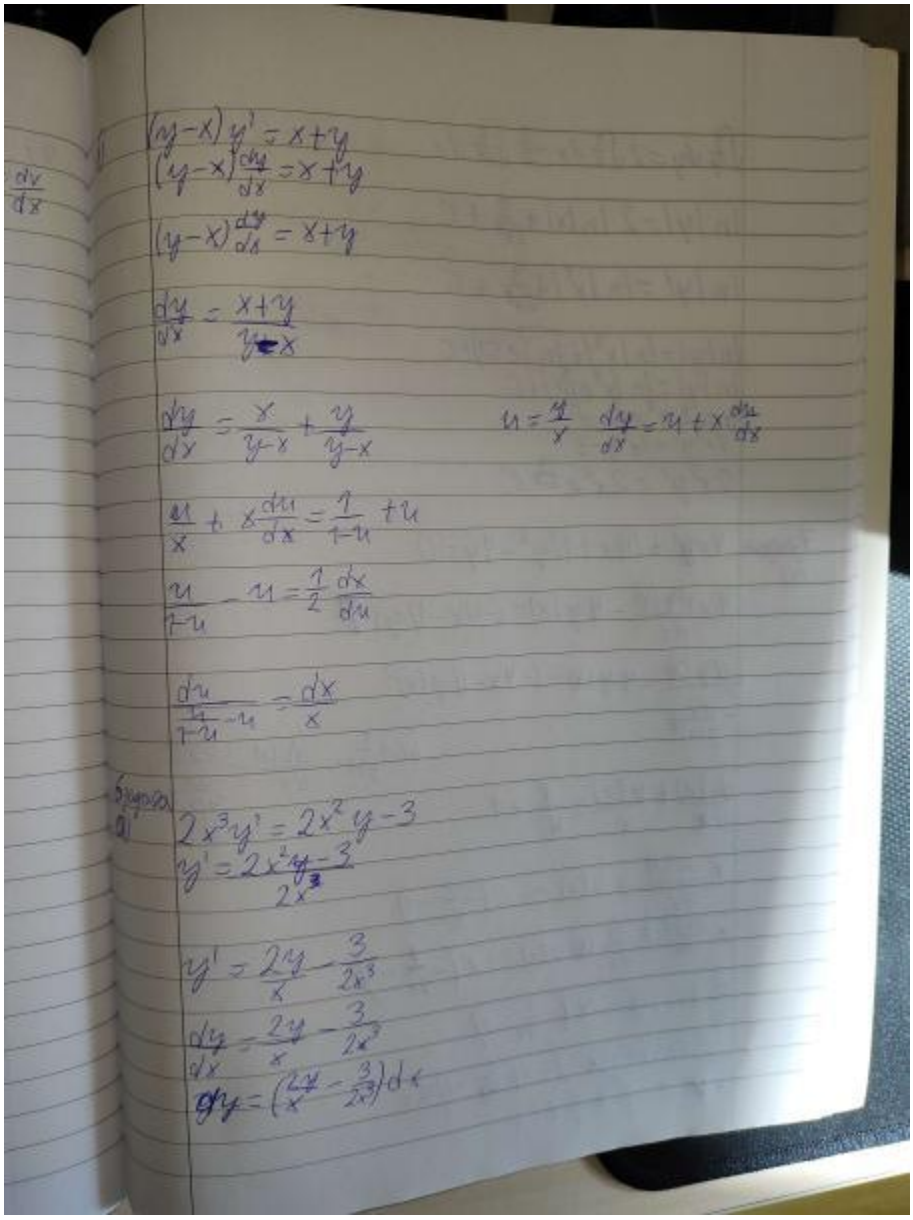
$\frac{v}{v-1}dv = \frac{x}{x(1+x)}dx$

$\int \frac{v}{v-1}dv = \int \frac{1}{1+x}dx$

б)

6 задача

а)



$(y-x)y' = x+y$

$(y-x)\frac{dy}{dx} = x+y$

$(y-x)\frac{dy}{dx} = x+y$

$\frac{dy}{dx} = \frac{x+y}{y-x}$

$\frac{dy}{dx} = \frac{x}{y-x} + \frac{y}{y-x}$ $\quad u = \frac{y}{x}$ $\quad \frac{dy}{dx} = u + x\frac{du}{dx}$

$\frac{1}{x} + x\frac{du}{dx} = \frac{1}{1-u} + u$

$\frac{u}{1-u} - u = \frac{1}{2}\frac{dx}{du}$

$\frac{du}{\frac{u}{1-u} - u} = \frac{dx}{x}$

$2x^3y' = 2x^2y - 3$

$y' = \frac{2x^2y - 3}{2x^3}$

$y' = \frac{2y}{x} - \frac{3}{2x^3}$

$\frac{dy}{dx} = \frac{2y}{x} - \frac{3}{2x^3}$

$dy = \left(\frac{2y}{x} - \frac{3}{2x^3}\right)dx$

$\int \frac{1}{y}dy = 2\int \frac{1}{x}dx - \frac{3}{2}\int \frac{1}{x^3}dx$

$\ln|y| = 2\ln|x| + \frac{3}{2x^2} + C$

$\ln|y| = \ln|x^2| + \frac{3}{2x^2} + C$

$\ln|y| = \ln|x^2| + \ln\left(e^{\frac{3}{2x^2}}\right) + C$

$\ln|y| = \ln\left(x^2 e^{\frac{3}{2x^2}}\right) + C$

$y = \pm x^2 e^{\frac{3}{2x^2}} e^C$

$2x^3y' = 2x^3y - 3$

$2x^3y' = 2x^2 e^{\frac{3}{2x^2}} C$

7 задача

а) $4xy' + (4x+1)y^2 - 4y = 0$

$4x\frac{dy(x)}{dx} - 4y(x) = (-4x-1)y(x)^2$

$4x\frac{dy(x)}{dx} - 4y(x) = (-4x-1)y(x)^2$

$-\frac{dx}{y(x)^2}$

$v(x)\frac{1}{y(x)}$ $\frac{dv(x)}{dx} = \frac{\frac{dy(x)}{-dx}}{\frac{y(x)^2}{y(x)^2}}$

$\frac{dv(x)}{dx} + \frac{v(x)}{x} = \frac{1}{4x} + 1$

$x\frac{dv(x)}{dx} + v(x) = -x\left(-\frac{1}{4x} - 1\right)$

$x\frac{dv(x)}{dx} + \frac{d}{dx}(x)v(x) = -x\left(-\frac{1}{4x} - 1\right)$

$\frac{d}{dx}(xv(x)) = -x\left(-\frac{1}{4x} - 1\right)$

$\int \frac{d}{dx}(xv(x))dx = \int -x\left(\frac{1}{4x} - 1\right)dx$

$xV(x) = \frac{x^2}{2} + \frac{x}{4} + C_1$

$$v(x) = \frac{x}{2} + \frac{C_1}{x} + \frac{1}{4}$$

$$y(x) = \frac{1}{v(x)} = \frac{4x}{2x^2 + x + 4C_1}$$

$$y(x) = \frac{4x}{2x^2 + x + C_1}$$