2.2. Курсова

1з. а) $\begin{cases} \dot{x} = -5x - 2y - 2z \\ \dot{y} = 10x + 4y + 2z \\ \dot{z} = 2x + y + 3z \end{cases}$

$(\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = -1)$

$A = \begin{pmatrix} -5 & -2 & -2 \\ 10 & 4 & 2 \\ 2 & 1 & 3 \end{pmatrix}$

$A - \lambda E = \begin{pmatrix} -5 & -\lambda-2 & -2 \\ 10 & 4-\lambda & -2 \\ 2 & 1 & 3-\lambda \end{pmatrix} \begin{matrix} -5-\lambda & -2 \\ 10 & 4-\lambda \\ 2 & 1 \end{matrix}$

$\boxed{\lambda_1 = 1}$

$(A - \lambda_1 E) \cdot h_1 = 0$

$\Rightarrow \begin{pmatrix} -6 & -2 & -2 \\ 10 & 3 & 2 \\ 2 & 1 & 2 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\begin{cases} \gamma = \alpha \\ 10\alpha + 3\beta + 2\gamma = 0 \\ \beta = -4\alpha \end{cases}$

$h_2 = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} \alpha \\ -4\alpha \\ \alpha \end{pmatrix} \Rightarrow$

$\Rightarrow x_1 = \begin{pmatrix} 1 \\ -4 \\ 1 \end{pmatrix} \cdot e^t$

$$\begin{cases} 6\alpha - 2\beta - 2\gamma = 0 \\ 10\alpha + 3\beta + 2\gamma = 0 \\ 2\alpha + \beta + 2\gamma = 0 \end{cases}$$

$$\begin{cases} -6\alpha - 2\beta - 2\gamma = 0 \\ 10\alpha + 3\beta + 2\gamma = 0 \\ \beta = -2\alpha - 2\gamma \end{cases}$$

$$\begin{cases} -2\alpha + 2\gamma = 0 \\ 10\alpha + 3\beta = 0 \\ \beta = -2\alpha - 2\gamma \end{cases}$$

$\boxed{\lambda_2 = 2}$

$(A - \lambda_2 E) \cdot h_2 = 0$

$$\begin{pmatrix} -4 & -2 & -2 \\ 10 & 2 & 2 \\ 2 & 1 & 1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} -4\alpha - 2\beta - 2\gamma = 0 \\ 10\alpha + 2\beta + 2\gamma = 0 \\ 2\alpha + \beta + \gamma = 0 \end{cases}$$

$$\begin{cases} 3\alpha = 0 \\ 10\alpha + 2\beta + 2\gamma = 0 \\ \beta = -\gamma \end{cases}$$

$$h_2 = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 0 \\ -\gamma \\ \gamma \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

$$\Rightarrow x_2 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \cdot e^{2t}$$

$\boxed{\lambda_3 = -1}$

$(A - \lambda_3 E) \cdot h_3 = 0$

$$\begin{pmatrix} -4 & -2 & -2 \\ 10 & 5 & 2 \\ 2 & 1 & 4 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} -4\alpha - 2\beta - 2\gamma = 0 \\ 10\alpha + 5\beta + 2\gamma = 0 \\ 2\alpha + \beta + 4\gamma = 0 \end{cases}$$

$$\begin{cases} 4\alpha - 2\beta - 2\gamma = 0 \\ 10\alpha + 5\beta + 2\gamma = 0 \\ \beta = -2\alpha - 4\gamma \end{cases}$$

$$\begin{cases} 6\gamma = 0 \\ 10\alpha + 5\beta + 2\gamma = 0 \\ \beta = -2\alpha \end{cases}$$

$$h_3 = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} \alpha \\ -2\alpha \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$$

$$\Rightarrow x_3 = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} \cdot e^{-t}$$

$$x = C_1 \cdot \begin{pmatrix} 1 \\ -4 \\ 1 \end{pmatrix} \cdot e^{t} + C_2 \cdot \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \cdot e^{2t} + C_3 \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} \cdot e^{-t}$$

2δ)

$$\begin{cases} \dot{x} = 3x - 6y + \frac{1}{\cos^3 3t} \\ \dot{y} = 3x - 3y \end{cases}$$

$$(A - \lambda E) = \begin{pmatrix} 3-\lambda & -6 \\ 3 & -3-\lambda \end{pmatrix} = 0 \Rightarrow$$

$$\Rightarrow (3-\lambda)(-3-\lambda) + 18 = 0$$

$$-9 - 3\lambda + 3\lambda + \lambda^2 = 0$$

$$\lambda^2 - 9 = 0$$

$$\lambda^2 = 9$$

$$\lambda_{1,2} = \pm 3$$

$\boxed{\lambda_1 = 3}$

$$\begin{pmatrix} 0 & -6 \\ 3 & -6 \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} -6\beta = 0 \\ 3\alpha - 6\beta = 0 \end{cases}$$

$$h_1 = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \alpha \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot e^{3t}$$

$\boxed{\lambda_2 = -3}$

$$\begin{pmatrix} 6 & -6 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} 6\alpha - 6\beta = 0 \\ 3\alpha = 0 \end{cases}$$

$$h_2 = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot e^{3t}$$

$$X_{хом} = C_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot e^{3t} + C_2 \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot e^{-3t}$$

$$\begin{cases} C_1' \cdot e^{3t} = 0 \\ C_2' \cdot e^{-3t} = \frac{1}{\cos^3 3t} \end{cases}$$

$C_1 = 0$

$$C_2 = \int \frac{e^{3t}}{\cos^3 3t} dt = \int e^{3t} dt + \int \frac{1}{\cos^3 3t} dt$$

$$= \frac{e^{3t}}{3} + \frac{\sin 3t}{6 \cos(3t)^2} - \frac{1}{12} \left(\ln \left| \frac{\sin(3t) - 1}{\sin(3t) + 1} \right| \right.$$

$$\eta = 0 \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot e^{3t} + \left(\frac{e^{3t}}{3} + \frac{\sin 3t}{6 \cos^3 3t} - \frac{1}{12} \left(\ln \left(\frac{\sin 3t - 1}{\sin 3t + 1} \right) \right) \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot e^{-3t} \right.$$

$X_{неодн} = X_{одн} + \eta$

3з. а) $\dot{x} = -2x - y + 37\sin t$
$\dot{y} = -4x - 5y$

$(A - \lambda E) = \begin{pmatrix} -2-\lambda & -1 \\ 4 & -5-\lambda \end{pmatrix} = 0$

$\det = (-2-\lambda)(-5-\lambda) - 4 = 0$

$10 + 2\lambda + 5\lambda + \lambda^2 - 4 = 0$

$\lambda^2 + 7\lambda + 6 = 0$

$D = 48 - 24 = 25 = 5^2$

$\lambda_1, \lambda_2 = \frac{-7 \pm 5}{2} \nearrow -1 \searrow -6$

$\boxed{\lambda_1 = -1}$

$\begin{pmatrix} -1 & -1 \\ -4 & -4 \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\begin{cases} -\alpha - \beta = 0 \\ -4\alpha - 4\beta = 0 \end{cases}$

$\alpha = -\beta$

$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} -\beta \\ \beta \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \cdot e^{-t}$

$\boxed{\lambda_2 = -6}$

$\begin{pmatrix} 4 & -1 \\ -4 & 1 \end{pmatrix} \cdot \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\begin{cases} 4\alpha - \beta = 0 \\ -4\alpha + \beta = 0 \end{cases}$

$\Rightarrow \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \alpha \\ 4\alpha \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \cdot e^{-6t}$

$y_{хом} = C_1 \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} \cdot e^{-t} + C_2 \cdot \begin{pmatrix} 1 \\ 4 \end{pmatrix} \cdot e^{-6t}$

$f1 = \sin t$
$f2 = 0$

$$F = \begin{pmatrix} f1 \\ f2 \end{pmatrix} = \begin{pmatrix} \sin t \\ 0 \end{pmatrix} \Rightarrow F = \begin{pmatrix} 0 \cos t + 37 \sin t \\ 0 \cos t + 0 \sin t \end{pmatrix} \cdot e^{0t}$$

$\alpha = 0 \quad \beta = 1$
$\alpha \pm i\beta = i$
$k = 0$
$m + k = 0$

$$\Rightarrow \eta = \begin{pmatrix} a \cos t + b \sin t \\ c \cos t + d \sin t \end{pmatrix} \cdot e^{0t}$$

$$\begin{cases} (a \cos t + b \sin t)' = -2(a \cos t + b \sin t) - (c \cos t + d \sin t) \\ (c \cos t + ed \sin t)' = -4(a \cos t + b \sin t) - 5(c \cos t + d \sin t) \end{cases}$$

$$\begin{cases} -a \sin t + b \cos t = -2(a \cos t + b \sin t) - (c \cos t + d \sin t) \\ -c \sin t + d \cos t = -4(a \cos t + b \sin t) - 5(c \cos t + d \sin t) \end{cases}$$

$$\begin{cases} b = -2a - c \\ a = 2b + d \\ d = -4a + 5c \\ c = 4b + 5d \end{cases}$$