2.1. Курсова

D = b.2 - 4.a.c

1з. а) $y'' - 6y' + 8y = 0$

$\lambda^2 - 6\lambda + 8 = 0$

$D = 36 - 32 = 4 = 2^2$

$\lambda_1, \lambda_2 = \frac{6 \pm 2}{2} \nearrow^{4} \searrow_{2}$

$\lambda_1 = 4 \quad \lambda_2 = 2$

$y = C_1 \cdot e^{4x} + C_2 \cdot e^{2x}$

2з. а) $y'' - 2y' + 10y = 0$

$\lambda^2 - 2\lambda + 10 = 0$

$D = -36$

$\lambda_1, \lambda_2 = \frac{2 \pm \sqrt{-36}}{2} = 1 \pm 3i$

$\lambda_1 = 1 + 3i$

$= e^{(1+3i)x}$

$e^x (\cos 3x + i \sin 3x)$

$= e^x \cdot \cos 3x + e^x \cdot i \sin 3x$

$y = C_1 \cdot e^x \cdot \cos 3x + C_2 \cdot e^x \cdot \sin 3x$

3з. б) $y'' + 2y' + y = \frac{1}{xe^x}$

$\lambda^2 + 2\lambda + 1 = 0$

$(\lambda + 1)^2 = 0$

$\lambda_1 = \lambda_2 = -1$

$y_{хом} = C_1 \cdot e^{-x} + C_2 x \cdot e^{-x}$

$$\begin{cases} C_1' e^{-x} + C_2' x e^{-x} = 0 \\ C_1' (e^{-x})' + C_2' (e \cdot x^{-1})' = 1 \end{cases}$$

$$\begin{cases} C_1' \cdot e^{-x} + C_2' x \cdot e^{-x} = 0 \\ -C_1' e^{-x} + C_2' \cdot e^{-x} - C_2' \cdot x \cdot e^{-x} = \frac{1}{x \cdot e^x} \end{cases}$$

$C_2' \cdot e^{-x} = \frac{1}{x \cdot e^x}$

$C_2 \cdot e^{-x} = \frac{e^{-x}}{x} \quad | : e^{-x}$

$C_2 = \frac{1}{x} \qquad C_1 = -\frac{1}{x} \cdot x = -1$

$C_2 = \int \frac{1}{x} dx = \ln x$

$C_1 = -\int dx = -x$

$y(x) = -x \cdot e^{-x} + \ln x \cdot x \cdot e^{-x}$

$y + e_{хом} = C_1 \cdot e^{-x} + C_2 \cdot x \cdot e^{-x} + (-x) \cdot e^{-x} + \ln x \cdot x \cdot e^{-x}$

4з. 2) $y'' - 4y' + 13y = -9\cos 2x - 8\sin 2x$

$\lambda^2 - 4\lambda + 13 = 0$

$D = 16 - 52 = -36$

$\lambda_1, \lambda_2 = \frac{4 \pm \sqrt{-36}}{2} = \frac{4 \pm 6i}{2} = 2 \pm 3i$

$\lambda_1 = 2 + 3i = e^{(2+3i)} = e^{2x} \cdot e^{3ix}$

$y = C_1 \cdot e^{2x} \cdot \cos 3x + C_2 \cdot e^{2x} \cdot \sin 3x$

$\left.\begin{matrix} \alpha = 0 \\ \beta = 2 \end{matrix}\right\}$ $0 + i2 = 2i \Rightarrow k = 0$

$q(x) = -9 \Rightarrow$ наєв $q(x) = 0 \Rightarrow m = 0$

$p(x) = -8 \Rightarrow$ наєв $p(x) = 0$

$P(x) = b \qquad Q(x) = a$

$\eta(x) = b \cdot \sin 2x + a \cdot \cos 2x$

$\eta'(x) = -2a \sin 2x + 2b \cos 2x$

$\eta''(x) = -4a \cos 2x - 4b \cdot \sin 2x$

$4a\cos 2x - 4b\sin 2x + 8a\sin 2x - 8b\cos 2x + 13a\cos 2x + 13b\sin 2x = -9\cos 2x$

$9a\cos 2x + 9b\sin 2x + 8a\sin 2x - 8b\cos 2x = -9\cos 2x - 8\sin 2x$

$(9a - 8b)\cos 2x + (9b + 8a)\sin 2x = -9\cos 2x - 8\sin 2x$

$$\begin{cases} 9a - 8b = 9 \\ 9b + 8a = -8 \end{cases} \Rightarrow \begin{matrix} a = -1 \\ b = 0 \end{matrix}$$

$$\eta(x) = 0 \cdot \cos 2x + 0 \cdot \sin 2x = -\cos 2x$$

$$y_{\text{неодн}} = C_2 \cdot e^{2x} \cdot \sin 3x + C_2^{2x} \cos 3x - \cos 2x$$