Data-Driven Problem Solving in Mechanical Engineering

Logistic Regression

Masoud Masoumi

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Department of Mechanical Engineering
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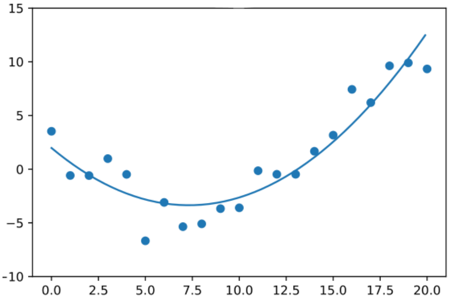
Regression

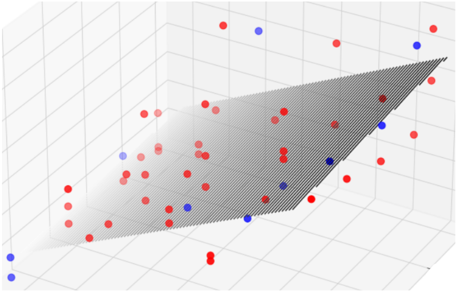
We can apply linear regression to classification problems by converting the class names of training examples to numbers, i.e. probabilities.

Recall Multiple **Linear Regression**

$f(\mathrm{x}) = \omega_o + \omega_1 x_1 + \omega_2 x_2 + \omega_3 x_3 + ... \rightarrow f(\mathrm{x}) = \mathrm{w}^T \mathrm{x}$

with $\mathrm{w}^T = [\omega_o, \omega_1, \omega_2, \omega_3, ...]$ and $\mathrm{x} = [1, x_1, x_2, x_3, ...]^T$





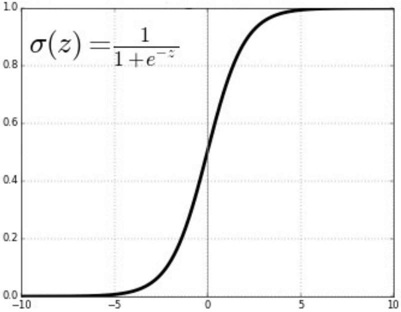
Regression to Logistic Regression

- In order to estimate the class of a data point, we need some sort of guidance on what would be the *most probable class* for that data point.

- Logistic regression produces a formula that predicts the probability of the class label as a function of independent variables.

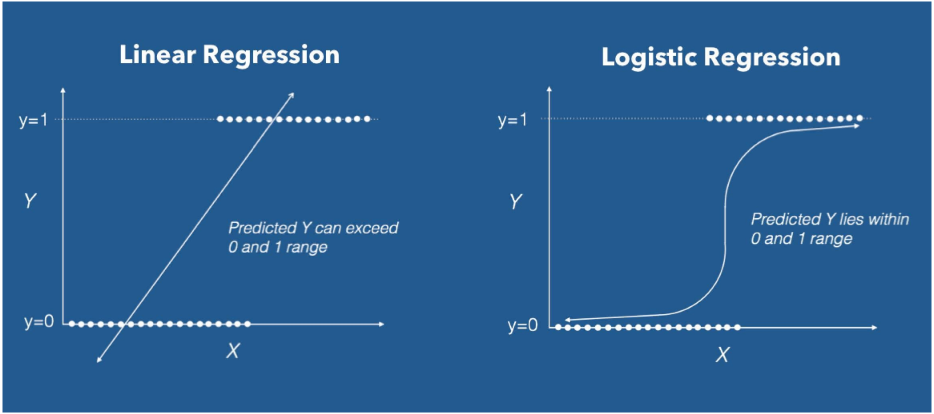
- In logistic regression, we need to output a probability, a value between 0 and 1. $P(y = 1|x)$ (Probability of y being class 1 given features x)

- One way to achieve this goal is to use logistic function or Sigmoid function $\sigma(z) = \frac{1}{1+e^{-z}}$, whose output is between 0 and 1.



Logistic Regression

Logistic regression fits Sigmoid function by taking the linear regression and transforming the numeric estimate into a probability $\sigma(\mathrm{w}^T x) = \frac{1}{1+e^{-\mathrm{w}^T x}}$



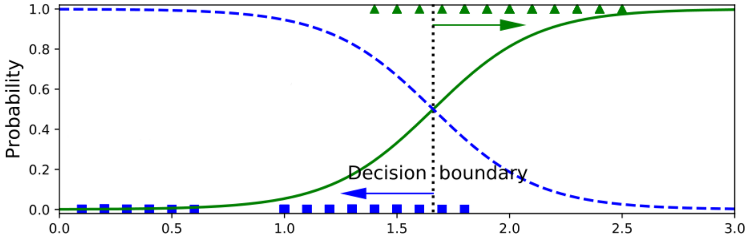
(ref: https://towardsdatascience.com/introduction-to-logistic-regression-66248243c148)

Logistic Regression

In other words and in statistical language, we are looking for the probability of dependent variable being a specific class, given the selected features.

Probability of y being class 1 given features x: $P(y=1|x) = \sigma(\mathrm{w}^T x) = \frac{1}{1+e^{-\mathrm{w}^T x}}$.

The prediction function returns a probability score between 0 and 1. In order to map this to a discrete class (true/false, cat/dog, faulty/not faulty, 1/0), we select a *threshold value* above which we will classify values into class 1 and below which we classify values into class 0.



(*modified from* Murphy, Kevin P. Probabilistic machine learning: an introduction. MIT press, 2022)

Model Evaluation

To evaluate our model logistic regression model, we introduce a new metric for classifiers where the predicted output is a probability value between 0 and 1.

$$-\begin{cases} \log a & y = 1, \\ \log(1-a) & y = 0. \end{cases}$$

For one prediction, we can define **Log Loss** as

$$-y \times log[\sigma(\mathrm{w}^T x)] - (1-y) \times log[1 - \sigma(\mathrm{w}^T x)]$$

For all m samples

Log Loss= $\frac{1}{m}\sum_{i=1}^{m} -y^{(i)} \times log[\sigma(\mathrm{w}^T x^{(i)})] - (1-y^{(i)}) \times log[1 - \sigma(\mathrm{w}^T x^{(i)})]$