\documentclass[11pt,a4paper]{article}

\usepackage{chngcntr}

\counterwithin{figure}{section}

\usepackage[T1]{fontenc} \usepackage{lmodern} \usepackage[utf8]{inputenc} \usepackage[english]{babel} \usepackage{amssymb} \usepackage{amsmath} \usepackage[round]{natbib} \usepackage{xcolor,graphicx}

\author{Zully Faralli, Marc Spörri} \title{Analysis and forecasting of the Swiss Rent Index Time Series, 1993 to 2018} \date{\today}

\begin{document}

\maketitle

**\section{Introduction}**

describe aims of your project and give an overview of the report. describe briefly the methods you have used to analyze the data.

\\

\\

copied from the proposal: The main goal of the project is to draw inference from the rent index time series and to predict their

future values and a future trend. In order to do that we have first to transform the data in a

stationary time series, either by fitting a polynomial trend, either by differencing. Afterwards we are

going to fit a model to our data. The model will be used to describe and interpret our data as well as

to predict future values.

**\section{Data Description}**

Describe the data and the context in which it has been gathered and explain. Give some general references about the data.

\\

\\

copied from the proposal: The data is provided by the Swiss Federal Statistical Office, implying 100 quarterly observations of

the Swiss rent index beginning by the 2nd quarter of the year 1993 and ending by the first quarter of

the year 2018. The rent index measures the inflation of the rented dwellings in Switzerland. It is the

most significant partial index of the Swiss consumer’s price index, representing a weighted share of

13 percent of this index. The data are collected quarterly, based upon a stratified random survey sampling

of around 10'000 lessors. The time series’ first observation (2nd quarter of the year 1993) will be the

reference value which is set to a base index of 100 and represents the weighted average rent at this

time in Switzerland (OFS, 2016: p. 20-23).

**\section{Data Analysis and Transformation into a Stationary Time Series}**

Describe your models carefully, give references, justify your choices. You can use the guidelines provided below.

\\1. Do you transform the data? If yes, give the transformation you used and explain this choice. 2. Does the series show a trend and a seasonality?

\\If yes, describe how you model them. Check the plot of the residuals.

\\3. Comment the autocorrelation and the partial autocorrelation of the residual series obtained after removing the trend and seasonality.

\\4. Model the residuals with an appropriate model. If you consider different models, explain strengths and weaknesses of each model.

\\5. Comment the diagnostig plots of the model(s) you chose. you can plot the standardized residuals, the normal qqplot, the ACF and PACF of the residuals, and the p-value of the Ljung-Box-Tests \citep{LjungBox78} for different lags to assess stationarity, independence and normality of the residuals.

\\6. By means of your model, make some predictions and give confidence intervals for the prediction. you can illustrate it by showing some plots.

\\

\\

The first step in any analysis of a time series is to plot the data in order to identify discontinuities or a sudden change of level in the series (book p. 23). As we can see from figure~\ref{fig:indiceloyers\_timeseries} there seems to be a clear positive trend.

\\In addition it may be advisable as well to analyze the series by breaking it into homogeneous segments \cite[p.~23]{bd02} . Let's have a look at the segmented plots 1993 to 2009 and 2009 to 2018 in figure~\ref{fig:indiceloyers\_test} and figure~\ref{fig:indiceloyers\_train}. We can see a slightly slower increase in the segment from 2009 to 2018 indicating that in the years from 1993 to 2009 the growth in rental prices was higher than in the years in the second's segment which begins from 2009. that can be explained by the big baisse in the early 90ies, starting from a lower initial point and having better conjunctural perspectives the increase was stronger, whilst from 2009 on the growth in rental prices slowed down, which can be very well explained by the US subprime crises beginning in the year 2008 followed by a long-taking global recession. However they are no sudden changes in level or huge outliers, therefore we can start to find a transformation into a stationary time series for our whole data.

These changes are due to the global economic environment. Otherwise there is nothing unusual about the time plot and there appears to be no need to do any data adjustments.

There is no evidence of changing variance, so we will not do a Box-Cox transformation.

In this section we want to produce a noise sequence with no apparent deviations from stationarity, meaning we want to transform our data in such a way that covariances between the observations are not depending on time and that we get zero mean expectation and constant variances \cite[pp.~14--23]{bd02}. \\

However, the objective is not to get a pure white noise sequence with no dependence among the residuals, since there would no further modelling to be done except to estimate their mean (which would be zero) and variance. The objective is too get a stationary series with some few significant dependence among the residuals anyhow, so we can look for a more complex stationary time series model for the noise that accounts for the dependence. Since dependence means that past observations of the noise sequence can assist in predicting future values this allows us to get a better prediction quality than with a pure white noise sequence \cite[p.~35]{bd02}. \\

In order to get our noise sequence we have to eliminate any trend and/or seasonal components from our data. In the next few subsections we will fit several models to get a stationary series. Afterwards we check our fitted models first for stationary by tests like the Augmented Dickie-Fuller-Test for the null hypothesis of a unit root of a univariate time series (with the alternative hypothesis of stationarity by consequence) \citep{adf} and the Kwiatkowski-Phillips-Schmidt-Shin-Test (KPSS-Test) for the null hypothesis that the observations are trend stationary \citep{kpss92}. Furthermore we test the estimated noise sequences numerically by examining some simple tests

for checking the hypothesis that the obtained residuals are values of independent and identically distributed random variables (as mentioned above: this is not our objective, otherwise our prediction work would be simply done by estimating their mean and variance). In a final step we check them visually by means of the autocorrelation-function plot as well as the partial auto-correlation-function in order to get sure that we are not modelling a white noise sequence on the one hand, and to get an idea of the orders of p and q, respectively for a possible ARMA(p,q)-model \cite[pp.~83--110]{bd02}.

Once we have found a good model to transform our data in a stationary series we can begin to estimate its parameters in section~\ref{Fitting and Testing the Model}.

\\

\begin{figure}[!ht]

\centering

**\includegraphics**[angle=0,

width=0.7\textwidth]{indiceloyers\_timeseries}

\caption{Swiss rent index, years 1993 to 2018**\label{fig:indiceloyers\_timeseries}**}

\end{figure}

As we can see in figure~\ref{fig:indiceloyers\_timeseries} a trend is obviously visible, however we cannot be that sure if it doesn't exist a seasonal component either. Therefore in the next step we check first whether they are possible seasonal components, afterwards we will fit several models to get rid of the trend.

\begin{figure}[!htb]

\centering

**\includegraphics**[angle=0,

width=0.7\textwidth]{indiceloyers\_test}

\caption{Swiss rent index, years 2009 to 2018**\label{fig:indiceloyers\_test}**}

\end{figure}

\begin{figure}[!htb]

\centering

**\includegraphics**[angle=0,

width=0.7\textwidth]{indiceloyers\_train}

\caption{Swiss Rent index, years 1993 to 2009**\label{fig:indiceloyers\_train}**}

\end{figure}

**\subsection{Check for Seasonality}**

Since the data are collected quarterly we use a linear regression model with d=4 dummy predictors to check for significant seasonal coefficients, meaning a dummy for every quarter (LN1-2, p. 36).

\begin{figure}[!htb]

\centering

**\includegraphics**[angle=0,

width=0.7\textwidth]{summary\_seasonmodel}

\caption{summary statistics of the fitted seasonal model**\label{fig:summary\_seasonmodel}**}

\end{figure}

As we can see from figure~\ref{fig:summary\_seasonmodel} none of the estimated seasonal coefficients are significant. We can be sure now that they are no seasonal impacts on our data, hence what remains is to eliminate the trend. In the absence of a seasonal component our model becomes the following \cite[p.~24]{bd02}:

\begin{equation}

X\_t = m\_t + Y\_t, \text{t = 1,...,n,}

\\where EY\_t = 0

\end{equation}

**\subsection{Trend Elimination by fitting polynomial models}**

As we have shown in the above subsection, we don't have to get rid of seasonal components, nevertheless we have to get rid of the obvious trend. Whilst the time series \ref{fig:indiceloyers\_timeseries} indicates a probable polynomial trend, especially a linear one, we are starting by fitting different polynomial trends by ordinary least squares estimation. We are going to fit a linear trend, a quadratic trend, a cubic trend as well as a logarithmic trend. The latter helps to transform a potentially exponential increase in the rent index into a linear trend, even though at first sight the time series \ref{fig:indiceloyers\_timeseries} looks more like a linear than an exponential trend, we are going to test for a logarithmic trend either.

\\

\begin{figure}[!htb]

\centering

**\includegraphics**[angle=0,

width=0.7\textwidth]{summary\_cubicmodel}

\caption{summary statistics of the fitted cubic model**\label{fig:summary\_cubicmodel}**}

\end{figure}

The coefficients in all 4 models are highly significant and with an $R^2$-Value of 98 to 99 percent (meaning that the models would be able to explain up to 98percent (99percent respectively) of the variance), the linear, the quadratic and the cubic model would explain our data extremely well. This can can be well seen at the cubic model's example in \ref{fig:summary\_cubicmodel}. The logarithmic model shows a bit less explanatory power with an $R^2$-Value of 0.72, even though its coefficient is highly significant as well. However we have to be careful with the interpretation of the p-values since this regression models assume independence of the observations whereas our purpose of the summary statistics of fitted linear models is a different one: to use residuals in order to construct a stationary time series (Brooks Davis p. xy).

**\subsubsection{Diagnostics of the fitted polynomial trends}**

\begin{figure}[!htb]

\centering

**\includegraphics**[angle=0,

width=0.5\textwidth]{resid\_linearmodel}

\caption{Residuals of a fitted linear model

**\label{fig:resid\_linearmodel}**}

\end{figure}

\begin{figure}[!htb]

\centering

**\includegraphics**[angle=0,

width=0.5\textwidth]{resid\_quadraticmodel}

\caption{Residuals of a fitted quadratic model

**\label{fig:resid\_quadraticmodel}**}

\end{figure}

\begin{figure}[!htb]

\centering

**\includegraphics**[angle=0,

width=0.5\textwidth]{resid\_cubicmodel}

\caption{Residuals of a fitted cubic model

**\label{fig:resid\_cubicmodel}**}

\end{figure}

\begin{figure}[!htb]

\centering

**\includegraphics**[angle=0,

width=0.5\textwidth]{resid\_logmodel}

\caption{Residuals of a fitted logarithmic model

**\label{fig:resid\_logmodel}**}

\end{figure}

Unfortunately the plots for polynomial trend elimination don't show any stationarity, as the series wanders up and down for quite some periods. as we can derive from the four figure~\ref{fig:resid\_linearmodel}, figure~\ref{fig:resid\_quadraticmodel}, figure~\ref{fig:resid\_cubicmodel} and figure~\ref{fig:resid\_logmodel}. The four plots show large covariances and their residuals are evidently depending on time $t$, meaning non-stationary.

\\

\begin{figure}[!htb]

\centering

**\includegraphics**[angle=0,

width=0.5\textwidth]{acf\_cubicmodel}

\caption{Sample autocorrelation function of the cubic model

**\label{fig:acf\_cubicmodel}**}

\end{figure}

Furthermore from the ACF-plot (figure~\ref{fig:acf\_cubicmodel}) and the PACF-plot (figure~\ref{fig:pacf\_cubicmodel}) exemplified on the cubic model, we can see that there are a lot of significant lags outside the 95-percent-confidence bounds of $+/-1.96sqrt{n}$. These bars don't die out quickly. So the $Rho\_hat$ can be a useful indicator of non-stationarity \cite[p.~21]{bd02}. Hence, we can conclude our covariances depend on time and therefore the series is non-stationary. similar time-depending patterns hold true for the autocovariance-functions of our other polynomial trend models: the linear, the quadratic and the logarithmic model.

\\In addition we get numerical evidence from three different tests checking for stationarity or independence of the observations, respectively. First the Ljung-Box test examines whether there is significant evidence for non-zero correlations at lags 1-40. Small p-values (i.e., less than 0.05) suggest that the residuals are independent. The Ljung-Box Hypothesis0 of independence has to be rejected for all four polynomial models so far (TODO show the 4 p-values). The Augmented Dickie-Fuller test and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test provide as well strong evidence for non-stationary for all 4 polynomial models (TODO show the 4 p-values). Even though we have to keep in mind the low power of Dickie-Fuller Test for our purposes given the fact this test assumes a AR-Process, about which we cannot be too sure yet.

\\ The following figure~\ref{fig:overview\_polynomial} shows an overview over the tests considering non-stationarity and independence of the observations.

Since the fitted polynomial models doesn't help us to give a stationary series we will consequently try to differentiate our data in the next subsection.

**\subsection{Trend Elimination by differencing}**

Since it is not possible to obtain a stationary time series by fitting polynomial trends we have to use different methods. Hence, we try to eliminate the trend by differencing the data at different lags (1, 2, 3 and 4) in order to generate a noise sequence and therefore get a stationary series \cite[p.~35]{bd02}).

The following figure~\ref{fig:diff1\_timeseries}, figure~\ref{fig:diff2\_timeseries}, figure~\ref{fig:diff3\_timeseries} and figure~\ref{fig:diff4\_timeseries} show the differenced series derived from the quarterly rents for each lag = 1, 2, 3 and 4.

\begin{figure}[!htb]

\centering

**\includegraphics**[angle=0,

width=0.5\textwidth]{diff1\_timeseries}

\caption{once-differenced rent index

**\label{fig:diff1\_timeseries}**}

\end{figure}

\begin{figure}[!htb]

\centering

**\includegraphics**[angle=0,

width=0.5\textwidth]{diff2\_timeseries}

\caption{twice-differenced rent index

**\label{fig:diff2\_timeseries}**}

\end{figure}

\begin{figure}[!htb]

\centering

**\includegraphics**[angle=0,

width=0.5\textwidth]{diff3\_timeseries}

\caption{triple-differenced rent index

**\label{fig:diff3\_timeseries}**}

\end{figure}

\begin{figure}[!htb]

\centering

**\includegraphics**[angle=0,

width=0.5\textwidth]{diff4\_timeseries}

\caption{quadruple-differenced rent index

**\label{fig:diff4\_timeseries}**}

\end{figure}

For the first two models we were succesfully able to generate stationary residuals which don't depend on time. However figure~\ref{fig:diff1\_timeseries} shows mostly patterns of a white noise which is not what we intended since in case of a white-noise-process prediction quality is poor (Expectation of $x\_n+1$ would be 0 in case of a mean-centered series). This white-noise patterns is as well indicated by the sample autocorrelation function of the once-differenced series where all lags up to 40 fall well within the bounds of $+/-1.96sqrt{98}$ (brooks davis p.39) (see figure~\ref{fig:diff1\_acf}) . To conclude, the once-differenced series is to versatile and contains too much randomness to get good predictions.

\begin{figure}[!htb]

\centering

**\includegraphics**[angle=0,

width=0.5\textwidth]{diff1\_acf}

\caption{auto-correlation function of the once-differenced series

**\label{fig:diff1\_acf}**}

\end{figure}

The Ljung-Box-Test \citep{LjungBox78} indicates a p-value of 0.35, meaning that we cannot reject the iid-Hypothesis $H\_0$ that there is independence between the observations. Therefore The Ljung-Box-Test emphasizes furthermore the independence of the values and the white-noise-patterns of the once-differenced series. Since white-noise would mean E0 and Variance sigma no more modelling would be necessary which is not what we want as we evoked in the introduction to this section

**\subsubsection{Diagnostics of the twice-differenced series}**

\begin{figure}[!htb]

\centering

**\includegraphics**[angle=0,

width=0.5\textwidth]{diff2\_acf}

\caption{auto-correlation function of the twice-differenced series

**\label{fig:diff2\_acf}**}

\end{figure}

\begin{figure}[!htb]

\centering

**\includegraphics**[angle=0,

width=0.5\textwidth]{diff2\_pacf}

\caption{Partial auto-correlation function of the twice-differenced series

**\label{fig:diff2\_pacf}**}

\end{figure}

In order to get better predictions and forecasting power than with pure white-noise-residuals seen in the once-differenced series, we now go for a twice-differenced model. Indeed our twice-differenced model doesn't show anymore iid-noisy patterns, indicating by the sample ACF (see figure~\ref{fig:diff2\_acf} which shows at least a correlation at lag1 and some other lags are touching the $+/-1.96sqrt{n=100}$ - 95percent-confidence-bounds. The ACF of the twice-differenced model shows only a few significant lags which die out quickly. This is a good sign, meaning that on one hand we have correlations which are needed to do prediction on the other hand the covariance is not too large to be time-dependant and therefore we can conclude our series is stationary.

Cross-validation with a test data set considering only the last 35 of the 100 observations (see figure~\ref{fig:diff2\_testset} above) confirms furthermore the non-stationary character of the twice-differenced-model

Box.test(d2.indiceloyers.test, lag=2, type="Ljung-Box") p-value of 0.033 suggests the data are (time-)independent

adf.test(d2.indiceloyers.test, alternative = "stationary", k=2) H0 of non-stationarity cannot be rejected which is not a problem, that could be due to lack of observations

kpss.test(d2.indiceloyers.test) with a p-value>0.05 the test data seems to be stationary aswell

the data of the first segment's (1993 to 2009) twice-differenced series show independent observations as well (see figure~\ref{fig:diff2\_trainingset}), however we can see a slightly slower increase in the segment from 2009 to 2018 indicating that in the years from 1993 to 2009 the growth in rental prices was higher than in the years in the second's segment which begins from 2009. that can be explained by the big baisse in the early 90ies, starting from a lower initial point and better conjunctural perspectives the increase was stronger, whilst from 2009 on the growth in rental prices slowed down, which can be very well explained by the US subprime crises beginning in the year 2008 followed by a long-taking global recession.

\begin{figure}[!htb]

\centering

**\includegraphics**[angle=0,

width=0.5\textwidth]{diff2\_trainingset}

\caption{twice-differenced series from 1993 to 2009

**\label{fig:diff2\_trainingset}**}

\end{figure}

\begin{figure}[!htb]

\centering

**\includegraphics**[angle=0,

width=0.5\textwidth]{diff2\_testset}

\caption{twice-differenced series from 2009 to 2018

**\label{fig:diff2\_testset}**}

\end{figure}

**\subsubsection{Diagnostics of the triple-/quadruple-differenced series}**

Differencing by lag 3 and 4 is not appropriate neither since there is too much autocorrelation showed by the ACF a lot of bars outside the $+/-1.96sqrt{98}$ - bounds.

To conclude, our transformation should be the twice-differenced series, since the once-differenced series is too white-noisy and the triple/quadruple-differenced series show too large covariances (time-dependence/nonstationarity). Finally to continue fitting and testing a model we center our twice-differenced series by subtracting the mean in order to...

**\section{Fitting and Testing the Model}** **\label{Fitting and Testing the Model}**

**\subsection{preliminary estimation and order selection}**

Having again a look on the ACF-plot (figure~\ref{fig:diff2\_acf) and on the PACF-plot (figure~\ref{fig:diff2\_pacf) of our centred twice-differenced model seen above we can see in the ACF-plot a highly significant bar at lag1 and and the bars at lag 2 and 3 are slightly significant aswell, lying sligthly outside the $+/-1.96sqrt{98}$ - bounds. The decreasing exponentially patterns suggests an auto-regressive progress.

The PACF shown in Figure 8.12 with 3 significant spikes is suggestive of an AR(3) model. So an initial candidate model is an ARIMA(3,0,0). There are no other obvious candidate models.

We fit an ARIMA(3,0,0) model along with variations including ARIMA(4,0,0), ARIMA(2,0,0), ARIMA(3,0,1), etc. Of these, the ARIMA(4,0,0) has a slightly smaller AICc value.

we should go with the ARIMA(4,0,0) since there is not much further gain in using an ARIMA(5,0,0), the first coefficients remain more or less the same and the 5th one is close to zero and therefore presumably not significant. on the other hand the coefficients in comparison to the ARIMA(3,0,0) model are quite different, indicating that there is a gain by adding an additional AR-term.

An ARIMA(4,0,0)model is furthermore suggested by the BURG-method, whose order of the fitted model is chosen by minimizing the AIC \cite[p.~145]{bd02}

(ar.burg allows two methods to estimate the innovations variance and hence AIC. Method 1 is to use the update given by the Levinson-Durbin recursion (Brockwell and Davis, 1991, (8.2.6) on page 242), and follows S-PLUS. Method 2 is the mean of the sum of squares of the forward and backward prediction errors (as in Brockwell and Davis, 1996, page 145). Percival and Walden (1998) discuss both. In the multivariate case the estimated coefficients will depend (slightly) on the variance estimation method.)

#The ACF plot of the residuals from the ARIMA(4,0,0) model shows all correlations within the threshold limits

#indicating that the residuals are behaving like white noise. A ljung-box test returns a large p-value, also suggesting

#the residuals are white noise.

rough estimate by yule-walker or burg

**\subsection{Modelling AIC and mse}**

owever, the analysis summary report shows that the model nonetheless performs quite well in the validation period, both AR coefficients are significantly different from zero, and the standard deviation of the residuals has been reduced from 1.54371 to 1.4215 (nearly 10percent) by the addition of the AR terms. Furthermore, there is no sign of a "unit root" because the sum of the AR coefficients (0.252254+0.195572) is not close to 1. (Unit roots are discussed on more detail below.) On the whole, this appears to be a good model.

In addition we will compare standard deviations of the models in case of several good candidates, we take the one with the smallest standard deviation.

From the visualization of the sample ACF and sample PACF of the twice-differenced series (see figure~\ref{fig:diff2\_acf} wfigure~\ref{fig:diff2\_pacf},respectively) we can guess an ARMA-process, since both plots ACF and PACF have the tendency to decrease exponentially. From the plots an ARMA(3,1) seems to be a suitable model for our data, if we take only the significant bars into account which lye clearly outside the bounds. However, from the ACF-plot we can see that the bars at lag 2, 3 or 4 are touching the bounds, hence by using a less conservative approach one could pledge as well for an ARMA(3,2), ARMA(3,3) or even ARMA(3,4)model.

If we want to be quite conservative and keep it simple we could go for an

ARMA(1,1) model, since only the first bar in the PACF-plot is of big importance and in the ACF-plot only the first bar lies clearly outside the borders (the following three only touching the bounds).

To conclude, from the visual analysis there is not a single exclusive candidate-model. Therefore we want to do further analyzes by taking into account the AICC and the mean squared errors before we choose a model.

cosmetic

Our sample PACF ahat (h) is significantly different than zero for all h smaller or equal than 2 and becomes negligible for h larger than 2. indicating that a suitable model for the could be p=2 so an AR(2) rpocess. . This is further emphasized by the plot where the sample PACF values beyond lag 2 are approximately i.i.d. with a N(0,1/n) distribution. Therefore in our supposed AR(2) process rouglhly 95 percent of the sample PACF values beyond lag 2 should be within the bound +/- 1.96/sqrt(n) which is in fact true according to the plot (no single bar after p=2 is outside the bound +/-1.96/sqrt(n). sample PACF satisfyise |alpha(h)|>1.96/sqrt(n) for h<=p and |alpha(h)| < 1.06/sqrt(n) for h>p, which suggest an AR(p=2) model for our data.

in other words: The fact that all of the PACF values beyond lag 3 fall within the bounds suggests the possible suitability of an AR(p=3) model for the mean corrected data set Xt=St-46.93. One simple way to estimate the parameters phi1 phi2 phi33 and sigma2 of such a model is to require tha the ACVF of the model at lags 0,1 and 2 should match the sample ACVF at those lags. Substituting the sample ACVF vlaues gammahat0 gammahat1 gammahat2 for gammma0 gamma1 and gamma 2 in the first thee equations of (3.2.5) and (3.2.6) and solving for phi1 phi2 phi3 and sigma2 gives the fitted model Xt-1.318xt-1+255Xtte-2+sxt-3=Zt wit hZt WN (this method of model fitting is called yule-walker estimation ) (brock davis p. 99)

Furthermore the choice of an AR(p) model is underlined by the fact that the ACF plot shows an exponentially decreasing patterns of |rho(h)| and PACF patterns with p=3 "spikes". Exponentially decrease in AR and alpha(h)=0 for h>p=3 in PACF are typical properties for an AR(p)-process.

In addition to the visualised evidence provided above further analytical evidence for an AR(1)-process is provided by the preliminary prediction by using the Yule-Walker-Algorithm for estimating roughly the AR-coefficients (phi). As we can see in table xy only the first estimated coefficient is a strong coefficient, all the others are close to zero and therefore not significant, emphasizing our assumption to go on with an AR(1)-Model.

The program used from the ITSMR package will search thorugh all the Yule-Walker AR(p) models, p=0,1,....,27, selecting the one with smallest AICC value. (explain AICC, corrected AIC) The minimum-AICC Yule-Walker AR model turns out to be the one defined by (eq 5.1.14) with p=1 and AICC value 74.541

In addition the estimated coefficient by Yule-Walker-Algorithm guarantees us that our model is causal. We can see that aswell in table x which shows indicating that the unit root is (slightly) outside the unit circle, with an value of 1.023714.

causality of a process (brock davis p. 85)

HannanRissanen-Algoritm shows strongly varying coefficients theta (q) for different fixed numbers of coefficients

which further underlines our assumption that an AR(p)-model suits our data well instead of an MA(q) or ARMA(p,q) process.

we can see bringing a q into the game doesn't bring much, this is aswell confirmed by the AICC which is 41 and heigher than 34

BrockwellDavis p. 193 a root near 1 of the autoregressive polynomial suggests that the data should be differenced before fitting an AMA model, whereas a root near 1 of the moving-average polynomial indicates that the data were overdifferenced.

**\subsection{Model Simulating}**

**If we simulate several models with our estimated coefficients we can see that the model ACF and PACF of the ARMA(1,1) suits our sample ACF and PACF best. And it suits them even better when we reduce the evidently overestimated MA to 0.5.**

**\section{Prediction of future values}**

**\section{Discussion}**

Discuss your results, the strengths and the limitations of your own analysis.

**\section{Conclusion}**

Conclude the report. Sketch further analyses that you could carry out if you would have more time.

**\section{quotation examples}**

Hello World. Mister Resnick wrote a beautiful book about Harry, see \citep{Resnick92}. If you want something more fancy, try \citet{Baddeley07}.

\\By looking at the plot we cannot definitely exclude that there might exist some seasonal patterns. To detect if a seasonality effect is really present in the series, we test it by modelling the seasonality and add it in the linear model.

A good numerical optimization algorithm is the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm. This algorithm requird an initial value which we obtained by the Hannam-Rinnanen-Alg. The BFGS-algorithm is a local optimization algorithm, meaning that it does not fid the global in minimum in general but instead a local minimum where the gradient is null. that's why we need a rough estimate by hannan-rinnanen to give te BFGS-algorithm a good inital vaue to get more secure that it will find the the global minimum and not only a loclal minimum instead.

\**bibliography{bibliography}**

\bibliographystyle{apa}

\end{document}