Detecting Additional Polarization Modes with eLISA

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Summary

Within the frame of Einstein's General Relativity, gravitational waves are expected to possess two tensorial polarizations, namely the well-known h_+ and $h_\times$ modes. Other metric theories of gravity however allow the existence of additional modes (two vector and/or two scalar modes), and the (non-)observation of those additional polarizations could put constraints on the validity of all existing theories, which would consequently provide a further test for General Relativity.

In its 2-arm-planned-configuration, eLISA only consists of one detector orbiting around the Sun, and we therefore investigate if there is a possibility to still detect and separate additional modes of a given gravitational wave signal.

Polarization modes

Perturbed metric corresponding to a propagating gravitational wave:

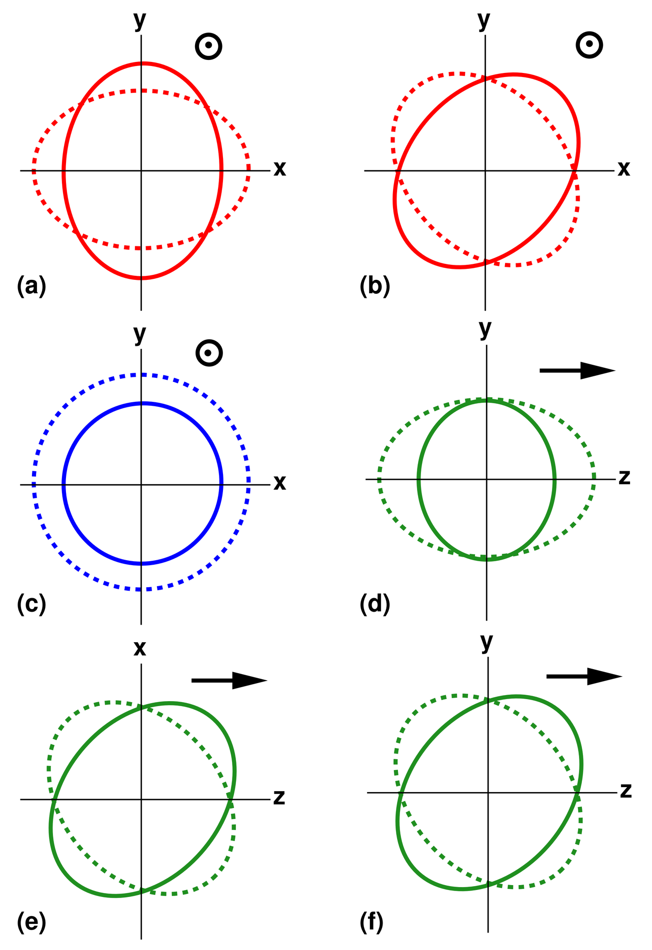
$$h_{ij}(\omega t - \mathbf{k}\cdot\mathbf{x}) = \sum_A h_A(\omega t - \mathbf{k}\cdot\mathbf{x}) e^A_{ij}$$

with $A = \times, +, b, l, x, y$ the six possible polarization modes and the following tensors (tensor, scalar and vector modes)

$$e^+_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad e^\times_{ij} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$e^b_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad e^l_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

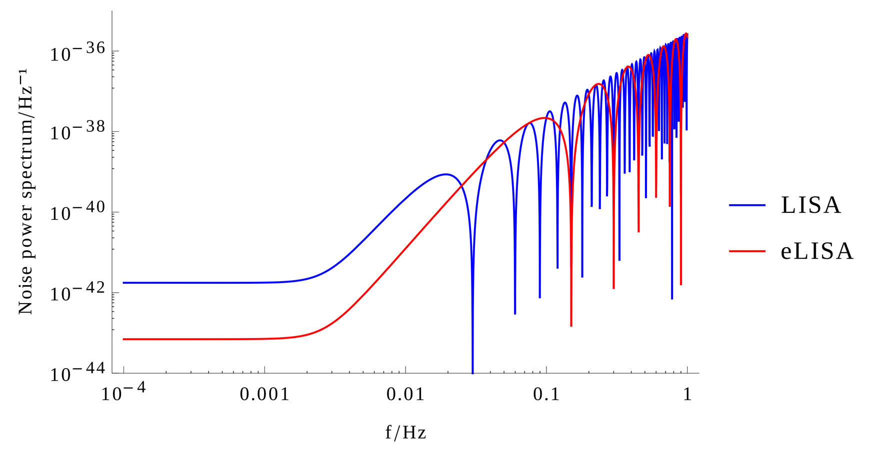
$$e^x_{ij} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad e^y_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

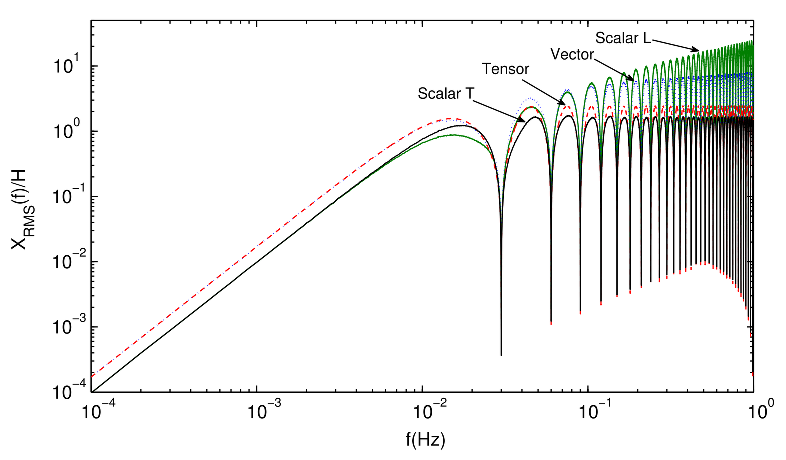


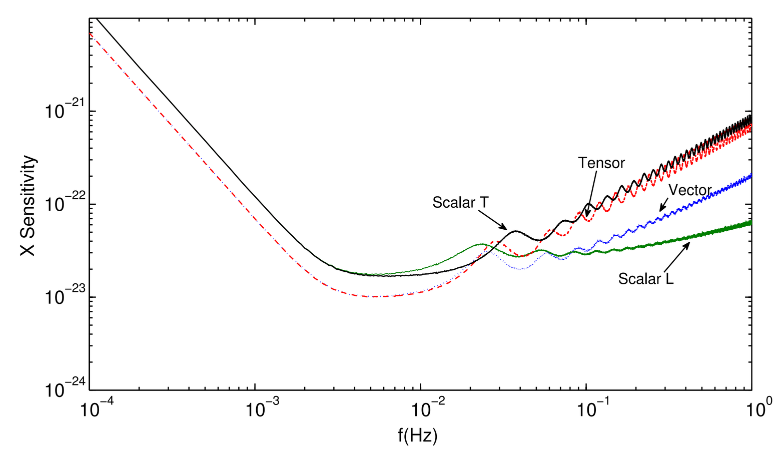
Sensitivity to additional modes

- Time-delay interferometric combinations, 2-arm configuration (4 beams)
- Noise power spectrum : $S_X(f)$
- Root-mean squared GW response : $X_{\mathrm{RMS}}(f)$
- Sensitivity : $\mathrm{SNR}\cdot\sqrt{S_X(f)B}/X_{\mathrm{RMS}}(f)$

  B : 1 cycle/year, SNR=5







- Similar behaviour between LISA and eLISA in the high-frequency regime (where they are more sensitive to (longitudinal) scalar modes)
- $X_{\mathrm{RMS}}(f)$ and the sensitivity are given for LISA (similar results expected with shorter arms; to be confirmed)

Stochastic GW background : network of detectors [1]

- GW stochastic background in the low frequency limit

$$h(t,\mathbf{x}) = \sum_A \int_{S^2} \mathrm{d}\hat{\mathbf{\Omega}} \int_{-\infty}^{\infty} \mathrm{d}f\ \tilde{h}_A(f,\hat{\mathbf{\Omega}}) e^{2\pi i f(t-\hat{\mathbf{\Omega}}\mathbf{x}/c)} F_A(\hat{\mathbf{\Omega}})$$

  with $A = +, \times, x, y, b, l$ all possible polarizations, F_A the antenna pattern function.

- Overlap reduction function (how much degree of correlation is preserved between detectors)

$$\gamma^M_{IJ}(f) = \frac{1}{\sin^2\chi}\left(\rho^M_1(\alpha) D^{ij}_I D^J_{ij} + \rho^M_2(\alpha) D^i_{I,k} D^{kj}_J \hat{d}_i \hat{d}_j + \rho^M_3(\alpha) D^{ij}_I D^{kl}_J \hat{d}_i \hat{d}_j \hat{d}_k \hat{d}_l\right)$$

  with $\rho^M_i = f(j_0(\alpha), j_2(\alpha), j_4(\alpha))$, $\hat{d}_i = \frac{\mathbf{x}}{|\mathbf{x}|}$, $\alpha = \frac{2\pi f|\mathbf{x}|}{c}$, $M = T, V, S$

- GW background energy density $\Omega^T_{\mathrm{gw}} = \Omega^+_{\mathrm{gw}} + \Omega^\times_{\mathrm{gw}}$ (similar for Ω^V_{gw} and Ω^S_{gw}), related to the one-sided power spectral density $S(|f|) \propto \langle \tilde{h}^*_A(f,\hat{\mathbf{\Omega}})\tilde{h}_{A'}(f',\hat{\mathbf{\Omega}}')\rangle$ by $\Omega^M_{\mathrm{gw}}(f) \propto f^3 S^A_h(f)$
- The tensor, vector and scalar modes can then be separately detected via

$$SNR^M \propto \int_0^\infty \mathrm{d}f \left[\frac{(\Omega^M_{\mathrm{gw}}(f))^2 \det \mathbf{F}(f)}{f^6 \mathcal{F}_M(f)}\right]^{(1/2)}$$

  where the elements of the (3×3)-matrix $\mathbf{F}$ are given by $F_{MM'} = \sum_{\text{det pairs}} \mathrm{d}t \frac{\gamma^M_{IJ}(t,f)\gamma^{M'}_i(t,f)}{P_I(f)P_J(f)}$

$\Rightarrow$ Valid for a network of independant detectors in space (3 LISA-like detectors would be sufficient), in the low frequency limit, and for a full polarized GWB

$\Rightarrow$ "Static" system (the relative position of the detectors in the network doesn't change)

Stochastic GW background : single detector [5]

- If the output data of the single detector is written as $h(t) + n(t)$, with $n(t)$ the noise, the autocorrelation of the signal reads

$$\langle \tilde{h}(f)\tilde{h}^*(f')\rangle = \frac{1}{2}\delta(f-f')S_h(|f|)$$

  with $S_h(|f|)$ the one-sided power spectral density; one can define S_n in a similar way for the noise.

- The maximum SNR given by this process is

$$SNR^2 = \frac{T}{2}\int_{-\infty}^{\infty} \mathrm{d}f \frac{S_h(|f|)^2}{[S_h(|f|) + S_n(|f|)]^2}$$

$\Rightarrow$ Also valid in the high-frequency limit, for a single detector, but for an unpolarized GWB

Possible solutions for eLISA

- Cross-correlating the TDI combinations of a LISA-like single detector also correlates the noise
- The separation of all modes however requires a network of detectors
- Work in progress
  - Letting eLISA evolve on its orbit and correlating the signals at different time
  - Correlating eLISA signal with future earth-based detectors around 1Hz

References

[1] Nishizawa A., Taruya A., Kawamura S., Phys. Rev. D **81**, 104043 (2010)

[2] Nishizawa A. et al., Phys. Rev. D **79**, 082002 (2009)

[3] Seto N., Phys. Rev. L **97**, 151101 (2006)

[4] Seto N., Taruya A., Phys. Rev. D **77**, 103001 (2008)

[5] Tinto M., Alves M.E., Phys. Rev. D **82**, 122003 (2010)

[6] Estabrook F.B., Tinto M., Armstrong J.W., Phys. Rev. D **62**, 042002 (2000)

[7] Tinto M., Armstrong J.W., arXiv:1205.4620v1 [gr-qc]

[8] Will C., Living Rev Relativity **9**, (2006), 3.