In [3]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import warnings
```

In [4]:
```
url = "https://raw.githubusercontent.com/selva86/datasets/master/USArrests.csv"
df = pd.read_csv(url)
```

In [7]:
```
df.head()
```

Out[7]:

	Murder	Assault	UrbanPop	Rape	State
0	13.2	236	58	21.2	Alabama
1	10.0	263	48	44.5	Alaska
2	8.1	294	80	31.0	Arizona
3	8.8	190	50	19.5	Arkansas
4	9.0	276	91	40.6	California

In [6]:
```
df.shape
```

Out[6]: (50, 5)

In [9]:
```
# Keep only the numeric columns used in classical demos
X = df[['Murder','Assault','UrbanPop','Rape']].copy()
states = df['Unnamed: 0'].rename('State') if 'Unnamed: 0' in df.columns else df.index
```

In [11]:
```
X.describe().T
```

Out[11]:

	count	mean	std	min	25%	50%	75%	max
Murder	50.0	7.788	4.355510	0.8	4.075	7.25	11.250	17.4
Assault	50.0	170.760	83.337661	45.0	109.000	159.00	249.000	337.0
UrbanPop	50.0	65.540	14.474763	32.0	54.500	66.00	77.750	91.0
Rape	50.0	21.232	9.366385	7.3	15.075	20.10	26.175	46.0

In [12]:
```
# Matrix of nonnegative values
A = X.to_numpy(dtype=float)

# Convert to relative frequencies (P) so CA works on profiles
grand_total = A.sum()
P = A / grand_total

# Row and column masses (marginals)
r = P.sum(axis=1, keepdims=True)        # (n_rows, 1)
c = P.sum(axis=0, keepdims=True)        # (1, n_cols)
```

In [13]:
```
# Expected under independence
rcT = r @ c   # outer product

# Diagonal mass matrices (as sqrt-inverses)
Dr_inv_sqrt = np.diag((r.flatten())**-0.5)
Dc_inv_sqrt = np.diag((c.flatten())**-0.5)

# Standardized residuals
S = Dr_inv_sqrt @ (P - rcT) @ Dc_inv_sqrt
```

In [14]:
```
U, s, Vt = np.linalg.svd(S, full_matrices=False)
eigvals = s**2                        # eigenvalues (principal inertias)
explained = eigvals / eigvals.sum()   # proportion of inertia

pd.DataFrame({
    'eigenvalue': eigvals,
    'explained_inertia_%': 100*explained
}).head(4)
```

Out[14]:

	eigenvalue	explained_inertia_%
0	4.501357e-02	8.353547e+01
1	6.065461e-03	1.125619e+01
2	2.806548e-03	5.208347e+00
3	1.293934e-32	2.401262e-29

In [15]:
```
# Row principal coordinates F and column principal coordinates G
F = Dr_inv_sqrt @ U @ np.diag(s)   # rows (states)
G = Dc_inv_sqrt @ Vt.T @ np.diag(s) # columns (crime types)

# Keep first two dimensions for plotting
F2 = F[:, :2]
G2 = G[:, :2]

row_coords = pd.DataFrame(F2, columns=['Dim1','Dim2'])
row_coords.insert(0, 'State', list(states))
col_coords = pd.DataFrame(G2, columns=['Dim1','Dim2'])
col_coords.insert(0, 'Variable', X.columns)

row_coords.head(), col_coords
```

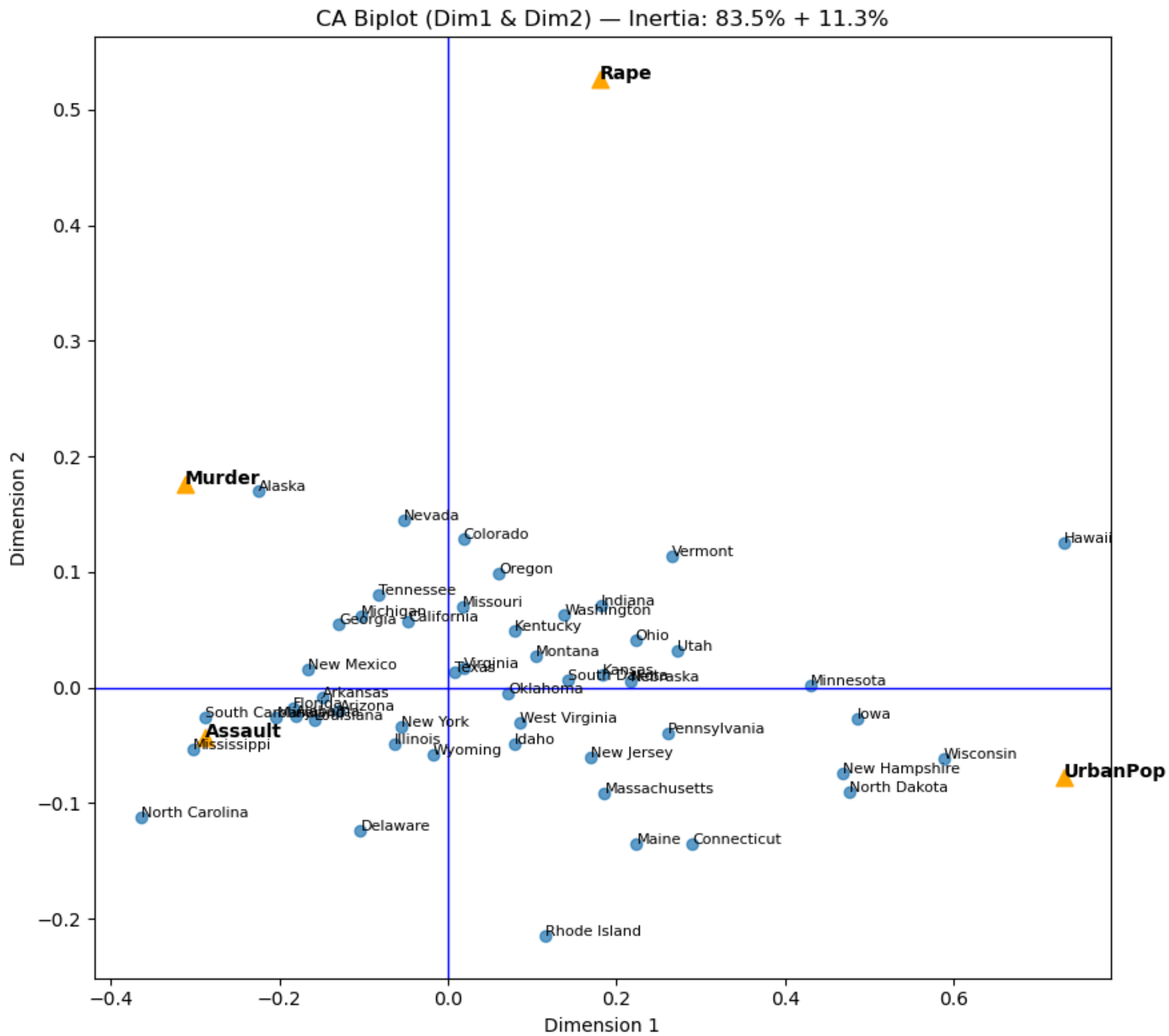
Out[15]:
```
(   State      Dim1      Dim2
 0      0 -0.181415 -0.024962
 1      1 -0.224989  0.170053
 2      2 -0.129295 -0.019609
 3      3 -0.149569 -0.008356
 4      4 -0.047561  0.057339,
   Variable      Dim1      Dim2
 0   Murder -0.151595  0.085575
 1  Assault -0.140128 -0.021127
 2 UrbanPop  0.354758 -0.037788
 3     Rape  0.087510  0.255170)
```

In [16]:
```python
# Row contributions to axis k: ctr_{ik} = r_i * F_{ik}^2 / λ_k
ctr_rows = (r * (F**2)) / eigvals
ctr_rows = pd.DataFrame(ctr_rows[:, :2], columns=['CTR_Dim1','CTR_Dim2'])
ctr_rows.insert(0, 'State', list(states))

# Column contributions to axis k: ctr_{jk} = c_j * G_{jk}^2 / λ_k
ctr_cols = (c.T * (G**2)) / eigvals
ctr_cols = pd.DataFrame(ctr_cols[:, :2], columns=['CTR_Dim1','CTR_Dim2'])
ctr_cols.insert(0, 'Variable', X.columns)

# COS² for rows/cols: share of a point's inertia carried by each axis
row_dist2 = (F**2).sum(axis=1, keepdims=True)
col_dist2 = (G**2).sum(axis=1, keepdims=True)
cos2_rows = (F2**2) / row_dist2
cos2_cols = (G2**2) / col_dist2

cos2_rows = pd.DataFrame(cos2_rows, columns=['COS2_Dim1','COS2_Dim2'])
cos2_rows.insert(0, 'State', list(states))
cos2_cols = pd.DataFrame(cos2_cols, columns=['COS2_Dim1','COS2_Dim2'])
cos2_cols.insert(0, 'Variable', X.columns)

ctr_rows.sort_values('CTR_Dim1', ascending=False).head(10), \
ctr_cols, \
cos2_rows.sort_values('COS2_Dim1', ascending=False).head(10), \
cos2_cols
```

Out[16]:
```
(    State  CTR_Dim1  CTR_Dim2
 10     10  0.138273  0.029829
 32     32  0.091632  0.064574
 48     48  0.076853  0.006248
 23     23  0.051762  0.011832
 39     39  0.050754  0.003038
 14     14  0.049927  0.001140
 22     22  0.048342  0.000004
 28     28  0.045851  0.008443
 33     33  0.036955  0.009820
 1       1  0.030983  0.131357,
    Variable  CTR_Dim1  CTR_Dim2
 0    Murder  0.014986  0.035440
 1   Assault  0.280751  0.047361
 2  UrbanPop  0.690649  0.058154
 3      Rape  0.013614  0.859046,
     State  COS2_Dim1  COS2_Dim2
 22     22   0.998177   0.000012
 26     26   0.997005   0.000664
 14     14   0.996802   0.003068
 40     40   0.996705   0.002039
 3       3   0.996338   0.003110
 35     35   0.990549   0.004465
 8       8   0.990266   0.009261
 48     48   0.986939   0.010811
 15     15   0.983697   0.003851
 39     39   0.981176   0.007915,
    Variable  COS2_Dim1  COS2_Dim2
 0    Murder   0.194279   0.061909
 1   Assault   0.971805   0.022090
 2  UrbanPop   0.988413   0.011215
 3      Rape   0.102889   0.874815)
```

In [25]:
```python
plt.figure(figsize=(9,8))

# Scale columns so both clouds fit nicely
scale = (np.abs(F2).max() / np.abs(G2).max()) if np.abs(G2).max() > 0 else 1.0
G2_plot = G2 * scale

# Ensure 'states' contains the actual names from the dataset
states = df['State']  # Column with state names

# Plot states (rows) with names instead of numbers
plt.scatter(F2[:,0], F2[:,1], alpha=0.7)
for i, name in enumerate(states):
    plt.text(F2[i,0], F2[i,1], name, fontsize=8)

# Plot variables (columns)
plt.scatter(G2_plot[:,0], G2_plot[:,1], marker='^', s=80, color="orange")
for j, var in enumerate(X.columns):
    plt.text(G2_plot[j,0], G2_plot[j,1], var, fontsize=10, fontweight='bold')

# Reference lines
plt.axhline(0, linewidth=1, color='blue')
plt.axvline(0, linewidth=1, color='blue')
```

```
# Titles and labels
plt.title(f"CA Biplot (Dim1 & Dim2) — Inertia: {explained[0]*100:.1f}% + {explained[1
plt.xlabel("Dimension 1")
plt.ylabel("Dimension 2")
plt.tight_layout()
plt.show()
```



Interpretaion

The correspondence analysis (CA) biplot illustrates the relationship between U.S. states and four crime statistics—Murder, Assault, Rape, and UrbanPop (percentage of urban population). The horizontal axis (Dimension 1) and vertical axis (Dimension 2) are the first two principal dimensions from the CA, together explaining 94.8% of the total variation (83.5% from Dimension 1 and 11.3% from Dimension 2). This high proportion means that the plot effectively captures most of the key information in the data.

In the biplot, blue points represent U.S. states, while orange triangles represent the crime variables. The proximity between points reflects similarity or association: states closer to a particular crime variable tend to have relatively higher values for that variable, whereas those farther away are less strongly related.

Several patterns emerge from the plot. UrbanPop, located in the lower right, is associated with states such as Hawaii, Wisconsin, Iowa, New Hampshire, and North Dakota, indicating higher urban population percentages in these states. Rape, positioned toward the upper right, is not extremely close to any one state but lies in the general direction of Vermont, Indiana, Ohio, and Utah, suggesting slightly elevated rates relative to other crimes. Murder, in the upper left quadrant, is most closely linked with Alaska, indicating a relatively higher murder rate there. Assault, found in the lower left quadrant, is near Mississippi and South Carolina, pointing to higher assault rates in these states.

States located near the center of the plot, such as Texas, Missouri, and Georgia, show no strong association with any single crime statistic, reflecting average or mixed values across the four variables. Overall, the quadrants of the biplot can be interpreted as follows: the top right corresponds to states leaning toward higher rape rates, the bottom right to those with higher urban population percentages, the top left to higher murder rates, and the bottom left to higher assault rates. This quadrant-based interpretation, combined with the proximity patterns, provides a clear summary of how different states relate to various crime characteristics.