

$$16) a) -1 + e^{-x} \cos(x+2) \neq 0$$

$$y'(x) = (-1 + e^{-x} \cos(x+2))' = -e^{-x} \cos(x+2) - e^{-x} \sin(x+2)$$

$$b) y' = 4x \ln x, \quad y(0) = 1$$

$$y(x) = 4 \int x \ln x dx = 2 \int \ln x d(x^2) = 2 \left(x^2 \ln x - \int \frac{x^2}{x} dx \right) = 2x^2 \ln x - x^2 + C$$

$$y(0) = C = 1 \Rightarrow \boxed{y(x) = 2x^2 \ln x - x^2 + 1}$$

$$11) y' \sin x + y \cos x + y^2 \sin^2 x = 0 \quad | \cdot \frac{1}{\sin x}, \quad x \neq 0$$

$$y' + \operatorname{ctg}(x) y = -\sin x y^2 \quad / \text{ y-p.e берем}$$

$$y=0 - \text{запасное решение}$$

$$\text{так } y \neq 0: \quad z = \frac{1}{y} \Rightarrow z' = -\frac{1}{y^2} y'$$

$$z' - \operatorname{ctg}(x) \cdot z = \sin x$$

$$1. \quad z' - \operatorname{ctg}(x) \cdot z = 0$$

$$\int \frac{dz}{z} = \int \operatorname{ctg}(x) dx \Rightarrow \ln|z| = \ln|\sin x| + \ln C \Rightarrow z = C \cdot \sin x$$

$$2. \quad C(x) = \int \frac{\sin x}{\sin x} dx = x + C \Rightarrow z = (x+C) \sin x \Rightarrow \boxed{\frac{1}{y} = (x+C) \sin x}$$

$$17) y'' + y' - 6y = 36x, \quad y(0) = 12, \quad y'(0) = -5$$

$$\lambda^2 + \lambda - 6 = 0 \Rightarrow D = 1 + 24 = 25 \Rightarrow \lambda_1 = \frac{-1 - \sqrt{25}}{2} = -3$$

$$y_0(x) = C_1 e^{-3x} + C_2 e^{2x}$$

$$\lambda_2 = \frac{-1 + 5}{2} = 2$$

$$y^*(x) = Ax + B \Rightarrow y^{*'} = A; \quad y^{*''} = 0$$

$$A - 6Ax - 6B = 36x \Rightarrow \begin{matrix} x^1 & | & -6A = 36 \\ x^0 & | & A - 6B = 0 \end{matrix} \Rightarrow \begin{matrix} A = -6 \\ B = -1 \end{matrix}$$

$$y(x) = C_1 e^{-3x} + C_2 e^{2x} - 6x - 1$$

$$y(0) = C_1 + C_2 = 13$$

$$5C_2 = 40 \Rightarrow C_2 = 8$$

$$y'(0) = -3C_1 + 2C_2 = -1$$

$$C_1 = 13 - 8 \Rightarrow C_1 = 5$$

$$\boxed{y(x) = 5e^{-3x} + 8e^{2x} - 6x - 1}$$

$$79) \begin{cases} \dot{x} = -7x - 4y \\ \dot{y} = -2x - 9y \end{cases}$$

$$\begin{cases} x(0) = -1 \\ y(0) = 2 \end{cases}$$

~~$$\ddot{x} = -7\dot{x} - 4\dot{y}$$~~

$$x = \frac{1}{2}(-\dot{y} - 9y)$$

$$\Rightarrow \frac{1}{2}(-\ddot{y} - 9\ddot{y}) = -\frac{7}{2}(-\dot{y} - 9y) - 4y$$

$$-\ddot{y} - 9\ddot{y} = 7\dot{y} + 63y - 8y$$

$$\ddot{y} + 16\dot{y} + 55y = 0$$

$$\lambda^2 + 16\lambda + 55 = 0 \Rightarrow \lambda_1 = -5; \lambda_2 = -11$$

$$\text{T.e. } y(t) = C_1 e^{-5t} + C_2 e^{-11t}$$

$$x = \frac{1}{2}(5C_1 e^{-5t} + 11C_2 e^{-11t} - 9C_1 e^{-5t} - 9C_2 e^{-11t}) = -2C_1 e^{-5t} + C_2 e^{-11t}$$

$$x(t) = -2C_1 e^{-5t} + C_2 e^{-11t}, \quad x(0) = -2C_1 + C_2 = -1 \quad C_1 = 1$$

$$y(t) = C_1 e^{-5t} + C_2 e^{-11t}, \quad y(0) = C_1 + C_2 = 2 \Rightarrow C_2 = 1$$

$$\Rightarrow \begin{cases} x(t) = -2e^{-5t} + e^{-11t} \\ y(t) = e^{-5t} + e^{-11t} \end{cases}$$