IT-24631
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Problem-1

Statement: If the set of odd numbers with binary operation <+> an abelian group or not explain the reason with necessary notations.

Answer: No, the set of odd integers under addition, denoted as $(O, +)$, is not a group (and hence not abelian). This is because it is not closed under the operation '+' and it does not contain an additive identity element.

- Set and operation: Let $O = \{2k+1 \mid k \in \mathbb{Z}\}$ be the set of all odd integers, and consider the binary operation '+' (addition) restricted to O.

- A group $\langle G, * \rangle$ must satisfy the closure, associativity, identity and inverse axioms. It is an abelian Group if operation is also commutative.

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Failure of closure: Take any two arbitrary element $a, b \in O$. Let $a = 2m+1$ and $b = 2n+1$ for some integers m and n.

Their sum is:

$a + b = (2m+1) + (2n+1) = 2m+2n+2$
$= 2(m+n+1)$. This is an even number. And therefore this is not in the set O. Since the set is not closed under addition, it fails a required Group axiom. The failure alone is sufficient to show that $\langle O, + \rangle$ is not a group.

Other axioms: Addition on integers is associative and commutative. However these property does not salvage the Group structure since the axiom for closure and identity already fail.

Conclusion: $\langle O, + \rangle$ cannot be a group. Hence it cannot be an abelian group.