Quantitative Risk Management
Assignment 9

Question 1: Let $(X_i)_{i\in\mathbb{N}}$ be i.i.d. with distribution function F, and let

$$M_n = \max\{X_1, \dots, X_n\}.$$

Recall that $F \in \mathrm{MDA}(H_\xi)$ means that there exist sequences $c_n \in \mathbb{R}$ and $d_n > 0$ such that

$$\mathbb{P}\left(\frac{M_n - c_n}{d_n} \le x\right) \longrightarrow H_\xi(x), \qquad n \to \infty,$$

where H_ξ is the Generalized Extreme Value (GEV) distribution with shape parameter ξ.

1. Exponential distribution: Let

$$F(x) = 1 - e^{-\beta x}, \qquad x \ge 0,\ \beta > 0.$$

   Show that $F \in \mathrm{MDA}(H_0)$ using the normalizing constants

$$c_n = \frac{\log n}{\beta}, \qquad d_n = \frac{1}{\beta}.$$

2. Pareto distribution: Let

$$F(x) = 1 - \left(\frac{\kappa}{\kappa + x}\right)^\alpha, \qquad x \ge 0,\ \alpha, \kappa > 0.$$

   Show that $F \in \mathrm{MDA}(H_{1/\alpha})$ using the normalizing constants

$$c_n = \kappa n^{1/\alpha} - \kappa, \qquad d_n = \frac{\kappa n^{1/\alpha}}{\alpha}.$$

Question 2: Suppose that the excess distribution of X over the threshold u is

$$F_u(x) = G_{\xi,\beta}(x), \qquad x \ge 0.$$

Show that the excess distribution over a higher threshold $v \ge u$ is

$$F_v(x) = G_{\xi,\, \beta + (v-u)\xi}.$$

Question 3: Download 5 years of Microsoft (MSFT) daily stock prices beginning on November 26, 2011.

1. Compute the negative log returns and reproduce the sample mean excess plot shown in the lecture.
2. Select a threshold $u = 0.01$ for the negative log returns and fit a Generalized Pareto Distribution (GPD) to the excesses above u using maximum likelihood estimation. The Matlab function `gppdf` may be useful.

Question 4: Consider a random variable with distribution function

$$F(x) = 1 - \left(\frac{\kappa}{\kappa + x^\theta}\right)^\lambda, \qquad x \ge 0,\ \lambda, \kappa, \theta > 0.$$

Show that F belongs to the maximum domain of attraction of H_ξ, and determine ξ in terms of the parameters of the distribution.