
FIN 417: QUANTITATIVE RISK MANAGEMENT

Rafiki's Notes

Rafael Barroso
Ingenierie Mathématique
École Polytechnique Fédérale de Lausanne
September 21, 2025

Contents

1	Basic notions	3
2	Standard risk measures and methods	10

1 Basic notions

We first start by exploring some basic notions of risk, and mainly define all of the financial jargon that is used throughout the course.

Financial risk and regulation

There are a lot of definitions of risk, but as for many things in life, no sentence can capture the full essence of a concept (especially in finance since all of the theory is constructed by phony, try-hardening beaurocrats i.e. beaurocs). As such, we will just list some of the most common definitions of risk:

- The Concise Oxford English Dictionary: ‘hazard, a chance of bad consequences, loss or exposure to mischance’.
- The course textbook (M.F.E. McNeil, R. Frey, P. Embrechts, Quantitative Risk Management: Concepts, Techniques and Tools, Princeton University Press, 2015): ‘any event or action that may adversely affect an organization’s ability to achieve its objectives and execute its strategies’.
- ISO 31073: ‘effect of uncertainty on objectives; can be positive, negative or both, can result in opportunities and threats’. This same guide also defines:
 - **Uncertainty**: state, even partial, of deficiency of information related to understanding or knowledge (‘randomness’).
 - **Objective**: result to be achieved.
 - **Risk management**: coordinated activities to direct and control an organization with regard to risk.

As you can see, pure jargon and buzzwords are used to define risk, which is not very helpful. In this course we define risk as the following notion.

Definition 1.1. Risk is the chance of financial loss due to uncertainty or ‘randomness’ (but what is randomness anyway? don’t even ask cuz’ I get erected).

So, given this definition, we may model and measure risk using probabilistic (i.g. random variables, distributions) and statistical (i.g. data) tools, which is what we will do in this course.

Definition 1.2. A *financial derivative* is a contract whose payoff is a function f of some underlying random variable U (e.g., stock price at time T). The whole industry is about analyzing, pricing, and hedging such contracts under uncertainty.

Mainly, we may thank daddy Kolmogorov for developing a reliable mathematical framework (using probabilistic axioms) to think about risk in a rigorous manner. Since this course is heavily focused on finance, we will mainly be concerned with financial risk, which, yoy guessed it, has a shit ton of variable definitions (each focusing on different aspects of finance).

Remark 1.3. Note that financial risk is generated by a future change in value of an asset.

Here are some of the main types of financial risk:

- **Market risk:** change in value due to changes in underlying components (such as stock, bond, or commodity prices).
- **Credit risk:** possibility of not obtaining future payments due to fuck ups of the counterparty.
- **Operational risk:** risk of loss due to inadequate or failed internal process, people (beaurocs in action!) and systems or from external events (fraud, earthquakes).
- **Liquidity risk:** not being able to buy or sell something quickly enough to minimize loss.
- **Model risk:** using an inadequate model for measuring risk.

Note that financial firms are not passive (i.e. defensive) towards risk, they confront it in order to gain return (risk tha biscuit!). Thus, banks and insurers are like risk function engineers. They take raw random variables (risky outcomes), apply transformations $f(X)$, and then allocate these transformed versions to those most willing to hold them. That process is what keeps financial markets running smoothly.

Remark 1.4. If a bank's portfolio value over a certain holding period is represented by the random variable X , then assessing the risk of this portfolio depends on finding the distribution function $F_X(x) = P(X \leq x)$. If $X = \sum_{k=1}^n w_k X_k$, where each X_i represents a separate investment, then we need a joint model for (X_1, \dots, X_n) .

So, we might ask the question **whada fuck is involved whilst managing risks?** Well, here are some of those things to think about:

- Determine an emergency ammount of money to absorb losses.
 1. for regulatory purposes (satisfy regulators) e.g. Bassel 3.
 2. economic capital purposes
- Making sure portfolios are well diversified.
- Optimizing portfolios.

History of Risk Management & What “Risk Measurement” Really Means

Ancient roots Buying an *option* (a contract that gives you the right, not the obligation, to buy/sell an asset at a preset price) is a way to *cap* your downside: you pay a fee (the premium) so that bad price moves hurt less. With that lens, risk management shows up

as early as ~1800 BC: fun fact, first known person to do this was Thales de Miletus while buying an option to rent all of the olive presses during a period in which he suspected olives would overproduce; and they did! Thales made a good fortune (cite veritasium's vlack-scholes video for this).

Modern leap: Markowitz and mean–variance (~1952). Before the 1950s, many investors chased *expected return* only. Harry Markowitz formalized the tradeoff between *mean* (expected return) and *variance* (volatility, a measure of dispersion). A portfolio is *mean–variance efficient* if, among all portfolios with the same expected return, it has the smallest variance (or, equivalently, for a fixed variance it has the largest expected return). The set of such portfolios traces the *efficient frontier*. This is the birth of quantitative portfolio construction (and yes, *gracias, daddy Kolmogorov*, for the probability axioms that make these expectations/variances a real thing).

Approaches to Measuring Portfolio Risk (and what each really computes)

Let V be portfolio value that depends on underlying *risk factors* (e.g., equity prices, rates, FX). Risk measurement is about quantifying how much V might drop over a horizon. Four standard approaches:

1. Notional-amount approach (book value sum).

Define risk as the (weighted) sum of *notional* amounts (face values) of positions. *Pros*: dead simple. *Cons*: ignores price dynamics, hedges, convexity, and nonlinearity (e.g., options). Treats \$100 in equity the same as \$100 in a deep out-of-the-money option—not great.

2. Factor-sensitivity measures (“how twitchy am I?”).

Compute the change in V for a small, prescribed shock to a chosen factor. In practice this means *Greeks* for options (Delta, Vega, Rho) or fixed-income *duration/convexity* for interest-rate risk. Formally, if $V = V(F)$ and F moves by ΔF , a first-order estimate is $\Delta V \approx \frac{\partial V}{\partial F} \Delta F$. Good for quick “what-if”s; limited for large, nonlinear moves.

3. Scenario-based risk (stress and narrative shocks).

Specify plausible or extreme *scenarios* (joint moves in several factors), reprice the portfolio under each, and record losses. The risk statistic might be the *maximum* loss across scenarios or the loss under a specific stress (e.g., “rates +200 bps, equities 20%, credit spreads +150 bps”). Useful when history is thin or tail events matter.

4. Loss-distribution-based measures (statistical view).

Model the *distribution* of the portfolio loss L over a horizon, then extract summary numbers. Two classics:

- **Value-at-Risk (VaR) at level α** : the smallest ℓ such that $\mathbb{P}(L \leq \ell) \geq \alpha$. “With 100 α % confidence, losses won’t exceed VaR.”
- **Expected Shortfall (ES) at α** : the conditional mean loss given that you breach VaR. Captures the *average* severity of tail losses beyond VaR.

These require either (i) a parametric model for returns, (ii) historical resampling, or (iii) full revaluation Monte Carlo.

What Risk Measures are *Used* For (and the regulatory lens)

(1) Determining risk capital (“the cushion”). A *risk measure* tells you how much capital you need to set aside as a buffer against *unexpected* losses to satisfy a regulator. This is the *regulatory capital* idea: hold enough equity so a bad-but-plausible shock doesn’t sink the firm.

(2) Management tool (limits and incentives). Internally, desks get *limits* based on risk measures (e.g., notional limits, VaR limits). Breaches trigger escalations, hedges, or deleveraging. This aligns day-to-day risk-taking with the firm’s appetite.

(3) Pricing insurance risk (reserve vs premium risk). For (re)insurers, total *insurance risk* splits into:

- **Reserve risk:** uncertainty in how *past* underwritten policies will develop (IBNR, claim severity inflation, legal lags).
- **Premium risk:** uncertainty in *future* business to be written (volume, pricing adequacy, catastrophe exposure).

Regulatory view (acceptability via capital add-ons). Abstractly, let L be future loss. A regulator defines an *acceptance set* \mathcal{A} (positions considered safe). A *risk measure* $\rho(L)$ is the *minimal cash* you must add so that the position becomes acceptable: $L - \rho(L) \in \mathcal{A}$. In plainer words: $\rho(L)$ is the capital add-on that makes the position pass the test.

Historical Shocks That Drove Regulation (and the risk lessons)

- **1970s oil crises.** Energy prices tanked due to geopolitical supply shocks. Lesson: commodity price risk can be massive and persistent; need stress testing and liquidity buffers.
- **End of Bretton Woods (early 1970s).** Transition from fixed to floating FX regimes made currency risk *volatile*. Lesson: FX settlement, hedging, and margining infrastructure matter.
- **Herstatt (1973).** A German bank failed mid-day; counterparties delivered one currency but did not receive the other due to time-zone mismatches. This *settlement risk* is still nicknamed *Herstatt risk*. Lesson: payment-versus-payment (PvP) mechanisms and intraday credit controls are crucial.
- **1987 U.S. stock crash (“Black Monday”).** A 20% one-day plunge; portfolio *insurance* strategies (dynamic hedging informed by options logic) amplified selling pressure. Lesson: procyclical hedging and liquidity spirals must be modeled, not assumed away.

- **Barings (1995).** Unauthorized derivatives bets by Nick Leeson blew up the bank (\$1.4bn loss). Lesson: model risk is not the only risk—*operational risk* (controls, limits, segregation of duties) is vital.
- **LTCM (1998).** Highly leveraged convergence trades unraveled; systemically risky unwind required a \$3.5bn private bailout. Lesson: correlation can jump, leverage bites, and “rare” events cluster.

Connecting back to the toolkit

Each approach above exists because different portfolios, data regimes, and decision horizons call for different diagnostics. Notional is rough but fast; factor sensitivities give local insight; scenarios explore narratives and tails; full loss-distribution methods provide *quantiles* and *tail means* that map cleanly into capital and limits. In practice, firms use them *together*: quick sensitivities for intraday steering, scenarios for governance, and distribution-based numbers for capital.

Role of regulators

In order to protect us mortals that are legally responsible for a debt or an obligation i.e. liability holders, financial institutions have been *nerfed* such that they have to meet certain **regulatory frameworks**. The **main goal of regulation** is to ensure that financial institutions hold sufficient **capital** to remain solvent i.e. to have enough assets that exceed their current liabilities. Some of the regulatory frameworks introduced in the lectures include:

- Basel (now Basel III/IV): Banking system.
- Solvency II: Insurance companies within EU.
- Swiss Solvency Test: Insurance companies in CH.

Basel Committee on Banking Supervision formed in 1974, under the Bank for International Settlements (BIS), due to the Herstatt incident (a big German bank that basically placed a big ass bet on the direction of the us dollar which had high volatility at the time and lost).

Remark 1.5. The Basel Committee does not have legal force but it formulates standards/best practices/guidelines, the Basel Accords, in expectation that government authorities will take steps to implement them.

Basel I (1988).

- **Core idea:** banks must hold capital proportional to their *risk-weighted assets* (RWA).
- **Limitations:** too crude; little sensitivity to actual credit quality, off-balance-sheet derivatives, or market risk.

Basel II (2004).

- **Three pillars:**
 1. **Minimum capital requirements:** refine RWA by introducing credit, market, and operational risk. Banks can use standardized formulas or (if approved) internal models (e.g., VaR for market risk).
 2. **Supervisory review:** regulators review banks' internal risk management, ensuring adequacy beyond formulas.
 3. **Market discipline:** disclosure requirements so investors/markets can exert pressure.
- **Motivation:** make capital more risk-sensitive, reward better risk management.
- **Criticism:** complexity, reliance on banks' own models (which may underestimate tail risk).

Basel III (2010, post-2008 crisis).

- **Trigger:** the global financial crisis exposed undercapitalization, hidden leverage, and liquidity mismatches.
- **Key enhancements:**
 - **Higher and better-quality capital.** Stricter definition of Tier 1 (common equity) and higher minimum ratios.
 - **Capital buffers.** Extra cushions:
 - * *Capital conservation buffer* (to absorb losses in downturns).
 - * *Countercyclical buffer* (to lean against credit booms).
 - **Leverage ratio.** A simple non-risk-weighted cap: equity/total assets must exceed a threshold (prevents gaming of risk weights).
 - **Liquidity standards.**
 - * *Liquidity Coverage Ratio (LCR)*: enough high-quality liquid assets to survive a 30-day stress.
 - * *Net Stable Funding Ratio (NSFR)*: stable funding over a one-year horizon.

(Ongoing) Basel IV. Not an official new accord but shorthand for final revisions to Basel III (e.g., standardized approaches, output floors for internal models). Goal: reduce variability of risk-weighted assets and improve comparability across banks.

Takeaway. The Basel Accords progressively tightened the link between measured risk and required capital/liquidity. Basel I was crude but pioneering; Basel II added sophistication (and complexity); Basel III reacted to crisis with higher, better capital and liquidity standards. The unifying theme: *banks must carry buffers big enough to absorb shocks and keep the system safe.*

Some challenges of Quantitative Risk Management (QRM)

Extreme values are a pain in the ass; they usually fuck up models whenever the normal distribution is assumed (and finance bros love to overuse normal distributions). **Concentration of risks** is also a big challenge since modeling the dependence of several extreme outcomes is hard, even harder to model when will many risk factors will make *furios* movements at the same time.

Another thig to consider is that of **scale** of a portfolio; they may sometimes be extremely large and a full calibration is essentially impossible; requires dimension reduction. Last but not least (amongst other), it is an **interdisciplinary field** and so, as technical people we sometimes have to deal with not so technical people (i.e. fucking beaurocs!).

What Quantitative Risk Management (QRM) Can Do & What It Cannot

QRM can:

- **Identify and quantify risks.** QRM provides a toolkit of quantitative models and metrics (e.g., Value-at-Risk, Expected Shortfall, stress tests) to measure credit, market, liquidity, and operational risks in a consistent way.
- **Monitor risks and provide early warning.** By continuously tracking key indicators (volatility spikes, widening credit spreads, liquidity dry-ups), QRM can flag emerging stress and give institutions time to react before losses escalate.
- **Support portfolio optimization.** Techniques such as diversification and hedging are guided by quantitative models that aim to reduce concentrated exposures and balance risk–return trade-offs (think back to Markowitz and the efficient frontier).

QRM cannot:

- **Predict Black Swans.** Models are built on historical data and probabilistic assumptions. They fail when the world produces *unprecedented* events (e.g., 9/11, Lehman’s collapse). Rare events outside model assumptions remain invisible until they strike.
- **Capture reputational risk.** The fallout from scandals, frauds, or loss of public trust (think Wells Fargo account scandal, Volkswagen emissions) cannot be meaningfully quantified by the usual statistical risk measures.
- **Substitute for governance.** Even perfect models do not fix weak governance structures, cultural blind spots, or reckless decision-making. A bank with poor oversight will blow up regardless of its VaR model.

Conclusion. QRM is indispensable as a *technical* discipline for measuring, monitoring, and steering risk. But its outputs must be embedded in a broader system of sound governance and a culture of risk awareness. Numbers alone are not enough: discipline and judgment are equally crucial.

2 Standard risk measures and methods

After all of the jargon, we get into the good stuff. This section focuses on developing the framework we'll use later on in order to do some risk management.

Value and Loss

We denote the value of a portfolio as V (mathematically, a stochastic process) where at each time t the value of the portfolio is defined as $V_t = V(t)$. Then, each random variable $V(t)$ is observable at time t .

Definition 2.1. For a given risk horizon Δ , we define the loss of a portfolio V over the period $[t, t + \Delta]$ as

$$\mathcal{L}(t, t + \Delta) = -(V_{t+\Delta} - V_t)$$

this is observable at time $t + \Delta$ but is 'random' at t . This distribution is called the *loss distribution*.

A large positive value of L indicates that the portfolio suffered a large decrease in value over $[t, t + \Delta]$. We also define the **negative** of the loss function, called the Profit-and-loss distribution i.e. $PnL(t, t + \Delta) = -\mathcal{L}(t, t + \Delta)$.

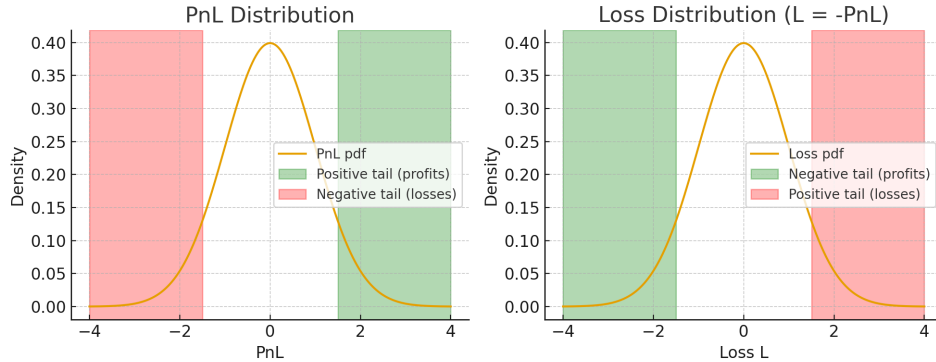


Figure 1: PnL v. Loss distribution

Remark 2.2. Much of risk management is focused on the negative tail of the PnL distribution, which is equivalent to the positive tail of the loss distribution.

Sometimes, we may not have a full understanding (or any at all) of \mathcal{L} . It is also important to know that we may sometimes model V as a function of several risk factors. This suggests the following notation:

- $V_t = f(t, Z_t)$: value of portfolio as function of underlying risk factors.
- $Z_t = (Z_{1,t}, \dots, Z_{d,t})$: underlying risk factors; often representing stock value, interest rate, etc.
- $f : \mathbb{R} \times \mathbb{R}^d \rightarrow \mathbb{R}$: measurable function.

- for V_t to be observable at time t , we must have $Z_{k,t}$ observable at $t \forall k$

The choice for each Z_k and f is a modeling issue.

Definition 2.3. We define the vector of risk factor changes as $X_{t+\Delta} = Z_{t+\Delta} - Z_t$ and so, we may redefine the loss distribution as

$$\begin{aligned}\mathcal{L}(t, t + \Delta) &= -(V_{t+\Delta} - V_t) = -(f(t + \Delta, Z_{t+\Delta}) - f(t, Z_t)) \\ \Rightarrow \mathcal{L} &= -(f(t + \Delta, X_{t+\Delta} + Z_t) - f(t, Z_t))\end{aligned}$$

The loss distribution is fully determined by the distribution of $X_{t+\Delta}$ and the function f . If f is differentiable, we sometimes consider a first-order Taylor approximation of the loss as

$$\mathcal{L}^\delta(t, t + \Delta) = - \left(\partial_t f(t, Z_t) \Delta + \sum_{i=1}^d \partial_{Z_i} f(t, Z_i) X_{i,t+\Delta} \right)$$

Remark 2.4. Note that this approximation is good whenever Δ is small enough. This may be observed in figure 2.

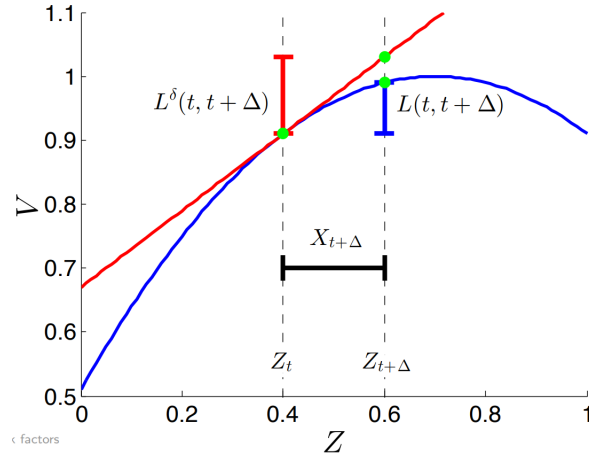


Figure 2: First order Taylor approximation of \mathcal{L}