FIN-417 - Quantitative Risk Management
Assignment 1

Question 0: It would be useful if you register for a WRDS account (if you have not already done so): https://wrds-www.wharton.upenn.edu/pages/

Question 1: This question refers to Example 1 from the lecture slides.

1. Take $d = 1$ and $\lambda_1 = 1$. Suppose $X_{1,t+\Delta}$ has mean zero and a standard deviation equal to 0.01, and $S_t = 100$. For each of the following distributions of $X_{1,t+\Delta}$, simulate 10,000 realizations of $L(t, t+\Delta)$ and plot the empirical distribution. Then compute the mean and standard deviation of $L(t, t+\Delta)$. Find the normal probability density function corresponding to this mean and standard deviation and plot it over the empirical distribution.

   (a) $X_{1,t+\Delta}$ is a scaled Student's t-distribution with 3 degrees of freedom.

   (b) $X_{1,t+\Delta}$ is a scaled Student's t-distribution with 10 degrees of freedom.

   (c) $X_{1,t+\Delta}$ is a scaled Student's t-distribution with 50 degrees of freedom.

   (d) $X_{1,t+\Delta}$ has a normal distribution.

   By "$X_{1,t+\Delta}$ is a scaled Student's t-distribution with ν degrees of freedom" we mean that $\alpha X_{1,t+\Delta}$ has the Student's t-distribution with ν degrees of freedom for some appropriate $\alpha \in \mathbb{R}$, such that $X_{1,t+\Delta}$ has the correct standard deviation. The Matlab functions `randn` and `trnd` will be useful for this question.

   Which of the resulting distributions of $L(t, t+\Delta)$ above are normal distributions? How do you know?

2. For each of the distributions from the previous part, state the exact probability distribution of $L^{\delta}(t, t+\Delta)$.

Question 2: This question refers to Example 2 from the lecture slides.

1. Take $S_t = 100$, $r_t = 0.05$, and $\sigma_t = 0.2$. Suppose that $X_{1,t+\Delta}$ has a normal distribution with mean zero and standard deviation 0.01, $X_{2,t+\Delta}$ has a normal distribution with mean zero and standard deviation 10^{-4}, and $X_{3,t+\Delta}$ has a normal distribution with mean zero and standard deviation 10^{-3}. Further, $X_{2,t+\Delta}$ is independent from the other two risk factor changes, but $X_{1,t+\Delta}$ and $X_{3,t+\Delta}$ have a correlation of -0.5. The Matlab function `mvnrnd` may be useful for this question.

   Let $T = 1$, $K = 100$, and $\Delta = 1/252$.

   Simulate 10,000 realizations of $L(t, t+\Delta)$ and plot the empirical distribution. Note that there is positive probability for $\sigma_{t+\Delta}$ to be negative given the assigned distribution. State whether you think this is a problem and why, and describe how you could circumvent it if necessary.

2. State the closed-form expressions for the following Greeks in Example 2: theta, delta, rho, and vega. For the same distributions as in the previous part, simulate 10,000 realizations of $L^{\delta}(t, t+\Delta)$ and plot the corresponding empirical distribution. Which of the three risk factors seems to contribute the most to $L^{\delta}(t, t+\Delta)$?

Question 3: Let $X \sim \mathcal{N}(\mu, \sigma^2)$. Derive the formula:

$$\mathbb{E}[e^X] = e^{\mu + \frac{1}{2}\sigma^2}.$$

Question 4: You hold a portfolio of two stocks whose prices are $S_{t,1}$ in Euros and $S_{t,2}$ in Swiss Francs, respectively. The CHF/EUR exchange rate is denoted as e_t, so that 1 CHF = e_t EUR at time t. The portfolio consists of λ_1 shares of asset 1 and λ_2 shares of asset 2. The domestic currency of the portfolio is EUR.
You decide to model the risk of the portfolio in terms of the stock price change and in terms of the exchange rate, so that the risk factors are $Z_{t,j} = \log S_{t,j}$, $j \in \{1, 2\}$ and $Z_{t,3} = \log e_t$.

1. Derive an expression for V_t, the value of the portfolio at time t in EUR.
2. Derive an expression for the loss L_{t+1} after one period.
3. Derive an expression for the linearised loss L^{δ}_{t+1}.
4. Express the previous result in terms of the portfolio weights w_1 and w_2 of the two stocks.

Question 5: Read *An Academic Response to Basel II*, available on Moodle. What are the issues with using VaR as a measure of risk according to the authors? What were the potential problems in the implementation of the measures proposed by Basel II?