Quantitative Risk Management
Assignment 3

Question 1: This question deals with a delta-hedged call option. The following are the Black-Scholes parameters for a European call option at time $t = 0$:

$$T = 0.5$$
$$r_t = 0.05$$
$$\sigma_t = 0.2$$
$$S_t = 100$$
$$K = 100$$

The portfolio consists of a long position in the call option and the corresponding position in the stock that makes the portfolio delta neutral. Let $\Delta = 1$day, $Z_1 = \log(S)$, and $Z_2 = \sigma$ (r is considered constant in this problem). The risk factor changes are normally distributed with mean zero. Their standard deviations over one day are 10^{-3} and 10^{-4}, and their correlation is -0.5.

1. Compute VaR_α, VaR_α^{mean}, and ES_α for $\alpha = 0.95$ and $\alpha = 0.99$ using the following methods:

    (a) Monte Carlo full revaluation with 10,000 simulations

    (b) Monte Carlo on the linearlized loss with 10,000 simulations

    (c) Variance-covariance method

    Do not neglect the time derivative in any linearization in this question.

Question 2: Let L have the Student t distribution with ν degrees of freedom. Derive the formula:

$$ES_\alpha(L) = \left(\frac{g_\nu(t_\nu^{-1}(\alpha))}{1 - \alpha} \right) \left(\frac{\nu + (t_\nu^{-1}(\alpha))^2}{\nu - 1} \right)$$

You will need to look up the probability density function of the distribution at hand.

Question 3: In this question you will perform an exercise similar to the backtesting example from the lecture slides.

1. Download 5 years worth of historical stock prices (not returns) of Intel, Yahoo, and Microsoft starting from March 10, 2011

    - Wharton WRDS $\rightarrow$ CRSP $\rightarrow$ Stock/Security Files $\rightarrow$ Daily Stock File.

2. Estimate VaR_α for each day in the period of March 11, 2013 to March 10, 2016. Do this with the following method for both $\alpha = 0.95$ and $\alpha = 0.99$.

    - Assume risk factor changes (log returns) have a multivariate normal distribution.
    - On each day that you estimate VaR_α, calibrate the mean and covariance of the risk factor changes to the previous two years of returns.

- Use the variance-covariance method to compute VaR_α at each day of interest. The portfolio consists of 100 shares of Intel stock, n shares of Yahoo stock, and m shares of Microsoft stock, where n and m are chosen so that the total value of the holdings on March 11, 2013 are equal between all three equities.

Question 4:

1. Suppose L has geometric distribution with parameter p. Here, we consider a geometric distribution that includes 0 in the support.

   (a) If $p = 0.5$, what is $VaR_{0.95}$?

   (b) Plot VaR_α for values of α ranging from 0.9 to 0.99 $(p = 0.5)$.

2. Suppose X and Y are independent with Poisson distributions with parameters $\lambda_X = 1$ and $\lambda_Y = 2$. Let $L = X + Y$.

   (a) Plot $VaR_\alpha(X)$ for values of α ranging from 0.9 to 0.99.

   (b) Plot $VaR_\alpha(Y)$ for values of α ranging from 0.9 to 0.99.

   (c) Plot $VaR_\alpha(L)$ for values of α ranging from 0.9 to 0.99.