Quantitative Risk Management
Assignment 6

Question 1: Determine whether the time series defined below are covariance stationary.

$$X_t = \begin{cases} \epsilon_t & t \,\text{odd} \\ \epsilon_{t+1} & t \,\text{even} \end{cases}, \quad (\epsilon_t)_{t\in\mathbb{Z}} \sim WN(0,\sigma^2);$$

$$X_t = \sum_{j=1}^{t} \epsilon_j, \quad (\epsilon_t)_{t\in\mathbb{N}} \sim WN(0,\sigma^2).$$

Question 2: Consider the MA(2) process

$$X_t = \epsilon_t + \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2}, \quad t \in \mathbb{Z},$$

where $(\epsilon_t)_{t\in\mathbb{Z}} \sim WN(0,\sigma^2)$.
Determine whether it is causal and covariance stationary. Compute the autocorrelation function.

Question 3: Recall that a causal ARMA process can be represented as

$$X_t = \sum_{i=0}^{\infty} \psi_i \epsilon_{t-i},$$

where

$$\sum_{i=0}^{\infty} |\psi_i| < \infty$$

and $(\epsilon_t)_{t\in\mathbb{Z}}$ is $WN(0,\sigma_\epsilon^2)$. Derive the autocorrelation function $\rho(h)$ for the process $(X_t)_{t\in\mathbb{Z}}$:

$$\rho(h) = \frac{\sum_{i=0}^{\infty} \psi_i\psi_{i+|h|}}{\sum_{i=0}^{\infty} \psi_i^2}.$$

Question 4: Recall that an $ARMA(1,1)$ process satisfies:

$$X_t - \phi X_{t-1} = \epsilon_t + \theta\epsilon_{t-1}.$$

As stated at the lecture, this process is causal if $|\phi| < 1$ and $\phi \neq -\theta$, meaning that it can be written as:

$$X_t = \sum_{i=0}^{\infty} \psi_i \epsilon_{t-i},$$

where the coefficients satisfy

$$\sum_{i=0}^{\infty} \psi_i z^i = \frac{1+\theta z}{1-\phi z}.$$

Find ψ_i in terms of ϕ and θ. Also compute the autocorrelation function $\rho(h)$ for this process.