Quantitative Risk Management
Assignment 11

Question 1: Consider a bond investor who expects the credit quality of bond issuer A to decrease. Develop a strategy to profit from this view by using CDS contracts with reference entity A and discuss the risks of the strategy.

Question 2: Consider the problem of modeling the default probability in Merton's model. Assume that the growth rate of assets μ_V is positive and that the initial asset value V_0 exceeds the liability B. Show that the probability of default at the horizon date T is an increasing function of the volatility σ_V.

Question 3: Suppose that assumptions of the Merton model hold and the dynamics of company's asset value V_t is given by

$$dV_t = V_t \mu_V \, dt + V_t \sigma_V \, dW_t,$$

where W_t is a Brownian motion with respect to the physical probability measure $\mathbb{P}$. Assume $T = 1, B = 50$, and $V_0 = 100$. Suppose that the short rate r and the market price of risk $\theta = \frac{\mu_V - r}{\sigma_V}$ are fixed real constants.

a) Express physical and risk-neutral default probabilities $p(\sigma_V) = \mathbb{P}(V_T < B)$ and $q(\sigma_V) = \mathbb{Q}(V_T < B)$ as functions of σ_V.

b) For $r = 5\%$, $\theta = -1, 0, 1$, and σ_V ranging over $(0, 1)$, plot the curves $p(\sigma_V)$, $q(\sigma_V)$ and $p(\sigma_V) - q(\sigma_V)$. Comment on the result.

Question 4: Consider the Merton model with a constant risk-free rate $r = 5\%$ and initial value of assets $V_0 = 100$.

a) Compute debt D_t and equity S_t values at time $t = 0$ if $\sigma_V = 0.25$, and firm's debt consists of a single zero-coupon bond with a notional of $B = 50$ to be reimbursed at date $T = 5$. Verify that the firm value V_t at time $t = 0$ is 100.

b) Plot credit spread surfaces $c(0, T)$ as functions of (x-axis) time to maturity T and (y-axis) asset volatility σ_V for leverages $L = [0.3, 0.6, 0.9]$. Comment on the result.