Quantitative Risk Management
Assignment 2

Question 1: You hold a long position in asset A and a short position in asset B. The long exposure is equal to 2 shares and the short one is equal to 1 share. The asset returns' annual volatilities are estimated to $\sigma_A = 20\%$ and $\sigma_B = 20\%$, while their correlation is $\rho_{A,B} = 50\%$. The mean effect is ignored in the following.

1. Assuming normality of simple returns, and for a holding period of one day, compute the 99% Gaussian VaR of the portfolio.
2. The worst scenarios for the portfolio loss in the last 250 days were found to be 0.05870, 0.05685, 0.054270, 0.052170, 0.049231. Compute the 99%-VaR using the method of historical simulation.

Question 2: You hold a portfolio which is long 4 shares of stock A and long 3 shares of stock B. The annualized volatilities of the assets are 25% and 20%, respectively, and the correlation of their returns is -20%. Their current prices are 100 CHF and 200 CHF.
You observe the 8 worst returns (in %) in the last 250 days and you report them in the following table.

	1	2	3	4	5	6	7	8
r_A	-3	-4	-3	-5	-6	3	1	-1
r_B	-4	1	-2	-1	2	-7	-3	-2

Compute the 97.5%-VaR and the 97.5%-ES for the daily loss of your portfolio.

Question 3: This question deals with a portfolio of three stocks. At time t, the stock prices are $S_{1,t} = 100$, $S_{2,t} = 50$, and $S_{3,t} = 25$. The portfolio consists of 1 share of S_1, 3 shares of S_2, and 5 shares of S_3. The risk factors are logarithmic prices and the risk factor changes have mean zero and standard deviations 10^{-3}, $2 \cdot 10^{-3}$, and $3 \cdot 10^{-3}$, respectively. The risk factors are independent.

1. Compute VaR_α, VaR_α^{mean}, and ES_α using Monte Carlo with 10,000 simulations. Do this for $\alpha = \{0.90, 0.91, \ldots, 0.99\}$. Use the following distributions for the risk factor changes:
   (a) For each $i \in \{1, 2, 3\}$, $X_{i,t+\Delta} \sim t(3, \mu, \sigma)$ for appropriate values of μ and σ
   (b) For each $i \in \{1, 2, 3\}$, $X_{i,t+\Delta} \sim t(10, \mu, \sigma)$ for appropriate values of μ and σ
   (c) For each $i \in \{1, 2, 3\}$, $X_{i,t+\Delta} \sim t(50, \mu, \sigma)$ for appropriate values of μ and σ
   (d) For each $i \in \{1, 2, 3\}$, $X_{i,t+\Delta}$ has a normal distribution

   and plot the results.
2. Comment on the following:
   (a) The value of VaR_α compared to VaR_α^{mean}.
   (b) The value of VaR_α and ES_α as compared between the four distributions. Are the results what you expected?

Question 4: Give an example that shows that VaR_α is not subadditive. That is, find two random variables L_1 and L_2 with a joint distribution such that the following does not hold:

$$VaR_\alpha(L_1 + L_2) \leq VaR_\alpha(L_1) + VaR_\alpha(L_2)$$