Geometrical Transformations

Geometrical transformations are operations that change the spatial relationship of pixels in an image. This includes changing position, size, orientation, and even the perspective of objects within an image.

These transformations map coordinates from one image space to another using mathematical models, typically represented by matrices.

Why Are Geometrical Transformations Important?

Use Case	Purpose
Object Alignment	Aligning objects for recognition, e.g., aligning faces before classification.
Image Augmentation	Creating varied training samples (e.g., rotation, scaling, flip) to improve deep learning model generalization.
Viewpoint Correction	Fixing tilted or skewed images using perspective correction.
Image Stitching	Warping and aligning overlapping parts of multiple images to create panoramas.
Medical Imaging	Mapping different scans to a common orientation for analysis.

Every image can be thought of as a 2D function:

$I\ (x, y)\ \rightarrow$ pixel intensity at coordinates (x, y)

When you transform an image, you're changing the position of each pixel according to a rule:

$$(x', y') = f(x, y)$$

Affine Transformations

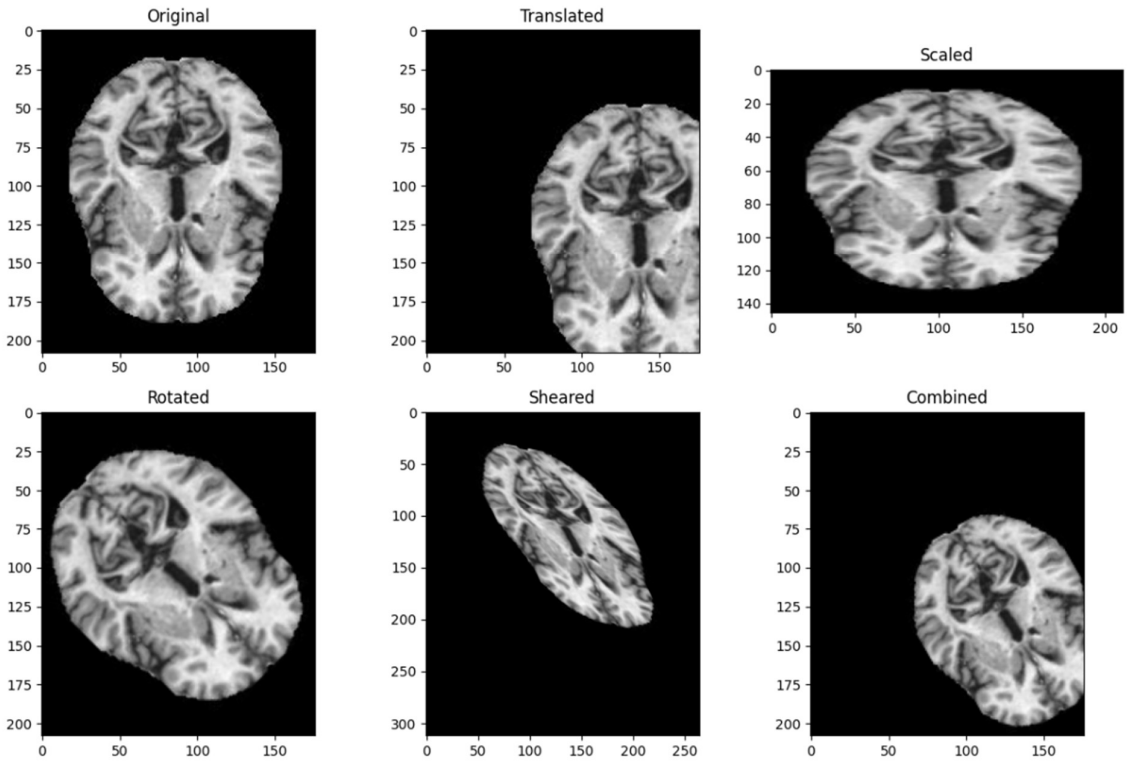
An Affine Transformation is a linear mapping method that preserves:

- Points
- Straight lines
- Parallelism

It does not preserve angles or lengths in general but maintains collinearity and ratios of distances.

It allows you to perform:

- Translation
- Scaling
- Rotation
- Shearing
- Any combination of the above



Perspective Transformation

A **Perspective Transformation** is a type of geometric transformation that simulates the change in viewpoint caused by moving the camera or object in 3D space. It maps points from one plane to another using a **homography matrix**.

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Unlike affine transformations, **lines remain straight**, but:

- **Parallel lines may converge**
- **Angles and relative proportions** can change

Think of photographing a rectangular piece of paper from an angle — it appears trapezoidal. Perspective transformation can correct or simulate this effect.

Applications

- Document Scanning
- Camera Calibration

- Planar Object Tracking
- Image Stitching

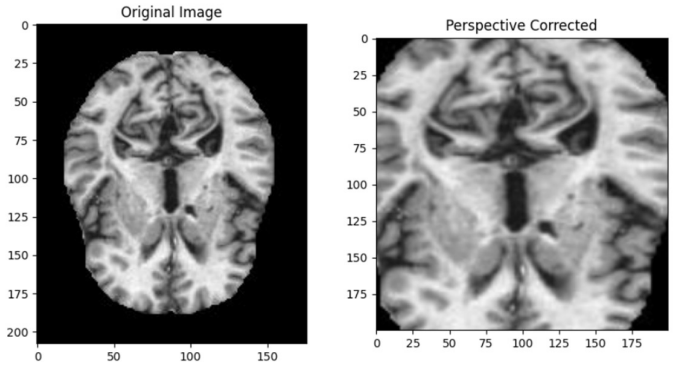


Image Warping

Image Warping is the process of mapping each pixel from the source image to a new location in the output image using a pixel-wise transformation.

Unlike affine or perspective transforms (which are global), warping functions can be nonlinear and highly flexible, enabling:

- Barrel/pincushion distortion correction
- Morphing between images
- Shape-based deformations
- Fish-eye or swirl effects