

1. If A is a square matrix such that $|A| = 5$, write the value of $|AA^T|$.
2. Find the distance between the planes $\vec{r} \cdot (2\hat{i} - 3\hat{j} + 6\hat{k}) - 4 = 0$ and $\vec{r} \cdot (6\hat{i} - 9\hat{j} + 18\hat{k}) + 30 = 0$.
3. If \vec{a} and \vec{b} are unit vectors, then what is the angle between \vec{a} and \vec{b} for $\vec{a} - \sqrt{2}\vec{b}$ to be a unit vector?
4. $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}$, Find $|AB|$.
5. if $A = \begin{bmatrix} 0 & 3 \\ 2 & -5 \end{bmatrix}$ and $KA = \begin{bmatrix} 0 & 4a \\ -8 & 5b \end{bmatrix}$, find the values of k and a
6. If vectors \vec{a} and \vec{b} are such that $|\vec{a}| = \frac{1}{2}$, $|\vec{b}| = \frac{4}{\sqrt{3}}$ and $|\vec{a} \times \vec{b}| = \frac{1}{\sqrt{3}}$, then find $|\vec{a} \cdot \vec{b}|$.
7. Find k , if $f(x) = \begin{cases} k \sin \frac{\pi}{2}(x+1), & x \leq 0 \\ \frac{\tan x - \sin x}{x^3}, & x > 0 \end{cases}$ is continuous at $x = 0$.
8. Differentiate $(\sin 2x)^x + \sin^{-1} \sqrt{3x}$ with respect to x
9. Differentiate $\tan^{-1} \left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right)$ with respect to $\cos^{-1} x^2$
10. Given that vectors \vec{a} , \vec{b} , and \vec{c} forms a triangle such that $\vec{a} = \vec{b} + \vec{c}$. find p, q, r, s such that area of the triangle is $5\sqrt{6}$, where $\vec{a} = p\hat{i} + q\hat{j} + r\hat{k}$, $\vec{b} = s\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - 2\hat{k}$.
11. There are two bags A and B . Bag A contains 3 white and 4 red balls whereas bag B contains 4 white and 3 red balls. three balls are drawn at random (*without replacement*) from one of the bags and are found to be two white and one red. find the probability that these were drawn from bag B .
12. Ishan wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth by 50 m, then its area will remain same, but

if length decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2 . using matrices, find the dimension of the plot. Also give the reason why he wants to donate plot for a school.

13. Solve the differential equation: $2ye^{\frac{x}{y}}dx + \left(y - 2xe^{\frac{x}{y}}\right)dy = 0$.

14. solve the differential equation : $(x + 1) \frac{dy}{dx} - y = e^{3x}(x + 1)^3$

15. Find :

$$\int [\log(\log x) + \frac{1}{(\log x)^2}]dx.$$

16. Prove that $2\sin^{-1}(\frac{3}{5}) - \tan^{-1}(\frac{17}{31}) = \frac{\pi}{4}$

17. Solve the equation for x: $\cos(\tan^{-1} x) = \sin(\cot^{-1} \frac{3}{4})$

18.

19. Find :

$$\int \frac{1 - \sin x}{\sin x(1 + \sin x)}, dx$$

20. Evaluate:

$$\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx$$

21. Evaluate :

$$\int_0^1 \cot^{-1}(1 - x + x^2)dx$$

22. Find equation of normal to the curve $ay^2 = x^3$ at the point whose x coordinate is am^2

23. Find the equation of the plane passing through the points $A(3, 2, 1)$, $B(4, 2, -2)$, and $C(6, 5, 1)$. Hence, find the value of λ for which the points $A(3, 2, 1)$, $B(4, 2, 2)$, $C(6, 5, 1)$, and $D(\lambda, 5, 5)$ are coplanar.

24. Find the coordinates of the point where the line $\vec{r} = (-\hat{i} - 2\hat{j} - 3\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 3\hat{k})$ meets the plane, which is perpendicular to the vector $\vec{n} = \hat{i} + \hat{j} + 3\hat{k}$ and at a distance of $\frac{4}{\sqrt{11}}$ from the origin.
25. Three numbers are selected at random (*without replacement*) from first six positive integers. If X denotes the smallest of three numbers obtained, find the probability distribution of X . Also find the mean and variance of the distribution
26. A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available costing ₹5 per unit and ₹6 per unit respectively. One unit of food F_1 contains 4 units of vitamins A and 3 units of minerals whereas one unit of food F_2 contains 3 units of vitamin A and 6 units of minerals. Formulate this as a linear programming problems. Find the minimum cost of diet that consists of mixture of these two foods and also meets minimum nutritional requirement
27. Using properties of determinants, prove that:
$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$
28. Using elementary row operations, find the inverse of the following matrix:
$$A = \begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}.$$
29. Find the equation of the plane containing two parallel lines $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{3}$ and $\frac{x}{4} = \frac{y-2}{-2} = \frac{z+1}{6}$. Also, find if the plane thus obtained contains the line $\frac{x-2}{3} = \frac{y-1}{1} = \frac{z-2}{5}$ or not.
30. Using integration find the area of the region $\{(x, y) : y^2 \leq 6ax \text{ and } x^2 + y^2 \leq 16a^2\}$
31. Let $f : N \rightarrow N$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f : N \rightarrow S$ is invertible (where S is range of f). Find the inverse of f and hence find $f^{-1}(31)$ and $f^{-1}(87)$.
32. determine the intervals in which the function $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$ is strictly increasing (or) strictly decreasing.

33. Find the maximum and minimum values of $f(x) = \sec x + \log \cos^2 x$, $0 < x < 2\pi$