1. If $A$ is a square matrix such that $\left|A\right| = 5$, write the value of $\left|AA^{\top}\right|$.

2. Find the distance between the planes $\left[r\right] \cdot \left(2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}\right) - 4 = 0$ and $\mathbf{r} \cdot \left(6\hat{\imath} - 9\hat{\jmath} + 18\hat{k}\right) + 30 = 0$.

3. If $\mathbf{a}$ and $\mathbf{b}$ are unit vectors, then what is the angle between $\mathbf{a}$ and $\mathbf{b}$ for $\mathbf{a} - \sqrt{2}\,\mathbf{b}$ to be a unit vector?

4. $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ and $B$= $\begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}$,Find $\left|AB\right|$.

5. if $A = \begin{bmatrix} 0 & 3 \\ 2 & -5 \end{bmatrix}$ and $KA = \begin{bmatrix} 0 & 4a \\ -8 & 5b \end{bmatrix}$,find the values of $k$ and $a$

6. If vectors $\mathbf{a}$ and $\mathbf{b}$ are such that—$\mathbf{a}| = \frac{1}{2}$, $|\mathbf{b}|= \frac{4}{\sqrt{3}}$ and $|\mathbf{a} \times \mathbf{b} = \frac{1}{\sqrt{3}}$,then find $|\mathbf{a} \cdot \mathbf{b}|$.

7. Find $k$, if$f(x) = \begin{cases} k \sin \frac{\pi}{2}(x+1), & x \leq 0 \\ \frac{\tan x - \sin x}{x^3}, & x > 0 \end{cases}$is continuous at $x = 0$.

8. Differentiate $(\sin 2x)^x + \sin^{-1} \sqrt{3x}$ with respect to $x$

9. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}\right)$with respect to $\cos^{-1} x^2$

10. Given that vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ forms a triangle such that$\mathbf{a}$=$\mathbf{b}$+$\mathbf{c}$. find $p$, $q$, $r$,$s$ such that area of the triangle is $5\sqrt{6}$, where $\mathbf{a} = \mathrm{p}\hat{\imath} + \mathrm{q}\hat{\jmath} + \mathrm{r}\hat{k}$,$\mathbf{b} = \mathrm{s}\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ and $\mathbf{c}$=$3\hat{\imath}$+ $\hat{\jmath}$ - $2\hat{k}$.

11. There are two bags $A$ and $B$.Bag $A$ contains 3 white and 4 red balls whereas bag $B$ contains 4 white and 3 red balls.three balls are drawn at random ($without replacement$)from one of the bags and are found to be two white and one red.find the probability that these were drawn from bag $B$.

12. Ishan wants to donate a rectangular plot of land for a school in his village.When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth by 50 m,then its area will remain same,but if length decreased by 10 m and breadth is decreased by 20 m,then its area will decrease by 5300 $m^2$.using matrices,find the dimension of the plot. Also give the reason why he wants to donate plot for a school.

13. Solve the differential equation: $2ye^{\frac{x}{y}}dx + \left(y - 2xe^{\frac{x}{y}}\right)dy = 0.$

14. solve the differential equation : $(x+1)\,\frac{dy}{dx} - y = e^{3x}(x+1)^3$

15. Find :

$$\int [\log\, [\log x] + \frac{1}{(\log x)^2}]dx.$$

16. Prove that $2sin^{-}1(\frac{3}{5}) - tan^{-}1(\frac{17}{31}) = \frac{\pi}{4}$

17. Solve the equation for x: $\cos\left(\tan^{-1} x\right) = \sin\left(\cot^{-1} \frac{3}{4}\right)$

18.

19. Find :

$$\int \frac{1 - \sin x}{\sin x(1 + \sin x)}, dx$$

20. Evaluate:

$$\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x}\, dx$$

21. Evaluate :

$$\int_0^1 \cot^{-1}(1 - x + x^2)dx$$

22. Find equation of normal to the curve $ay^2 = x^3$ at the point whose x coordinate is $am^2$

23. Find the equation of the plane passing through the points $A\,(3, 2, 1)$, $B\,(4, 2, -2)$, and $C\,(6, 5, 1)$. Hence, find the value of $\lambda$ for which the points $A\,(3, 2, 1)$, $B\,(4, 2, 2)$, $C\,(6, 5, 1)$, and $D\,(\lambda, 5, 5)$ are coplanar.

24. Find the coordinates of the point where the line $\mathbf{r} = (-\hat{\imath} - 2\hat{\jmath} - 3\hat{k}) + \lambda\left(3\hat{\imath} + 4\hat{\jmath} + 3\hat{k}\right)$ meets the plane, which is perpendicular to the vector $\mathbf{n} = \hat{\imath} + \hat{\jmath} + 3\hat{k}$ and at a distance of $\frac{4}{\sqrt{11}}$ from the origin.

25. Three numbers are selected at random (*withoutreplacement*) from first six positive integers.If X denotes the smallest of three numbers obtained,find the probability distribution of X.Also find the mean and variance of the distribution

26. A diet is to contain at least 80 units of vitamin A and 100 units of minerals.Two foods $F_1$ and $F_2$ are available costing ₹5 per unit and ₹6 per unit respectively.One unit of food $F_1$ contains 4 units of vitamins A and 3 units ofminerals whereas one unit of food $F_2$ contains 3 units of vitamin A and 6 units of minerals.Formulate this as a linear programming problems.Find the minimum cost of diet that onsists of mixture of these two foods and also meets minimum nutritional requirement

27. Using properties of determinants, prove that: $\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)($

28. Using elementary row operations, find the inverse of the following matrix:$A = \begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$.

29. Find the equation of the plane containing two parallel lines $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{3}$ and $\frac{x}{4} = \frac{y-2}{-2} = \frac{z+1}{6}$. Also,find if the plane thus obtained contains the line $\frac{x-2}{3} = \frac{y-1}{1} = \frac{z-2}{5}$ or not.

30. Using integration find the area of the region $\{(x, y) : y^2 \leq 6axandx^2 + y^2 \leq 16a^2\}$

31. Let $f : N \to N$ be a function defined as $f(x) = 4x^2 + 12x + 15$.Show that $f : N \to S$ is invertible( where S is range of f).Find the iverse of f and hence find $f^{-1}(31)$ and $f^{-1}(87)$.

32. determine the intervals in which the function $f(x)=x^4-8x^3+22x^2-24x+21$ is strictly increasing (or) strictly decreasing.

33. Find the maximum and minimum values of $f(x) = \sec + \log \cos^2 x, 0 < x < 2\pi$