1. Let ABC be a triangle with incentre I.A point P in the interior of the triangle satisfies

$$\angle PBA + \angle PCA = \angle PBC + \angle PCB$$

Show that $AP \geq AI$,and that equality holds if only if $P = I$.

2. Let P be a regular 2006-gon. A diagonal of P is called good if its endpoints divide the boundary of P into two parts, cach composed of an odd mumber of sides of P. The sides of Pare also called good. Suppose P has been dissected into triangles by 2003 diagonals, no two of which have a common point in the interior of P. Find the maximum number of isosceles triangles having two good sides that could appear in such a configuration.

3. Let determine the least real number M such that the inequality
$\left|ab\left(a^2 - b^2\right) + bc\left(b^2 - c^2\right) + ca\left(c^2 - a^2\right)\right| \leq M\left(a^2 + b^2 + c^2\right)^2$
holds for all real numbers $a, bandc$.

4. Determine all pairs (x,y) of integers such that

$$1 + 2^x + 2^{x+1} = y^2$$

5. Let P (x) be a polynomial of degree n > 1 with integer coefficients and let k be a positive integer. Consider the polynomial Q(x) = P(P(...P(P(x))...)), where P occurs k times. Prove that there are at most a integers t such that Q(1) = 1.

6. Assign to each side b of a convex polygon P the maximum area of a triangle that has b as a side and is contained in P. Show that the sum of the areas assigned to the sides of P is at least twice the area of P.