FAST DECOUPLED LOAD FLOW

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ABSTRACT

This paper describes a simple, very reliable and extremely fast load-flow solution method with a wide range of practical application. It is attractive for accurate or approximate off- and on-line routine and contingency calculations for networks of any size, and can be implemented efficiently on computers with restrictive core-store capacities. The method is a development on other recent work employing the MW-Θ/MVAR-V decoupling principle, and its precise algorithmic form has been determined by extensive numerical studies. The paper gives details of the method's performance on a series of practical problems of up to 1080 buses. A solution to within 0.01 MW/MVAR maximum bus mismatches is normally obtained in 4 to 7 iterations, each iteration being equal in speed to $1\frac{1}{2}$ Gauss-Seidel iterations or 1/5th of a Newton iteration. Correlations of general interest between the power-mismatch convergence criterion and actual solution accuracy are obtained.

INTRODUCTION

Load-flow calculations are performed in system planning, operational planning and operation/control. The choice of a solution method for practical application is frequently difficult. It requires a careful analysis of the comparative merits and demerits of the many available methods[1] in such respects as storage, speed and convergence characteristics, to name but the most obvious, and to relate these to the requirements of the specific application and computing facilities. The difficulties arise from the fact that no one method possesses all the desirable features of the others. For routine solutions Newton's method[2] has now gained widespread popularity. However it is limited for small-core applications where the weakly-convergent Gauss-Seidel-type method is the most economical, and it is not as fast as newer methods for approximate repetitive solutions such as in AC security monitoring[4,5].

Numerical methods are generally at their most efficient when they take advantage of the physical properties of the system being solved. Hence, for example, the exploitation of network sparsity by ordered elimination and skilful programming in the Newton and other methods has been a very important advance[3]. More recently, attention has been given to the exploitation of the loose physical interaction between MW and MVAR flows in a power system, by mathematically decoupling the MW-Θ and MVAR-V calculations[4-11].

The method described in this paper is a rational integration of some of these ideas. It combines many of the advantages of the existing "good" methods. The algorithm is simpler, faster and more reliable than Newton's method, and has lower storage requirements for entirely in-core solutions. Using a small number of core-disk block transfers its core requirements are similar to those of the Gauss-Seidel method. The method is equally suitable for routine accurate load flows as for outage-contingency evaluation studies performed on- or off-line.

NOTATION

$\Delta P_k + j\Delta Q_k$ = complex power mismatch at bus k, where

$$\Delta P_k = P_k^{sp} - V_k \sum_{m\varepsilon k} V_m (G_{km}\cos\Theta_{km} + B_{km}\sin\Theta_{km}) \quad (1)$$

$$\Delta Q_k = Q_k^{sp} - V_k \sum_{m\varepsilon k} V_m (G_{km}\sin\Theta_{km} - B_{km}\cos\Theta_{km}) \quad (2)$$

$P_k^{sp} + jQ_k^{sp}$ = scheduled complex power at bus k

Θ_k , V_k = voltage angle, magnitude at bus k

Θ_{km} = $\Theta_k - \Theta_m$

$G_{km} + jB_{km}$ = (k,m)th element of bus admittance matrix [G] + j[B]

$\Delta\Theta$, ΔV = voltage angle, magnitude corrections

$m\varepsilon k$ signifies that bus m is connected to bus k, including the case m=k, and [ ] signifies vector or matrix.

$\max|\Delta P|, \max|\Delta Q|$ = largest absolute element of [ΔP], [ΔQ]

$\max|e_v|, \max|e_s|$ = largest absolute bus-voltage magnitude error, branch MVA-flow error, respectively.

DERIVATION OF BASIC ALGORITHM

The well-known polar power-mismatch Newton method[2] is taken as a convenient and meaningful starting point for the derivation. The Newton method is the formal application of a general algorithm for solving nonlinear equations, and constitutes successive solutions of the sparse real jacobian-matrix equation

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ J & L \end{bmatrix} \begin{bmatrix} \Delta\Theta \\ \Delta V/V \end{bmatrix} \quad (3)$$

The first step in applying the MW-Θ/MVAR-V decoupling principle is to neglect the coupling submatrices [N] and [J] in (3), giving two separated equations

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$$[\Delta P] = [H]\ [\Delta\Theta] \quad (4)$$

$$[\Delta Q] = [L]\ [\Delta V/V] \quad (5)$$

where $H_{km} = L_{km} = V_k V_m (G_{km}\sin\Theta_{km} - B_{km}\cos\Theta_{km})$ for $m \neq k$

$$H_{kk} = -B_{kk}V_k^2 - Q_k \quad \text{and} \quad L_{kk} = -B_{kk}V_k^2 + Q_k$$

Equations (4) and (5) may be solved alternately as a decoupled Newton method[11], re-evaluating and retriangulating [H] and [L] at each iteration, but further physically-justifiable simplifications may be made. In practical power systems the following assumptions are almost always valid:

$$\cos\Theta_{km} \approx 1; \quad G_{km}\sin\Theta_{km} << B_{km}; \quad Q_k << B_{kk}V_k^2 ,$$

so that good approximations to (4) and (5) are:

$$[\Delta P] = [V.B'.V]\ [\Delta\Theta] \quad (6)$$

$$[\Delta Q] = [V.B''.V]\ [\Delta V/V] \quad (7)$$

At this stage of the derivation the elements of the matrices [B'] and [B"] are strictly elements of [-B]. The decoupling process and the final algorithmic forms are now completed by:

(a) omitting from [B'] the representation of those network elements that predominantly affect MVAR flows, i.e. shunt reactances and off-nominal in-phase transformer taps

(b) omitting from [B"] the angle-shifting effects of phase shifters

(c) taking the left-hand V terms in (6) and (7) on to the left-hand sides of the equations, and then in (6) removing the influence of MVAR flows on the calculation of [ΔΘ] by setting all the right-hand V terms to 1 p.u. Note that the V terms on the left-hand sides of (6) and (7) affect the behaviours of the defining functions and not the coupling

(d) neglecting series resistances in calculating the elements of [B'], which then becomes the DC-approximation load-flow matrix. This is of minor importance, but is found experimentally to give slightly improved results.

With the above modifications the final fast decoupled load-flow equations become

$$[\Delta P/V] = [B']\ [\Delta\Theta] \quad (8)$$

$$[\Delta Q/V] = [B'']\ [\Delta V] \quad (9)$$

Both [B'] and [B"] are real, sparse and have the structures of [H] and [L] respectively. Since they contain only network admittances they are constant and need to be triangulated once only at the beginning of the study. [B"] is symmetrical so that only its upper triangular factor is stored, and if phase shifters are absent or accounted for by alternative means [B'] is also symmetrical.

The immediate appeal of (8) and (9) is that very fast repeat solutions for [ΔΘ] and [ΔV] can be obtained using the constant triangular factors of [B'] and [B"]. These solutions may be iterated with each other in some defined manner towards the exact solution, i.e. when [ΔP/V] and [ΔQ/V] are zero. A brief account of the alternative decoupled algorithms investigated is given in Appendix 1.

APPLICATION TO PRACTICAL LOAD FLOW SOLUTIONS

Iteration scheme

Of the iteration strategies tried, undoubtedly the best scheme for all applications is to solve (8) and (9) alternately, always using the most recent voltage values. Each iteration cycle comprises one solution for [ΔΘ] to update [Θ] and then one solution for [ΔV] to update [V], termed here the (1Θ,1V) scheme. Separate convergence tests are used for (8) and (9) with the criteria:

$$\max|\Delta P| \le c_p\ , \ \max|\Delta Q| \le c_q \quad (10)$$

The flow diagram of the process is given in Fig. 1. The convergence-testing logic permits the calculation to terminate after a [ΔΘ] solution (called a ½ iteration). It is also possible, though unusual in practice, to terminate after more than one consecutive [ΔΘ] or [ΔV] solution if [ΔQ] or [ΔP] respectively do not need converging further.

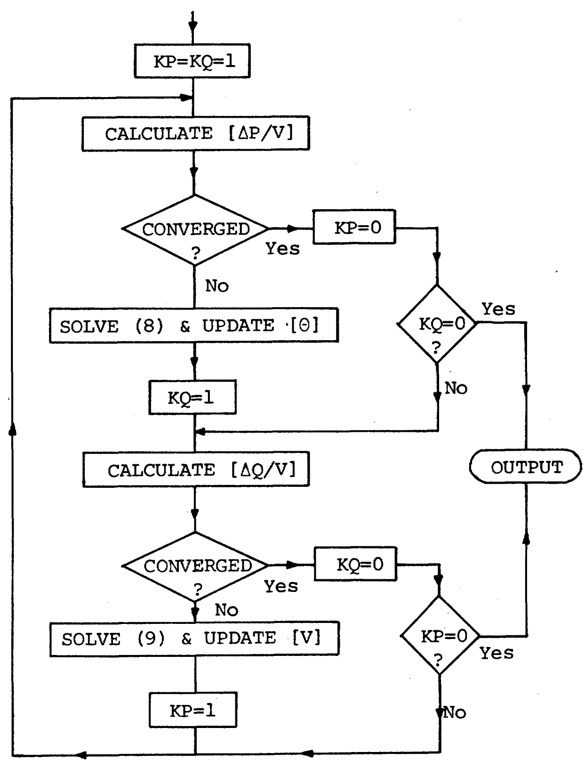

Fig. 1 Flow diagram of the iteration scheme

Convergence characteristics

A typical convergence pattern for the method is shown in Fig. 2, in terms of the largest mismatches at every half iteration. Each solution of (8) and (9) produces rapid reduction of [ΔP] and [ΔQ] respectively towards the exact solution of the load-flow problem.

The effect of Θ-changes on the MVAR flows is shown by the increase produced in $\max|\Delta Q|$ after each solution of (8). In this and most other problems the effect of V-changes on MW flows is less pronounced, because the active losses due to MVAR flows are normally smaller than the reactive losses due to MW flows. Although this suggests that faster overall convergence could be achieved by adopting a (2Θ,1V) iteration scheme, this idea (among others) was not successful in any of the systems studied. Overconverging [ΔP] at any stage produces a severe increase in [ΔQ], thereby slowing down the overall convergence rate.

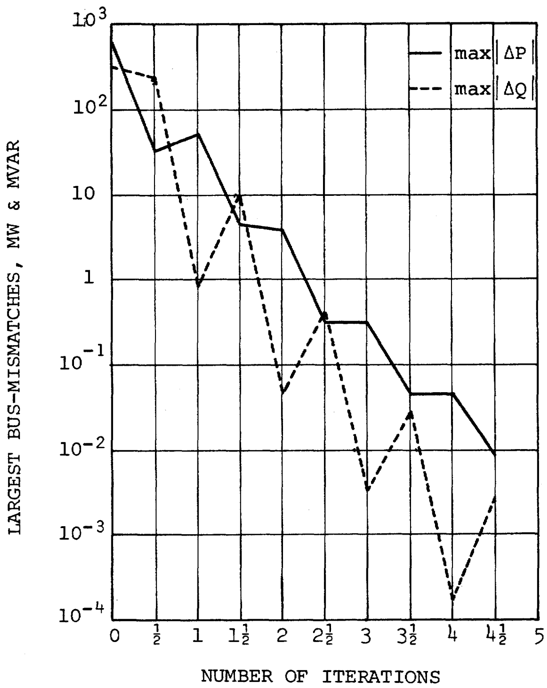

Fig. 2 Typical convergence pattern of the method (IEEE 118-bus system)

Algorithms (8) and (9) each have geometric convergence, as explained more fully in Appendix 2. This is not as fast as Newton's quadratic rate, but is more than compensated for by the much-faster iteration speed. Unlike Newton's method, the fast decoupled method has not failed on any feasible problem.

Accuracy requirements

The question as to what convergence tolerances c_p and c_q in (10) give an acceptably-accurate load-flow solution is relevant not only to the present method but more generally. Overconvergence, and therefore unnecessary extra iterations, can be avoided by correlating the solution errors with the bus MW and MVAR mismatches. Since power flows, voltage magnitudes and losses are the primary objectives of a load-flow calculation, an investigation of their errors was included in the numerical tests performed.

Numerical results

The studies carried out on the fast decoupled method included unadjusted, adjusted and sequential-outage solutions. In all cases $\max|\Delta P|$ and $\max|\Delta Q|$ were both monitored at every half iteration. In each case the process was iterated to the "exact" (very accurate) solution. By storing the voltages, branch MVA flows and losses at each stage of the calculation a post-solution history of the largest errors in these quantities during the iterations was constructed.

The load-flow problems used were chosen as deliberately-severe tests. With the exception of the IEEE Standard Test Systems, the problems are practical systems from seven countries supplied by industry because they have presented convergence difficulties. The systems contain a wide variety of features, and cover the range of voltage levels from EHV to distribution. They include long EHV line and cable circuits, capacitive series branches, shunt capacitors and reactors, and very small and very large series impedances and X/R ratios. Some networks are highly radial and others are highly meshed, and the proportion of PV buses varies from 0 - 45%. In some cases the voltage regulation is nearly 0.3 p.u.

Table I gives results for unadjusted base-case solutions of the test problems using the normal flat voltage start. Details of the errors are shown for the first three iterations. The largest bus voltage-magnitude error $\max|e_V|$ at each iteration is given in percent. The largest branch-flow error $\max|e_S|$ at each iteration is given both in MVA and as a percentage of the branch flow. Except while the solution is still very inaccurate, the errors in the total system MW and MVAR losses are insignificant, and are not presented. Also, the $\max|e_S|$ occurs on one of the most heavily-loaded lines or transformers, and is nearly all MW-flow error.

The close correspondence between $\max|\Delta P|$ and $\max|e_S|$ gives a good guide to the specification of c_p in (10). $\mathrm{Max}|\Delta Q|$ and $\max|e_V|$ do not correlate so closely, but by iteration 3 they are seen to be of the same order as each other. Depending on the purpose of the load-flow study, most of the solutions shown in Table I are sufficiently accurate after 3 iterations, and even after 2 iterations in several cases. The number of iterations required is seen not to be a function of problem size.

Apart from the results shown, the fast decoupled program has been used on other problems. Noteworthy among these is a very difficult 43-bus system which fails with Newton's method and takes 22 (1Θ,1V) iterations for an accurate solution. Otherwise, the 27-bus African system of Table I takes the largest number of iterations of any problems tried. The method has also solved a 1080-bus system (see the section on Solution Speed and Storage).

ADJUSTED SOLUTIONS

The most common adjustments in routine load-flow solutions are single-criterion controls such as on-load transformer taps, phase-shifters and area interchanges, and generator Q limits and load-bus V limits. Conventionally, adjustments are made in between or during iteration cycles by some simple easily-programmable parameter- or injection-changing error-feedback logic or by bus-type switching[12]. These schemes usually work satisfactorily with the slowly-converging load-flow methods when the system is sufficiently well-conditioned. With a powerful method such as Newton the number of extra iterations due to the adjustments can be large relative to the 3 or 4

TABLE I ERRORS DURING FAST DECOUPLED LOAD-FLOW SOLUTION

No. of buses in system	After Iteration 1					After Iteration 2					After Iteration 3					Accurate Solution		
	max $\|\Delta P\|$ MW	max $\|\Delta Q\|$ MVAR	max $\|e_V\|$ %	max $\|e_S\|$ MVA	max $\|e_S\|$ %	max $\|\Delta P\|$ MW	max $\|\Delta Q\|$ MVAR	max $\|e_V\|$ %	max $\|e_S\|$ MVA	max $\|e_S\|$ %	max $\|\Delta P\|$ MW	max $\|\Delta Q\|$ MVAR	max $\|e_V\|$ %	max $\|e_S\|$ MVA	max $\|e_S\|$ %	max $\|\Delta P\|$ MW	max $\|\Delta Q\|$ MVAR	no. of iters
13	87	2.5	0.72	105	13	11	0.4	0.08	9	1	1.4	0.04	0.00	1.4	0.2	0.002	0.001	$5\frac{1}{2}$
14*	46	1.1	1.08	11	57	2	0.1	0.05	1	3	0.1	0.00	0.00	0.1	0.0	0.005	0.000	4
19	19	8.5	3.39	28	13	6	0.9	0.55	7	9	1.6	0.14	0.10	1.8	2.5	0.006	0.003	$6\frac{1}{2}$
22	63	1.6	0.53	31	498	1	0.1	0.06	1	1	0.1	0.01	0.01	0.1	0.2	0.002	0.005	$4\frac{1}{2}$
27	211	10.5	2.16	158	24	39	0.9	0.97	47	7	13.6	0.46	0.38	17.5	1.9	0.003	0.007	$10\frac{1}{2}$
30*	17	0.9	0.68	10	5	0.4	0.0	0.04	1	0.5	0.05	0.00	0.00	0.1	0.1	0.006	0.000	4
38	22	4.3	0.94	15	31	2	0.4	0.07	2	2	0.2	0.04	0.02	0.1	0.1	0.004	0.007	$4\frac{1}{2}$
57*	24	1.0	1.24	9	9	1	0.1	0.29	1	0.7	0.1	0.01	0.03	0.1	0.01	0.004	0.006	$4\frac{1}{2}$
107	204	10.1	1.10	165	8	19	1.3	0.22	9	10	4.8	0.24	0.06	2.4	2.6	0.007	0.006	$7\frac{1}{2}$
118*	58	0.9	0.37	34	726	3	0.1	0.02	2	3	0.3	0.00	0.00	0.3	0.5	0.009	0.003	$4\frac{1}{2}$
125	257	12.5	1.56	77	20	6	0.8	0.28	3	3	0.9	0.11	0.06	0.5	0.5	0.007	0.001	6
180	225	7.1	1.56	153	21	9	0.7	0.29	4	2.6	1.8	0.13	0.06	0.9	0.6	0.002	0.004	$6\frac{1}{2}$
205	20	1.4	3.52	25	14	5	0.2	0.84	4	4.5	1.1	0.05	0.21	0.8	0.9	0.003	0.004	$6\frac{1}{2}$

* IEEE Standard Test System

required for an unadjusted solution. For Q and V limits there seems to be no way of avoiding these extra iterations.

The Newton formulation can be modified to give automatic solutions for controlled parameters[13-15], but the benefits of this can be outweighed by the introduction of new problems. Not the least of these in some cases is the substantially-increased programming detail, for what is already a complicated method if efficiently sparsity-programmed. An organisation may find that it is expensive or impractical to incorporate these types of modifications into an existing Newton program to cater for the individual and perhaps changing requirements of their load-flow studies.

With its very fast iteration time and reliable convergence, the new method encourages the retention of the well-tried conventional adjustment schemes. Adjustments will be made before or after the solution of (8) or (9) according to whether they primarily affect MW flows or MVAR flows, respectively. Although the adjustments largely dictate the total number of iterations to convergence, the absolute increase in computing time compared with the unadjusted solution is not great. As a typical example, consider an unadjusted solution that takes 6 iterations. Adjustments which add say, 6 extra iterations, represent little more time than one Newton iteration. This is insignificant for solutions requiring non-minimal input/output.

On-load transformer tap changing

A suitable adjustment algorithm for an in-phase off-nominal tapping t_i p.u. controlling the voltage of bus k to V_k^{sp} p.u. is

$$t_i^{(new)} - t_i^{(old)} = \pm \alpha (V_k - V_k^{sp}) \tag{11}$$

where α is an empirical error-feedback factor. Applied in between Newton iterations, this algorithm works reliably, and for well-conditioned problems $\alpha = 1$ gives the most rapid convergence.

When used with the new load-flow method, (11) is applied after calculating [V]. The choice of α and the total number of iterations for the solution are similar to those for Newton's method, since the tap convergence rate is the main determining factor in both methods. A (1Θ,2V) iteration scheme is inferior to the standard (1Θ,1V) scheme.

Most important, the convergence rate is affected little by reflecting the transformer π circuit parameter changes into [B"] at each tap correction. Unless better initial tap estimates are available, [B"] is therefore formed and factored at the start of the study assuming that all controlled taps are nominal, and it remains constant throughout the solution.

As an example, the fast decoupled method converges to within 0.01 MW/MVAR maximum bus mismatches in 11, 9 and $6\frac{1}{2}$ iterations for three different cases on a practical 22-bus system with 11 controlled transformers, 4 radially connected and 7 in network loops. This includes the enforcement of tap limits and corrections to the nearest physical tap settings after initial convergence. Using (11) and the same $\alpha = 1$, the three cases require 10, 12 and 11 iterations, respectively, with Newton's method.

The development of single-criterion controls in the fast decoupled program has as yet been limited to in-phase transformer taps. It is conjectured that with other controls it will also be found to be unnecessary to reflect parameter changes into [B'] or [B"].

Generator Q limits

Once a load-flow solution is moderately converged, any PV-bus Q-limit violations can be corrected. Provision must be made for subsequent interactive effects, i.e. MVARs backing off limits and new Q violations.

Two approaches are available. In the first, each violating bus is explicitly converted to PQ type so that the MVAR output is held at the limiting value. The bus remains a PQ type during the rest of the solution unless at some stage it can be re-converted to a PV type at the original voltage magnitude without the violation. In the fast decoupled program, converting any number of bus types at one time involves retrianulating [B"].

The second approach is to correct the voltage of each violating PV bus k by an amount ΔV_k at each following iteration to reduce the error $\Delta Q_k = (Q_k^{limit} - Q_k)$ to zero. The convergence of this process is rapid when an approximate sensitivity factor S_k relating ΔV_k and ΔQ_k is used thus:

$$\Delta V_k = S_k \, \Delta Q_k / V_k \qquad (12)$$

If S_k is defined according to (9) it is the diagonal element corresponding to bus k in the inverse of matrix [B"] augmented by the previously-absent row and column for bus k. Appendix 3 shows that S_k can be calculated easily and very rapidly without retriangulating [B"]. For an operational network each S_k may be stored permanently and updated only for significant system configuration changes. The correction (12) ceases to be applied if at some stage in the solution the value of V_k is restored to or goes beyond its original value.

Both approaches have been programmed and tested, and are equally effective. The sensitivity method usually takes more iterations, but against this it requires no retriangulations for limit enforcement and back-off, and is simpler programming-wise. Using a 0.01 MW/MVAR tolerance on bus-mismatches and Q-limit enforcement, typical results for the IEEE 30-bus and 118-bus systems are quoted. With two buses in the 30-bus system violating their Q-limits by 17% and 6% after initial convergence, the solution takes $7\frac{1}{2}$ iterations by approach 1, and 9 iterations by approach 2. For the 118-bus system, with 12 buses initially violating their limits by an average of 12% and a maximum of 20%, the solutions take $7\frac{1}{2}$ and $12\frac{1}{2}$ iterations respectively.

V-limits on PQ buses are handled in the inverse manner, using either of the two approaches.

APPLICATION TO OUTAGE STUDIES

Sequential branch- and generator-outage load-flow calculations are performed off-line for planning and operational planning studies and on-line or semi-on-line for operation/control, to evaluate system performance and particularly security following credible outage contingencies. In all these applications a large and time-consuming number of load flows are solved consecutively and automatically after a solution of the base-case problem, and branch flows, bus voltages and generator MVAR outputs are monitored to detect insecure and unsatisfactory conditions. For these purposes very accurate solutions are not normally required.

The approximate decoupled approach has already been developed successfully for the outage-study application [4,5]. The present method can also be employed efficiently without retriangulating the base-case [B'] or [B"] at any stage.

Transmission network outages

Line and transformer outages are simulated by an adaptation of the inverse-matrix modification technique applied to [B'] and [B"]. The experiments have shown that it is only necessary to simulate the removal of series transmission elements from these matrices. Shunt capacitors and reactors, line charging capacitance, and the shunt branches of off-nominal-tap transformer equivalent circuits can remain in [B"] without affecting convergence noticeably. All outages must of course be reflected correctly in the calculation of [ΔP/V] and [ΔQ/V].

For the outage of a series branch two non-sparse vectors [X'] and [X"] must be calculated, each requiring one repeat solution using the factors of [B'] and [B"], respectively. After each solution of (8), [Δθ] is corrected by an amount

$$-c' \, [X'] \, [M'] \, [\Delta\theta] \qquad (13)$$

where c' is a scalar and [M'] is a highly-sparse row vector containing one or two elements. This correction requires about n multiplication-additions. Similarly, after each solution of (9), [ΔV] is corrected by an amount

$$-c'' \, [X''] \, [M''] \, [\Delta V] \qquad (14)$$

The details of this technique are given in Appendix 4, including the case of multiple outages. The scheme can equally be used for branch switch-in operations if desired.

Generator outages

A generator outage requires as input data the redistribution of scheduled MW generation to compensate for the loss of supply. It is usual to cater for the partial loss of generation at a plant, in which case it is only necessary to reflect the changes in scheduled MW in [ΔP/V]. The complete loss of a plant means that the net MVAR of the relevant bus must be fixed by either of the methods used in enforcing generator Q limits.

Since only one bus is involved, an alternative is to augment the upper-triangular factors of [B'] and [B"] by an extra column each. This is exactly the same calculation as performed in obtaining the sensitivity factor S_k used in the Q-limit adjustments, and is described in Appendix 3.

Experience with outages

Tests were conducted on several systems, including a series of 35 line and transformer single-outage cases on the heavily-loaded 180 bus problem which is part of the main British transmission system. Ten of the outaged branches were chosen as the least-loaded ones in the base-case, and 25 were chosen as the most loaded ones, with base-case branch power flows of up to 2370 MW and 240 MVAR.

It was found that the (1θ,1V) iteration scheme remains the best in this application, and it was confirmed that using the base-case solution to start the outage calculations is on the average distinctly better than using the normal flat voltage start each time. The worst branch MVA-flow error at each itera-

TABLE II RESULTS FOR 35 SINGLE-BRANCH OUTAGE CASES ON 180-BUS SYSTEM

No. of (1Θ,1V) iterations for max. branch-flow error of 3%	No. of outage cases in groups	ERRORS IN GROUPS							
		max|ΔP| MW		max|ΔQ| MVAR		max|e_v| %		max|e_s| MVA	
		largest	average	largest	average	largest	average	largest	average
Group 1, 1 iteration	21	49	13	1.8	0.3	0.13	0.05	23	8
Group 2, 2 iterations	12	10	4	0.8	0.3	0.62	0.18	17	5
Group 3, 3 iterations	2	1	0.7	1.3	0.7	0.09	0.05	0.1	0.06

tion was obtained as before. Since this always seems to occur on one of the most heavily-loaded branches, the performance of the method for security-monitoring can be meaningfully interpreted even though branch MVA-rating data was not available.

In the tests, one (1Θ,1V) iteration was adequate in the majority of cases to give a maximum branch-flow error of an arbitrarily-chosen 3%, and in two cases three iterations were needed. Table II gives a breakdown of the 35 cases.

SOLUTION SPEED AND STORAGE

Central to the fast decoupled method is the use of efficient sparsity-programmed routines for the triangulation of the sparse, real, symmetric matrices [B'] and [B"] by Crout elimination and the subsequent solutions of (8) and (9) by forward and backward substitutions[3]. If, as is becoming more widespread, such routines are available as standard packages, the method is not difficult to program efficiently.

At the beginning of the load-flow study the system buses are re-ordered to avoid excessive fill-up in the table of factors during the triangulation of [B']. It can be shown that this bus ordering remains equally suitable for [B"] with the PV buses by-passed in the ordering list. If the system is very large or if consecutive load-flow solutions are to be calculated then it is advantageous to minimise fill-up as far as possible by using a good ordering scheme[16]. Since [B'] and [B"] remain unchanged, triangulation routines that perform the ordering during elimination are at no great disadvantage.

The triangulation routine is programmed to take account of matrix symmetry so that only the upper-triangular factors of [B'] and [B"] are stored for use in the solution routine. Using dynamic re-ordering according to row sparsity, the ordering, triangulation and solution times each vary roughly linearly with network size[3]. For a typical well-developed network with n buses and b branches the number of elements in the upper-triangular factor of [B'] is about 3(n+b)/2. Depending on the number and location of PV buses, the triangulation and solution processes for [B"] are faster and require less storage than for [B'], in some cases considerably so.

The calculations of the vectors [ΔP/V] and [ΔQ/V] can each be performed rapidly in a single sweep of the branch admittance list. For each series-branch connecting buses k and m, it can be seen from (1) and (2) that where appropriate, the terms $V_kV_mG_{km}$ and $V_kV_mB_{km}$ can be used directly in accumulating both ΔP_k and ΔP_m, or ΔQ_k and ΔQ_m. Also, it is noted that $\sin\Theta_{km} = -\sin\Theta_{mk}$ and $\cos\Theta_{km} = \cos\Theta_{mk}$. These trigonometrical functions are calculated using the approximations $\sin\Theta \approx \Theta-\Theta^3/6$ and $\cos\Theta \approx 1-\Theta^2/2+\Theta^4/24$, which have been found to give final load-flow results indistinguishable from those obtained with the accurate function evaluations. If storage is not critical, the sine and cosine terms calculated during the construction of [ΔQ/V] can be stored for use in the next construction of [ΔP/V]. From a flat voltage start, each sine and cosine term can be initialised to 0 and 1 respectively for the first solution of (8).

During the experiments the constituent parts of the solutions were timed on the CDC 7600 computer. Results are given for a 1080 bus, 1862 branch system with 41 PV buses, and the IEEE 118 bus, 186 branch system with 53 PV buses, using non-optimised FORTRAN compilation. The inferior 'static' bus ordering scheme was used and the sine and cosine terms were not stored.

	Time, seconds 1080-bus	Time, seconds 118-bus
Bus ordering	0.062	0.007
formation & triangulation of [B']	0.635	0.028
formation & triangulation of [B"]	0.515	0.007
calculation of [ΔP/V] & convergence testing	0.059	0.006
solution of (8) & updating [Θ]	0.063	0.005
calculation of [ΔQ/V] & convergence testing	0.053	0.005
solution of (9) & updating [V]	0.050	0.001

For the 1080-bus problem, the solution to within 0.01MW and MVAR bus-mismatch tolerances from a flat voltage start required 7(1Θ,1V) iterations plus two additional (1Θ) iterations in the final stages. Excluding input/output, the total time was 3.2 seconds of which 1.98 seconds were for the iterative process. For the 118-bus problem these times were 0.13 and 0.09 seconds respectively. The iteration times could be reduced considerably by using a better ordering scheme and more realistic mismatch tolerances.

The storage requirements of the fast decoupled method are about 40% less than those of Newton's method. This saving is reduced somewhat if the sine and cosine terms are stored, and for outage cases where vectors [X'] and [X"] are needed. Each of the parts of the solution given in the above timing list is conveniently performed in a separate program subroutine. Apart from simple subroutine overlaying, core storage can be economised by reading certain selected vectors from disk when they are required and writing them back after the relevant subroutine. These block transfers are performed a limited number of times during a solution, and should not degrade the method's speed too severely. Using this scheme, the core storage requirements of the method are about the same as those of the Gauss-Seidel method.

Network tearing for load-flow calculations is not usually economical in computing speed when ordered elimination is used for solving the sparse network equations. However, if a piecewise approach is desirable for other reasons, e.g. storage, any of the available Y-matrix decomposition or diakoptical methods can be applied to give a series of smaller constant [B'] and [B"] matrices. Note that since branches connected to PV buses are absent from [B"], a certain amount of network tearing is already inherent in this matrix.

CONCLUSIONS

The fast decoupled method offers a uniquely attractive combination of advantages over the established methods, including Newton's, in terms of speed, reliability, simplicity and storage, for conventional load flow solutions. The basic algorithm remains unchanged for a variety of different applications. Given a set of good ordered elimination routines, the basic program is easy to code efficiently, and its speed and storage requirements are roughly proportional to system size. A useful feature of the method is the ability to reduce its core-storage requirements to approximately those of The Gauss-Seidel method with a small number of core-disk block transfers. The method performs well with conventional adjustment algorithms, and solves network outage security-check cases usually in one or two iterations. It is computationally suitable for optimal load-flow calculations, and developments in this area are to be reported separately.

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APPENDIX 1

Alternative decoupled versions investigated

This appendix gives a very brief outline of the large number of experimental studies conducted on variants of the decoupled approach. Each version was used on all or a selected group of the load-flow test problems of Table I. In nearly all cases the problems were solved successfully by the different versions, demonstrating the general effectiveness of decoupling the angle and magnitude calculations. The object of the studies was then to find the version that best combines consistent and rapid success with other computational advantages.

Initially, the methods of refs. 6, 7 and 11 were compared[17]. The Despotovic and Decoupled Newton methods converge well but require re-triangulations. Uemura's method is not very satisfactory on some practical systems because MVAR conditions are not excluded from the "P-Θ" matrix. The idea in ref. 18 of using the current mismatch [ΔI] in (9) :

$$[\Delta I] = [B''] \, [\Delta V] \,, \qquad (15)$$

and a variant in which each V_k is divided by $\cos\Theta_k$, were less successful than anticipated. The other idea in ref. 18 of using the constant matrix [B'] in the angle calculation of refs. 4 and 6 gives good convergence.

Having established this latter point, attention was devoted to the decoupled Newton approximations (6) and (7) which can be re-written

$$[\Delta P/V] = [B'] \, [V.\Delta\Theta] \qquad (16)$$

$$[\Delta Q/V] = [B''] \, [\Delta V] \qquad (17)$$

A question at this stage was whether the V terms on the left-hand sides of (16) and (17) and on the right of (16) should be omitted. The seven resulting combinations of Θ and V algorithms were compared with each other on all the test systems, and the combination (8) with (9) emerged as the best, although one or two of the others were not much worse. (8) and (9) and other versions can be rearranged to give direct solutions for [Θ] and [V] instead of [$\Delta\Theta$] and [ΔV], but there is no convergence difference unless further approximations are made. The [$\Delta\Theta$] and [ΔV] forms were preferred, in spite of a few extra arithmetic operations, because this form directly retains the familiar ΔP and ΔQ mismatch quantities for convergence testing.

All of the above tests were carried out using a (1Θ,1V) iteration scheme, flat voltage starts, and no adjustments. Having finalised on the algorithms (8) and (9), the problems of Table I were solved using each of the following schemes: (a) (2Θ,1V), (b) (2Θ,1V) once and subsequently (1Θ,1V), (c) (2Θ,2V) once and subsequently (1Θ,1V), (d) (1V,1Θ), and (e) (3Θ,2V). None of these schemes was as good as the simple (1Θ,1V) approach, which is also best for transformer-tap adjustment solutions, and branch outage solutions.

Several different attempts at acceleration, including block successive over-relaxation and a heuristic approach based on testing sign-changes in the bus mismatches, proved to be unrewarding.

For all the branch-outage studies, the removal of line-charging and transformer π-equivalent shunt elements from [B"] was found to affect the convergence only minimally, provided that they are correctly removed in the Calculations of [ΔP/V] and [ΔQ/V], and similarly for shunt capacitor or inductor outages. However, failing to remove the outaged series elements from [B"] and [B'] is totally unsuccessful. Appendix 4 shows how these outages are carried out, and also that there is no extra effort involved in removing transformer π-circuit shunts.

APPENDIX 2

Notes on the convergence of the fast decoupled method

The iteration process of the fast decoupled method has three distinct components, each with its own convergence characteristics: (a) the solution of [ΔP/V] = O for [Θ] using algorithm (8), (b) the solution of [ΔQ/V] = O for [V] using algorithm (9), and (c) the interactive effects of V-changes and Θ-changes on the defining functions [ΔP/V] and [ΔQ/V] respectively.

Algorithms (8) and (9) are both Newton-like, except that instead of re-evaluating the true Jacobian-matrix tangent-slopes to the left-hand functions at each iteration, fixed approximated tangent-slopes [B'] and [B"] are used. The algorithms therefore correspond to the generalised 'fixed-tangent' or 'constant-slope' method[19] which has geometric convergence. For reasonably-behaved functions, this method is very reliable and if, as in the present application, the fixed tangent-slopes correspond closely to the Jacobian matrix at the initial point (for a flat voltage start), the initial convergence is very rapid. The process does not 'home in' as fast as the quadratic Newton method as the exact solution is approached, but load-flow solutions are rarely required to very high accuracy.

The fixed-tangent method is not thrown off-course when it encounters 'humps' in the defining functions, whereas Newton's method tends to be mis-directed even to the extent of divergence from the desired solution in such cases. In the decoupled load-flow problem, the changing V-values during the solution have the effect of causing sometimes-oscillatory shifts in the shapes of the multi-dimensional surfaces $[\Delta P/V]$ as functions of $[\theta]$, and likewise for θ-values on $[\Delta Q/V]$ as functions of $[V]$. Since $[B']$ does not represent MVAR conditions it corresponds to fixed tangent-directions that take no cognisance of these shifts and are therefore not affected by V-oscillations. Likewise for $[B'']$ in relation to MW conditions.

APPENDIX 3

Calculation of sensitivity factor

The matrix factors of $[B'']$ are expressed in the symmetric form, so that (9) is

$$[\Delta Q/V] = [U]^t [D]^{-1} [U] [\Delta V] \tag{18}$$

where $[D]^{-1}$ is a diagonal matrix and $[U]$ is upper-triangular with unit diagonal elements. $[U]^t$ is not stored. The solution of (9) is then performed according to the following three steps: (a) a forward-substitution process on $[\Delta Q/V]$ and $[U]^t$, giving an intermediate vector $[F]$; (b) the trivial matrix multiplication $[G] = [D][F]$; (c) a backward-substitution process on $[G]$ and $[U]$ to give $[\Delta V]$.

If a new PQ-type bus is created in the network, an extra row and column must be added to $[B'']$. Letting the order of $[B'']$ be $(n-1)$, then the new PQ bus is ordered n, i.e. last, and the enlarged matrix is

$$[B'']^+ = \left[\begin{array}{c|c} B'' & C_1 \\ \hline C_1^t & C_2 \end{array}\right] \tag{19}$$

To obtain the matrix factors of $[B'']^+$ it is only necessary to enlarge the existing factors by adding an nth column to $[U]$ and a diagonal element to $[D]$.

Replacing $[\Delta Q/V]$ in step (a) above by the sparse $[C_1]$, $[G]$ in step (b) gives the new sparse column of $[U]$:

$$[G] = [U_{1n}, U_{2n}, \ldots, U_{n-1,n}]^t \tag{20}$$

where $U_{nn} = 1$. The new element D_{nn} is given by

$$D_{nn} = (C_2 - \sum_{j=1}^{n-1} U_{jn}^2/D_{jj})^{-1} \tag{21}$$

= the sensitivity factor S_k used for Q-limits and generator outages.

Using the normal solution routine, this calculation takes about the same time as a solution of (9). By reprogramming steps (a) and (b) to take account of the high sparsity of $[C_1]$ and $[G]$, the calculation becomes extremely fast.

The process can be expressed in more formal matrix notation using the 'multi-product of inverse' approach and is easily generalised for any non-symmetric matrix. In the present case, the process can be used for the original triangular factorisation of $[B']$ and $[B'']$, adding one column at a time.

APPENDIX 4

Branch outage calculation

Let either (8) or (9) be represented in the base-case problem as the equation

$$[R] = [B_o] [E_o] \tag{22}$$

for which a solution

$$[E_o] = [B_o]^{-1} [R] \tag{23}$$

can be obtained using the factors of $[B_o]$. In the most general case, the outage of a line (neglecting charging capacitance) or transformer can be reflected in $[B_o]$ by modifying two elements in row k and two in row m. The new outage matrix is then

$$[B_1] = [B_o] + b[M]^t [M] \tag{24}$$

where:

b = line or nominal transformer series admittance

[M] = row vector which is null except for $M_k = a$, and $M_m = -1$

a = off-nominal turns ratio referred to the bus corresponding to row m, for a transformer

= 1 for a line.

Depending on the types of the connected buses, only one row, k or m, might be present in $[B']$ or $[B'']$, in which case either M_k or M_m above is zero, as appropriate. If both the connected buses are PV or slack, then $[B'']$ requires no modification.

It can be shown that

$$[B_1]^{-1} = [B_o]^{-1} - c[X][M][B_o]^{-1} \tag{25}$$

where:

$$c = (1/b+[M][X])^{-1} \quad \text{and} \quad [X] = [B_o]^{-1}[M]^t \tag{26}$$

The solution vector $[E_1]$ to the outage problem is :

$$[E_1] = [B_1]^{-1} [R] \tag{27}$$

and from (23), (25) and (27) we have

$$[E_1] = [E_o] - c[X] [M] [E_o] \tag{28}$$

Hence, the solution to the base-case problem is easily corrected, as in (13) and (14). $[X]$ is calculated at the beginning of the outage case by a repeat solution using the factors of $[B_o]$ with $[M]^t$ as the independent vector.

The above procedure can be applied recursively for multiple simultaneous outages. The solution vector [E] is corrected successively as the effect of each branch outage is introduced one at a time. For a set of outaged branches 1.....m, the recursion is

$$[E_i] = [E_{i-1}] - c_i[X_i][M_i][E_{i-1}] \text{ for } i=1...m \quad (29)$$

where $[M_i]$ is defined as before for the outaged branch introduced ith in the sequence. The scalars c_i and the vectors $[X_i]$ are calculated at the beginning of the case from the recursive algorithm:

$$[X_i] = [B_o]^{-1} [M_i]^t$$

$$[X_i] = [X_i] - c_j[X_j][M_j][X_i] \text{ for } j=1...i-1 \quad (30)$$

$$c_i = (1/b_i - [M_i] [X_i])^{-1}$$

all for i = 1...m . If any $\frac{1}{c_i} = 0$ a split network is indicated.

Although this scheme avoids the retriangulation of the network matrices, it is faster for at most three simultaneously-outaged branches.

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Discussion

E. Hobson (The Capricornia Institute of Advanced Education; Rockhampton 4700; QLD, Australia): The authors have made significant changes to previously described decoupled load flow methods in order to improve solution speed and to permit rapid reliability assessments to be made. I am grateful to them for giving me the opportunity to experiment with their methods before publication.

My comments refer to experience gained whilst incorporating some of the authors' techniques into an operational interactive minicomputer load flow program written in Basic and employing the Gauss Seidel algorithm [1]. This program was designed primarily for small distribution authorities where versatility was considered more important than speed.

The claim that core requirements are similar to those of the Gauss Seidel method was found to be approximately true. Increased complexity of programming and the necessity to store the factorised forms of both B' and B" matrices of equations (8) and (9) led to an increase in core requirements of approximately 2:1. This increase proved embarrassing on an 8k PDP8 computer when attempting to replace the Gauss Seidel algorithm with the new decoupled method. In order to fit the new method onto the PDP8, certain changes were made to the authors' approach. These modifications permitted the replacing of matrices B' and B" with a single matrix B, comprising the negated reactances of the nodal admittance matrix. For PV buses, the reactive mismatches of equation (9) were put to zero, and no adjustments made to V. Simple pre-ordering was considered adequate for the size of networks contemplated, hence no re-numbering scheme was employed.

With the above modifications, solutions were obtained for a range of small distribution networks. In general the authors' claims were substantiated. Each iteration took approximately 1.5 times a Gauss Seidel iteration, and overall solution time for 'difficult' networks improved by 10:1 or more. The number of iterations was greater than for pure Newton Raphson, but no speed comparisons were possible since different computers were involved. Convergence was appreciably slowed and divergence problems were sometimes introduced in the simplified method if in-phase transformer taps departed more than about 5% from nominal settings. This was presumably because the effects of tap changing were being incorporated into the $P - \theta$ coupling. A further divergence problem arose with a very weak 9 bus radial system including a line with R/X approaching 2. Convergence was achieved by making R/X closer to unity. Removal of series resistive elements from B', as described by the authors, eliminated the convergence problem but did not significantly reduce the number of iterations. Convergence could not be achieved by removing series resistive elements from the single matrix B.

After experimenting with the authors' decoupled methods I conclude that they have much to recommend them in terms of speed, storage and simplicity in comparison with existing methods.

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Manuscript received August 1, 1973.

T. E. Dy Liacco and K. A. Ramarao (Cleveland Electric Illuminating Company, Cleveland, Ohio 44101): This paper, which is a model of clear exposition, reflects the thoroughness and extent of the efforts made by Dr. Stott and his colleagues in developing and testing decoupled load flow methods. For the past two years we have been planning on using a decoupled load flow for both system operation and system planning but have not had time to try out various techniques. We did try Dr. Stott's first method[11] on our system with very good results but could not pursue further experimentation with it. The authors have done us and certainly many others in the industry a great service by doing much of the testing for us.

The important result of all this work is the demonstration that a decoupled load flow using a fixed susceptance matrix requiring only one initial triangularization is more efficient and reliable than other existing methods. This idea had always been appealing to us since it was first proposed by Dr. Uemura[7]. We believe that the industry should try out this approach extensively in order to gain as wide an experience as possible. The author's refinement of the decoupling process by removing all shunt reactances from the B' matrix is a remarkable idea. Have the authors tried Uemura's method with the modified B'? Does the inclusion of the voltage magnitudes in the forcing functions make a significant difference in overall speed or convergence characteristics to make it worthwhile?

The authors state that the current mismatch idea[18] did not work out too well. Since we had more than just a passing interest in the current mismatch formulation, what specifically was its weakness?

We compliment and thank the authors for an excellent paper.

Manuscript received July 30, 1973.

A. K. Laha and K. E. Bollinger (University of Saskatchewan, Saskatoon, S7N OWO Canada): The authors are to be complimented for an interesting paper. In solving the load flow problem there is always a trade-off of speed and storage. The formal Newton method has demonstrated distinct advantages over other iterative schemes. This paper is a welcome addition to the different modifications of Newton's method, to accelerate the speed of convergence, which are already published in the literature. Others will no doubt further test the idea outlined in this paper thereby confirming the authors' claim about its relative superiority over other methods. In the meantime we would like to comment on some of the salient points of the paper.

Although the idea of neglecting the off-diagonal matrices N and J was first suggested by Carpentier as early as 1963, this was later found to suffer from weak convergence and poor reliability. As we understand, the authors' decoupled method is a modification of equations (4) and (5) in the sense that they introduced the $(1\theta, 1v)$ successive iteration scheme, which in turn, increases the stability property of the solution. The modified Newton method[1] which we are developing at present, although completely mathematically different from the authors' method, follows the idea of a successive iteration scheme. The modifications suggested in equations (6) – (9) of the authors' paper essentially increase the speed of computation time, but does not affect the convergence properties of the algorithm nor will it affect the total number of iterations. Thus this algorithm apparently contradicts one of the authors' earlier comments regarding the algorithm[2] that "this is also found not to be very satisfactory for most problems and the reason is that equation (4) (which is same as equation (5) of this paper) is a relative unstable algorithm at some distance from the exact solution." It may be interesting to know whether the authors' comment regarding the fast convergence property is based solely on the results as shown in Table 1 and figure 2 of this more recent paper or is there any mathematical justification behind it.

In Dr. Stott's earlier paper[2], he introduced the idea of current mismatch. Since that method and the one outlined in this paper are competitive to each other, we would appreciate it if the authors could give us a relative comparison of both of the methods from the point of view of speed, convergence characteristics, storage requirement etc. and which one, the authors think, is superior from an overall point of view. One may note that the the 27th bus system, the two methods take 3 and 10½ iterations respectively to arrive at the correct solution.

We fully agree with the authors' suggestion to approximate Cosine and Sine trigonmetric functions by the first few terms of the Series, as it definitely increases the speed of computation. One should, however, use this approximation with caution as this may give errors if the angle is quite large. These are cases where the phase angle of some buses are 60° or so out of phase with that of the slack bus.

Again we would like to commend the author on a well written important paper.

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Manuscript received August 6, 1973.

W. O. Stadlin and B. F. Wollenberg (Leeds & Northrup Company, North Wales, Pennsylvania): The authors have performed an invaluable service in their evaluation of alternate approximate forms of the Newton method. In order to further conserve computer memory we would welcome the authors comments as to the feasibility of setting [B"] equal to [B']. Shunt reactances and line charging would then be considered as part of the Q load and the corresponding MVAR value of these shunt elements would be updated as part of the Q mismatch calculation based on the previously calculated voltage. Secondly, if such a load flow is part of an optimal power flow package can approximate incremental losses be derived using similar decoupling techniques as used in the solution – or must one revert to the normal Jacobian?

Manuscript received August 2, 1973.

B. Stott and **O. Alsac**: We are very grateful to the discussers for their valuable comments and questions.

The main aim of the work reported in the paper was to identify a single and preferably simple version of the decoupled load-flow approach that uses constant network matrices and performs well, in terms of speed and reliability, over a wide range of problems and applications. From the many alternatives studied, the version embodied in eqs. (8) and (9) emerged as the best according to these criteria. Other variants differing slightly in form can perform very satisfactorily on specific load-flow problems. Two of the chief points to come out of the work, the first of which replies to the relevant questions of Messrs. Dy Liacco and Ramarao, can be stated as follows.

1. $[\Delta Q/V]$ is very much better than $[\Delta Q]$ as the defining function of the MVAR-V problem. It can be seen from eq. (2) that $\Delta Q_i/V_i$ has only one term, Q_i^{sp}/V_i, that is non-linear in V. This term is significant only for impractically-small values of V_i and vanishes on no-load. Hence, for given values of θ obtained from the solution of eq. (8), each solution of eq. (9) for V is thoroughly reliable and normally very accurate. In contrast, it is of secondary importance whether $[\Delta P]$ or $[\Delta P/V]$ is used for the MW-θ problem, since the non-linearities here are due mainly to the sine and cosine terms in eq. (1).

2. Since shunt susceptances do not appear in $[\Delta P]$, nor in the Newton matrix [H], large shunts represented in [B'] can cause poor convergence of the MW-θ problem: we encountered this difficulty when originally testing Uemura's version. Correspondingly, failure to represent such shunts in [B"] can give convergence difficulties of the MVAR-V problem.

We thank Mr. Hobson for experimenting with the method at an early stage, and for the very useful ideas and information that he has presented. His results go some way towards answering Messrs. Stadlin and Wollenberg about the possibility of using a single B matrix. This is clearly an attractive prospect, but will be suitable only for some power systems, bearing in mind our point 2 above. The ideal application of a single B matrix is on lower-voltage networks with little or no shunt compensation and with one infeed point, where [B'] and [B"] are very similar or identical to each other. For instance, electrified railway networks exhibit this property, and we have used a single B matrix with complete success for many hundreds of load flows on these systems, in which the voltage magnitudes under extreme loading conditions go down to less than 0.3 per-unit. Mr. Hobson's scheme for accommodating PV buses is analogous to that used in standard impedance-matrix load flow (setting ΔQ_i to zero is the same as using the calculated value of Q_i in place of Q_i^{sp}). This will detract from the accuracy of the solution for V at each application of eq. (9). However, this may not matter too much – as indicated in Fig. 2 of the paper, V convergence is usually well ahead of θ convergence during the iterations, and some slowing-down of the former may be tolerable, except for fast approximate load flows taking one or two iterations. In practice, we think that it will rest with individual users to determine whether a single B matrix is adequate for their systems. The difficult 9-bus problem mentioned by Mr. Hobson, with solution voltages down to 50%, could be solved accurately by the normal (B' and B") method in 17 iterations.

Messrs. Laha and Bollinger have referred to what is an interesting apparent anomaly between the performances of eq. (9) and the decoupled Newton eq. (5). The answer lies in our point 1 above. In reference 11 the difficulties experienced by Newton's method with the non-linearities in the defining function $[\Delta Q]$ were overcome by using the more-linear current-mismatch formulation. In hindsight, the simple expedient of dividing each ΔQ_i by V_i would presumably have been just as good. At this stage therefore, we have no reason to advocate the use of decoupled current-mismatch formulations. Lack of success of the approximated current-mismatch idea[18], mentioned by Messrs. Dy Liacco and Ramarao, was because approximating each relevant Jacobian-matrix element $(G_{ik}\sin\theta_k+B_{ik}\cos\theta_k)$ by B_{ik} is not valid for large bus angles. The approximations in the present method have no such limitations, since it is only the transmission angle θ_{ik} across each line that is assumed not to be large. Likewise, in connection with the point made by Messrs. Laha and Bollinger, it is each $\sin\theta_{ik}$ and $\cos\theta_{ik}$ as per eqs. (1) and (2) that are computed using truncated series. The errors are extremely small for practical systems even up to 30-degree transmission angles.

In answer to the same discussers, we do not believe that the Newton (NM) or decoupled Newton (DNM) methods are at all competitive with the fast decoupled method (FDM) for the vast majority of conventional load-flow applications. The computation per iteration of DNM is only slightly less than that of NM, whereas for FDM it is five times less, assuming very good programming in each case. Acceptable accuracy can frequently be obtained from a flat start with FDM in 2 or 3 iterations, as seen from Table I of the paper – the fact that NM gives higher accuracy is seldom of practical importance. If the number of iterations required by NM is 3 – 4, the break-even point in computing time is 15 – 20 iterations for FDM. The "constant slope" convergence mechanism of FDM inhibits divergence for difficult problems including cases whose solution voltages are very low, with the effect that the method is well-behaved even if a feasible solution is not possible. In certain difficult cases FDM converges rather slowly by its normal standards, such as with the 27-bus, 43-bus and 9-bus problems mentioned in the paper or the discussion, but has a very high reliability in excess of other methods. Probably only NM with the starting process of reference 10 is comparably reliable. Storage-wise, FDM requires a little more than DNM but less than NM.

The application of FDM for rapid sequential outage evaluations, such as in AC security monitoring, seems to be quite promising. For each outage case, a single calculation of the angles from eq. (8) can be made initially. This is normally much more accurate than a pure DC load flow, since it starts from the base-case solution. At this stage the MW flows are calculated and if none of them exceeds defined limits, the process proceeds to the next outage case. Otherwise, the FDM solution is continued to higher accuracy.

The subject of Messrs. Stadlin and Wollenberg's second question has been of great interest to us. After developing our original Newton-based optimal load flow[20], an obvious improvement was to replace the NM load flow at each gradient step by FDM. To produce a program containing no matrix triangulations during the solution, we conducted many studies on eight different medium-size systems (up to 118 buses) to determine how the linear λ-calculation can be accomplished using the factored [B'] and [B"] matrices, and how its accuracy affects the overall optimisation. We formulated a decoupled iteration scheme exactly equivalent to FDM with $[\Delta\theta]$ and $[\Delta V]$ in eqs. (8) and (9) replaced by $[\Delta\lambda_p]$ and $[\Delta\lambda_q]$ respectively, and different left-hand sides, such that exact values of $[\lambda_p]$ and $[\lambda_q]$ could be obtained by iterating to convergence. It was found that if the last λ-values are stored for use as starting values, a single iteration of this process at each gradient step gives the accurate optimal solution obtained by the original Newton program, usually in the same or slightly larger number of steps.[21]

We conclude by thanking the discussers again for their contributions and kind remarks.

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