Gradual Deformation Method

1 Introduction

Gradual Deformation Method (GDM) is an History Matching Method (HMM) (or Calibration) from the family of deformation Technique.

1.1 Objetif

The general objective of HMM is the conditioning the model to data, which is formulate as an optimization problem. An objectif function is created to measure the misfit and the target of the method is to minimized it. A Good technique should :

- Respect spacial continuity and structure (2-pts or multi-point)
- Have me most regular OF (use of Gradient Based (GB) technique improve computation effort)
- Cover the whole solution space

1.2 Other technique

- Swaping Value, Genetic Operation (Deutsch, 1993) But they violate spacial continuity.
- simulated annealing or genetic algorithms
- Conditional simulation (Oliver et al., 1997) : Very slow OF convergence
- Pilot Point Method (De Marsily et al., 1984) (use of conditioning kriging (Journel and Huijbregts, 1978)): + : minimial nb of parm., use of GB opt. Algo. - : degradation of spacial structure when pilote point too close.
- Visualizing Uncertainty (Srivastava and Others, 1994) (generate lager grid solution and moving the grid in the model) pb when large model

1.3 History and Developement

1. Hu (2000) explain in detail GDM.
2. Hu et al. (2001) show application for iterative calibration of Sequential (not necessarly Gaussian) Simulation
3. Le Ravalec-Dupin and Noetinger (2002) look at gradient based gradual deformation with uncertainty.
4. Hu (2002) generalized GDM to dependant relalizations. He looked at conditioning before deforme and combine correlated field.
5. Hu and Le Ravalec-Dupin (2004) explore a gradient based compounding procedure to explore faster the space

Other paper include : Hu and Blanc (1998); Roggero and Hu (1998); Le Ravalec-Dupin et al. (2000) and Ravalec-Dupin Book's.

2 Basic Description

2.1 Gaussian related Stochastic models

$Z(\mathbf{x})$ is the physical property of interest at location $\mathbf{x}$. Let $Y(\mathbf{x})$ be a stationary standard Gaussian ($\mathcal{N}(0,1)$) random function. G and F are the distribution function of Y and Z respectively.

$$Z(\mathbf{x}) = F^{-1}G[Y(\mathbf{x})]$$

The idea behind the equation is the build the field from a standard normally distributed field. For exemple, if Z is the hydraulic conductivity (K), $F(Z)$ would be the $\log(K)$, and $G(Y)$ would simply be: $m + \sigma Y$. Such that :

$$Z(\mathbf{x}) = \exp(m + \sigma Y)$$

2.2 Gradual Deformation

The idea is to deform a field Y_1 into $Y(\theta)$ using another field Y_2 and a parameter θ. What is magical is that Y will preserve the std normal distribution of $Y1$. It's important to note that Y_1 and Y_2 are std normally distributed ($\mathcal{N}(0,1)$) (also called standard gaussian or white noise)

$$Y(\theta) = Y_1 \cos(\theta) + Y_2 \sin(\theta)$$

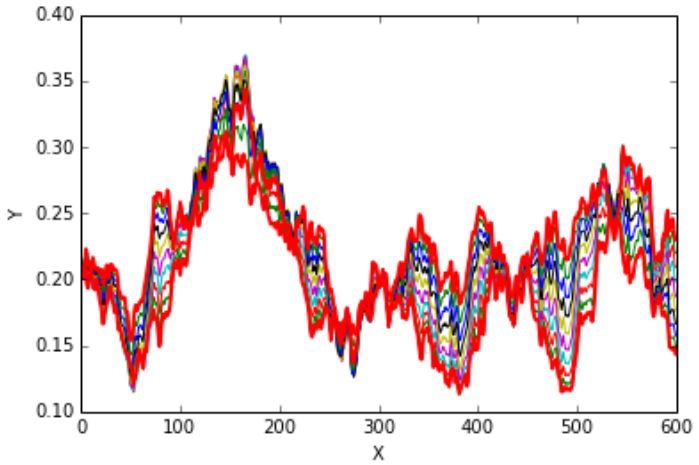

Figure 1: Exemple of 1D Gradual deformation. Thick red line are Y_1 and Y_2. Other line are deformation with various θ value

An graphical interpretation of gradual deformation is to view the equation as an ellipse equation, when Y dimension is two.

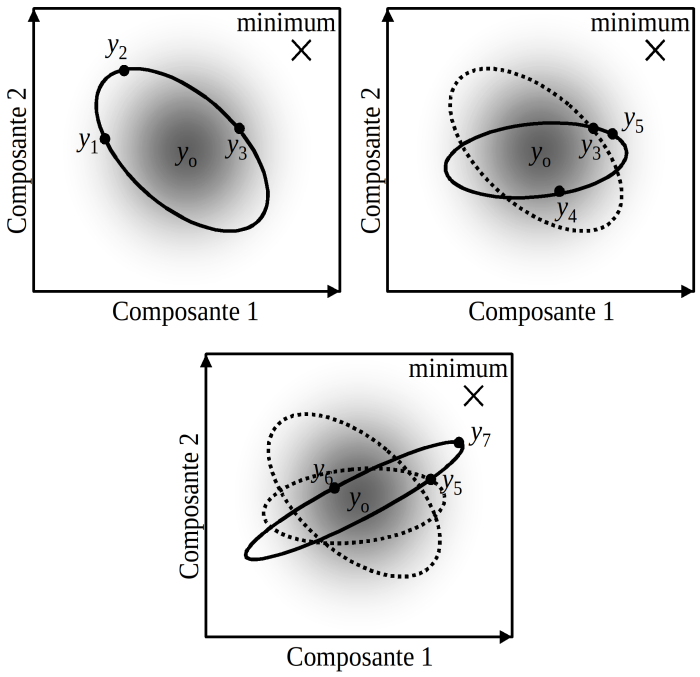

Figure 2: Graphical illustration of Gradual Deformation

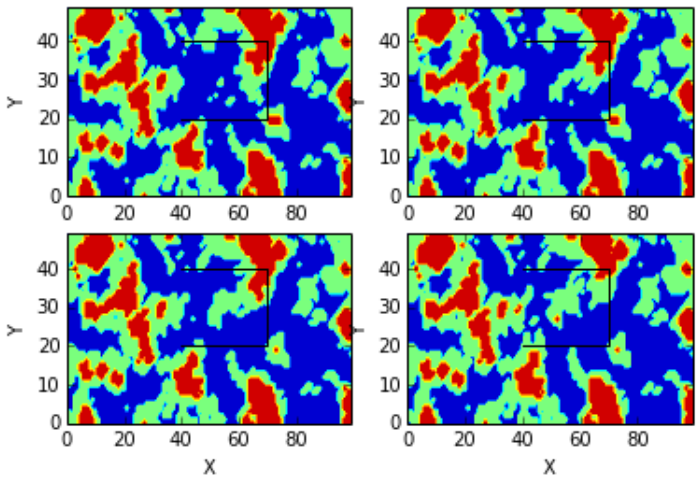

Figure 3: Exemple of 2D Local Gradual deformation.

3 Extension

3.1 Structural parameter

Often the hard data are not enough to give the structure (variance, covariance, mean...) and *a priori* distribution need to be assumed. The Trick here is to transform the field $Y(\theta)$ into a new field $X(\theta) = L[Y(\theta)]$ which impose the structure (covariance). Algo such as Cholesky (LU decomposition), FFT-MA or moving average can do that

3.2 Conditioning Kriging

Incorporation of "hard data" can done using conditional kriging. The integration of this data is on the field Y not Y_1 or Y_2

$$Y_c(\mathbf{x}) = Y_{dk}(\mathbf{x}) + [Y(\mathbf{x})Y_k(\mathbf{x})]$$

Where Y_c is the resulting simulation, Y_{dk} is the kriged data, Y the unconditionned simulation and Y_k the krigeage of the result of uncondition simulation (Z) at the known data point. Ying and Gomez-Hernandez, 2000 propose to combine directly conditional realization before deformation. (reframe in Hu (2002))

3.3 Multidimentional Gradual Deformation

see Hu (2000) or Le Ravalec-Dupin and Noetinger (2002). GDM can be extend to more than two gradual deformation.

$$Y(\theta_i, i \in [1,S]) = \prod_{i=1}^{S} Y_1 \cos(\theta_i) + \sum_{i=1}^{S} \sin(\theta_i) \prod_{j=i+1}^{S} Y_{i+1} \cos(\theta_j)$$

3.4 Indicator variable

can be used for binary variable...

3.5 Local and Regionalized

see Hu (2000).

3.6 Way to combine them

see Hu and Le Ravalec-Dupin (2004). GDM is often loop to improve at each iteration the solution. Three way of combining exist:

$$Y_{n+1}(\theta) = Y_n(\theta)\cos(\theta) + U_{new}\sin(\theta)$$

1. $Y_{n+1} = GDM(Y_n, U)$: (fig A)

2. $Y_{n+1} = GDM(Y_n, U_{1,2...})$: (fig B) use the Multidimentional GDM

3. $Y_{n+1} = GDM(Y_n, GBC(U_{1,2...}))$: (fig C). Use a linear combination of U_i (Gradient Based Compounding) such that the gradient search is the closest to the gradient search direction. This require the OF to be differentiable.

See graph to understand. The method on the right converge very quickly to a local optima without finding the whole space. The second method was then proposed, it uses multidimensional gradual deformation. It is also slow.

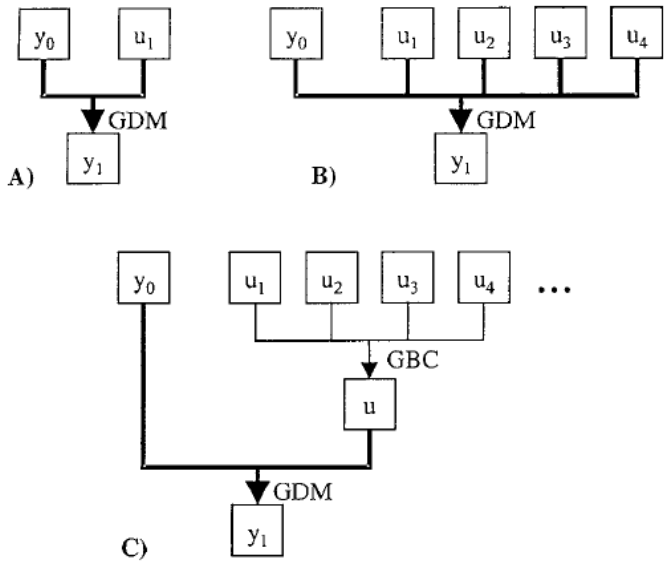

Figure 4: Three way to combine realizations

4 General Utilisation

4.1 History Matching

As usual, minimize an objectif function (OF)

$$J(\theta) = \frac{1}{p}\sum_{i=1}^{p} \omega_i (f_i(Z(Y(\theta))) - f_i^{obs})^2$$

where an function f depend on the field Z and therefore on the gaussian field deformated $Y(\theta)$. The optimization procedure is to find θ_{opt} which minimize O.

Efficient optimization procedure require the OF to be derivable in regard to θ (Gradient Based Method). Otherwise, other method such as Golden Search (require monotonic function). Hu (2000) suggest not to limit θ to $[0, \pi/2]$ (his period) as optimization search often exclude search at their limit (Golden search by instance).

4.2 Advance Optimization: Prior constraint

(see (Le Ravalec-Dupin and Noetinger, 2002)) This optimization problem is often ill posed (not one unique solution, measurement error, model error...). Extra information can be add to the OF to help convergence (Regularization, Neuman, 1973). The model (p) can have prior estimate p_0 to be constraint to. α is giving the relative importance of this prior estimate

$$J(p) = \sum_i \omega_i (f_i - f_i^{obs})^2 + \alpha \sum_i v_i (p_i - p_{0,i})^2$$

4.3 Iterative Calibration of Sequential (not necessary Gaussian) Simulation

see Hu et al. (2001). In sequential simulation, the order to visit each cell is randomly generated (U). At each location i, $z(u_i)$ is sampled from a distribution conditioned by the previously simulated point and hard data ($pdf(Z_i = z_i)$). In order make the sampling easier, cumulative distributed function (cdf) is build. z_i is obtain with $z_i = CDF^{-1}(v_i)$ where v_i is sampled in a uniform distribution [0,1]. From this point of view, sequential simulation can be view as transforming a uniform sampled points V into a field Z. As GDM require a std normally dist. field (Y) to be deform, V need to be generated from Y. This is easily done using the normal cumulative distribution function (normcdf). We end up with a similar optimization problem (calibration) as before : field Y to deform to get better OF.

4.4 Uncertainty

see Le Ravalec-Dupin and Noetinger (2002); Le Ravalec-Dupin et al. (2000)

4.5 Combining Dependant Realization

see Hu (2002). Y_i are still (multi-) gaussian random function. They have a covariance function $C(h)$ and their cross-covariance function is proportional to $C(h)$

$$C_{ij}(h) = E[Y_i(x)Y_j(x+h)] = r_{ij}C(h)$$

where r_{ij} stand for the correlation coef. between Y_i and Y_j. The covariance of a linear combinaison $Y(x) = \sum_i \alpha_i Y_i$ will therefore be :

$$\begin{aligned} C_Y(h) &= E[Y(x)Y(x+h)] \\ &= E\left[\left(\sum_i \alpha_i Y_i(x)\right)\left(\sum_j \alpha_j Y_j(x+h)\right)\right] \\ &= E\left[\left(\sum_i \sum_j \alpha_i\alpha_j Y_i(x)Y_j(x+h)\right)\right] \\ &= C(h)\sum_{i,j} r_{ij}\alpha_i\alpha_j \end{aligned}$$

To maintain the same covariance, we need $\sum_{i,j} r_{ij}\alpha_i\alpha_j = 1$. This can be shown to be a generalisation of GDM for dependant realization. One idea in his paper was to transform each realization to force a mean of zero and standard deviation of 1 : $Y_i^* = \frac{Y_i - \bar{Y}_i}{\sigma_{Y_i}}$

5 Limitation

- 1st order stationary (mean cst) required
- Only applicable for Gaussian-related stochastic model
- if search long enough : global minima is achieved.
- In theory, spacial structure is preserve. But in practise, because each simulation is not perfect (exact mean, variance and covariance) spatial statistical structure is not preserve. This become a serious problem when the number of realisation is higher than the number of grid cells (see Hu (2002))
- Preserve only variogram structure (covariance) not multi-point
- for linear problems, GDM's optimizations converge exponentially to the global minimum but not for non linear.
- Something about getting accurate estimate... using a prior constraint Le Ravalec-Dupin and Noetinger (2002)

6 To go futher

- Grdual Pilote Point (book)
- Probability Perturbation Method

References

De Marsily, G., Lavedan, G., Boucher, M., and Fasanino, G. (1984). Interpretation of interference tests in a well field using geostatistical techniques to fit the permeability distribution in a reservoir model. *Geostatistics for natural resources characterization, Part*, 2:831–849.

Deutsch, C. V. (1993). Conditioning reservoir models to well test information. In *Geostatistics Tr{ó}ia92*, pages 505–518. Springer.

Hu, L. Y. (2000). Gradual deformation and iterative calibration of Gaussian-related stochastic models. *Mathematical Geology*, 32(1):87–108.

Hu, L. Y. (2002). Combination of dependent realizations within the gradual deformation method. *Mathematical Geology*, 34(8):953–963.

Hu, L. Y. and Blanc, G. (1998). Constraining a reservoir facies model to dynamic data using a gradual deformation method. In *6th European Conference on the Mathematics of Oil Recovery*.

Hu, L. Y. and Le Ravalec-Dupin, M. (2004). An improved gradual deformation method for reconciling random and gradient searches in stochastic optimizations. *Mathematical Geology*, 36(6):703–719.

Hu, L. Y., Le Ravalec-Dupin, M., and Blanc, G. (2001). Gradual deformation and iterative calibration of truncated {Gaussian} simulations. *Petroleum Geoscience*, 7(4):25–30.

Journel, A. G. and Huijbregts, C. J. (1978). *Mining geostatistics*. Academic press.

Le Ravalec-Dupin, M. and Noetinger, B. (2002). Optimization with the gradual deformation method. *Mathematical Geology*, 34(2):125–142.

Le Ravalec-Dupin, M., Noetinger, B., and Hu, L. Y. (2000). The FFT moving average (FFT-MA) generator: An efficient numerical method for generating and conditioning Gaussian simulations. *Mathematical Geology*, 32(6):701–723.

Oliver, D. S., Cunha, L. B., and Reynolds, A. C. (1997). Markov chain Monte Carlo methods for conditioning a permeability field to pressure data. *Mathematical Geology*, 29(1):61–91.

Roggero, F. and Hu, L. Y. (1998). Gradual deformation of continuous geostatistical models for history matching. In *SPE annual technical conference*, pages 221–236.

Srivastava, R. M. and Others (1994). The interactive visualization of spatial uncertainty. *SPE*, 27965:87–95.