Kalman-Filter

1 Introduction

1.1 Historical Development

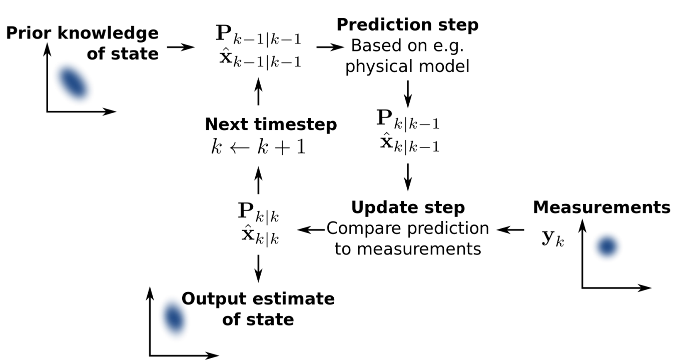
- The filter is named after Hungarian Rudolf E. Klmn, although Thorvald Nicolai Thiele and Peter Swerling developed a similar algorithm earlier.
- Stanley F. Schmidt is credited for the first implementation of Kalman-filter to the problem of trajectory estimation for the Apollo program.

1.2 Overview

Kalman filtering, (or linear quadratic estimation (LQE)), operates recursively on streams of noisy input data to produce a statistically optimal estimate of the underlying system state.

The algorithm works in a two-step process:

1. *Prediction step*: the Kalman filter produces estimates of the current state variables $\hat{\mathbf{x}}_{k|k-1}$, along with their uncertainties $\hat{\mathbf{P}}_{k|k-1}$.
2. *Update step*: Once the outcome of the next measurement ($\hat{\mathbf{y}}_k$) is observed, these estimates are updated ($\hat{\mathbf{x}}_{k|k}$ and $\hat{\mathbf{P}}_{k|k}$).



2 Kalman-Filter

2.1 Description

The Kalman filter model assumes the true state to be based on linear dynamic systems discretized in the time domain:

$$\mathbf{x}_k = \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{B}_k \mathbf{u}_k + \mathbf{w}_k$$

where:

- $\mathbf{F}_k$ is the state transition model.
- $\mathbf{B}_k$ is the control-input model.
- $\mathbf{u}_k$ is the control vector.
- $\mathbf{w}_k$ is the process noise of covariance $\mathbf{Q}_k$. $\mathbf{w}_k \sim \mathcal{N}(0, \mathbf{Q}_k)$

At time k, observation of $\mathbf{x}_k$ is mad according to

$$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k$$

where:

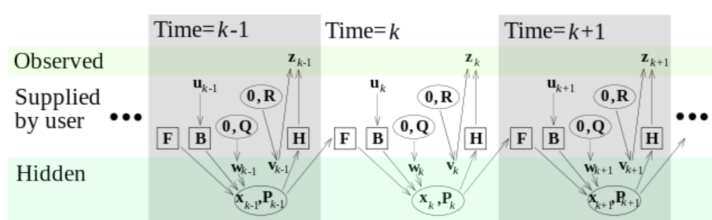
- $\mathbf{H}_k$ is the observation model
- $\mathbf{v}_k \sim \mathcal{N}(0, \mathbf{R}_k)$ is the observation noise of covariance $\mathbf{R}_k$.

1. *Prediction step*:

A priori state estimate	$\hat{\mathbf{x}}_{k|k-1} = \mathbf{F}_k \hat{\mathbf{x}}_{k-1|k-1} + \mathbf{B}_k \mathbf{u}_k$
A priori estimate cov.	$\mathbf{P}_{k|k-1} = \mathbf{F}_k \mathbf{P}_{k-1|k-1} \mathbf{F}_k^{\mathrm{T}} + \mathbf{Q}_k$

2. *Update step*:

Measurement residual	$\tilde{\mathbf{y}}_k = \mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_{k|k-1}$
Residual covariance	$\mathbf{S}_k = \mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^T + \mathbf{R}_k$
Optimal Kalman gain	$\mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k^T \mathbf{S}_k^{-1}$
A posteriori state estimate	$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \tilde{\mathbf{y}}_k$
A posteriori estimate cov.	$\mathbf{P}_{k|k} = (I - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k|k-1}$



2.2 Practical Application

- Unknown stochastic signals as input can seriously degrade the filter performance. Robust Control is then used.
- One major practical challenge is finding the covariance of the noise ($\mathbf{Q}_k$ and $\mathbf{R}_k$). One of the more promising approaches is the Autocovariance Least-Squares (ALS) technique that uses the time-lagged autocovariances of routine operating data.

3 Extension

3.1 Recursive Bayesian Estimation

Recursive Bayesian estimation, also known as a Bayes filter, is a general probabilistic approach for estimating an unknown probability density function recursively over time using incoming measurements and a mathematical process model.