Hydrogeologie

1 Porosity

$V_t = V_s + V_p$	Total volume
$V_p = V_a + V_w$	Pore volume
$\Phi = V_p/V_t$	Total porosity
$S = V_w/V_p$	Water saturation
$\theta = V_w/V_t$	water content
$\theta = \Phi S$	
$S_e = \frac{\theta-\theta_s}{\theta_r-\theta_s}$	Effective saturation
$r_h = \frac{A}{P}$	Hydraulic radius
$T = \left(\frac{L_a}{L}\right)^2$	Tortuosity
$S_p = \frac{S}{m}$	Specific surface area

with,

A cross sectional area of flow
P wetted perimeter
F Electrical formation factor
S Surface area
m masse

- The porosity generated at the genesis of the rock (organogensis, clastic sedimentation) is called "Primary" and the one resulting from its geological history as the "secondary" (eg. tectonic or chemical process).
- The interconnected porosity consider only connected pore (electrical current and fluid can flow amongst them). Not very well define: depend on the scale, and gradual connection...
- Effective porosity is related to pore allowing free fluids.
- Influence of grain properties: (1) $d_{50} \nearrow \Rightarrow \Phi \searrow$ (theoretical independant), (2) grain sorting $\nearrow \Rightarrow \Phi \nearrow$ and (3) rounder grain $\nearrow \Rightarrow \Phi \searrow$
- Porosity increase with depth (or pressure to be more precise). Often model with an exponential equation $\Phi(z) = \Phi_0 \exp -Bz$

2 Hydraulic Conductivity

Hydraulic conductivity has first be discover with the very famous Darcy's law which found a propotionallity bewteen the discharge and the head difference:

$$Q = K\frac{\Delta P}{\Delta L}$$

It has since be derived from Navier-Stokes equation :

$$\boldsymbol{q} = -\frac{\kappa}{\mu}\left(\boldsymbol{\nabla} p - \rho \boldsymbol{g}\right)$$

Note κ is the intrinsic permeability of the medium [L^2] while K is the hydraulic conductivity depending on the fluid [L/T] and can be related with:

$$K = \frac{\kappa \rho_w g}{\mu}$$

This is for saturated medium otherwise, the saturated and relative K are used:

$$K = K_r K_s$$

Allen Hazen (1893?) derived an empirical formula:

$$K_s = C d_{10}$$

where d_{10} is the effective grain size for which 10% is finer
Navier Stockes Equations Hagen-Poiseuille Equation Couette Flow
The Kozeny-Carman equation comes from Poiseuille Equation

$$v = \frac{\Delta(P)}{\eta L}\frac{\phi^3}{KS^2(1-\phi)^2}$$

And where:

$$\kappa = \frac{d^2}{180}\frac{\phi^3}{(1-\phi)^2}$$

where d is Grain size.
a Tube factor shape.
F Electrical formation factor

3 Capillary Equations

Young-Lapace Equation:

$$\Delta p = -\gamma \dot{\nabla n} = 2\gamma H$$

Jurin's Law

$$\psi = -\frac{2\gamma\cos\theta}{\rho_w g r}$$

γ surface tension.
H mean curvature.
ψ capillary head.
θ contact angle.
r radius of the tube

4 Soil moisture characteristic

Van Genuchten Equation (1980):

$$S_e = \frac{1}{[1+\alpha\psi^n]^m}$$

Mualem's Equation (1976)

$$K_r = \sqrt{S_e}\left[\frac{\int_0^{S_e} dS_e/|\psi|}{\int_0^1 dS_e/|\psi|}\right]^2$$

γ surface tension.
H mean curvature.
ψ capillary head.
θ contact angle.
r radius of the tube

5 Electrical Properties

5.1 General definition

$$\sigma^* = \frac{1}{\rho^*} = i\omega\varepsilon^*$$

$$\sigma^* = |\sigma|e^{i\phi_\sigma} = \sigma' + i\sigma''$$

$$\varepsilon = \varepsilon_r\varepsilon_0 = (1+\chi)\varepsilon_0$$

$$\kappa = \varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

5.2 Permittivity

Refractive Index model (RI) for sat. and unsat.

$$\sqrt{\varepsilon_{r,eff}} = n\sqrt{\varepsilon_{r,w}} + (1-n)\sqrt{\varepsilon_{r,s}}$$

$$\sqrt{\varepsilon_{r,eff}} = \theta\sqrt{\varepsilon_{r,w}} + (n-\theta)\sqrt{\varepsilon_{r,a}} + (1-n)\sqrt{\varepsilon_{r,s}}$$

5.3 Electrical Conductivity

Archie's empirical law (1942) for sat. and unsat.

$$\sigma_{eff,sat} = \frac{\sigma_w}{F}(+\sigma_{surface})$$

$$\sigma_{surface} = \begin{cases} \frac{BQ_v}{F} & \begin{cases} B = \alpha[1-\beta\exp(-\gamma\sigma_w)] \\ B = 1.93m/(1+0.7/\sigma_m) \end{cases} \\ \frac{\Sigma_s S_p}{f} \end{cases}$$

$$\sigma_{eff} = \sigma_{eff,sat} S^d$$

$$\sigma_{eff} = \begin{cases} \sigma_{eff,sat} S^d & \text{for } S > 0.2 \\ \sigma_w n^m S^d & \text{if } \sigma_{surface} = 0 \\ (\sigma_w + BQ_v/S)\frac{S^d}{F} & \text{(Waxman and Smits,1968)} \\ \sigma_w \theta^m & \text{for/if } d = m, \sigma_{surface} = 0 \\ \sigma_w \theta T_c(\theta) + \sigma_{surface} & Tc(\theta) = a\theta + b \end{cases}$$

In a homogenous soil with a one-point current

$$V = \frac{\rho I}{2\pi r}$$

5.4 Induced Polarisation

$\phi_\rho = \tan^{-1}(\rho''/\rho')$	CR phase.
$PFE = 100\frac{\rho(\omega_1)-\rho(\omega_0)}{\rho(\omega_0)}$	percent frequency effect
$M = \frac{V_s}{V_p} = \frac{1}{V_p(t_1-t_0)}\int_{t_0}^{t_1} V(t)dt$	chargeability

$$\sigma^* = \frac{1}{F}(\sigma_w + BQ_v) + i\frac{\lambda Q_v}{Fn}$$

Relation KC type where S_p is replace with a power law of σ''

$$K_s = \frac{a}{FS_p^c} = \frac{a}{F(b\sigma''^p)^c}$$

Or Hazen type

$$K_s = a(\sigma'')^b$$

5.5 Coomplex Conductivtity

Havriliak-Negami (HN)

$$\varepsilon^*(\omega) = \left(\varepsilon_\infty + \frac{\sigma_{dc}}{i\omega}\right)\frac{\Delta\varepsilon}{\left(1+(i\omega\tau_0)^{1-\alpha}\right)^\beta}$$

Constnat Phase-Angle (CPA)

$$\sigma^*(\omega) = (\sigma_{dc} + i\omega\varepsilon_\infty) + \sigma_0(\omega/\omega_0)^p e^{ip\pi/2}$$

The Bruggeman-Hanai-Sen (BHS) effective medium model

$$\varepsilon^* = \varepsilon_w^* n^m \left(\frac{1-\varepsilon_s^*/\varepsilon_w^*}{1-\varepsilon_s^*/\varepsilon^*}\right)^m$$

$$\varepsilon_w^* = \varepsilon_{r,w}\varepsilon_0 + i\omega\sigma_w$$

5.6 Legend

$\sigma^*, \lvert\sigma\rvert, \phi_\sigma, \sigma', \sigma''$	conductivity (complex).
$\rho^*, \lvert\rho\rvert, \phi_\rho, \rho', \rho''$	complex resistivity.
$\varepsilon^*_{,r,0}$	permittivity (relative, vaccum).
$\kappa = \varepsilon_r$	dielectric constant
$\omega = 2\pi f = 1/\tau$	angular frequency, relaxation time
χ	electric susceptibility.
$F = T/n = n^{-m}$	Electrical formation factor
Q_v	cation exchange capacity (clay content)
B	Ionic conductance of Q_v
f	parameter of tortuosity
m	cementation index (1.3¡2.0)
Σ_s	specific surface conductance
f	parameter of tortuosity (F)
d	saturation index (usally ¿m)
λ	effective img. conductance
$\Delta\varepsilon = \varepsilon_{static} - \varepsilon_\infty$	dielectric increment

6 How to get to Poisson Law

6.1 Basic equation

$\boldsymbol{F_1} = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\boldsymbol{\hat{r}_{21}}}{\lvert\boldsymbol{r_{21}}\rvert^2}$,	Coulomb's law
$\mathbf{E} \equiv \frac{\mathbf{F}_q}{q}$	Definition of electric field
$V = -\int_C \mathbf{E}\cdot d\boldsymbol{\ell}$	Electrical potential
$I = \frac{dq}{dt}$	Intensity or current
$R = V/I$	Ohm's law for resistance
$I = \int \mathbf{J}\cdot d\mathbf{A}$	Current Flux
$\mathbf{J} = \sigma\mathbf{E}$	Reformulation of Ohm's law
$R = \rho\frac{\ell}{A}$	Pouillet's Law
$\Phi_E = \iint_S \mathbf{E}\cdot d\mathbf{S}$	Definition of Electric Flux
$\lambda_q = \frac{dq}{d\ell}, \sigma_q = \frac{dq}{dS}, \rho_q = \frac{dq}{dV}$	Density charges
$\boldsymbol{D} = \varepsilon_0\boldsymbol{E} + \boldsymbol{P}$	Electric displacement field

6.2 From Coulomb to Poisson

From Coulomb's law and the definition of Electric field

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0 r^2}$$

And we get Gauss Law

$$\Phi_E = \oint\!\!\!\oint_S \frac{q}{4\pi\varepsilon_0 r^2}\cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$

And Using the Divergence Theorem (in/out = change inside), we can get the differential form of Gauss Law

$$\iiint_V (\nabla\cdot\mathbf{E})\, dV = \oint\!\!\!\oint_S (\mathbf{E}\cdot\mathbf{n})\, dS.$$

$$\nabla\cdot\mathbf{E} = \frac{\rho_q}{\varepsilon_0}$$

An finally with $\mathbf{E} = -\nabla\cdot V$, we get Poisson's equation

$$\nabla^2 V = -\frac{\rho_q}{\varepsilon_0}$$

Continuity equation apply to charge density :

$$\nabla\cdot\mathbf{J} = -\frac{\partial\rho_q}{\partial t}$$

And with $I = \frac{dq}{dt}$, $\mathbf{J} = \sigma\mathbf{E}$ and with linear charge density $\rho_q = \frac{dq}{dV}$:

$$\nabla \cdot (\sigma \mathbf{E}) = -\frac{\partial}{\partial t}\left(\frac{dq}{dV}\right)$$

With a constant (over time) point charge q at the origine $\rho_q(r) = q\delta(r)$

$$\nabla \cdot (\sigma \nabla V) = -I\delta(r)$$

7 Electromagnetism

7.1 Basic definition

$\mathbf{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\boldsymbol{\ell}\times\hat{\mathbf{r}}}{r^2}$	Magnetic Field (Biot-Savart law)
$\mathbf{F} = q\mathbf{v}\times\mathbf{B}$	Lorentz force
$\mathbf{v}$	Particle's velocity
μ_0	Magnetic constant
$\Phi_B = \iint \mathbf{B}\cdot d\mathbf{S}$	Magnetic flux
$\mathcal{E} = -\frac{d\Phi_\mathrm{B}}{dt}$	Electromotive force (EMF) (Faraday Law)
$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$	Maxwell-Faraday equation

7.2 Ampere law

Ampere's law relates magnetic fields to electric currents that produce them:

$$\oint_C \mathbf{B}\cdot \mathrm{d}\boldsymbol{\ell} = \mu_0 \iint_S \mathbf{J}\cdot \mathrm{d}\mathbf{S} = \mu_0 I_\mathrm{enc} \quad \text{or} \quad \nabla\times\mathbf{B} = \mu_0\mathbf{J}$$

Current in a material is affected by magnetization (electrons remain bound to their respective atoms, but behave as if they were orbiting the nucleus in a particular direction, creating a microscopic current) and polarization (positive and negative bound charges can separate over atomic distances).

$$\mathbf{J} = \mathbf{J}_\mathrm{f} + \mathbf{J}_\mathrm{M} + \mathbf{J}_\mathrm{P}$$

In order to take this effect to account and only count for the free current, the H magnetic field is introduced $\mathbf{H} \equiv \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$ with (M: magnetization). And Ampere Law become :

$$\oint_C \mathbf{H}\cdot \mathrm{d}\boldsymbol{\ell} = \iint_S \mathbf{J}_\mathrm{f}\cdot \mathrm{d}\mathbf{S} = I_\mathrm{f,enc} \quad \text{or} \quad \nabla\times\mathbf{H} = \mathbf{J}_\mathrm{f}$$

7.3 Issue with Ampere law and derivation of MaxwellAmpre equation

In vector calculus, $\nabla\cdot(\nabla\times\mathbf{B}) = 0$ therefore according Ampere law $\nabla\cdot\mathbf{J} = 0$. But in genearale $\nabla\cdot\mathbf{J} = -\frac{\partial\rho}{\partial t}$. The Displacement current ($J_\mathbf{D} = \frac{\partial\mathbf{D}}{\partial t}$) was added by Maxwell :

$$\oint_C \mathbf{H}\cdot \mathrm{d}\boldsymbol{\ell} = \iint_S \left(\mathbf{J}_\mathrm{f} + \frac{\partial}{\partial t}\mathbf{D}\right)\cdot \mathrm{d}\mathbf{S} \quad \text{or} \quad \nabla\times\mathbf{H} = \mathbf{J}_\mathrm{f} + \frac{\partial}{\partial t}\mathbf{D}$$

$$\nabla\times\mathbf{B} = \left(\mu_0\mathbf{J} + \mu_0\epsilon_0\frac{\partial}{\partial t}\mathbf{E}\right)$$

7.4 hello

CSEM main equation result by combining Ampere-Maxwell, Ohm and Faraday laws into the damped wave equation

$$\nabla^2\mathbf{B} - \mu_0\sigma\frac{\partial\mathbf{B}}{\partial t} - \mu_0\epsilon\frac{\partial^2\mathbf{B}}{\partial t^2} = \mu_0\nabla\times\mathbf{J}_S$$

where

8 Seismology

A	Amplitude
$\lambda = 1/k$	Wavelength (1/wave nb) (distence)
$T = 1/f$	Period (1/freq.)(time)
$V = f\lambda = 1/Tk$	Propagation speed
$\frac{\partial^2 u}{\partial x^2} = \frac{1}{V^2}\frac{\partial^2 u}{\partial t^2}$	Wave equation
$V_p = \sqrt{\frac{K+4/3}{\rho}}$	P-wave velocity
$V_s = \sqrt{\frac{\mu}{\rho}}$	S-wave velocity
$K = \rho\frac{\mathrm{d}P}{\mathrm{d}\rho}$	bulk modulus
ρ	density
E	Module d'elasticity
$\sigma = E\varepsilon$	Module de traction
$\mu = G_s\ \varepsilon$	Shear modulus
$\nu = \frac{(l_0-l)/l_0}{(L-L_0)/L_0}$	Coefficient poisson
$\mu = \frac{E}{2(1+\nu)}$.	

9 Geostat

9.1 Basic Definition

$\Pr[X\le x] = F_X(x) = \int_{-\infty}^{x} f_X(u)\,du$	Prob. and cum. density fu
$\mathrm{E}[X] = \int_{-\infty}^{\infty} x\,f(x)\,dx.$	Expected value
$\mathrm{Var}(X) = \mathrm{E}\left[(X-\mu)^2\right]$	Variance
$\mathrm{Cov}(X,Y) = \mathrm{E}\left[(X-\mathrm{E}[X])(Y-\mathrm{E}[Y])\right]$	Covariance
$2\gamma(h) = \mathrm{var}[Z(x) - Z(x+h)]$	Variogramme
$C(h) = \mathrm{cov}(Z(x), Z(x+h)).$	Covariogramme
$C(0) = C(h) - \gamma(h)$	Variance