Electricity in the Soil

1 Electrical Law

1.1 Basic Definition

$\boldsymbol{F_1} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\boldsymbol{\hat{r}_{21}}}{\lvert\boldsymbol{r_{21}}\rvert^2}$,	Coulomb's law
$\mathbf{E} \equiv \frac{\mathbf{F}_q}{q}$	Definition of electric field
$V = -\int_C \mathbf{E} \cdot \mathrm{d}\boldsymbol{\ell}$	Electrical potential
$I = \frac{dq}{dt}$	Intensity or current
$R = V/I$	Ohm's law for resistance
$I = \int \mathbf{J} \cdot d\mathbf{A}$	Current Flux
$\mathbf{J} = \sigma\mathbf{E}$	Reformulation of Ohm's law
$R = \rho\frac{\ell}{A}$	Pouillet's Law
$\Phi_E = \iint_S \mathbf{E} \cdot d\mathbf{S}$	Definition of Electric Flux
$\lambda_q = \frac{dq}{d\ell}, \sigma_q = \frac{dq}{dS}, \rho_q = \frac{dq}{d}$	Density charges
$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P}$	Electric displacement field

1.2 From Coulomb to ...

From Coulomb's law and the definition of Electric field

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0 r^2}$$

And we get Gauss Law

$$\Phi_E =_S \frac{q}{4\pi\varepsilon_0 r^2} \cdot \mathrm{d}\mathbf{S} = \frac{q}{\varepsilon_0}$$

And Using the Divergence Theorem (in/out = change inside), we can get the differential form of Gauss Law

$$\iiint_V (\nabla \cdot \mathbf{E})\, dV =_S (\mathbf{E} \cdot \mathbf{n})\, dS.$$

$$\nabla \cdot \mathbf{E} = \frac{\rho_q}{\varepsilon_0}$$

An finally with $\mathbf{E} = -\nabla \cdot V$, we get Poisson's equation

$$\nabla^2 V = -\frac{\rho_q}{\varepsilon_0}$$

Continuity equation apply to charge density :

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_q}{\partial t}$$

And with $I = \frac{dq}{dt}$, $\mathbf{J} = \sigma\mathbf{E}$ and with linear charge density $\rho_q = \frac{dq}{dV}$:

$$\nabla \cdot (\sigma\mathbf{E}) = -\frac{\partial}{\partial t}\left(\frac{dq}{d}\right)$$

With a constant (over time) point charge q at the origine $\rho_q(r) = q\delta(r)$

$$\nabla \cdot (\sigma\nabla V) = -I\delta(r)$$