

# A Prediction Model of Humidity Sensor Response

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**Abstract –** Many modern algorithms that process sensor information need both current sensor output values and some prediction of these values in future moments. In the current work, an approach to building a predictive low order model of the SHT31-DIS humidity sensor is discussed. Assuming that the sensor can be presented with a standard first-order output error model plus delay, it is shown that the measurements obtained from the sensor can be predicted one step ahead with an error of less than 5%RH. When estimating the parameters of the model, a mean square criterion and measurements traceable to the standard relative humidity scale OIML R 121 were used.

**Keywords –** identification, humidity sensor, output error model, prediction model, step response.

## I. INTRODUCTION

The Kalman filter is one of the most well-known state estimators, and many studies and publications have been dedicated to it [1,2]. So it is no surprise that it can be found in many applications used to track process or object variables such as temperature, pressure, humidity, etc. [3-6].

Figure 1 shows a simplified block diagram of the algorithm implementing the filter, and Table 1 contains the equations that describe the separate steps in the linear case. The Kalman filter loop has two main phases: prediction and correction. During the first phase, a model of the observed process is used to calculate the new a priori state (prediction)  $\hat{x}(k|k-1)$ . The correction phase takes the measurement  $y(k)$  and corrects the prediction.

When drawing the filter equations, it is assumed that the measurement  $y(k)$  is obtained using the non-inertial (zero order) linear sensor. However, real sensors have a finite response time and cannot react immediately to a sudden change in the observed quantity. Ignoring the dynamics of the sensor and the resulting change in the measurement  $y(k)$  can have a significant effect on the estimation process. To solve this problem, the Kalman filter must be augmented with an appropriate prediction model of the sensor response. In this paper, we consider an approach to determining such a first-order model of a sensor used for monitoring the air humidity.

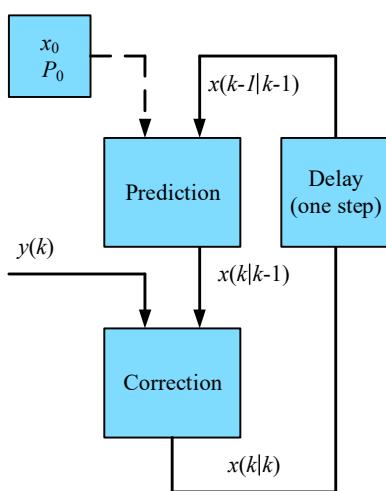


Fig. 1. The Kalman filter algorithm.

TABLE 1. KALMAN FILTER EQUATIONS.

Phases	Equations
Prediction	$\hat{x}(k k-1) = F\hat{x}(k-1 k-1)$ $P(k k-1) = FP(k-1 k-1)F^T + Q$ $E(k) = y(k) - H\hat{x}(k k-1)$
Correction	$K(k) = P(k k-1)H^T(HP(k k-1)H^T + R)^{-1}$ $\hat{x}(k k) = \hat{x}(k k-1) + K(k)E(k)$ $P(k k) = (I - K(k)H)P(k k-1)$

## Variables of the equation

- $k$  - discrete time
- $\hat{x}(k|k)$  - a posteriori state estimate
- $\hat{x}(k|k-1)$  - predicted state estimate
- $P(k|k)$  - a posteriori estimate covariance matrix
- $P(k|k-1)$  - predicted estimate covariance matrix
- $F$  - state transition matrix
- $H$  - observation matrix
- $K(k)$  - Kalman gain
- $E(k)$  - Innovation
- $Q$  - covariance of the process noise
- $R$  - covariance of the observation noise
- $I$  - identity matrix
- $x_0, P_0$  - initial state and covariance

## II. INPUT-OUTPUT MODELLING

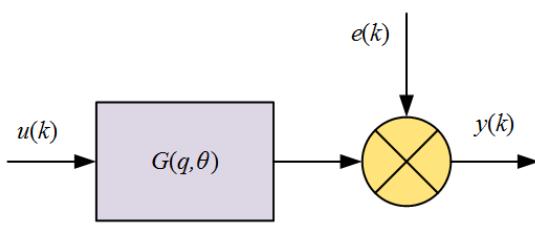


Fig. 2. Output error model of humidity sensor.

In this work, it is accepted that a linear discrete model with output error (OE) structure can present the behaviour of the humidity sensor (HS):

$$y(k) = G(q, \theta)u(k) + e(k) \quad (1)$$

where  $u(k)$  is the relative humidity,  $y(k)$  is the relative humidity measured by the sensor,  $G(q, \theta)$  is a discrete transfer function,  $q$  is a backward shift operator  $q^{-1}u(k) = u(k-1)$ ,  $e(k)$  is white noise with zero mean value accounting random changes in the output. The block diagram of the sensor described by equation (1) is depicted in Figure 2. The discrete transfer function  $G(q, \theta)$  is defined by the equation:

$$G(q, \theta) = \frac{(b_0 + b_1 q^{-1} + \dots + b_{n_b} q^{-n_b})q^{-n_k}}{1 + f_1 q^{-1} + \dots + f_{n_f} q^{-n_f}} \quad (2)$$

where  $f_1, \dots, f_{n_f}$  and  $b_0, b_1, \dots, b_{n_b}$  are the parameters of the model,  $\theta = (f_1, \dots, f_{n_f}, b_0, b_1, \dots, b_{n_b})$  is the parameter vector,  $n_b$  and  $n_f$  indicate the order of the polynomials in the numerator and denominator respectively. The parameter  $n_k$  is the time delay between output  $y(k)$  and input  $u(k)$ .

One of the main advantages of the OE structure compared to ARX and BJ is the ability to transform equation (1) directly into a prediction model. Let  $\hat{y}(k|k-1)$  denote the estimate of the sensor output  $y(k)$  one-step ahead, and  $U^{k-1} = \{u(1), u(2) \dots u(k-1)\}$  and  $Y^{k-1} = \{y(1), y(2) \dots y(k-1)\}$  are the input-output data up to time  $(k-1)$ . The best estimate  $\hat{y}(k|k-1)$  in the mean-square sense is given by the equation:

$$\hat{y}(k|k-1) = \arg \min_{\hat{y}} E[(\hat{y} - y)^2 | U^{k-1}, Y^{k-1}]$$

It can easily find the minimum by setting the derivative

$$\begin{aligned} \frac{d}{d\hat{y}} E[(\hat{y} - y)^2 | U^{k-1}, Y^{k-1}] \\ = E[2(\hat{y} - y) | U^{k-1}, Y^{k-1}] \\ = 2(\hat{y} - E[y | U^{k-1}, Y^{k-1}]) \end{aligned}$$

to zero:

$$\hat{y}(k|k-1) = E[y | U^{k-1}, Y^{k-1}] \quad (3)$$

According to (3), the estimate  $\hat{y}(k|k-1)$  is determined by the conditional expectation of the output, given  $U^{k-1}$  and  $Y^{k-1}$ . From (1) and (3), taking into account that  $e(k)$  is white

noise with zero mean  $E[e(k)|U^{k-1}, Y^{k-1}] = 0$ , we get a model predicting the output of the sensor one-step ahead:

$$\hat{y}(k|k-1) = G(q, \theta)u(k) \quad (4)$$

The Kalman filter is a data processing algorithm with computational complexity  $O(n^3)$  [7,8]. Since the obtained step responses exhibit no overshoot, this naturally raises the question of representing the sensor behavior by a standard first-order linear model.

$$G(q, \theta) = \frac{b_0 q^{-n_k}}{1 + f_1 q^{-1}}, n_k = 1, 2, \dots \quad (5)$$

The solution of this problem is of great practical importance in the context of the computational complexity of the algorithm implementing the Kalman filter. Any increase in the order of the sensor model (respectively the dimension of the state vector) would lead to a serious increase in the number of computational operations that must be performed.

From (1) and (5) for the prediction model of the HS it can be written:

$$\hat{y}(k) + f_1 \hat{y}(k-1) = b_0 u(k - n_k) \quad (6)$$

In the steady state, the final value  $u_s$  of the input variable and the final value  $y_s$  of the output variable in (6) remain unchanged:

$$\begin{aligned} y_s &= \hat{y}(k) = \hat{y}(k-1) \\ u_s &= u(k - n_k) \end{aligned} \quad (7)$$

and immediately from (6) and (7) the equation describing the relationship between them is obtained:

$$y_s = \frac{b_0}{1 + f_1} u_s \quad (8)$$

## III. EXPERIMENTAL DATA

The general practice of most manufacturers is to present the dynamics of HS with transient characteristics obtained by a step change in humidity [9]. Following this approach, experiments were organized and performed in which step responses of humidity sensor SHT 31-DIS were taken [10]. One part of the recordings represent the response of the sensor when the humidity increases from 12% RH to 59% RH, and another when it decreases from 59% RH to 12% RH. The duration of each recording is about 53 min, enough time for the output value of the HS to reach 99,5% of its final value. Figure 3 shows two of the characteristics obtained, and further information on how this part of the study was conducted can be found in [11].

An important element of the pre-processing of experimental data is to bring them to the standard scale of relative humidity described in [12]:

$$\begin{aligned} u^*(k) &= \frac{u(k) - u_b}{u_e - u_b} (A - B) + B, \%PH \\ y^*(k) &= \frac{y(k) - y_b}{y_e - y_b} (A - B) + B, \%PH \end{aligned} \quad (9)$$

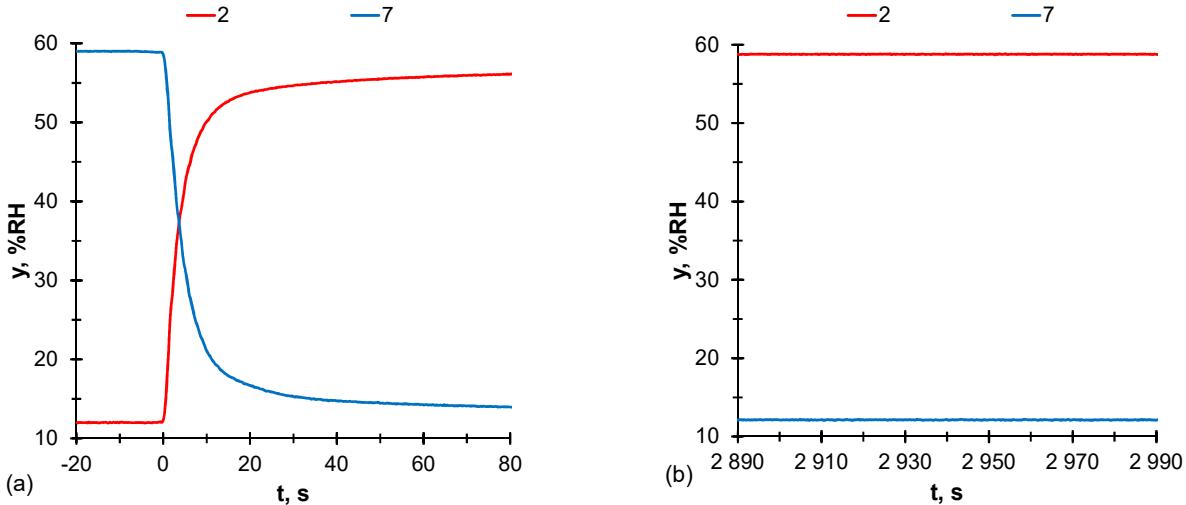


Fig. 3. The beginning (a) and end (b) of the step response of the humidity sensor. The red and blue curves represent the results obtained during the second and seventh experiments.

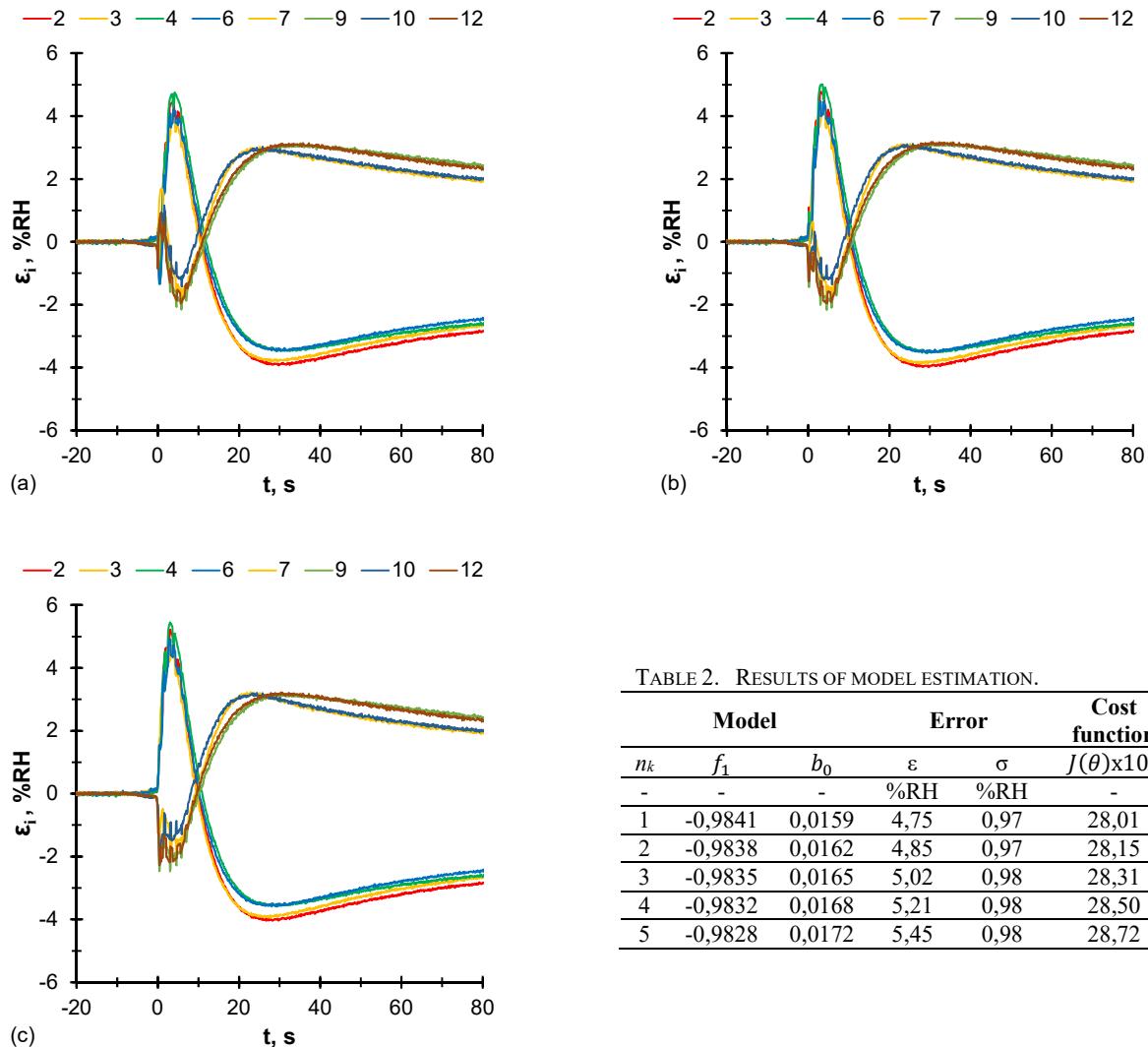


Fig. 4. Absolute prediction error for different realizations and  $n_k = 1$  (a),  $n_k = 3$  (b) and  $n_k = 5$  (c).

TABLE 2. RESULTS OF MODEL ESTIMATION.

$n_k$	$f_1$	$b_0$	Error		Cost function $J(\theta) \times 10^5$
			$\varepsilon$	$\sigma$	
-	-	-	%RH	%RH	-
1	-0,9841	0,0159	4,75	0,97	28,01
2	-0,9838	0,0162	4,85	0,97	28,15
3	-0,9835	0,0165	5,02	0,98	28,31
4	-0,9832	0,0168	5,21	0,98	28,50
5	-0,9828	0,0172	5,45	0,98	28,72

where  $u_b$ ,  $u_e$  and  $y_b$ ,  $y_e$  are the equilibrium values of the input and output values before and after the end of the transient, A and B are the values of the relative humidity of the reference points used to reproduce  $u_b$  and  $u_e$ . This pre-processing makes it possible to eliminate some of the biases in the experimental data and to ensure the same weight of the individual sensor responses when estimating the parameter vector  $\theta$  [13]. Using the variables (9), equations (6) and (8) are transformed to:

$$\hat{y}^*(k) + f_1 \hat{y}^*(k-1) = (1 + f_1) u^*(k - n_k) \quad (10)$$

$$b_0 = 1 + f_1. \quad (11)$$

#### IV. ESTIMATION OF PREDICTION MODEL PARAMETERS

The parameter vector  $\theta = (f_1, b_0)$  of the model (10) is estimated by minimizing the cost function computed directly from the prediction error:

$$J(\theta) = \frac{1}{8N} \sum_{i=1}^8 \sum_{k=1}^N (y_i^*(k) - \hat{y}^*(k, \theta))^2 \quad (12)$$

Aligned data from eight different realizations were used: four of them are transients obtained by increasing and four by decreasing the input humidity. The sizes of each realization and sampling interval are  $N = 32 \cdot 10^3$  and 100 ms, respectively. The optimization problem (12) was solved using the Generalized reduced gradient algorithm [14,15]. Thus, the values of the coefficients  $f_1$  and  $b_0$  were found for different time delays  $n_k$ . Results can be seen in Figure 4 and Table 2. Figures 4a, 4b and 4c show that the absolute prediction errors

$$\varepsilon_i = y_i^*(k) - \hat{y}^*(k, \hat{\theta})$$

of the models in all three cases are similar. In Table 2,  $\varepsilon$  is the largest positive or negative error  $\varepsilon_i$ , and  $\sigma$  is the maximum of root mean squared prediction error:

$$\sigma_i = \sqrt{\frac{1}{N} \sum_{k=1}^N \varepsilon_i^2}$$

The maximum root mean squared prediction error of the different models is approximately the same and equal to  $\approx 1\%RH$ . The maximum absolute error is the smallest ( $\approx 5\%$ ) in the model with a time delay  $n_k = 1$ . This gives reason to consider it the best.

#### V. CONCLUSION

Kalman's filter is a well-known and commonly used data processing algorithm. When this data is obtained from sensors, the performance of the filter can be improved by

adding an appropriate sensor model to the prediction model describing the observed process. As long as the algorithm implementing the filter has computational complexity  $O(n^3)$ , it is desirable to use models of the lowest possible order. In the current work, an approach to building a predictive low order model of the SHT31-DIS humidity sensor is discussed. Assuming that the sensor can be presented with a standard first-order output error model plus delay, it is shown that the measurements obtained from the sensor can be predicted one step ahead with an error of less than 5% RH. When estimating the parameters of the model, a mean square criterion and measurements traceable to the standard relative humidity scale OIML R 121 were used.

#### REFERENCES

- [1] D. Simon, "Optimal state estimation: Kalman, H infinity, and nonlinear approaches.", John Wiley & Sons, p.552, 2006.
- [2] Y. Bar-Shalom, X. Li, and T. Kirubarajan, "Estimation with applications to tracking and navigation: theory algorithms and software," John Wiley & Sons, p.584, 2004.
- [3] K. Eom, S. Lee, Y. Kyung, C. Lee, M. Kim, and K. Jung, "Improved kalman filter method for measurement noise reduction in multi sensor rfid systems," Sensors, 11(11), 10266-10282, 2011.
- [4] T. Hasfjord, "Design and implementation of a Kalman Filter based estimator for temperature control," MS thesis. Institutt for teknisk kybernetikk, 2014.
- [5] S. Das, N. Deka, N.Sinha, N., S. Dhar, D. Bhattacharjee and S. Gupta, "Environmental monitoring using sensor data fusion," In 2012 International Conference on Radar, Communication and Computing (ICRCC), IEEE, pp. 83-86, December 2012.
- [6] C. Castello, J. New, and M. Smith, "Autonomous correction of sensor data applied to building technologies using filtering methods," 2013 IEEE Global Conference on Signal and Information Processing. IEEE, pp. 121-124, 2013.
- [7] A. Valade, P. Acco, P. Grabolosa, J. Fourniols, "A study about Kalman filters applied to embedded sensors," Sensors 17.12 (2017): 2810.
- [8] N. Assimakis, M. Adam, and A. Douladiris. "Information filter and kalman filter comparison: Selection of the faster filter," Information Engineering. Vol. 2. No. 1, pp.1-5, 2012.
- [9] Sensirion, "Testing Guide For SHTxx Relative Humidity & Temperature Sensor Series," Sensirion, Version 1.3, pp.5, 2018.
- [10] Sensirion, "Datasheet Humidity Sensor SHT3x Digital. Sensirion," Version 6, pp.22, 2019.
- [11] P. Nikovski, B. Markov, and N. Doychinov, "Sampling Time Variation of the EK-H5 Based Humidity Data Acquisition System," X National Conference with International Participation (ELECTRONICA). IEEE, pp.137-139, 2019.
- [12] OIML, "The scale of relative humidity of air certified against salt solutions," Organization Internationale De Metrologie Legale OIML R 121, 1996.
- [13] P. Nikovski, "Assessment of bias removal techniques used in case of sensor parametric modelling," IOP Conference Series: Materials Science and Engineering. Vol. 618. No. 1. IOP Publishing, 2019.
- [14] K. Deb, "Optimization for engineering design: algorithms and examples," PHI Learning, 2004.
- [15] J. Arora, Introduction to Optimum Design, Academic Press, p.880,2011.