MAST90104: A First Course in Statistical Learning

Week 2 Workshop/Lab

Workshop questions

1. (a) Find the eigenvalues, and an associated eigenvector for each eigenvalue, of the matrix

$$A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}.$$

   (b) Find an orthogonal matrix P such that $P^T AP$ is diagonal.

   (c) Write down $P^T AP$ for the P given in part (b).

2. Let

$$A = \begin{bmatrix} 1 & 4 & 3 \\ -2 & 0 & 2 \\ 4 & 4 & 0 \end{bmatrix}.$$

   (a) Write down the trace of A.

   (b) Are the columns of A linearly independent? Justify your answer.

   (c) Find the rank of A.

3. Show that if X is of full rank, then

$$I - X(X^T X)^{-1} X^T$$

   is an idempotent matrix.

4. Consider the matrix

$$X = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$$

   (a) Show that X is idempotent.

   (b) What is the rank of X ?

5. Prove that for any matrix A

$$r(A) = r(A^T) = r(A^T A).$$

   You may use the fact that pre- or post-multiplying by a non-singular matrix does not change the rank.

Practical questions

1. Use R to find the number of integers that are divisible by 17 between 1 and 500

2. Suppose that `queue <- c("Steve", "Russell", "Alison", "Liam")` and that `queue` represents a supermarket queue with Steve first in line. Using R expressions update the supermarket queue as successively:

   (a) Barry arrives;

   (b) Steve is served;

   (c) Pam talks her way to the front with one item;

   (d) Barry gets impatient and leaves;

   (e) Alison gets impatient and leaves.

For the last case you should not assume that you know where in the queue Alison is standing.

Finally, using the function `which(x)`, find the position of Russell in the queue.

Note that when assigning a text string to a variable, it needs to be in quotes.

3. The table below is taken from a clinic's database, that records the patients' name, age, and their waiting time. Create an R *data frame* with these information. Find the patient(s) with the longest waiting time

Name	Age	Waiting time
Ron	23	5
Steve	24	7
Barry	20	2
Louise	30	3
Ann	25	5
Kristen	24	4
Emma	21	6

4. Let

$$A = \begin{bmatrix} 2 & 4 & 0 \\ 7 & 6 & 8 \\ 3 & 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 2 \\ -1 & -1 & 0 \\ 4 & 0 & 1 \end{bmatrix}$$

   (a) Give R expression that return A and B
   (b) Use R to compute AB, B^TA
   (c) Use R to find $det(A)$ and $r(B)$

5. Use R to create 3 vectors

$$\mathbf{x}_1 = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}, \quad \mathbf{x}_2 = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{x}_3 = \begin{bmatrix} 5 \\ -4 \\ 6 \end{bmatrix}$$

   (a) Create matrix $\mathbf{A} = \begin{bmatrix} \mathbf{x}_1 & \mathbf{x}_2 & \mathbf{x}_3 \end{bmatrix}$ (*Hint: use function cbind*)
   (b) Create a vector of length 3, call it $\mathbf{z}$
   (c) Add $\mathbf{z}$ to $\mathbf{A}$ as the last row (*Hint: use function rbind*)

6. Write a program to read in a square matrix and return its trace. *Hint: We first need to check whether the input is a square matrix.*