

**Instruction:** You must show all your work clearly for credit. Partial credit will only be given to meaningful answers. You will be graded according to your approach to the problems, mathematical rigor, and quality of your solutions.

1. (10) Using the inductive algorithm as illustrated in class to compute  $C(9, 7)$ . You must show the construction of your table clearly for credit. **Warning:** Do not perform any computation that is not required for the computation of  $C(9, 7)$ .
2. (15) Given a sequence of integers  $S = \langle 2, -3, 5, -2, 8, -4, 11, -5, 4, -2, 6, -3 \rangle$ . Using the divide-and-conquer algorithm as illustrated in class to compute a maximum subsum sequence for  $S$ . You must show the computation of the leftSum, rightSum, and crossSum clearly for credit.
3. (15) Given a sequence of integers  $S = \langle 2, -3, 5, -2, 8, -4, 11, -5, 4, -2, 6, -3 \rangle$ . Using the inductive algorithm as illustrated in class to compute a maximum subsum sequence for  $S$ . You must show your computation for each iteration clearly for credit.
4. (20) Given a matrix chain with five matrices and dimension vector  $(5, 10, 8, 20, 15, 18)$ . By using the technique of dynamic programming, compute an optimal order for computing the product  $M_1 * M_2 * M_3 * M_4 * M_5$ . You must show the computations of  $m_{i,j}$ ,  $t_{i,j}$ , and the construction of the optimal order clearly as discussed in class for credit.
5. (20) Given a sequence of 5 records with keys  $x_i$ ,  $x_1 \leq x_2 \leq \dots \leq x_5$ , a key  $x$ , and the probability  $\Pr(x = x_i) = p_i$ ,  $1 \leq i \leq n$ , with  $p_1 = 0.20$ ,  $p_2 = 0.25$ ,  $p_3 = 0.05$ ,  $p_4 = 0.35$ ,  $p_5 = 0.15$ . Construct an optimal binary search tree  $T$  such that the average number of comparisons in finding  $x$  in  $T$  is minimized. You must show the computations of  $c_{i,j}$ ,  $t_{i,j}$ , and the construction of the optimal binary search tree clearly as discussed in class for credit.
6. (20) Given 5 turkeys  $\{x_1, \dots, x_5\}$  together with their corresponding weights and cash values as follow:

	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$
$c(x)$	1	16	14	22	20
$w(x)$	1	2	3	6	7

If the maximum weight one can carry is 10, compute an optimal selection using dynamic programming so as to maximize the total cash values. You must show the computation of  $\text{cost}[i, K]$  in a table and explain how the optimal selection can be constructed from the table clearly as discussed in class for credit.

**Remark:** Turn in Problems 1, 4, and 5 for grading.