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Assignment - 2

Probablity Distribution

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① Given) $n = 10 \quad a = 7$

Bernoulli distribution as n is finite.

Sol:

p (Probablity of getting a head) $= \frac{1}{2} = p$

$$q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$$

$$P(X \geq a) = \sum_{r=a}^{n} {}^{n}C_{r}\, p^{r} q^{n-r}$$

$$\therefore P(X \geq 7) = \sum_{r=7}^{10} {}^{10}C_{r} \left(\frac{1}{2}\right)^{r} \left(\frac{1}{2}\right)^{10-r}$$

$$= \sum_{r=7}^{10} {}^{10}C_{r} \left(\frac{1}{2}\right)^{10}$$

$$= {}^{10}C_{7}\left(\frac{1}{2}\right)^{10} + {}^{10}C_{8}\left(\frac{1}{2}\right)^{10} + {}^{10}C_{9}\left(\frac{1}{2}\right)^{10} + {}^{10}C_{10}\left(\frac{1}{2}\right)^{10}$$

$$= \left(\frac{1}{2}\right)^{10}\left[\frac{10!}{7! \times 3!} + \frac{10!}{8! \times 2!} + \frac{10!}{9! \times 1!} + \frac{10!}{10! \times 0!}\right]$$

$$= \left(\frac{1}{2}\right)^{10}\left[\frac{10 \times 9 \times 8}{3 \times 2} + \frac{10 \times 9}{2} + 10 + 1\right]$$

$$= \left(\frac{1}{2}\right)^{10}\left[120 + 45 + 11\right]$$

$$= \left(\frac{1}{2}\right)^{10}(176) = 0.171875.$$

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(2) Given) $p = 0.75$ $q = 0.25$ $n = 6$

$a = 5$

Sol: Bernoulli's distribution.

a) $P(X \geq 5) = \sum_{r=5}^{6} {}^6C_r \left(\frac{75}{100}\right)^r \left(\frac{25}{100}\right)^{6-r}$

$= \sum_{r=5}^{6} {}^6C_r \left(\frac{3 \times 25}{100}\right)^r \left(\frac{25}{100}\right)^{6-r}$

$= \sum_{r=5}^{6} {}^6C_r (3)^r \left(\frac{25}{100}\right)^r \cdot \left(\frac{25}{100}\right)^{6-r}$

$= \sum_{r=5}^{6} {}^6C_r \left(\frac{25}{100}\right)^6 (3)^r$

$= \left(\frac{25}{100}\right)^6 \left[{}^6C_5 (3)^5 + {}^6C_6 (3)^6 \right]$

$= \left(\frac{1}{4}\right)^6 \left[\frac{6!}{5! \times 1} (3)^5 + \frac{6!}{6! \times 0!} \times (3)^6 \right]$

$= \frac{6 \times 3^5 + (3)^6}{(4)^6}$

$= \frac{1458 + 729}{4096} = \frac{2187}{4096} = 0.5266$

b) $P(X < 5) = 1 - P(X \geq 5)$

$= 1 - 0.5266$

$= 0.4734$

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(3) Given) $n=100$ $a=10$ $p=0.05$ $q=1-p=0.95$

sol: p is small but n is large

→ Poisson's Distribution.

$P(X>10)$

$= 1 - P(X \leq 10)$

$= 1 - \sum_{r=0}^{10} \frac{e^{-\lambda} \lambda^r}{r!}$

$\lambda = np = 100 \times 0.05 = 5$

$\therefore P(X>10) = 1 - \sum_{r=0}^{10} \frac{e^{-5} (5)^r}{r!}$

$= 1 - e^{-5} \sum_{r=0}^{10} \frac{(5)^r}{r!}$

$= 1 - e^{-5} \left[\frac{5^0}{0!} + \frac{5^1}{1!} + \frac{5^2}{2!} + \frac{5^3}{3!} + \frac{5^4}{4!} + \frac{5^5}{5!} + \frac{5^6}{6!} + \frac{5^7}{7!} + \frac{5^8}{8!} + \frac{5^9}{9!} + \frac{5^{10}}{10!} \right]$

$= 1 - e^{-5} [1 + 5 + 12.5 + 20.83 + 26.041 + 26.041 + 21.70 + 15.50 + 9.688 + 5.38 + 2.69]$

$= 1 - e^{-5} [146.37]$

$= 1 - 0.986$

$= 0.014$

(4) sol: $\lambda = \frac{390}{520} = 0.75$ (mean = no. of typo errors per page).

Poisson's probablity law,

$P(X=r) = \frac{e^{-\lambda} \lambda^r}{r!}$

$$= \frac{e^{-0.75}(0.75)^r}{r!}$$

$$\Rightarrow P(X=0) = \frac{e^{-\lambda}\lambda^0}{0!} = \frac{e^{-0.75} \times 1}{1}$$

For 5 random pages,

$$\Rightarrow [P(X=0)]^5 = [e^{-0.75}]^5$$
$$= 0.023501$$

⑤ Given) $\mu = 65$, $\sigma = 5$.

sol: Normal distribution

$$P(X>75) = P\left(\frac{X-\mu}{\sigma} > \frac{75-\mu}{\sigma}\right)$$
$$= P\left(\frac{X-65}{5} > \frac{75-65}{5}\right)$$
$$= P(Z>2)$$
$$= 1 - F(2)$$
$$= 1 - \left(\frac{1}{\sqrt{2\pi}} \times e^{-2^2/2}\right)$$
$$= 1 - \left(\frac{1}{\sqrt{2\pi}} e^{-2}\right)$$
$$= 1 - 0.0539$$
$$= 0.9461$$

$$q = 1-p = 0.0539$$

$$P(\text{atleast } 1) = P(X \geq 1)$$
$$= {}^3C_1 p^1 q^2 + {}^3C_2 p^2 q^1 + {}^3C_3 p^3 q^0$$
$$= {}^3C_1 (0.9461)(0.0539)^2$$
$$+ {}^3C_2 (0.895)(0.0539)$$
$$+ {}^3C_3 (0.848)$$
$$(1)$$

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$$= 3(0.9461)(0.0539)^2 + 3(0.895)(0.0539) + (0.846)$$

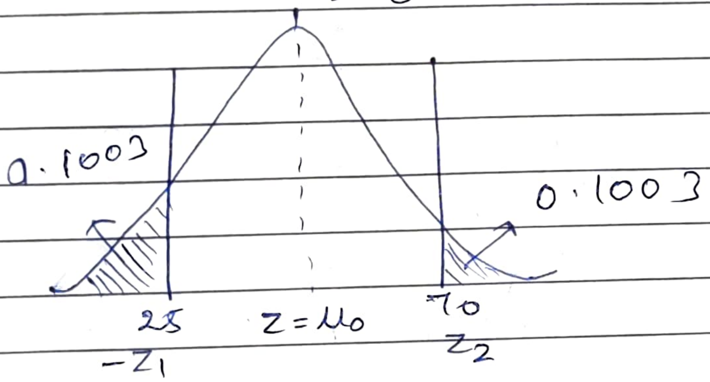
$$= 0.9989.$$

(6) Given Normal distribution

$$P(X < 25) = 0.1003$$
$$P(X < 70) = 0.8997$$

Sol:

$$P(X>70) = 1 - P(X<70)$$
$$= 1 - 0.8997$$
$$= 0.1003$$



$$X = 25 \Rightarrow \quad z = \frac{25-\mu}{\sigma} = -z_1 \text{ (left)}$$

$$X = 70 \Rightarrow \quad z = \frac{70-\mu}{\sigma} = z_2$$

$$P(0 < z < z_2) = 0.5 - 0.1003 = 0.3997$$
$$P(0 < z < z_1) = 0.5 - 0.1003 = 0.3997$$

$$\therefore \boxed{\begin{aligned} z_2 &= 1.28 \\ z_1 &= 1.28 \end{aligned}}$$ (using z table)

$$\therefore \frac{70-\mu}{\sigma} = 1.28 \rightarrow (1) \qquad \frac{25-\mu}{\sigma} = -1.28 \rightarrow (2)$$

$$\frac{70-\mu-25+\mu}{\sigma} = 2.56 \quad ((1)+(2))$$

$$\frac{45}{\sigma} = 2.56 \Rightarrow \boxed{\sigma = 17.57}$$

$$\frac{70-\mu}{17.57} = 1.28$$

$$\boxed{\mu = 47.52}$$

(7) Given) ~~Expo~~ Gamma Distribution.

$$\lambda = \frac{1}{10000} \qquad k = 2$$

Sol:

$$f(x) = \frac{\lambda e^{-\lambda x}(\lambda x)^{k-1}}{\Gamma k}$$

$$\Gamma k = (k-1)!$$

$$f(x) = \frac{\frac{1}{10000} \times e^{-x/10000} \times \left(\frac{x}{10000}\right)}{\Gamma 2}$$

$$= \left(\frac{1}{10000}\right)^2 e^{-x/10000}$$

Given excess of 20000.

$$\therefore P(X > 30000) = P(X > 30000 - 20000)$$

$$P(X > 10000) = \int_{10000}^{\infty} \frac{x}{(10000)^2} e^{-x/10000} dx$$

$$= \frac{1}{(10000)^2} \int_{10000}^{\infty} x e^{-x/10000} dx$$

Let $u = x \qquad dv = e^{-x/10000}$

$du = dx \qquad v = -10000\, e^{-x/10000}$

$$\int u\,dv = uv - \int v\,du$$

$$= -x \times 10000\, e^{-x/10000} + \int 10000 \int e^{-x/10000} dx$$

$$= -10000 \times x \times e^{-x/10000} + 10000\left[-10000 \times e^{-x/10000}\right]$$

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$$\Rightarrow -10000\left[0-\left(e^{-1}(10000)+10000\times e^{-1}\right)\right]$$

$$\Rightarrow 10000\left(10000\left(e^{-1}+e^{-1}\right)\right)$$

$$\Rightarrow (10000)^2\left(2e^{-1}\right)$$

$$\Rightarrow \frac{1}{\cancel{(10000)^2}}\times\cancel{(10000)^2}\left(2e^{-1}\right)$$

$$\Rightarrow \left(2e^{-1}\right).$$

(8) Given) Exponential.

Mean $= 40000 = \frac{1}{\lambda}$

Sol:

$$\boxed{\lambda = \frac{1}{40000}}$$

a) $P(X \geq 20000) = \int_{20000}^{\infty} f(x)\,dx = \int_{20000}^{\infty} \lambda e^{-\lambda x}\,dx$

$$= \int_{20000}^{\infty} \frac{1}{40000}\times e^{-x/40000}\,dx$$

$$= \frac{1}{40000}\left[-40000\,e^{-x/40000}\right]_{20000}^{\infty}$$

$$= -e^{\infty/40000} + e^{-\cancel{20000}/\cancel{40000}}$$

$$= e^{-1/2} = 0.606.$$

b) $P(X \leq 30000) = \int_{0}^{30000} \frac{1}{40000}\times e^{-x/40000}\,dx$

$$= \frac{1}{40000}\left[-40000\, e^{-x/40000}\right]_0^{30000}$$

$$= -e^{-3/4}+1$$

$$= 1-e^{-3/4}$$

$$= 1-0.472$$

$$= 0.528$$

(9) $f(x) = \alpha\beta\, x^{\beta-1} \cdot e^{-\alpha x^\beta} \; ; \; x>0$

$$f(x) = 2\alpha x e^{-\alpha x^2}$$

$$P(X>5) = e^{-0.25}$$

$$\Rightarrow \int_5^\infty 2\alpha x e^{-\alpha x^2} dx = e^{-0.25}$$

$$e^{-25\alpha} = e^{-0.25}$$

$$\therefore \boxed{\alpha = \frac{1}{100}}$$

$$\therefore f(x) = \left(\frac{2}{100}\right) x e^{-x^2/100}$$

$$\text{Mean} = \alpha^{-1/\beta} \cdot \Gamma\left(\frac{1}{\beta}+1\right)$$

$$= \left(\frac{1}{100}\right)^{-1/2} \cdot \Gamma\left(\frac{3}{2}\right) = 10 \times \frac{1}{2}\Gamma\left(\frac{1}{2}\right)$$

$$= 5\sqrt{\pi}$$

$$\text{Variance} = \alpha^{-2/\beta}\left[\Gamma\left(\frac{2}{\beta}+1\right) - \left(\Gamma\left(\frac{1}{\beta}+1\right)\right)^2\right]$$

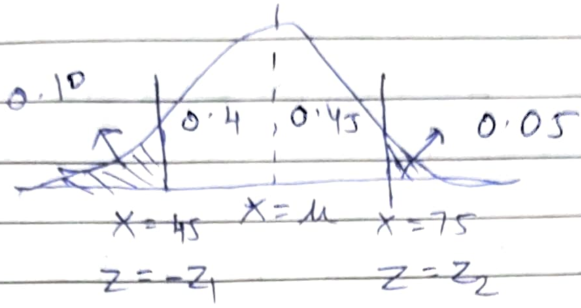
$$= 100\left(1-\frac{\pi}{4}\right)$$

(10) Normal Distribution

Sol: $P(X<45) = 0.10$ (failed)

$P(X>75) = 0.05$ (distinction)

where X denotes the marks of 100.



$X = 45 \Rightarrow Z = \dfrac{45-\mu}{\sigma} = -Z_1$

$X = 75 \Rightarrow Z = \dfrac{75-\mu}{\sigma} = Z_2$

$P(0<Z<Z_2) = 0.5 - 0.05 = 0.45$

$P(0<Z<Z_1) = 0.5 - 0.10 = 0.40$

From Z table;

$Z_1 = 1.29 \qquad Z_2 = 1.65$

$\therefore \dfrac{45-\mu}{\sigma} = -1.29 \rightarrow ①$

$\therefore \dfrac{75-\mu}{\sigma} = 1.65 \rightarrow ②$

② − ①;

$\dfrac{75-\cancel{\mu}-45+\cancel{\mu}}{\sigma} = 2.94$

$\dfrac{30}{\sigma} = 2.94$

$\boxed{\sigma = 10.204}$

$\therefore 75-\mu = 29.98$

$\boxed{\mu = 45.02}$

∴ Probablity that a candidate is placed in first division ⇒ (lies b/w 60 & 75).

$$p = P(60 < x < 75)$$

$$= P\left(\frac{60 - 45.02}{10.27} < z < \frac{75 - 45.02}{10.27}\right)$$

$$= P(1.45 < z < 2.919)$$

$$= P(1.45 < z < 0) + P(0 < z < 2.919)$$

$$= P(0 < z < -1.45) + P(0 < z < 2.919)$$

$= (0.55$ -