Università
della
Svizzera
italiana

Faculty
of Informatics

Numerical Algorithms

Fall 2020

Assignment 12

November 26, 2020

Exercise 1 [5 points]

Show that the product of two column-stochastic matrices is also column-stochastic.

Hand in your proof.

Exercise 2 [5 points]

Consider a web with $n = 15$ pages and $m = 34$ links, as specified in `links.txt` (the first line contains n and m, and the next m lines each contain two numbers i and j, which correspond to a link from page i to page j). Write a program that reads this file and carries out the efficient "Google power iteration" that we discussed in class, that is,

$$x_{k+1} = (1-\mu)Ax_k + \frac{\mu}{n}e, \qquad \text{where } e = (1, 1, \ldots, 1)^T,$$

for $\mu = 0.15$, using the initial guess $x_0 = \frac{1}{n}e$, until the difference between x_k and x_{k+1} is less than 10^{-8} in absolute value for each component, that is, $\|x_{k+1} - x_k\|_\infty < 10^{-8}$. After how many iterations does this happen? What is the resulting ranking of the web pages? How many iterations does it take to get to this ranking? How does the result change if page 14 adds a links to itself?

Hand in your code, the output of your program, and the answers to the questions.

Solutions must be returned online or in class on December 3, 2020