Università
della
Svizzera
italiana

Faculty
of Informatics

Numerical Algorithms

Fall 2020

Assignment 3 (only for students of the 6 ECTS course) September 24, 2020

Exercise 1 [5 points]

Let $A = \left(\begin{smallmatrix} a & b \\ b & c \end{smallmatrix}\right)$ be a symmetric, real-valued 2×2 matrix with $b \neq 0$. Show that the eigenvalues of A are

$$\lambda_1 = \frac{a+c+\sqrt{(a-c)^2+4b^2}}{2} \qquad \text{and} \qquad \lambda_2 = \frac{a+c-\sqrt{(a-c)^2+4b^2}}{2},$$

with corresponding (non-unit) eigenvectors

$$w_1 = \begin{pmatrix} \lambda_1 - c \\ b \end{pmatrix} \qquad \text{and} \qquad w_2 = \begin{pmatrix} \lambda_2 - c \\ b \end{pmatrix}.$$

Hand in your proof.

Note that it is not enough to verify that these are indeed the correct eigenvalues and eigenvectors. Instead, you should derive them, following some basic considerations.

Exercise 2 [5 points]

Consider the two 2D point sets $P = \{p_1, p_2, \dots, p_6\}$ with

$$p_1 = \begin{pmatrix} -2.4 \\ -1.6 \end{pmatrix}, p_2 = \begin{pmatrix} 0.5 \\ -0.7 \end{pmatrix}, p_3 = \begin{pmatrix} -3.0 \\ 0.3 \end{pmatrix}, p_4 = \begin{pmatrix} -1.1 \\ 0.9 \end{pmatrix}, p_5 = \begin{pmatrix} -3.6 \\ 2.2 \end{pmatrix}, p_6 = \begin{pmatrix} -0.7 \\ 3.1 \end{pmatrix},$$

and $Q = \{q_1, q_2, \dots, q_6\}$ with

$$q_1 = \begin{pmatrix} -0.3 \\ 0.5 \end{pmatrix}, q_2 = \begin{pmatrix} 1.6 \\ -1.8 \end{pmatrix}, q_3 = \begin{pmatrix} 1.3 \\ 1.7 \end{pmatrix}, q_4 = \begin{pmatrix} 2.5 \\ 0.2 \end{pmatrix}, q_5 = \begin{pmatrix} 2.8 \\ 3.0 \end{pmatrix}, q_6 = \begin{pmatrix} 4.7 \\ 0.6 \end{pmatrix}.$$

Write a program that computes the best *rigid transform* to match them. That is, minimize the function

$$f(R,t) = \sum_{i=1}^{6} \|p_i - Rq_i - t\|^2 \tag{1}$$

to find the best *rotation* $R \in \mathbb{R}^{2\times 2}$ and translation $t \in \mathbb{R}^2$. Print out R, t, $f(R,t)$, and the transformed points $Rq_i + t$, $i = 1, \dots, 6$.

Hand in your code and the output.

Solutions must be returned online or in class on October 1, 2020