Numerical Algorithms

Fall 2020

Assignment 6 (only for students of the 6 ECTS course) October 15, 2020

Exercise 1 [10 points]

Consider the $n+1$ interpolation points $P_0, \ldots, P_n$ from Exercise 2 of Assignment 4 and the uniform knot vector $T = (t_0, \ldots, t_{m+7})$ for $m = n-1$ with quadruple end knots, that is,

$$t_0 = t_1 = t_2 = t_3 = 0, \qquad t_{i+3} = i, \quad i = 1, \ldots, m, \qquad t_{m+4} = t_{m+5} = t_{m+6} = t_{m+7} = m+1.$$

Compute the cubic B-spline curve $F_n \colon [t_0, t_{m+7}] \to \mathbb{R}^2$ over T that interpolates the P_i at t_{i+3}, that is,

$$F_n(t_{i+3}) = P_i, \qquad i = 0, \ldots, m+1$$

and satisfies the natural end conditions

$$F''(t_3) = F''(t_{m+4}) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

Plot the three curves F_{10}, F_{20}, and F_{40} into three different pictures, together with the curve f from which the interpolation points were sampled (cf. Exercise 2 of Assignment 4), using $M+1$ sample points for each curve, where $M = 10000$.

Note: for plotting the curves, you can use the code of the de Boor algorithm provided on *iCorsi*.

Hint: use the derivative formula for B-spline curves (twice) and the de Boor algorithm to figure out how to express $F''(t_3)$ and $F''(t_{m+4})$ in terms of the D_i.

Make sure that your code is efficient and uses only $O(n)$ operations, by taking advantage of the fact that it needs to solve a tridiagonal system

$\longrightarrow$ see `https://en.wikipedia.org/wiki/Tridiagonal_matrix_algorithm`

Describe how you derived your code, and hand in your code and the three pictures with the plots.

Bonus Exercise [5 points]

Given the uniform knot vector $T = (t_0, \ldots, t_8)$ with $t_i = i$ and the control points

$$D_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad D_1 = \begin{pmatrix} 6 \\ 0 \end{pmatrix}, \quad D_2 = \begin{pmatrix} 0 \\ 12 \end{pmatrix}, \quad D_3 = \begin{pmatrix} 12 \\ 12 \end{pmatrix}, \quad D_4 = \begin{pmatrix} 12 \\ 12 \end{pmatrix},$$

consider the cubic B-spline curve $F(t) = \sum_{i=0}^{4} D_i N_i^3(t)$. This curve has two cubic pieces: $F_0 \colon [3,4] \to \mathbb{R}^2$ and $F_1 \colon [4,5] \to \mathbb{R}^2$. Compute the Bézier control points $C_{0,0}, \ldots, C_{0,3}$ and $C_{1,0}, \ldots, C_{1,3}$ for the curves F_0 and F_1 and verify, using the A-frame condition, that they join indeed with C^2-continuity.

Solutions must be returned online or in class on October 22, 2020