Università
della
Svizzera
italiana

Faculty
of Informatics

Numerical Algorithms

Fall 2020

Assignment 5 (only for students of the 6 ECTS course) October 8, 2020

Exercise 1 [5 points]

Let $n \in \mathbb{N}$ and $B_i^n(s) = \binom{n}{i}(1-s)^{n-i}s^i$ for $i = 0, \dots, n$ be the i-th Bernstein polynomial of degree n over the standard interval $[0, 1]$. Show that

$$(B_i^n)'(s) = n(B_{i-1}^{n-1}(s) - B_i^{n-1}(s))$$

for $i = 0, \dots, n$, where we follow the convention that $B_{-1}^n(s) \equiv 0$ and $B_{n+1}^n(s) \equiv 0$ for any $n \in \mathbb{N}$.

Exercise 2 [5 points]

If we define the polygon (blue) with vertices

$$P_0 = \begin{pmatrix} 200 \\ 200 \end{pmatrix},$$

$$P_1 = \begin{pmatrix} 200 \\ 500 \end{pmatrix},$$

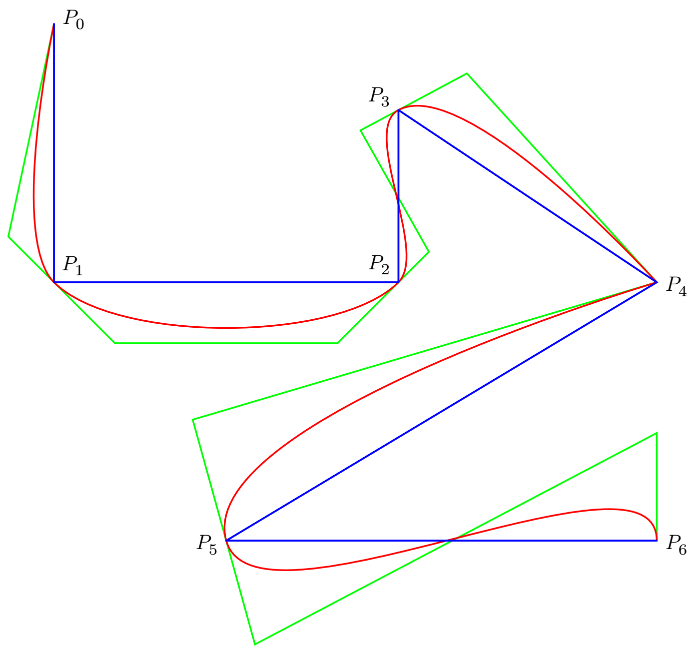
$$P_2 = \begin{pmatrix} 600 \\ 500 \end{pmatrix},$$

$$P_3 = \begin{pmatrix} 600 \\ 300 \end{pmatrix},$$

$$P_4 = \begin{pmatrix} 900 \\ 500 \end{pmatrix},$$

$$P_5 = \begin{pmatrix} 400 \\ 800 \end{pmatrix},$$

$$P_6 = \begin{pmatrix} 900 \\ 800 \end{pmatrix}$$



in *Adobe Illustrator* and convert the points P_1, P_2, P_3, P_5, P_6 from *corner* to *smooth*, then we get a cubic Bézier spline (red) with control points (green)

$$C_{0,0} = \begin{pmatrix} 200 \\ 200 \end{pmatrix}, \quad C_{0,1} = \begin{pmatrix} 200 \\ 200 \end{pmatrix}, \quad C_{0,2} = \begin{pmatrix} 146.97 \\ 446.97 \end{pmatrix}, \quad C_{0,3} = \begin{pmatrix} 200 \\ 500 \end{pmatrix},$$

$$C_{1,0} = \begin{pmatrix} 200 \\ 500 \end{pmatrix}, \quad C_{1,1} = \begin{pmatrix} 270.71 \\ 570.71 \end{pmatrix}, \quad C_{1,2} = \begin{pmatrix} 529.29 \\ 570.71 \end{pmatrix}, \quad C_{1,3} = \begin{pmatrix} 600 \\ 500 \end{pmatrix},$$

$$C_{2,0} = \begin{pmatrix} 600 \\ 500 \end{pmatrix}, \quad C_{2,1} = \begin{pmatrix} 635.36 \\ 464.64 \end{pmatrix}, \quad C_{2,2} = \begin{pmatrix} 555.92 \\ 323.59 \end{pmatrix}, \quad C_{2,3} = \begin{pmatrix} 600 \\ 300 \end{pmatrix},$$

$$C_{3,0} = \begin{pmatrix} 600 \\ 300 \end{pmatrix}, \quad C_{3,1} = \begin{pmatrix} 679.47 \\ 257.47 \end{pmatrix}, \quad C_{3,2} = \begin{pmatrix} 900 \\ 500 \end{pmatrix}, \quad C_{3,3} = \begin{pmatrix} 900 \\ 500 \end{pmatrix},$$

$$C_{4,0} = \begin{pmatrix} 900 \\ 500 \end{pmatrix}, \quad C_{4,1} = \begin{pmatrix} 900 \\ 500 \end{pmatrix}, \quad C_{4,2} = \begin{pmatrix} 361.09 \\ 659.52 \end{pmatrix}, \quad C_{4,3} = \begin{pmatrix} 400 \\ 800 \end{pmatrix},$$

$$C_{5,0} = \begin{pmatrix} 400 \\ 800 \end{pmatrix}, \quad C_{5,1} = \begin{pmatrix} 433.37 \\ 920.46 \end{pmatrix}, \quad C_{5,2} = \begin{pmatrix} 900 \\ 675 \end{pmatrix}, \quad C_{5,3} = \begin{pmatrix} 900 \\ 800 \end{pmatrix}.$$

Figure out which strategy *Adobe* has adopted for defining the tangents at the points that are converted to *smooth* and how they handle the situation at points that remain *corners*.

Describe the strategy and how you found it in detail, either with words or formulas.

Solutions must be returned online or in class on October 15, 2020