Numerical analysis: Assignment 8

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Exercise 1

$$x^* = \arg\min_{x \in \mathcal{R}^n} ||b - Ax|| \tag{1}$$

A solution to Equation 1 is also a solution to:

$$\frac{\partial}{\partial x} ||b - Ax|| \tag{2}$$

By expanding the norm and taking the derivative and setting it to zero:

$$\begin{aligned} \frac{\partial}{\partial x} ||b - Ax|| = \frac{\partial}{\partial x} < b - Ax, b - Ax >= \\ = \frac{\partial}{\partial x}(b - Ax)^2 = \frac{\partial}{\partial x} b^2 + (Ax)^2 - 2b^T Ax = 2A^T Ax - 2A^T b = 0 \\ \implies A^T Ax = A^T b \end{aligned} \tag{3}$$

Which is the normal equation.

Exercise 2

The code has been implemented in *exercise2.py*. The output of my code is reported in Lst. 1 and in Fig. 1 :

Listing 1: Code output

```
To reach an error of 1e-07 it took 255 iterations
Optimal parameter values : [45.20164   19.809147  -5.4695835 69.520905 ]
Model parameters:  [-2.86684774 -4.09136652  5.33633989]
```

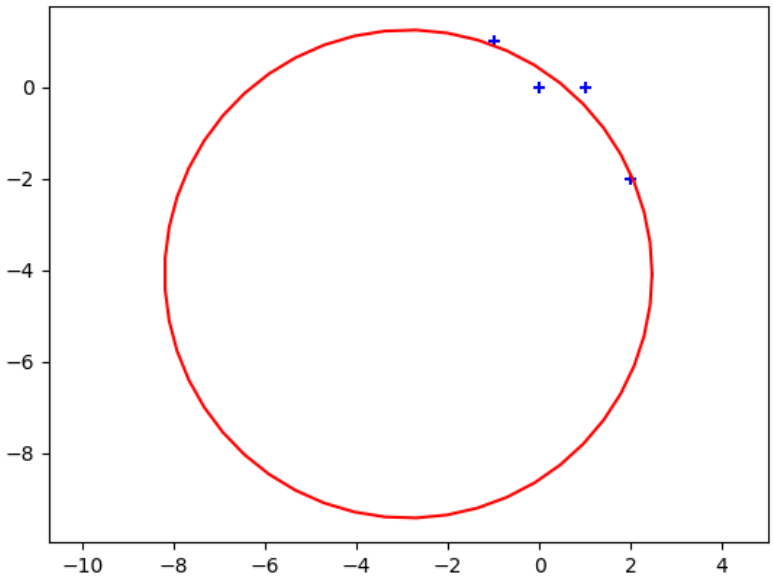

Figure 1: Result parametric circle model F