Numerical Algorithms

Fall 2020

Assignment 11

November 19, 2020

Exercise 1 [5 points]

For $n > 1$, let ω denote the n-th root of unity $\omega = e^{2\pi i/n}$. Show that

$$1 + \omega + \omega^2 + \omega^3 + \cdots + \omega^{n-1} = 0.$$

Hand in your proof.

Exercise 2 [5 points]

For $n \in \mathbb{N}$, let $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$ be two vectors in $\mathbb{R}^n$ such that

$$\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i = 0, \tag{1}$$

and let P be the closed planar polygon with n vertices $p_i = (x_i, y_i)$, $i = 1, \ldots, n$. Clearly, the barycentre $\bar{p}$ of P is the origin, $\bar{p} = (0, 0)$. Now consider the midpoint algorithm *with normalization*, which starts with the polygon $P^{(0)} = P$ and iteratively creates a sequence of polygons $P^{(k)}$ with n vertices $p_i^{(k)} = (x_i^{(k)}, y_i^{(k)})$, $i = 1, \ldots, n$ by computing

$$u^{(k+1)} = M_n x^{(k)} \qquad x^{(k+1)} = u^{(k+1)} / \|u^{(k+1)}\|_2,$$
$$v^{(k+1)} = M_n y^{(k)} \qquad y^{(k+1)} = v^{(k+1)} / \|v^{(k+1)}\|_2,$$

for $k = 0, 1, 2, \ldots$, where M_n is the averaging matrix that we defined in the lecture.

Implement this algorithm and run it for $n = 50$ and random initial points $p_i \in [-1, 1]^2$ satisfying (1). Plot $P^{(k)}$ for $k = 50, 100, 150, \ldots, 1000$ for three different random initial polygons.

Hand in your code and the output of your program.

Solutions must be returned online or in class on November 26, 2020