Università
della
Svizzera
italiana

Faculty
of Informatics

Numerical Algorithms

Fall 2020

Assignment 10 (only for students of the 6 ECTS course) November 12, 2020

Exercise 1 [5 points]

Consider the $m = 4$ circles in the plane with centres $C_1 = (x_1, y_1) = (-2, 0)$, $C_2 = (x_2, y_2) = (2, 1)$, $C_3 = (x_3, y_3) = (2, -1)$, $C_4 = (x_4, y_4) = (0, 2)$ and radii $R_1 = 2$, $R_2 = 1$, $R_3 = 1$, $R_4 = 1$, respectively. Implement a weighted version of the (classical) Gauss–Newton algorithm (with approximate first derivative) to find the point $P = (x, y)$ that minimizes the weighted sum of squared distances between P and the circles, which can also be seen as the weighted 2-norm of the residuals,

$$\sum_{i=1}^{m} w_i r_i(x, y)^2,$$

with $r_i(x, y) = \|P - C_i\| - R_i$. Run the algorithm for the initial weights $w_1 = \cdots = w_m = 1$ and the initial guess $P^{(0)} = (0.5, 0.5)$ up to convergence (say, 10 digits accuracy). After finding the optimal P, re-set the weights to $w_i = 1/|r_i(x, y)|$, $i = 1, \ldots, m$ and find again the optimal P for these weights. Iterate this procedure until it converges (say, 10 digits accuracy) and print out P, the weights $w_1, \ldots, w_n$, and the 1-norm of the residuals,

$$\sum_{i=1}^{m} |r_i(x, y)|,$$

after each iteration (of the re-weighting procedure, not after each iteration of the weighted Gauss–Newton algorithm). What do you observe?

Hand in your code and the output of your program.

Solutions must be returned online or in class on November 19, 2020