Università della Svizzera italiana

Faculty of Informatics

Numerical Algorithms

Fall 2020

Assignment 4

October 1, 2020

Exercise 1 [5 points]

Let $x_0, \dots, x_n$ be $n+1$ distinct real values and

$$\ell(x) = (x - x_0)(x - x_1) \dots (x - x_n).$$

Show that

$$\ell'(x_j) = \prod_{k=0, k \neq j}^{n} (x_j - x_k)$$

for any $j = 0, \dots, n$.

Hand in your proof.

Exercise 2 [5 points]

Consider the curve

$$f\colon [-1, 1] \to \mathbb{R}^2, \qquad f(t) = \frac{25t^2 + 2}{25t^2 + 1} \begin{pmatrix} \sin\left(\frac{\pi}{2} t\right) \\ \cos\left(\frac{\pi}{2} t\right) \end{pmatrix}$$

and the $n+1$ interpolation points

$$P_i = f(t_i), \qquad i = 0, \dots, n,$$

for equidistant parameter values

$$t_i = \frac{2i}{n} - 1, \qquad i = 0, \dots, n.$$

Now let $P_n\colon [-1, 1] \to \mathbb{R}^2$ be the interpolating polynomial curve of degree at most n with $P(t_i) = P_i$, $i = 0, \dots, n$. Write three programs that respectively use (a) Neville's algorithm, (b) the barycentric formula, and (c) the Newton form for evaluating P_n, and plot the four curves f, P_{10}, P_{20}, and P_{40} into one picture, using $m + 1$ sample points for each curve, where $m = 10000$. That is, compute the points

$$f(s_j), \qquad s_j = \frac{2j}{m} - 1, \qquad j = 0, \dots, m,$$

and draw the polygonal line $[f(s_0), f(s_1), \dots, f(s_m)]$, and similarly for the three interpolating polynomials. Which of the three programs is the fastest?

Hand in the code of the three programs and the three pictures with the plots.

Solutions must be returned online or in class on October 8, 2020