Università della Svizzera italiana | Faculty of Informatics | **Institute of Computational Science ICS**

Introduction to Ordinary Differential Equations – Fall Semester 2019

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Exercise Sheet 2

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It is mandatory to explain carefully how the exercises have been solved. The solutions must be delivered on the platform Icorsi3, in the corresponding folder.

Exercise

Let us consider the following initial value problems ($T > 0$):

$$\begin{cases} y' = f(t,y) \quad t \in (0,T] \\ y(0) = y_0 \end{cases} \qquad \begin{cases} \tilde{y}' = f(t,\tilde{y}) + \eta(t) \quad t \in (0,T] \\ \tilde{y}(0) = y_0 + \eta_0 \end{cases}$$

where the second system is a perturbation of the first one. The error is given by the function $\eta(t)$ with initial value η_0.
You have to show a bound for the difference of the two solutions $\tilde{y} - y$. Since these functions can have vector-values, then a proper estimate of their distance must be a scalar function. For this reason, you have to bound $\|\tilde{y}(t) - y(t)\|$ by a quantity which does not depend on $y(t)$ or $\tilde{y}(t)$, even though it will depend on $\eta(t)$.

(a) Write the solutions $y(t)$ and $\tilde{y}(t)$ in integral form.

(b) Please provide the conditions on f, η and η_0 such that existence is satisfied (state Peano's hypotheses).

(c) Write the difference between the two integral solutions.

(d) Compute the norm of the difference (left and right-hand sides).

(e) Bound the right-hand side by means of sum of norms, assuming:

$$\eta_0, |\eta(t)| \leq \epsilon \quad \forall t \in [0,T]$$

Here explain carefully which properties you use (linearity, triangle inequality, Cauchy-Schwarz and so on).

(f) Apply's Gronwall's lemma to obtain the final result.