Università della Svizzera italiana | Faculty of Informatics | **Institute of Computational Science ICS**

Introduction to Ordinary Differential Equations – Fall Semester 2019

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Exercise Sheet 3

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It is mandatory to explain carefully how the exercises have been solved. The solutions must be delivered on the platform Icorsi3, in the corresponding folder.

Exercise 1

Find the maximal interval of the following IVP:

$$\begin{cases} y' = 1 + y^2 \\ y(0) = 0 \end{cases}$$

(a) Verify the local existence and uniqueness.

(b) Find the solution of the problem by separation of variables.

(c) Is the solution globally defined? Find the maximal interval.

Exercise 2

Use the theorem for global existence to find the maximal interval of the following IVP:

$$\begin{cases} y'' + \sin(y) = 0 \\ y(t_0) = y_0 \end{cases}$$

(a) Cast the equation into a system.

(b) Show that the corresponding energy, as sum of kinetic and potential energy, is constant.

(c) Find a lower bound for the potential energy.

(d) Find an upper bound for the norm of the solution, using the above results.

(e) Find the maximal interval of the problem.