Lecture Summary: Jan. 13, 2023

• The fitted (or predicted) value for Y: $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 x$. It is a fitted value if Y is observed, and predicted value if Y is unobserved.

• Properties of LS estimates:

1. $\mathrm{E}(\hat{\beta}_0) = \beta_0$, $\mathrm{E}(\hat{\beta}_1) = \beta_1$; in other words, the LS estimators are unbiased.

2. The fitted (or predicted) value for Y_i, $\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$, satisfies $\mathrm{E}(\hat{Y}_i) = \beta_0 + \beta_1 x_i = \mathrm{E}(Y_i)$. Thus, $\hat{Y}_i$ is an unbiased estimator of $\mathrm{E}(Y_i)$.

• Estimation of σ^2, the variance of the regression errors, ϵ_i, (which is the same as the variance of Y_i):

$$s^2 = \frac{\sum_{i=1}^n \hat{\epsilon}_i^2}{n-2} = \frac{1}{n-2}\sum_{i=1}^n (Y_i - \hat{Y}_i)^2, \tag{1}$$

where $\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$, and $\hat{\epsilon}_i$ is the residual.

• Interpretation of the denominator $n-2$:

(i) As the degree of freedom (d.f.): $\hat{Y}_i$ is an estimator of $\mathrm{E}(Y_i) = \beta_0 + \beta_1 x_i$, in which two parameters (β_0 and β_1, are estimated). Therefore, two degrees of freedom are subtracted from n.

(ii) Theoretically, it can be shown that, with formula (1), one has $\mathrm{E}(s^2) = \sigma^2$; in other words, s^2 is an unbiased estimator of σ^2.

• Regression with normal errors: Under the normality assumption, the distribution of Y_i is normal; the mean is $\mathrm{E}(Y_i) = \beta_0 + \beta_1 x_i$; the variance is $\mathrm{var}(Y_i) = \sigma^2$.

• Maximum likelihood estimation:

<u>Idea</u>: Find the parameter, θ, that makes what happens (that is, what is observed—your data) most likely to happen (that is, the probability to happen is highest).