Lecture Summary: Jan. 25, 2023

• c.i. for a mean response, $\mathrm{E}(Y) = \beta_0 + \beta_1 x$:

A $100(1-\alpha)\%$ c.i. for $\mathrm{E}(Y)$ is

$$\hat{Y} \pm t_{n-2}\left(1 - \frac{\alpha}{2}\right) \text{s.e.}(\hat{Y}),$$

where $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 x$ and

$$\text{s.e.}(\hat{Y}) = \sqrt{\text{MSE}\left\{\frac{1}{n} + \frac{(x-\bar{x})^2}{\sum_i (x_i - \bar{x})^2}\right\}}.$$

• prediction interval for a future observation, $Y = \mathrm{E}(Y) + \epsilon = \beta_0 + \beta_1 x + \epsilon$, where Y is the future observation, and ϵ is the new error:

A $100(1-\alpha)\%$ prediction interval (p.i.) for Y is

$$\hat{Y} \pm t_{n-2}\left(1 - \frac{\alpha}{2}\right) \text{p.s.e.}(\hat{Y}),$$

where $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 x$ and

$$\text{p.s.e.}(\hat{Y}) = \sqrt{\text{MSE}\left\{1 + \frac{1}{n} + \frac{(x-\bar{x})^2}{\sum_i (x_i - \bar{x})^2}\right\}}.$$

The 1 in the expression of p.s.e corresponds to the variance of ϵ, which is σ^2. If the variance of ϵ is different, appropriate change needs to be made [e.g., if $\text{var}(\epsilon) = \sigma^2/2$, change the 1 to 1/2].