Lecture Summary: March 17, 2023

• More on probit link

Indicator variable: $1_A = 1$ if even A happens, and $1_A = 0$ otherwise. $Y = 1_{(u \geq 0)}$, where $u = x'\beta + \epsilon$ and $\epsilon \sim N(0, 1)$. Then, we have

$$p = \mathrm{P}(Y = 1) = \mathrm{P}(u \geq 0) = \mathrm{P}(-\epsilon \leq x'\beta) = \Phi(x'\beta),$$

where $\Phi(x) = \mathrm{P}(Z \leq x)$ for $Z \sim N(0, 1)$. It follows that

$$\Phi^{-1}(p) = x'\beta.$$

This explains the probit link, Φ^{-1}.

• Binomial response

$Y \sim \text{Binomial}(k, p)$, where k is the total number of independent trials, p is the probability of success in a single trial.

Example: In the Challenger risk analysis, $k = 6$ (total number of primary o-rings) and p is the probability that an o-ring is distressed.

Logistic regression model: $\text{logit}(p) = x'\beta$.

Probit regression model: $\Phi^{-1}(p) = x'\beta$.

• Review of final exam topics