Lecture Summary: Jan. 20, 2023

• Testing hypothesis about β_1: The null hypothesis is $\mathrm{H}_0 : \beta_1 = \beta_{10}$, where β_{10} is a specified value (e.g., $\beta_{10} = 0$); the alternative can be $\mathrm{H}_1 : \beta_1 \neq \beta_{10}$, or $\mathrm{H}_1 : \beta_1 > \beta_{10}$, or $\mathrm{H}_1 : \beta_1 < \beta_{10}$.

In this case, the test statistic is

$$t = \frac{\hat{\beta}_1 - \beta_{10}}{\text{s.e.}(\hat{\beta}_1)}, \tag{1}$$

where $\hat{\beta}_1$ is the LS estimate of β_1, and

$$\text{s.e.}(\hat{\beta}_1) = \sqrt{\frac{\text{MSE}}{\sum_i (x_i - \bar{x})^2}}$$

with $\text{MSE} = s^2$.

Assuming normality. Then, we have the following
Fact: The distribution of

$$T = \frac{\hat{\beta}_1 - \beta_1}{\text{s.e.}(\hat{\beta}_1)}$$

is t_{n-2}, where β_1 is the true β_1 (slope) of the regression.

Thus, the distribution of t is t_{n-2} under the null hypothesis.

• Making a decision (in testing $H_0 : \beta_1 = \beta_{10}$)

If the alternative is $H_1 : \beta_1 \neq \beta_{10}$, reject H_0 if $|t| > t_{n-2}(1 - \alpha/2)$;

if the alternative is $H_1 : \beta_1 > \beta_{10}$, reject H_0 if $t > t_{n-2}(1 - \alpha)$;

if the alternative is $H_1 : \beta_1 < \beta_{10}$, reject H_0 if $t < -t_{n-2}(1 - \alpha)$.

• Type I and Type II errors

Type I: Reject H_0 when it is true;

Type II: Accept H_0 when it is false.

• Level of significance α = upper bound for the probability of Type I error (computed under H_0). For example, if $\alpha = 0.05$, the probability of rejecting H_0 is 0.05, if H_0 is true.

- p-value

It is the observed level of significance, that is, the actual probability that the test statistic is as extreme as observed given that the null hypothesis holds.