Lecture Summary: Feb. 3, 2023

• Test for normality

Correlation test: I. Compute the coefficient of correlation between the ordered residuals and their expected values. The latter are given by

$$\sqrt{\text{MSE}}z\left(\frac{k-0.375}{n+0.25}\right), \quad k=1,\ldots,n,$$

where $z(p)$ is the pth quantile of the standard normal distribution, that is, $\text{P}[Z <= z(p)] = p$, where Z has the standard normal distribution.

II. Compare the coef. of correlation on I with the critical value from Table B.6, if the coef. of correlation exceeds the critical value, accept the normality assumption.

Shapiro-Wilk's test, implemented in R: shapiro.test(). If the p-value is $\leq \alpha$, reject the normality assumption.

• Test for constant variance

1. Divide the residuals into two parts according to residual pattern (or no pattern).

Let $\hat{\epsilon}_{i1}, i = 1, \ldots, n_1$ be the residuals for the first part, and $\hat{\epsilon}_{i2}, i = 1, \ldots, n_2$ be the residuals for the second part, where $n_1 + n_2 = n$.

Compute $m(\hat{\epsilon}_1)$ = median of $\hat{\epsilon}_{i1}, i = 1, \ldots, n_1$, and $m(\hat{\epsilon}_2)$ = median of $\hat{\epsilon}_{i2}, i = 1, \ldots, n_2$.

2. Compute $d_{i1} = |\hat{\epsilon}_{i1} - m(\hat{\epsilon}_1)|, i = 1, \ldots, n_1$, and $d_{i2} = |\hat{\epsilon}_{i2} - m(\hat{\epsilon}_2)|, i = 1, \ldots, n_2$.

3. Compute

$$t_{\text{BF}} = \frac{\bar{d}_1 - \bar{d}_2}{s\sqrt{n_1^{-1} + n_2^{-1}}},$$

where $\bar{d}_1 = n_1^{-1}\sum_{i=1}^{n_1} d_{i1}$ and $\bar{d}_2 = n_2^{-1}\sum_{i=1}^{n_{21}} d_{i2}$, and

$$s^2 = \frac{\sum_{i=1}^{n_1}(d_{i1} - \bar{d}_1)^2 + \sum_{i=1}^{n_2}(d_{i2} - \bar{d}_2)^2}{n-2}.$$

4. Test $\mathrm{H}_0 : \sigma_1^2 = \sigma_2^2$ vs $\mathrm{H}_1 : \sigma_1^2 \neq \sigma_2^2$ (or $\sigma_1^2 > \sigma_2^2$; or $\sigma_1^2 < \sigma_2^2$), where σ_1^2 is the variance of the errors corresponding to the first part of the data, and σ_2^2 is the variance of the errors corresponding to the second part.

$t_{\mathrm{BF}} \sim t_{n-2}$ under H_0. Given α (say, $\alpha = 0.05$), use the critical value (or p-value) to test H_0.