Lecture Summary: Feb. 13, 2023

• Bonferroni's method for simultaneous inference

If A, B are events, then

$$\mathrm{P}(A \text{ and } B) \geq 1 - \mathrm{P}(\text{not } A) - \mathrm{P}(\text{not } B).$$

• Simultaneous confidence interval for β_0 and β_1

Let $A = \{I_0 \text{ covers } \beta_0\}$, $B = \{I_1 \text{ covers } \beta_1\}$, where

$$I_j = \hat{\beta}_j \pm t_{n-2}\left(1 - \frac{\alpha}{4}\right) \text{s.e.}(\hat{\beta}_j), \quad j = 0, 1,$$

then $\mathrm{P}(\text{not } A) = \alpha/2$, $\mathrm{P}(\text{not } B) = \alpha/2$. Thus, the $100(1-\alpha)\%$ simultaneous c.i.'s for β_0, β_1 are

$$I_0 = \hat{\beta}_0 \pm t_{n-2}\left(1 - \frac{\alpha}{4}\right) \text{s.e.}(\hat{\beta}_0),$$

$$I_1 = \hat{\beta}_1 \pm t_{n-2}\left(1 - \frac{\alpha}{4}\right) \text{s.e.}(\hat{\beta}_1).$$

They satisfy

$$\mathrm{P}(\beta_0 \in I_0 \text{ and } \beta_1 \in I_1) \geq 1 - \alpha.$$

• In general, if there are g simultaneous c.i.'s ($g = 2$ in the above special case) such that each has coverage probability $1 - \alpha/g$, the combined c.i.'s have a family coverage probability $1 - \alpha$.

• Simultaneous confidence intervals (s.c.i's) for $\mathrm{E}(Y_h) = \beta_0 + \beta_1 x_h$, $h \in G, |G| = g$ ($|G|$ denotes cardinality; Example: $G = \{1, 4, 9\}$, then $g = 3$).

The $100 \times (1-\alpha)\%$ s.c.i.'s have the following form:

Working-Hotelling method: $\hat{Y}_h \pm W \times \text{s.e.}(\hat{Y}_h)$, where $\hat{Y}_h = \hat{\beta}_0 + \hat{\beta}_1 x_h$,

$$\text{s.e.}(\hat{Y}_h) = \sqrt{\text{MSE}\left\{\frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_i (x_i - \bar{x})^2}\right\}},$$

and $W = \sqrt{2F_{2,n-2}(1-\alpha)}$; or

Bonferroni method: $\hat{Y}_h \pm B \times \text{s.e.}(\hat{Y}_h)$, where $B = t_{n-2}(1 - \frac{\alpha}{2g})$.

• Simultaneous prediction intervals (s.p.i's) for Y_h corresponding to x_h, $h \in G, |G| = g$.

The $100 \times (1-\alpha)\%$ s.p.i.'s have the following form:

Scheffé's method: $\hat{Y}_h \pm S \times \text{p.s.e.}(\hat{Y}_h)$, where $\hat{Y}_h$ is the same as above, and

$$\text{p.s.e.}(\hat{Y}_h) = \sqrt{\text{MSE}\left\{1 + \frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum_i (x_i - \bar{x})^2}\right\}},$$

and $S = \sqrt{gF_{g,n-2}(1-\alpha)}$; or

Bonferroni method: $\hat{Y}_h \pm B \times \text{p.s.e.}(\hat{Y}_h)$, where $B = t_{n-2}(1 - \frac{\alpha}{2g})$.