Lecture Summary: Jan. 18, 2023

• Maximum likelihood estimation (continued):

Implementation: Suppose that Y is the observed data whose probability density function is $f(y|\theta)$, where θ is a vector of unknown parameters.

When y is replaced by the observed data, Y, and θ considered as a (vector-valued) variable, $f(Y|\theta)$, considered as a function of θ, is called the likelihood.

θ is then estimated by the maximizer of the likelihood, denoted by $\hat{\theta}$. This is called the maximum likelihood estimator (MLE).

In the case of linear regression with normal errors, the least squares (LS) estimators of β_0, β_1 are the same as the MLEs of β_0, β_1, but the MLE of σ^2, $\hat{\sigma}^2$, is different from s^2:

$$s^2 = \frac{1}{n-2}\sum_{i=1}^{n}\hat{\epsilon}_i^2, \quad \hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}\hat{\epsilon}_i^2,$$

where $\hat{\epsilon}_i = Y_i - \hat{Y}_i$ ($\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$) is the residual.

• Hypothesis testing: General principles

The null hypothesis is usually set as the opposite of the objective that one tries to prove. The idea is to look for evidence from the data that can reject the null hypothesis, and therefore support the objective.

Implementation:

1. A test statistic, that is, a statistic whose distribution is known under the null hypothesis.

2. The null hypothesis has to make a difference.

• Testing hypothesis about β_1: The null hypothesis is $\mathrm{H}_0 : \beta_1 = \beta_{10}$, where β_{10} is a specified value (e.g., $\beta_{10} = 0$); the alternative can be $\mathrm{H}_1 : \beta_1 \neq \beta_{10}$, or $\mathrm{H}_1 : \beta_1 > \beta_{10}$, or $\mathrm{H}_1 : \beta_1 < \beta_{10}$.