


Hands On 3 Questions/Answers

Part 1:

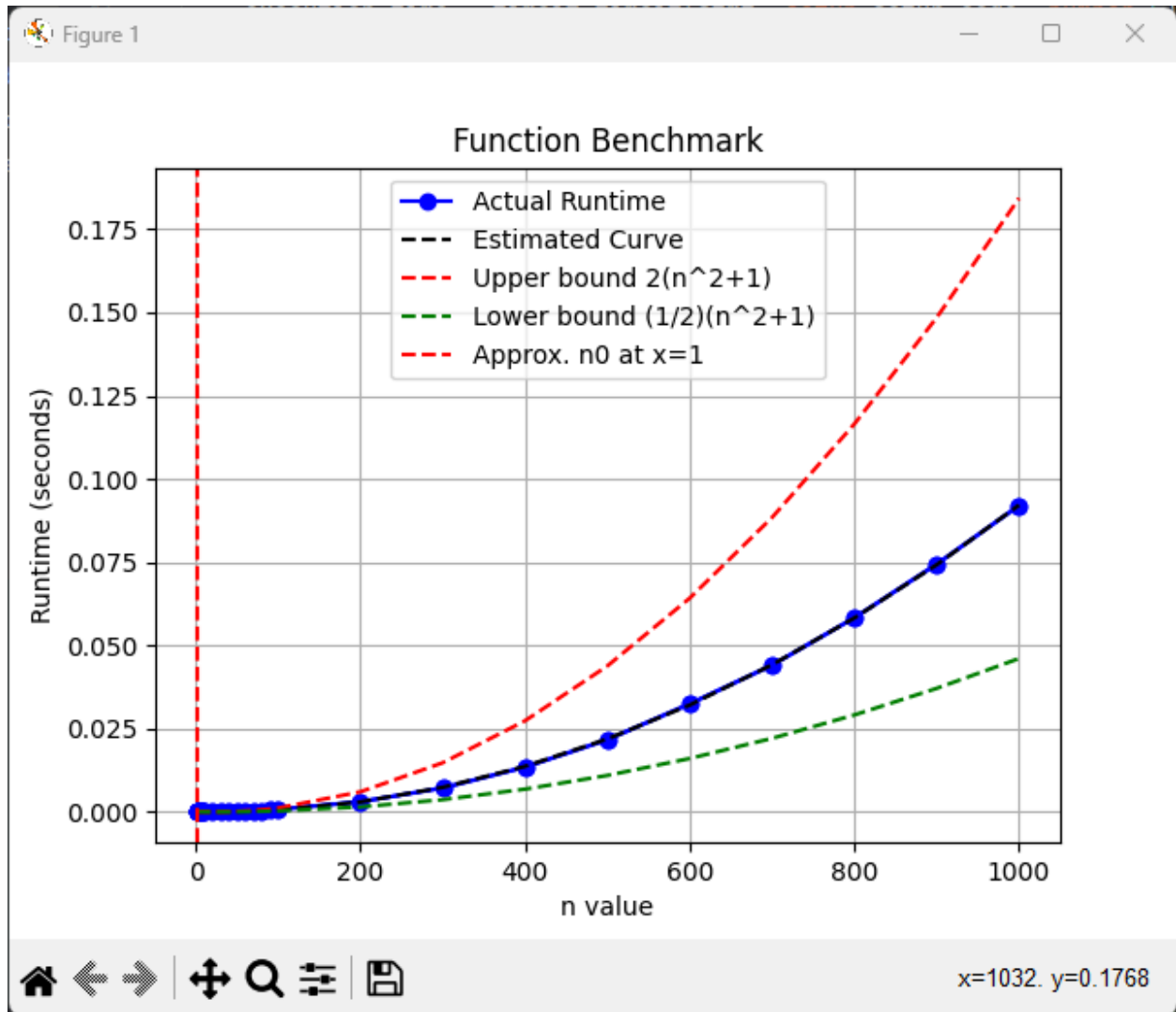
$$\begin{array}{l} x = 1 \\ \text{for } i = 1 : n \\ \quad \text{for } j = 1 : n \\ \quad \quad x = x + 1 \end{array}$$



$$\begin{aligned} T(n) &= 1 + \sum_{i=1}^n \sum_{j=1}^n 1 \\ &= 1 + \sum_{i=1}^n (n - i + 1) \Rightarrow T(n) = 1 + n^2 \\ &= 1 + \sum_{i=1}^n n \\ &= 1 + n^2 \end{aligned}$$

So the runtime should be $1 + n^2$ or $\theta(n^2)$.

Part 2:



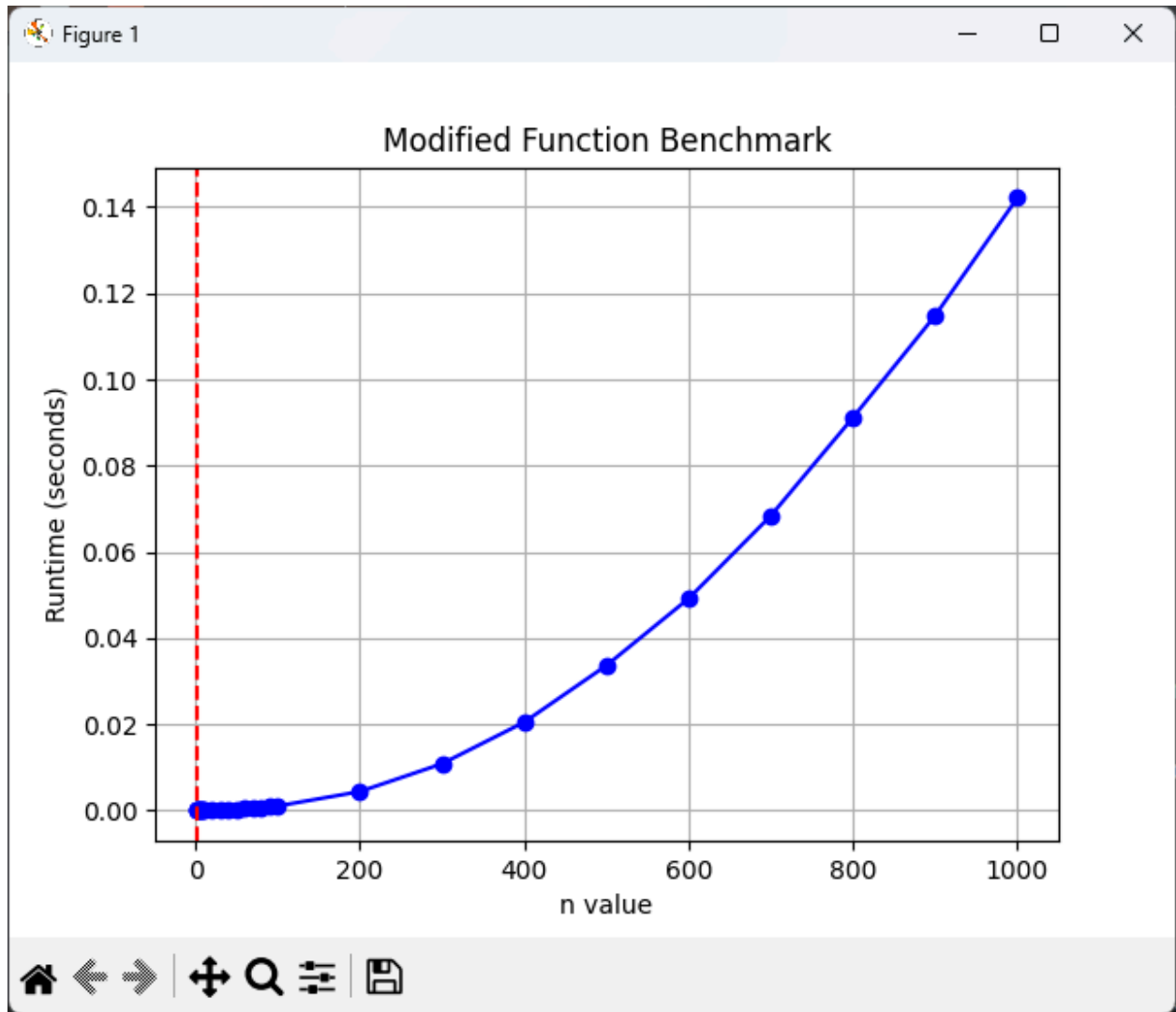
Part 3: I chose the upper bound to be $2(n^2+1)$ and the lower bound to be $(1/2)(n^2+1)$ where the estimate curve fits inside between them.

The values are $O(n^2)$, $\Omega(n^2)$, $\theta(n^2)$.

Part 4: n_0 should be at $x=1$ where the runtime is very close to 0. I picked this value because without the n^2 in $1+n^2$, 1 is left over.

Part 4(for modified function):

Yes this will increase the time the function takes.



Part 5:

Yes it will affect the results from number 1 by increasing it. Although it will still be a polynomial and in theta form the runtime will still be $\theta(n^2)$.

$x = 1$	<u>Time</u>
$y = 1$	1
for $i = 1 : n$	$\sum_{i=1}^n$
for $j = 1 : n$	$\sum_{j=1}^n$
$x = x + 1$	1
$y = i + j$	1



$$T(n) = 1 + 1 + \sum_{i=1}^n \sum_{j=1}^n 2$$

$$= 2 + \sum_{i=1}^n (n - 1 + 2)$$

$$= 2 + \sum_{i=1}^n (n + 1)$$

$$= 2 + n^2 + n$$



$$T(n) = n^2 + n + 2$$

Part 6:

Mergesort shown here:

```
Test array before: [5, 2, 4, 7, 1, 3, 2, 6]
Sorted: [2, 5]
Sorted: [4, 7]
Sorted: [2, 4, 5, 7]
Sorted: [1, 3]
Sorted: [2, 6]
Sorted: [1, 2, 3, 6]
Sorted: [1, 2, 2, 3, 4, 5, 6, 7]
Test array after: [1, 2, 2, 3, 4, 5, 6, 7]

Process finished with exit code 0
```