Department of Mathematics

Academic Year 2023-2024 (Even Semester 2024)

Date	**02/12/2024**	Time	**09:30 AM to 11:30 AM**
TEST	**CIE - II**	Maximum Marks	**10+50**
Course Title	**LINEAR ALGEBRA AND PROBABILITY THEORY**	Course Code	**MA231TC**
Semester	**III**	Programs	**CS, CD, CY, IS**

Instructions

1. **Answer all questions.**
2. **Mathematics handbook of Second-year B.E. programme is allowed.**

PART - A

Sl. no.	Questions	M	BT	CO
1	Given $A = [2 \quad 1 \quad 3]$, then singular values of A is/are ________.	1	1	1
2	If 0 is an eigenvalue of a symmetric matrix $A_{3\times3}$ repeated twice, then $Nullity(A) =$_____.	1	2	2
3	If X is a discrete random variable with $E[X] = 2$ and $E[X^2] = 5$, then $Var(X+1) =$_____.	1	2	2
4	Let $F(x) = \frac{x^2+k}{25}$, $x = 0, 1, 2, 3$ be the cumulative distribution of a random variable X. The value of k is ______.	1	2	2
5	Let $C\left[0, \frac{\pi}{4}\right]$ be the inner product space of continuous functions over $\left[0, \frac{\pi}{4}\right]$ with respect to the inner product defined by $\langle f, g\rangle = \int_0^{\pi/4} f(x)g(x)dx,\ \forall f,\ g \in C\left[0, \frac{\pi}{4}\right]$. Determine $\lVert f \rVert$ where $f(x) = \cos(x)$.	2	3	3
6	If $\begin{bmatrix}2\\3\end{bmatrix}$ and $\begin{bmatrix}3\\-2\end{bmatrix}$ are the eigen values of a symmetric matrix $A_{2\times2}$ with respect to the eigen value 1 and 2 respectively, then $A =$________	2	2	2
7	Determine c that renders $f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < c \\ 0, & \text{elsewhere} \end{cases}$ a valid density function.	2	3	2

PART – B

Sl. no.	Questions	M	BT	CO
1	Given the matrix A such that the column vectors are linearly independent and for $X \in \mathbb{R}^4$: $$A = \begin{bmatrix}1 & -1 & 4\\1 & 4 & -2\\1 & 4 & 2\\1 & -1 & 0\end{bmatrix},\ X = \begin{bmatrix}1\\2\\1\\1\end{bmatrix}$$ i) Obtain an orthonormal basis for column space of A ii) Find the shortest distance from X to $Col(A)$.	10	3	3
2a	Let $u_1 = \begin{bmatrix}2/3\\1/3\\-2/3\end{bmatrix}$, $u_2 = \begin{bmatrix}-1/\sqrt{5}\\2/\sqrt{5}\\0\end{bmatrix}$, $u_3 = \begin{bmatrix}4/\sqrt{45}\\2/\sqrt{45}\\5/\sqrt{45}\end{bmatrix}$ and $X = \begin{bmatrix}2\\3\\-1\end{bmatrix}$ i) Show that $B = \{u_1, u_2, u_3\}$ form orthonormal basis for $\mathbb{R}^3$. ii) Obtain the coordinate vector of X with respect to the basis B.	4	3	3

2b	Given $A = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$ i) Obtain a diagonal matrix D and an orthogonal matrix X such that $A = XDX^T$. ii) Hence determine the eigenvalues and eigenvectors of $B = A^2 - 2I$.	6	3	3
3	Obtain the singular value decomposition of the matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}$.	10	4	4
4a	Two balls are chosen randomly from an urn containing 8 white, 4 black, and 2 orange balls. Suppose that we win Rs. 200 for each black ball selected and we lose Rs. 100 for each white ball selected. Let X denotes our winnings. Determine i) The possible values of X, and the probabilities associated with each value. ii) The expected winning amount. iii) The cumulative distribution of X.	6	2	2
4b	Measurements of scientific systems are always subject to variation, some more than others. There are many structures for measurement error, and statisticians spend a great deal of time modeling these errors. Suppose the measurement error X of a certain physical quantity is decided by the density function $f_X(x) = \begin{cases} k(3 - x^2), & -1 \leq x \leq 1 \\ 0, & \text{Otherwise} \end{cases}$ i) Determine k that renders $f_X(x)$ a valid density function. ii) Find the probability that a random error in measurement is less than $1/2$.	4	2	2
5a	The probability density function of a random variable X is given by $f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 2 - x, & 1 \leq x \leq 2. \\ 0, & \text{Otherwise.} \end{cases}$ i) Determine the cumulative density function $F(x)$ ii) Find $P(X \geq 1.5)$.	5	2	2
5b	Given the function $p(x, y) = cxy$ for $x = 1, 2, 3;\ y = 1, 2, 3.$ i) Determine c such that $p(x, y)$ is the joint probability mass function of random variables X and Y. ii) Find the marginal distribution of X and Y. iii) Obtain $E[XY]$.	5	1	1

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BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

Marks Distribution	Particulars	CO1	CO2	CO3	CO4	L1	L2	L3	L4	L5	L6
	Test Max Marks	5	15	20	10	5	15	20	10	--	--
	Quiz Max Marks	1	7	2	-	1	5	4	-	-	-