Advanced Encryption Standard (AES)

Outline

- Overview of the AES algorithm
- Internal structure of AES
  - Byte Substitution
  - Shift rows
  - MixColumns
  - Key Addition
  - Key schedule
- Decryption

Overview of the AES algorithm

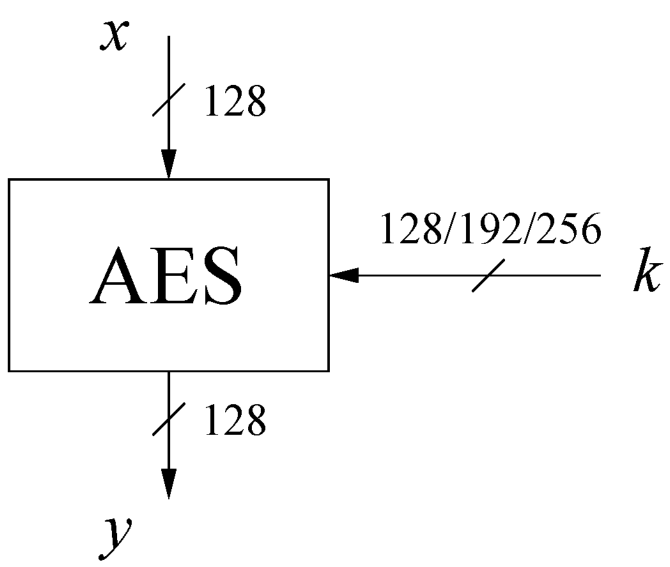
Some Basic Facts

- AES is the most widely used symmetric cipher today
- The algorithm for AES was chosen by the US *National Institute of Standards and Technology* (NIST) in a multi-year selection process
- The requirements for all AES candidate submissions were:
  - Block cipher with **128-bit block size**
  - **Three supported key lengths**: 128, 192 and 256 bit
  - Security relative to other submitted algorithms
  - **Efficiency** in software and hardware implementation

Chronology of the AES Selection

- The need for a new block cipher announced by NIST in January, 1997
- 15 candidates algorithms accepted in August, 1998
- 5 finalists announced in August, 1999:
  - *Mars* – IBM Corporation
  - *RC6* – *RSA* Laboratories
  - *Rijndael* – J. Daemen & V. Rijmen
  - *Serpent* – Eli Biham et al.
  - *Twofish* – B. Schneier et al.
- In October 2000, *Rijndael* was chosen as the AES
- AES was formally approved as a US federal standard in November 2001

AES Overview

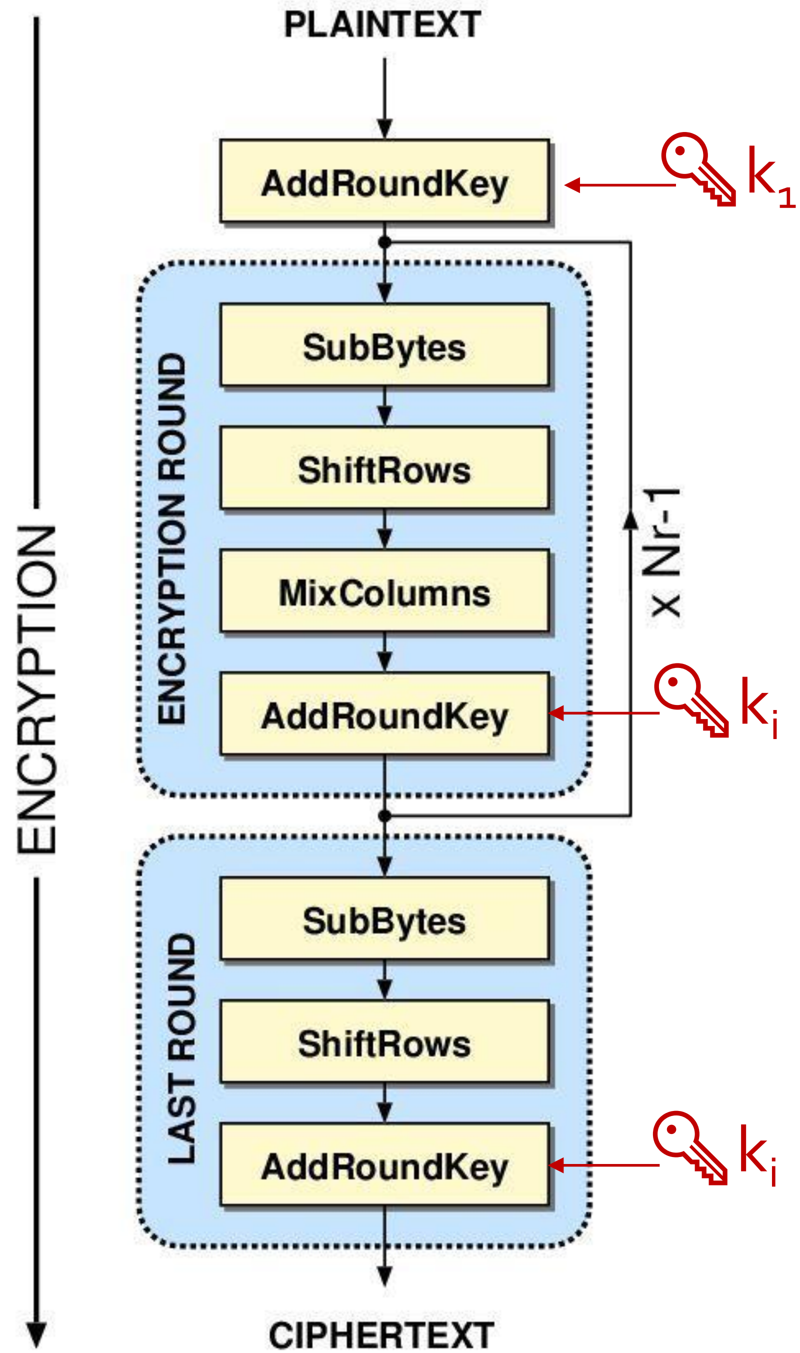


The number of rounds depends on the chosen key length:

Key length (bits)	Number of rounds
128	10
192	12
256	14

AES Overview

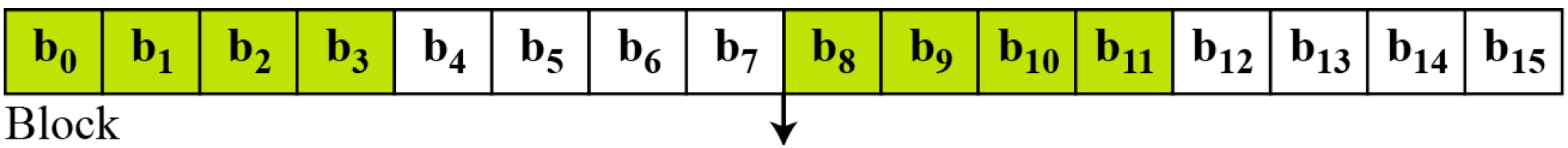
- An **iterative** rather than Feistel cipher
- Operates on entire data block in every round
- 10/12/14 rounds depending on the key size.
- Each round consists of Confusion and Diffusion operations
- Note: In the last round, the MixColumns tansformation is omitted



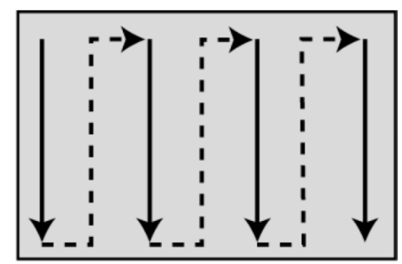
Internal structure of AES

Block to state

- AES is a byte-oriented cipher
- State = Block of bytes that are currently being worked on
- Arranged in 4 x 4 Matrix of **bytes**



$$\text{State} \begin{bmatrix} b_0 & b_4 & b_8 & b_{12} \\ b_1 & b_5 & b_9 & b_{13} \\ b_2 & b_6 & b_{10} & b_{14} \\ b_3 & b_7 & b_{11} & b_{15} \end{bmatrix}$$



Insertion and extraction flow

with $b_0, \ldots, b_{15}$ denoting the **16-byte** input of AES arranged in a 4x4 matrix

Block to state - example

Text	A	E	S	U	S	E	S	A	M	A	T	R	I	X	Z	Z
Hexadecimal	00	04	12	14	12	04	12	00	0C	00	13	11	08	23	19	19

$$\begin{bmatrix} 00 & 12 & 0C & 08 \\ 04 & 04 & 00 & 23 \\ 12 & 12 & 13 & 19 \\ 14 & 00 & 11 & 19 \end{bmatrix} \text{ State}$$

SubBytes = Byte Substitution

- Each value of the state is replaced with the corresponding S-Box value => bytewise S-Box substitution
- E.g. HEX 14 would get replaced with HEX FA

$$\boxed{14} \xrightarrow[\text{Column 4}]{\text{Row 1}} \boxed{FA}$$

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	63	7C	77	7B	F2	6B	6F	C5	30	01	67	2B	FE	D7	AB	76
1	CA	82	C9	7D	FA	59	47	F0	AD	D4	A2	AF	9C	A4	72	C0
2	B7	FD	93	26	36	3F	F7	CC	34	A5	E5	F1	71	D8	31	15
3	04	C7	23	C3	18	96	05	9A	07	12	80	E2	EB	27	B2	75
4	09	83	2C	1A	1B	6E	5A	A0	52	3B	D6	B3	29	E3	2F	84
5	53	D1	00	ED	20	FC	B1	5B	6A	CB	BE	39	4A	4C	58	CF
6	D0	EF	AA	FB	43	4D	33	85	45	F9	02	7F	50	3C	9F	A8
7	51	A3	40	8F	92	9D	38	F5	BC	B6	DA	21	10	FF	F3	D2
8	CD	0C	13	EC	5F	97	44	17	C4	A7	7E	3D	64	5D	19	73
9	60	81	4F	DC	22	2A	90	88	46	EE	B8	14	DE	5E	0B	DB
A	E0	32	3A	0A	49	06	24	5C	C2	D3	AC	62	91	95	E4	79
B	E7	C8	37	6D	8D	D5	4E	A9	6C	56	F4	EA	65	7A	AE	08
C	BA	78	25	2E	1C	A6	B4	C6	E8	DD	74	1F	4B	BD	8B	8A
D	70	3E	B5	66	48	03	F6	0E	61	35	57	B9	86	C1	1D	9E
E	E1	F8	98	11	69	D9	8E	94	9B	1E	87	E9	CE	55	28	DF
F	8C	A1	89	0D	BF	E6	42	68	41	99	2D	0F	B0	54	BB	16

SubBytes Example

$$\text{State}\begin{bmatrix} 00 & 12 & 0C & 08 \\ 04 & 04 & 00 & 23 \\ 12 & 12 & 13 & 19 \\ 14 & 00 & 11 & 19 \end{bmatrix} \xrightarrow{\text{SubByte}} \begin{bmatrix} 63 & C9 & FE & 30 \\ F2 & F2 & 63 & 26 \\ C9 & C9 & 7D & D4 \\ FA & 63 & 82 & D4 \end{bmatrix}\text{State}$$

Shift Rows

- Performs **Left Circular Shift** of the state matrix row:
- This is not a bit wise shift. The circular shift just moves each byte one space over.

Input matrix

B_0	B_4	B_8	B_{12}
B_1	B_5	B_9	B_{13}
B_2	B_6	B_{10}	B_{14}
B_3	B_7	B_{11}	B_{15}

Output matrix

B_0	B_4	B_8	B_{12}	no shift
B_5	B_9	B_{13}	B_1	← one position left shift
B_{10}	B_{14}	B_2	B_6	← two positions left shift
B_{15}	B_3	B_7	B_{11}	← three positions left shift

MixColumns

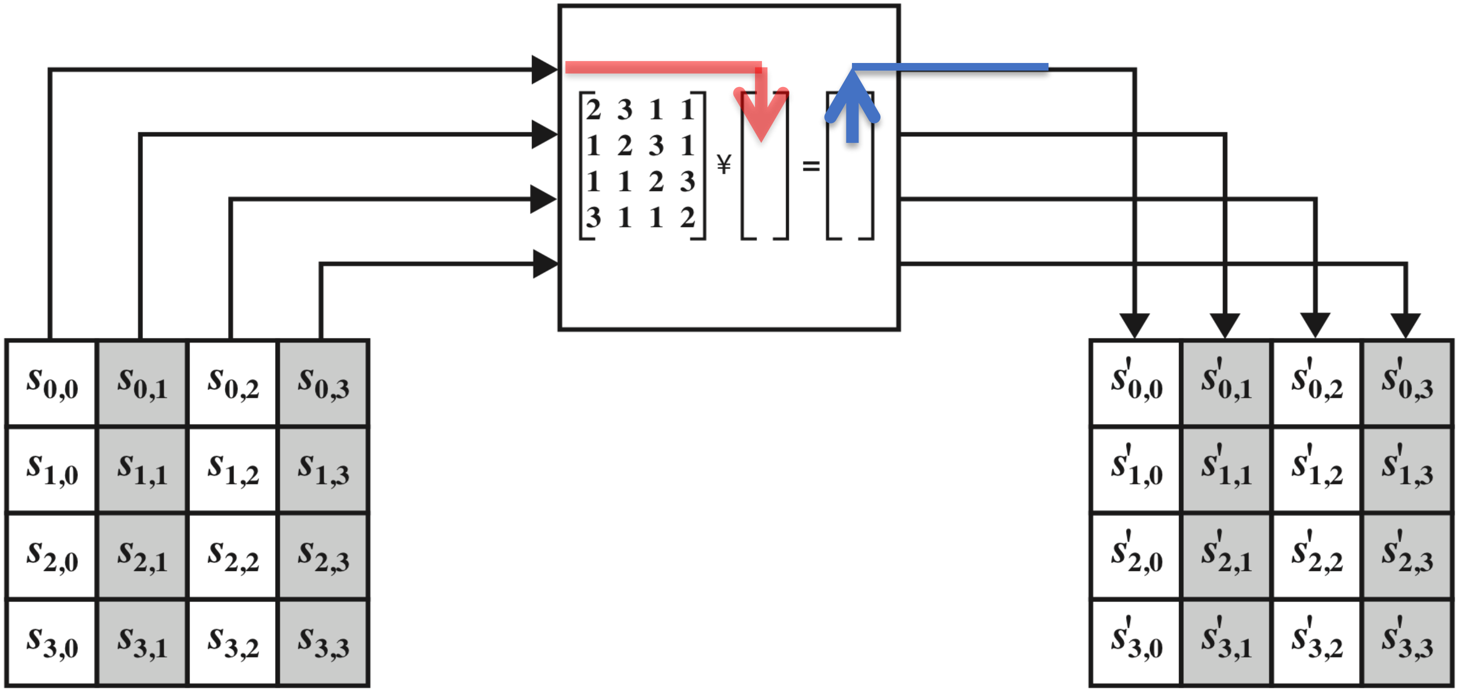
- The MixColumns transformation operates at the column level. It transforms each column of the state to a new column in the next state
- Each 4-byte column is considered as a vector and multiplied by a fixed 4x4 matrix, e.g.,

$$\begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 & 1 \\ 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 3 \\ 3 & 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$d_0 = 2 \bullet b_0 \oplus 3 \bullet b_1 \oplus 1 \bullet b_2 \oplus 1 \bullet b_3$$
$$d_1 = 1 \bullet b_0 \oplus 2 \bullet b_1 \oplus 3 \bullet b_2 \oplus 1 \bullet b_3$$
$$d_2 = 1 \bullet b_0 \oplus 1 \bullet b_1 \oplus 2 \bullet b_2 \oplus 3 \bullet b_3$$
$$d_3 = 3 \bullet b_0 \oplus 1 \bullet b_1 \oplus 1 \bullet b_2 \oplus 2 \bullet b_3$$

MixColumns Transformation

$$s'_{0,0} = 2 \cdot s_{0,0} + 3 \cdot s_{1,0} + 1 \cdot s_{2,0} + 1 \cdot s_{3,0}$$



- The MixColumns transformation operates at the column level. It transforms each column of the state to a new column.
- Each 4-byte column is considered as a vector and multiplied by a fixed 4x4 matrix.

Add Round Key

- **State matrix ⊕ Round key matrix**

- Inputs:
  - 16-byte state matrix C
  - 16-byte subkey k_i

- Output: $C \oplus k_i$

- The round keys are generated by the key schedule

AES Key Scheduling

- Subkeys are derived recursively from the original 128/192/256-bit input key

- Each round has 1 subkey, plus 1 subkey at the beginning of AES

Key length (bits)	Number of subkeys
128	11
192	13
256	15

AES Key Scheduling

- Takes 128-bits (16-bytes) key and **expands** into array of 44 32-bit words
- 11 subkeys are stored in W[0]…W[3], W[4]…W[7], … , W[40]…W[43]

Round	*Words*			
Pre-round	$\mathbf{w}_0$	$\mathbf{w}_1$	$\mathbf{w}_2$	$\mathbf{w}_3$
1	$\mathbf{w}_4$	$\mathbf{w}_5$	$\mathbf{w}_6$	$\mathbf{w}_7$
2	$\mathbf{w}_8$	$\mathbf{w}_9$	$\mathbf{w}_{10}$	$\mathbf{w}_{11}$
. . .	. . .			
N_r	$\mathbf{w}_{4N_r}$	$\mathbf{w}_{4N_r+1}$	$\mathbf{w}_{4N_r+2}$	$\mathbf{w}_{4N_r+3}$

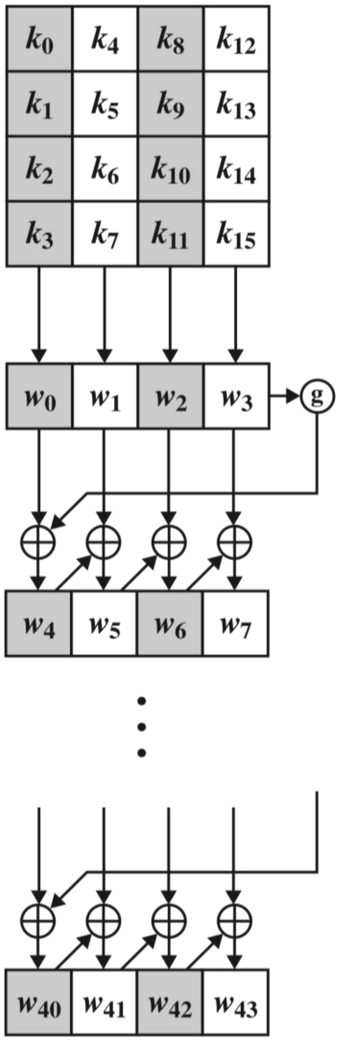
AES Key Expansion

- First subkey *W[0]…W[3]* is the original AES key
- Constructing subsequent groups of 4 words based on the *Previous Word* (W_{i-1}) & *4th back Word* (W_{i-4})

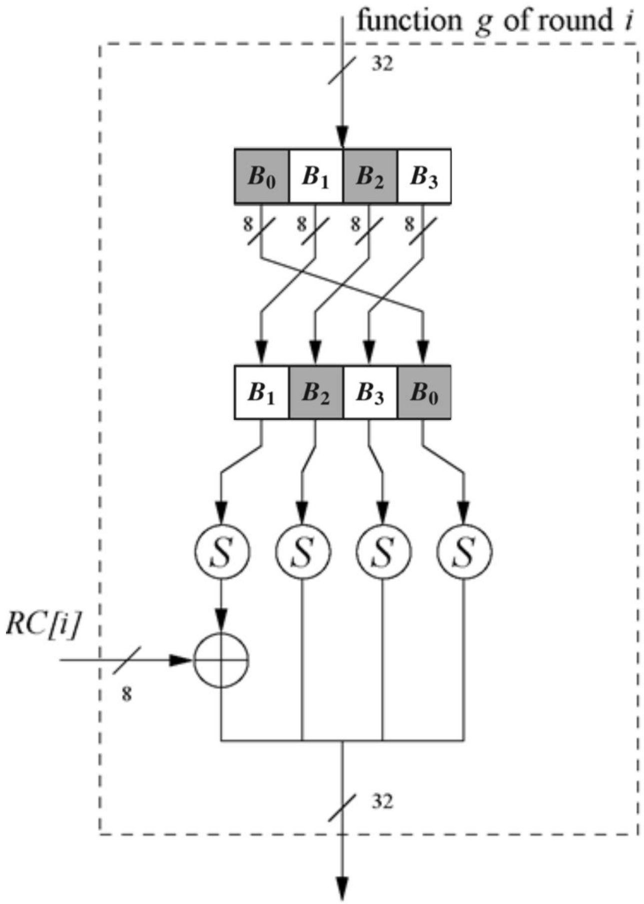
$$W_i = W_{i-1} \oplus W_{i-4}$$

- **1st word** in each group gets a "special treatment" using function **g** before XOR'ing the *4th back Word* (W_{i-4})

$$W_i = g(W_{i-1}) \oplus W_{i-4}$$



(a) Overall algorithm



(b) Function g

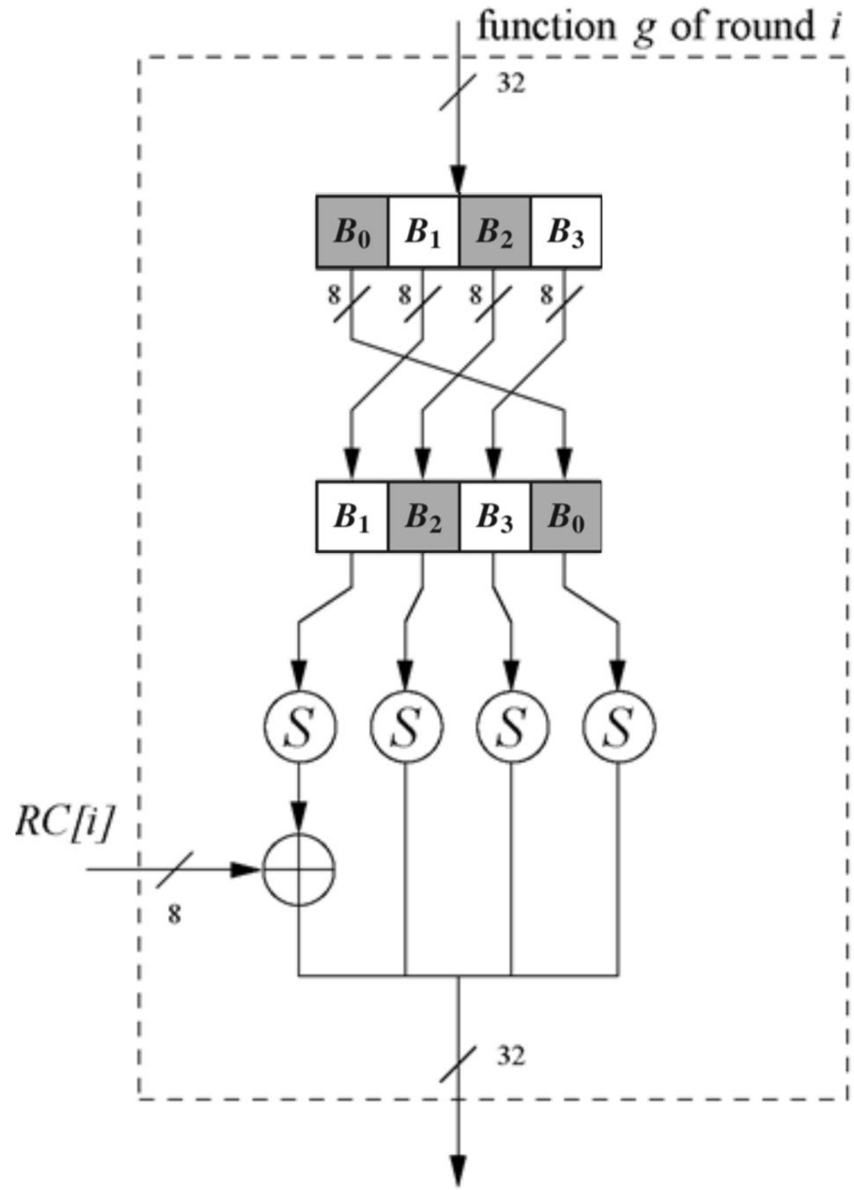
Rotate -> S-box -> XOR a constant

Key Expansion - 1st Word "special treatment"

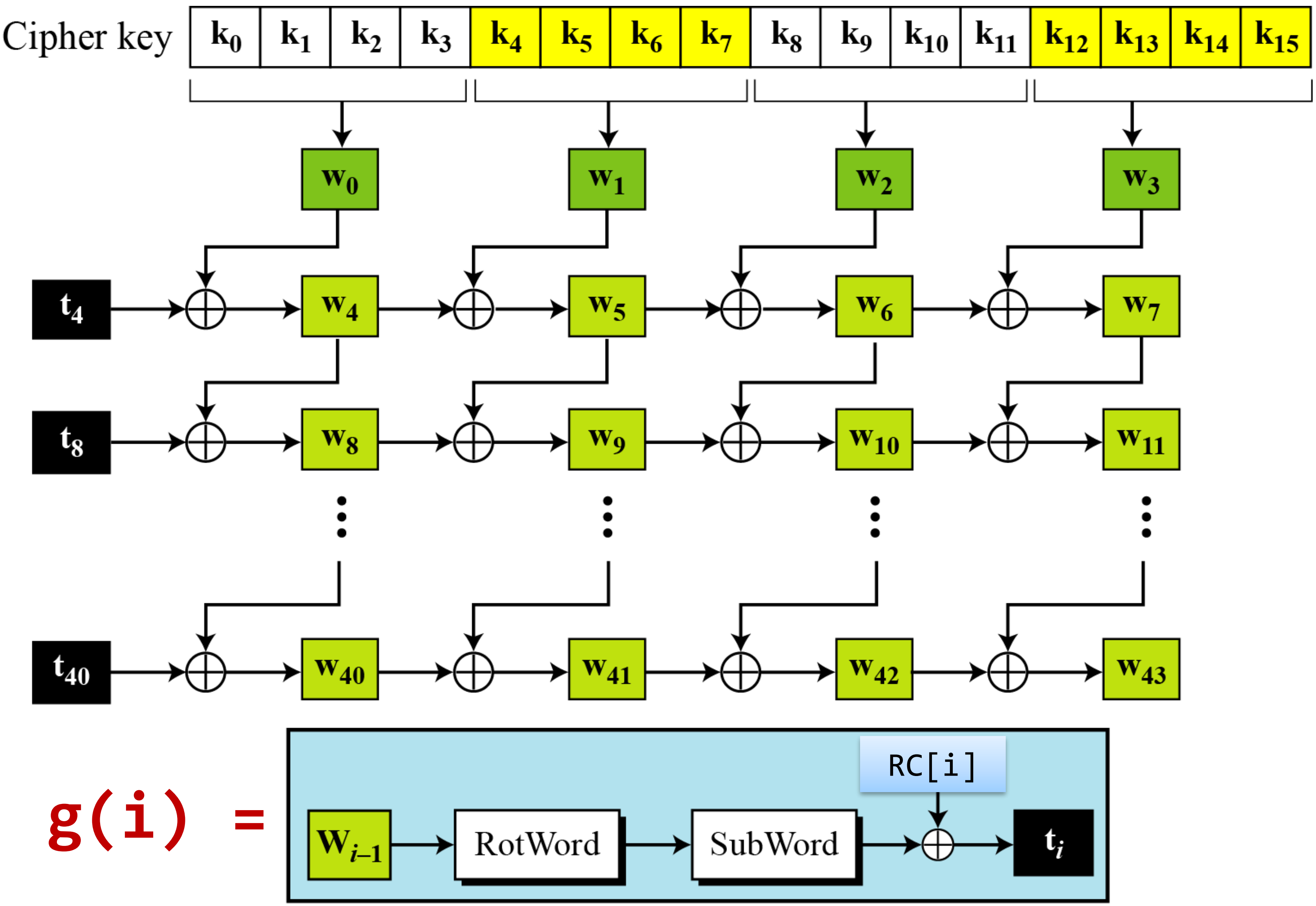
- Function *g* rotates its four input bytes and performs a **bytewise** S-Box substitution

- Leftmost byte is XORed with a **Round Constant** (RC):

Rcon Constants (Base 16)			
Round	Constant(Rcon)	Round	Constant(Rcon)
1	01 00 00 00	6	20 00 00 00
2	02 00 00 00	7	40 00 00 00
3	04 00 00 00	8	80 00 00 00
4	08 00 00 00	9	1B 00 00 00
5	10 00 00 00	10	36 00 00 00



Key Expansion Scheme – Another View



Making of t_i (temporary) words $i = 4\ N_r$

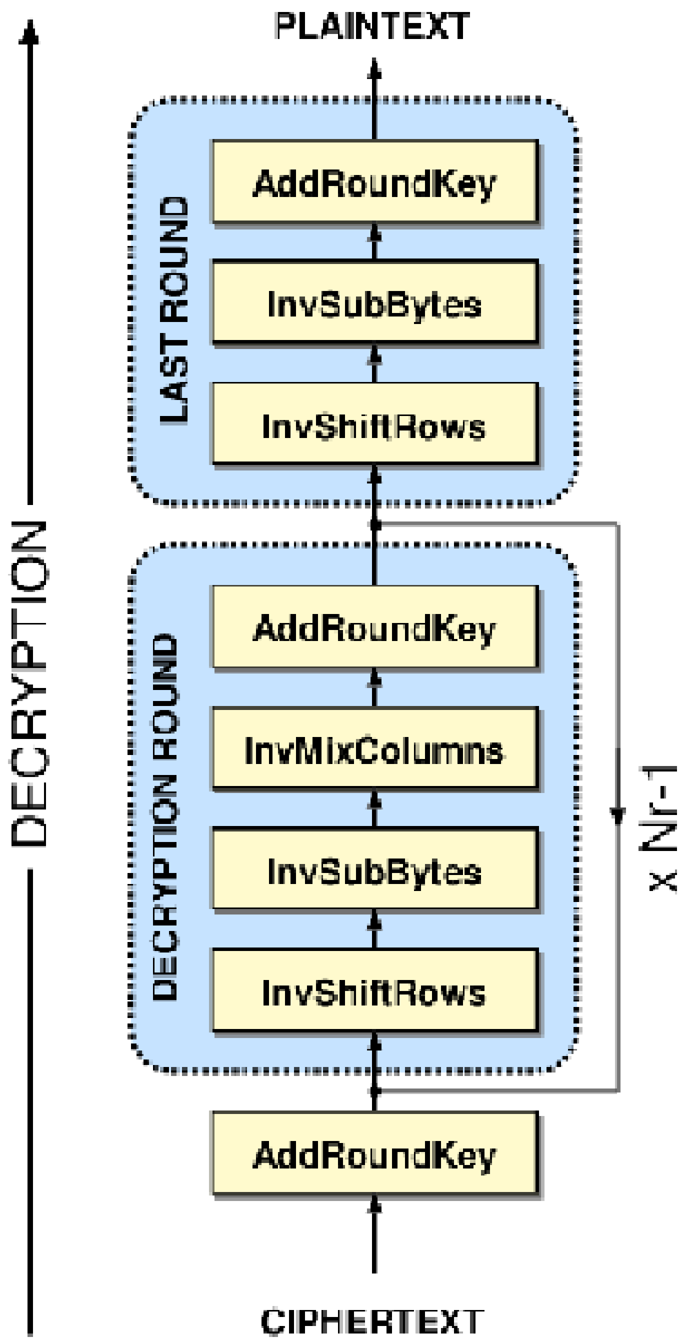
Example - First Roundkey

- Key in Hex (128 bits): 54 68 61 74 73 20 6D 79 20 4B 75 6E 67 20 46 75
- $w[0] = (54, 68, 61, 74), w[1] = (73, 20, 6D, 79), w[2] = (20, 4B, 75, 6E), w[3] = (67, 20, 46, 75)$
- $g(w[3])$:
  - circular byte left shift of $w[3]$: $(20, 46, 75, 67)$
  - Byte Substitution (S-Box): $(B7, 5A, 9D, 85)$
  - Adding round constant $(01, 00, 00, 00)$ gives: $g(w[3]) = (B6, 5A, 9D, 85)$
- $w[4] = w[0] \oplus g(w[3]) = (E2, 32, FC, F1)$:

0101 0100	0110 1000	0110 0001	0111 0100
1011 0110	0101 1010	1001 1101	1000 0101
1110 0010	0011 0010	1111 1100	1111 0001
E2	32	FC	F1

- $w[5] = w[4] \oplus w[1] = (91, 12, 91, 88)$, $w[6] = w[5] \oplus w[2] = (B1, 59, E4, E6)$, $w[7] = w[6] \oplus w[3] = (D6, 79, A2, 93)$
- first roundkey: E2 32 FC F1 91 12 91 88 B1 59 E4 E6 D6 79 A2 93

Decryption



Decryption

- AES is not based on a Feistel network

$\Rightarrow$ All layers must be inverted for decryption:

- ShiftRows → **Inv ShiftRows**
- MixColumn → **Inv MixColumn**
- Byte Substitution → **Inv Byte Substitution**
- Key Addition layer uses XOR
- Subkeys are needed in reversed order

Inv ShiftRows

- All rows of the state matrix B are shifted to the opposite direction:

Input matrix

B_0	B_4	B_8	B_{12}
B_1	B_5	B_9	B_{13}
B_2	B_6	B_{10}	B_{14}
B_3	B_7	B_{11}	B_{15}

Output matrix

B_0	B_4	B_8	B_{12}	no shift
B_{13}	B_1	B_5	B_9	→ one position right shift
B_{10}	B_{14}	B_2	B_6	→ two positions right shift
B_7	B_{11}	B_{15}	B_3	→ three positions right shift

Inv MixColumn

- The MixColumns operation has the following inverse (numbers are decimal):

$$\begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 14 & 11 & 13 & 9 \\ 9 & 14 & 11 & 13 \\ 13 & 9 & 14 & 11 \\ 11 & 13 & 9 & 14 \end{bmatrix} \begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

Or:

$$\begin{aligned} b_0 &= 14 \bullet d_0 \oplus 11 \bullet d_1 \oplus 13 \bullet d_2 \oplus 9 \bullet d_3 \\ b_1 &= 9 \bullet d_0 \oplus 14 \bullet d_1 \oplus 11 \bullet d_2 \oplus 13 \bullet d_3 \\ b_2 &= 13 \bullet d_0 \oplus 9 \bullet d_1 \oplus 14 \bullet d_2 \oplus 11 \bullet d_3 \\ b_3 &= 11 \bullet d_0 \oplus 13 \bullet d_1 \oplus 9 \bullet d_2 \oplus 14 \bullet d_3 \end{aligned}$$

InvSubByte

- During decryption each value in the state is replaced with the corresponding inverse of the S-Box
- For example HEX D4 would get replaced with HEX 19

		y															
		0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
	0	52	09	6a	d5	30	36	a5	38	bf	40	a3	9e	81	f3	d7	fb
	1	7c	e3	39	82	9b	2f	ff	87	34	8e	43	44	c4	de	e9	cb
	2	54	7b	94	32	a6	c2	23	3d	ee	4c	95	0b	42	fa	c3	4e
	3	08	2e	a1	66	28	d9	24	b2	76	5b	a2	49	6d	8b	d1	25
	4	72	f8	f6	64	86	68	98	16	d4	a4	5c	cc	5d	65	b6	92
	5	6c	70	48	50	fd	ed	b9	da	5e	15	46	57	a7	8d	9d	84
	6	90	d8	ab	00	8c	bc	d3	0a	f7	e4	58	05	b8	b3	45	06
	7	d0	2c	1e	8f	ca	3f	0f	02	c1	af	bd	03	01	13	8a	6b
x	8	3a	91	11	41	4f	67	dc	ea	97	f2	cf	ce	f0	b4	e6	73
	9	96	ac	74	22	e7	ad	35	85	e2	f9	37	e8	1c	75	df	6e
	a	47	f1	1a	71	1d	29	c5	89	6f	b7	62	0e	aa	18	be	1b
	b	fc	56	3e	4b	c6	d2	79	20	9a	db	c0	fe	78	cd	5a	f4
	c	1f	dd	a8	33	88	07	c7	31	b1	12	10	59	27	80	ec	5f
	d	60	51	7f	a9	19	b5	4a	0d	2d	e5	7a	9f	93	c9	9c	ef
	e	a0	e0	3b	4d	ae	2a	f5	b0	c8	eb	bb	3c	83	53	99	61
	f	17	2b	04	7e	ba	77	d6	26	e1	69	14	63	55	21	0c	7d

AES Security

- **Brute-force attack:** Due to the key length of 128, 192 or 256 bits, a brute-force attack is not possible
- **Analytical attacks:** There is no known analytical attack.
- **Side-channel attacks:**
  - Several side-channel attacks have been published
  - Note that side-channel attacks do not attack the underlying algorithm but its implementation

Summary

- AES is a modern block cipher which supports three key lengths of 128, 192 and 256 bit. It provides excellent long-term security against brute-force attacks.
- AES has been studied intensively since the late 1990s and no attacks have been found.
- AES is not based on Feistel networks. Its basic operations use Galois field arithmetic and provide strong diffusion and confusion.
- AES is part of numerous open standards such as IPsec or TLS, in addition to being the mandatory encryption algorithm for US government applications. It seems likely that the cipher will be the dominant encryption algorithm for many years to come.
- AES is efficient in software and hardware.

Resources

- Crypto Tool 2

https://www.cryptool.org/en/cryptool2

- AES Wikipedia page

https://en.wikipedia.org/wiki/Advanced_Encryption_Standard