Asymmetric Cryptography

Current Cryptographic Standards

Encryption
(Confidentiality)

Digital Signatures
Authentication / Non Repudiation
Message Integrity

Symmetric Key Algorithm

Stream Cipher
RC4
SEAL
WAKE
A5
PKZIP

Block Cipher
AES, DES, 3DES
RC2,RC5,RC6
Blowfish, Twofish,
Serpent
Mars, Cast, Idea

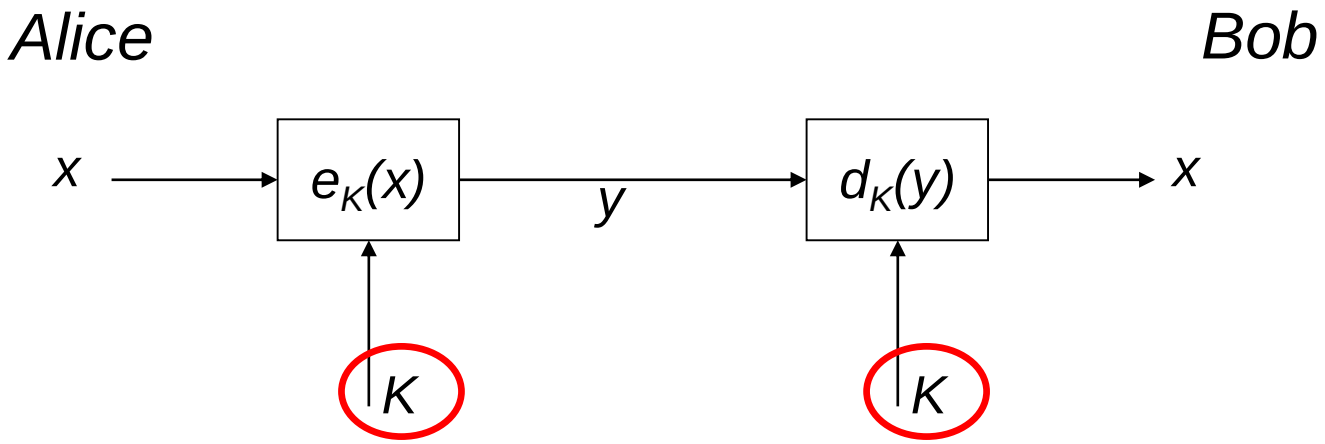
Public Key
Asymmetric Algorithm

Discrete Log
DSA
ECC
Diffe-Helman
ElGamal

Factoring
RSA
LUC

Message Digest
Hash Algorithm
MD2
MD5
SHA
SHA-1
SHA-2
SHA-3
Ripe-MD160

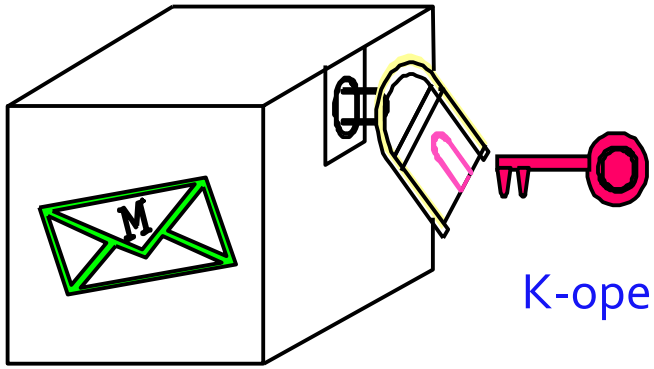
Cryptography revisited



Two properties of symmetric (secret-key) crypto-systems:

- The **same secret key *K*** is used for encryption and decryption
- Encryption and Decryption are very similar (or even identical) functions

Symmetric Cryptography: Analogy



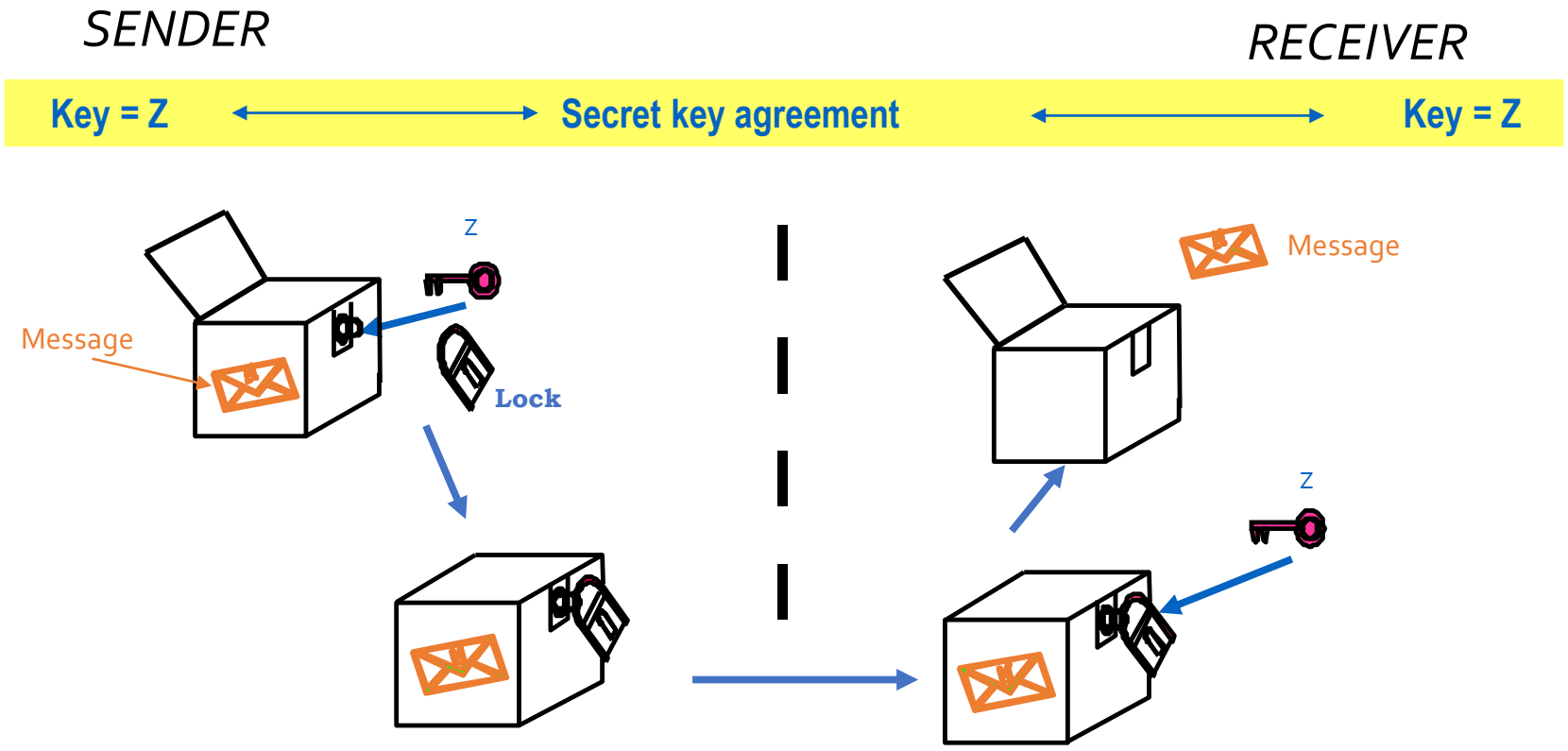
K-open = K-close

Safe with a strong lock, only Alice and Bob have a copy of the key

- Alice encrypts → locks message in the safe with her key
- Bob decrypts → uses his copy of the key to open the safe

- Open and close using shared secret keys
- Secret key agreement required !

Symmetric Cryptography: Analogy



Symmetric Cryptography Shortcomings

- Key distribution problem: The secret key must be **transported securely**
- Number of keys: In a network, each pair of users requires an individual key

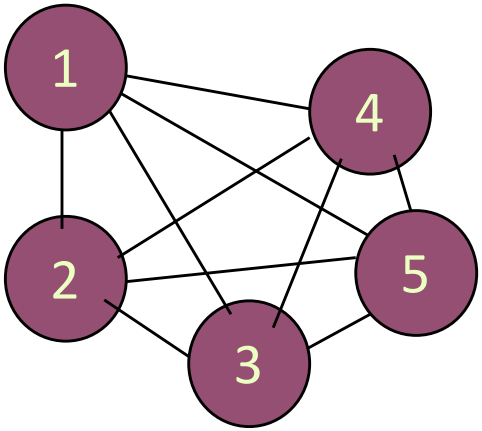
→ *n* users in the network require $\frac{n \cdot (n-1)}{2}$ keys, each user stores *(n-1)* keys

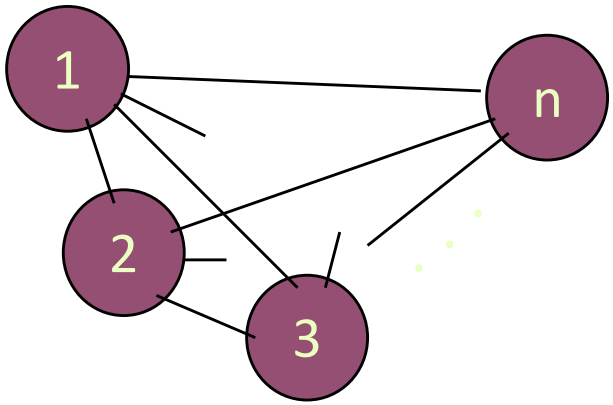
- No support for message **integrity**: verifying that a message comes intact from the sender
- No support for "**non-repudiation**"
  - **Example**: Alice can claim that she never ordered a TV on-line from Bob (he could have fabricated her order)

6

Symmetric Cryptography Shortcomings

Question: How many secret-keys needed to be exchanged in order to set up a system of n-users?





10 key-exchanges for 5 users

$\frac{n\,(n-1)}{2}$ keys for n users

For 10 000 users we need 50 million key-exchanges !

Public-Key system **drops out** the secret key-agreement completely

Idea behind Asymmetric Cryptography

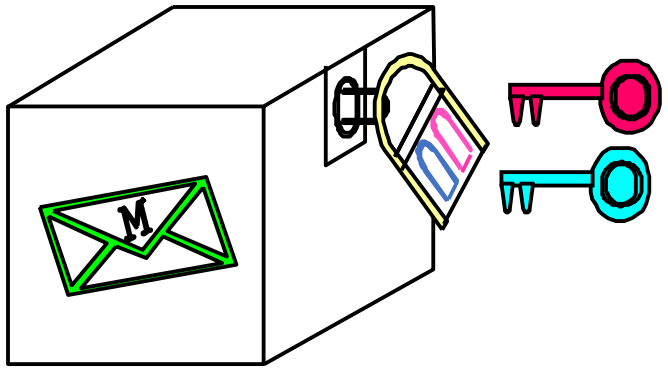
New Idea:

Use the “good old mailbox“ principle:

Everyone can drop a letter

But: Only the owner has the correct key to open the box

Asymmetric Cryptography: Analogy

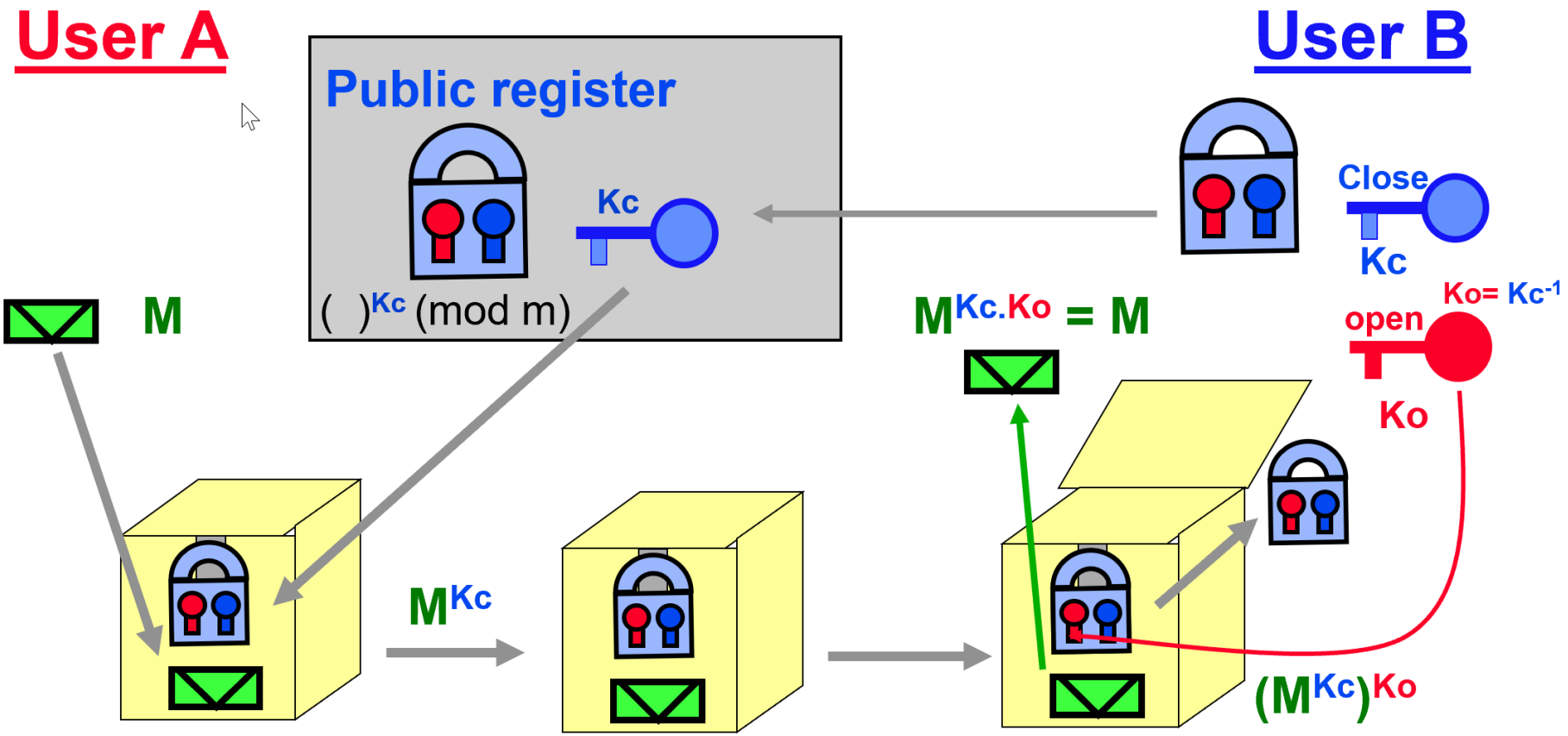


K-open

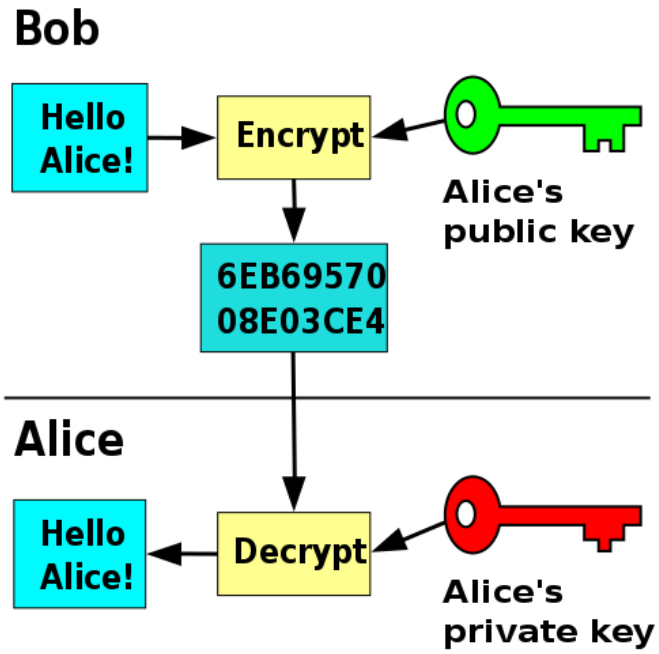
K-secret

- Open and close with <u>different keys</u>!!
- <u>No Secret</u> Key Agreement required

Asymmetric Cryptography: Analogy
User B gets a secured message from A)



Asymmetric Cryptography



→ **Key Distribution Problem** solved

→ Anyone can encrypt messages using the public key, but only the holder of the **paired private key** can decrypt. Security depends on the secrecy of the private key.

→ During the key generation, a key pair K_{pub} and K_{pr} is computed

Public Key Cryptography Applications

The asymmetric cryptography (e.g., RSA) are mainly used for:

- **Key Distribution** without a pre-shared secret (key)
- **Digital Signatures** to provide message integrity and non-repudiation

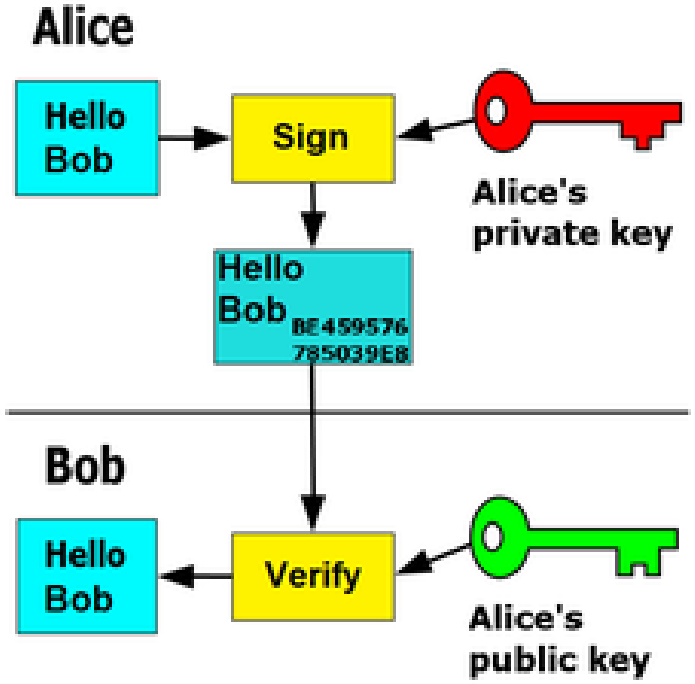
Rarely used for:

- **Encryption** because it is computationally very intensive (1000 times slower than symmetric Algorithms!)

Applications for Public-Key Cryptosystems

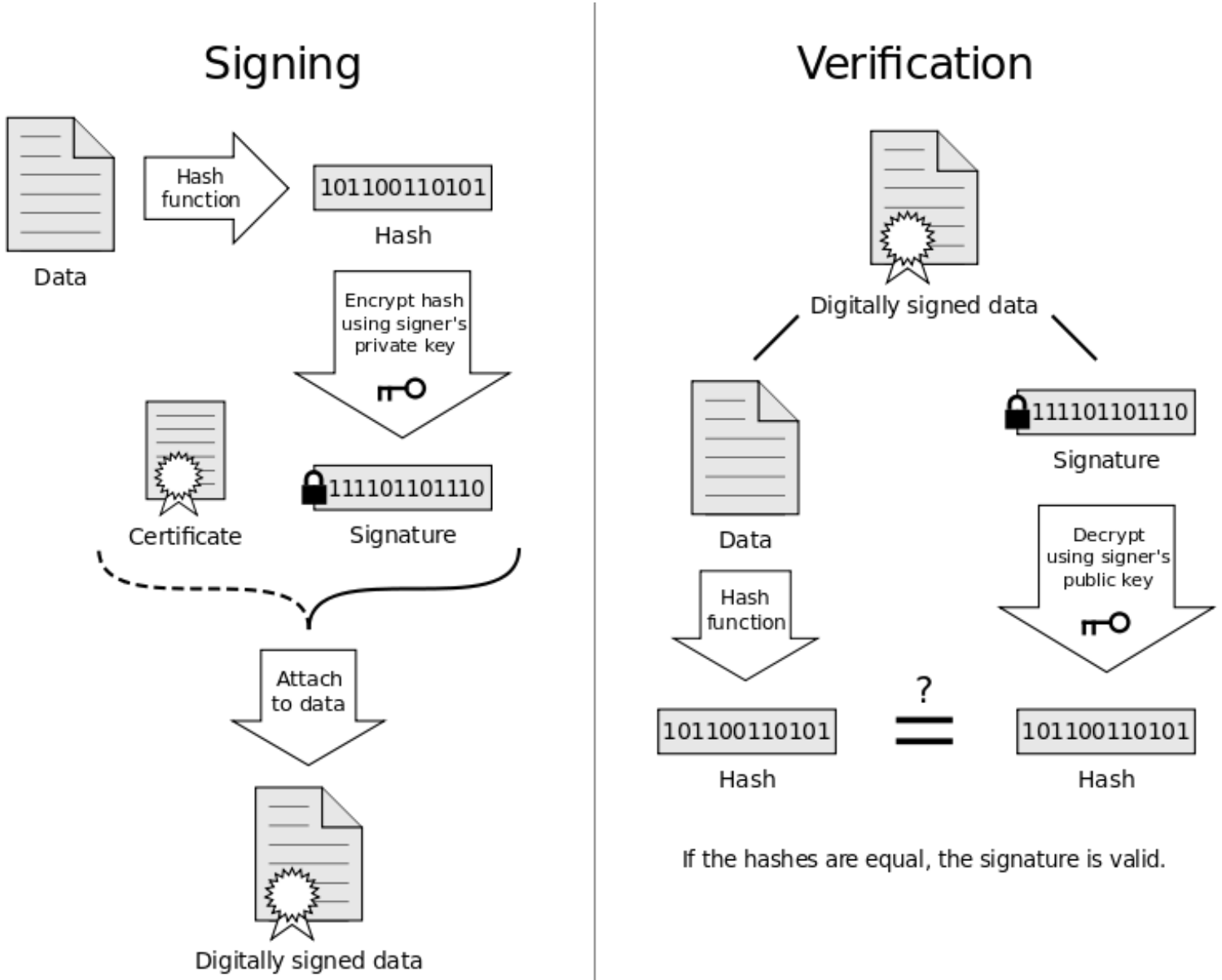
Algorithm	Encryption/Decryption	Digital Signature	Key Exchange
RSA	Yes	Yes	Yes
Elliptic Curve	Yes	Yes	Yes
Diffie-Hellman	No	No	Yes

Digital Signatures



- 1) Alice signs a message with her private key
- 2) Bob can verify that Alice sent the message (i.e., non-repudiation) and that the message has not been modified (i.e., integrity)

Public-Key Signature Process



Basic Key Transport Protocol 1/2

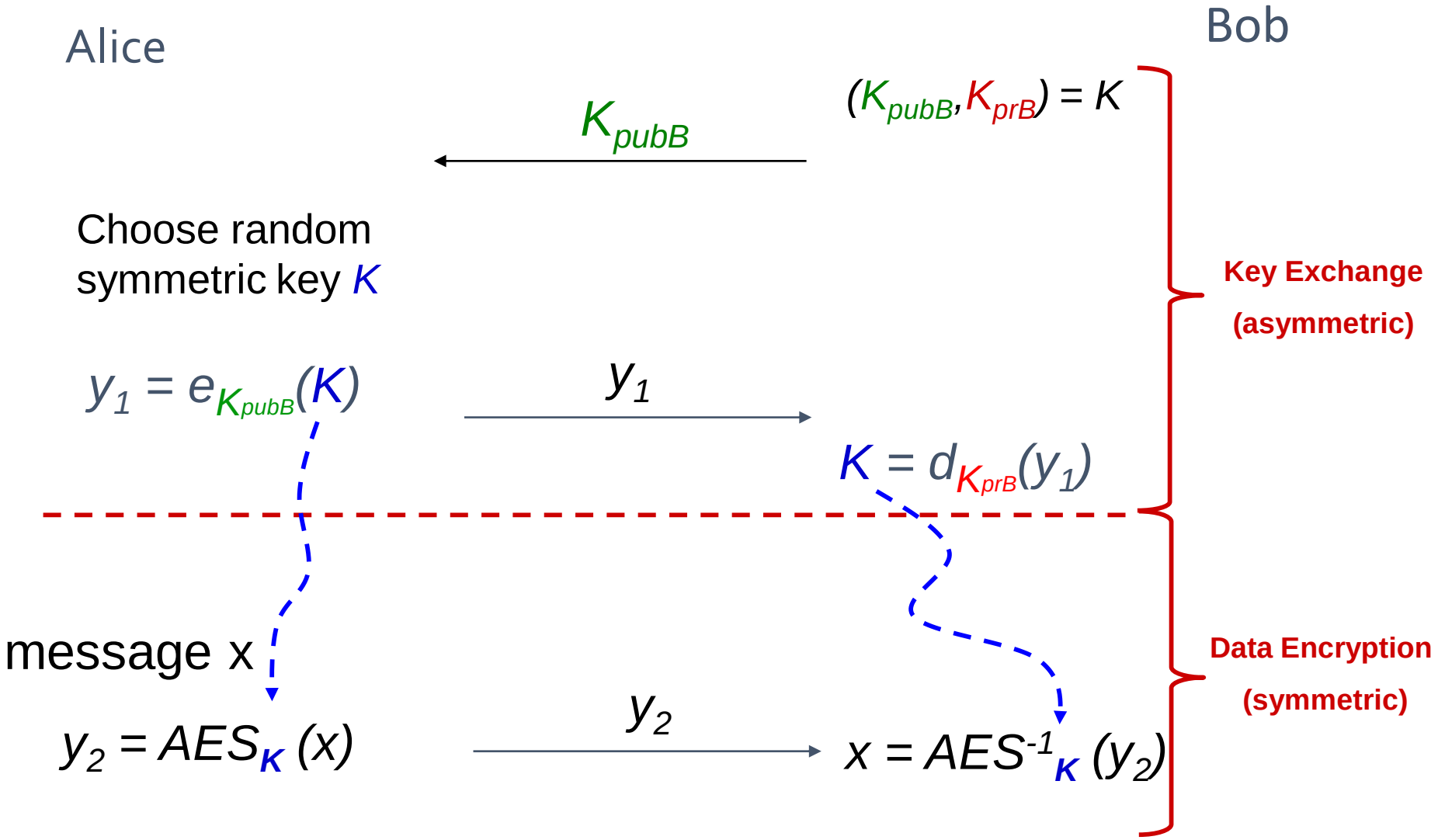
In practice: **Hybrid systems**, incorporating asymmetric and symmetric algorithms

1. **Key exchange** (for symmetric schemes) and **digital signatures** are performed with (slow) **asymmetric** algorithms

2. **Encryption** of data is done using (fast) symmetric ciphers, e.g., **block ciphers or stream ciphers**

Basic Key Transport Protocol 2/2

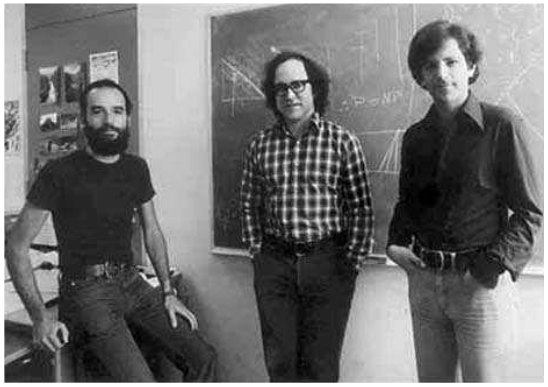
Example: Hybrid protocol with AES as the symmetric cipher

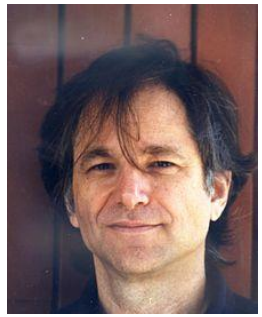


Key Exchange with Public Key Crypto

- Alice creates a secret key, encrypts it with Bob's public key and sends it off
- Bob decrypts the message with his private key
- Use shared key for further communication
- This is how many applications work
- Could encrypt/decrypt using public key cryptography but it is slow

RSA Cryptosystem





Rivest **Shamir** **Adleman**

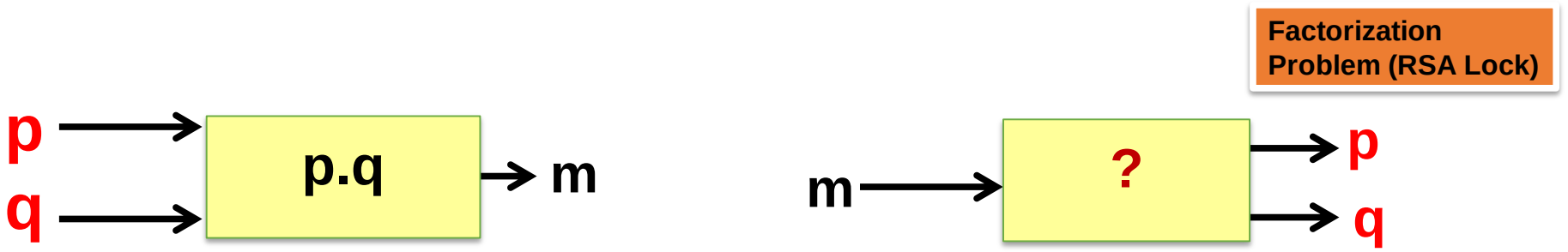
- Developed by Rivest, Shamir, and Adleman in 1977
- Until now, RSA is the most widely use asymmetric cryptosystem although elliptic curve cryptography (ECC) becomes increasingly popular
- RSA is mainly used for two applications: Key exchange & Digital signatures

RSA One-way Function (Lock)

Asymmetric schemes are based on a "**one-way function**" ***f()***:

- Computing $y = f(x)$ is computationally easy
- Computing the inverse $x = f^{-1}(y)$ is computationally infeasible
- One way functions are based on **mathematically hard problems**
- RSA security relies on the difficulty of factoring large integers
  - Multiplying two primes is easy (e.g., 1889 x 3547 = 6,700,283)
  - Given a composite integer *n*, find its prime factors is mathematically hard!

(e.g., It is hard to find $Prime_1$ and $Prime_2$ such that $Prime_1$ x $Prime_2$ = 6,700,283)



Encryption and Decryption

- Encryption and decryption are simply exponentiations

Definition

Given the public key $(n,e) = k_{pub}$ and the private key $d = k_{pr}$ we write

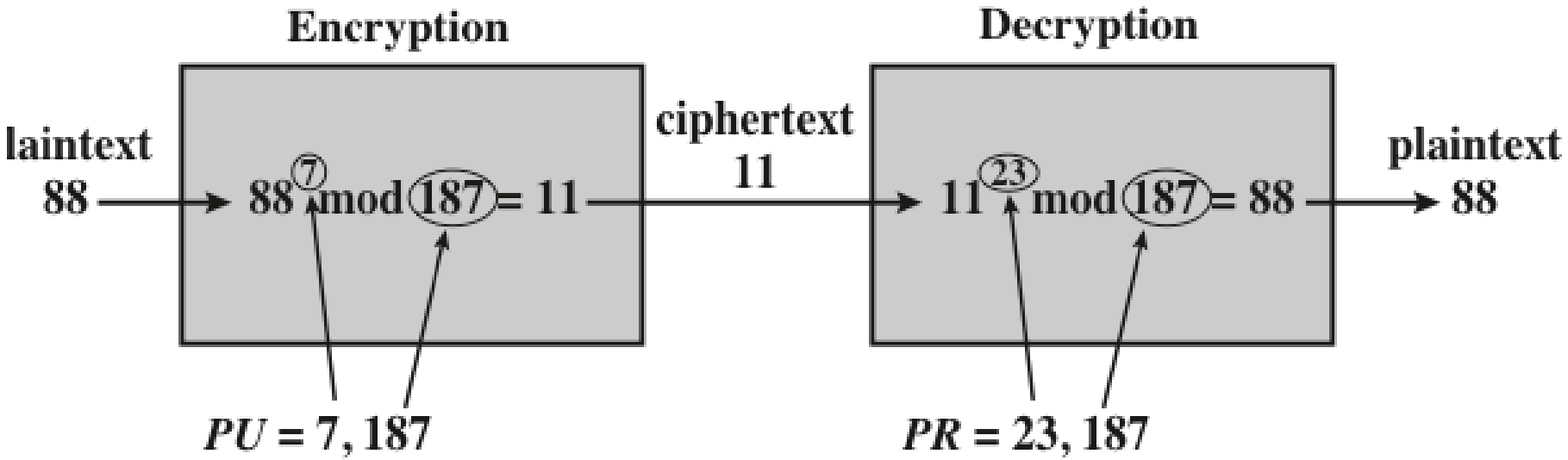
$$c = e_{k_{pub}}(m) = m^e \bmod n$$

$$m = d_{k_{pr}}(c) = c^d \bmod n$$

$e_{k_{pub}}()$ is the encryption operation and $d_{k_{pr}}()$is the decryption operation.

- In practice **d** and **n** are very long integer numbers (≥ 1024 bits)
- The security of the scheme relies on the fact that it is hard to derive the private key **d** given the public-key (n, e)

RSA Example


Encryption
Decryption
laintext
88
88 7 mod 187 = 11
ciphertext
11
11 23 mod 187 = 88
plaintext
88
PU = 7, 187
PR = 23, 187

Key Generation

- RSA has set-up phase during which the private and public keys are computed

Algorithm: RSA Key Generation

Output: public key: $k_{pub} = (n, e)$ and private key $k_{pr} = d$

1. Choose two large primes p, q
2. Compute $n = p * q$
3. Compute $\Phi(n) = (p-1) * (q-1)$
4. Select the public exponent e such that $1 < e < \Phi(n)$ *and does not share any factor with* $\Phi(n)$
5. Compute the private key d such that $(d * e) \bmod \Phi(n) = 1$

the extended **Euclidean algorithm** is used to compute d.

6. **RETURN** $k_{pub} = (n, e)$, $k_{pr} = d$

Φ is called Phi

Example: RSA with small numbers

ALICE

Message ***m = 4***

BOB

1. Choose *p* = 3 and *q* = *11*
2. Compute *n* = *p* * *q* = *33*
3. *Φ(n) = (3-1) * (11-1) = 20*
4. Choose *e* = *3*
5. *d* ≡ e^{-1} ≡ *7 mod 20 = 7*

K_{pub} = (33,3)

c= m^e = 4^3 mod 33 = 31

c = 31

c^d = 31^7 mod 33 = ***4***

Asymmetric Key Cryptography – RSA Encryption Algorithm

- Message: $m = 3$
- Choose 2 random, prime numbers: $p = 19,\ q = 13$
- $n = pq,\ n = 247$
- Choose a random # to be e (encryption key): $e = 7$
- $\Phi(n) = (p-1)(q-1) = 216$
- Compute d (decryption key) (private key)

  $d * e \bmod \Phi(n) = 1$ **(need to solve for d)**

  $d = 31$ (using Extended Euclidean Algorithm)
- Public key = $(n,e) = (247,7)$
- To encrypt: $c = m^e \bmod n$ ➔ $c = 3^7 \bmod 247$ ➔
  $c = 211$ (ciphertext)
- To decrypt: $m = c^d \bmod n$ ➔ $m = 211^{31}$ mode 247 ➔
  $m = 3$ (plaintext)

Attacks and Countermeasures

- **Brute force key search**
  - using exhaustive search for factoring of n in order to obtain Φ(n)
  - Can be prevented using a sufficiently large modulus *n*
  - The current factoring record is 664 bits. Thus, it is recommended that *n* should have a bit length between 1024 and 3072 bits
- Implementation attacks such **Side-channel analysis**
  - Exploit physical leakage of RSA implementation (e.g., power consumption, etc.)
  - Timing attacks on running of decryption can Infer operand size based on time taken

Summary

- RSA is the most widely used public-key cryptosystem
- RSA is mainly used for key transport and digital signatures
- RSA relies on the fact that it is hard to factorize *n*
- Currently 1024-bit cannot be factored, but progress in factorization could bring this into reach within 10-15 years. Hence, RSA with a 2048 or 3076 bit modulus should be used for long-term security

References

- RSA Wikipedia page

https://en.wikipedia.org/wiki/Public-key_cryptography

https://en.wikipedia.org/wiki/RSA_(cryptosystem)