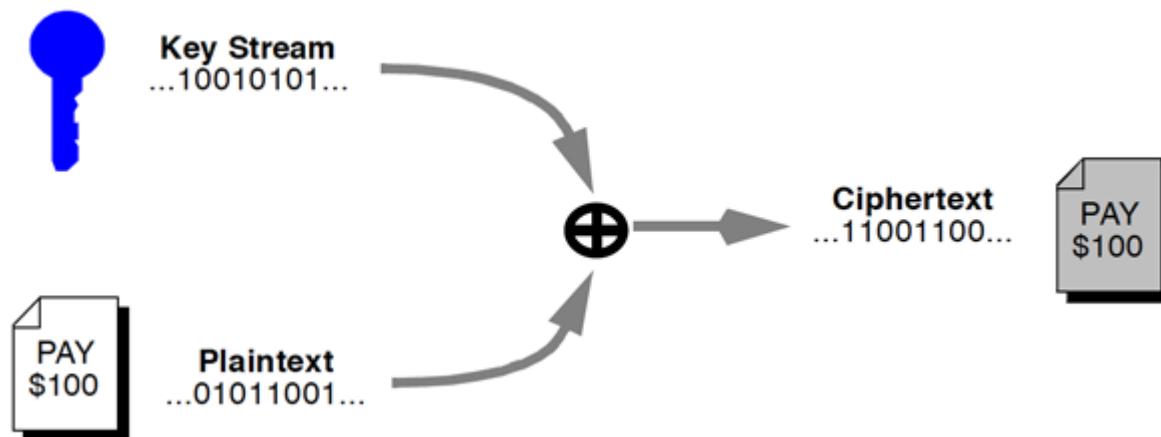


# Stream Ciphers



# Outline

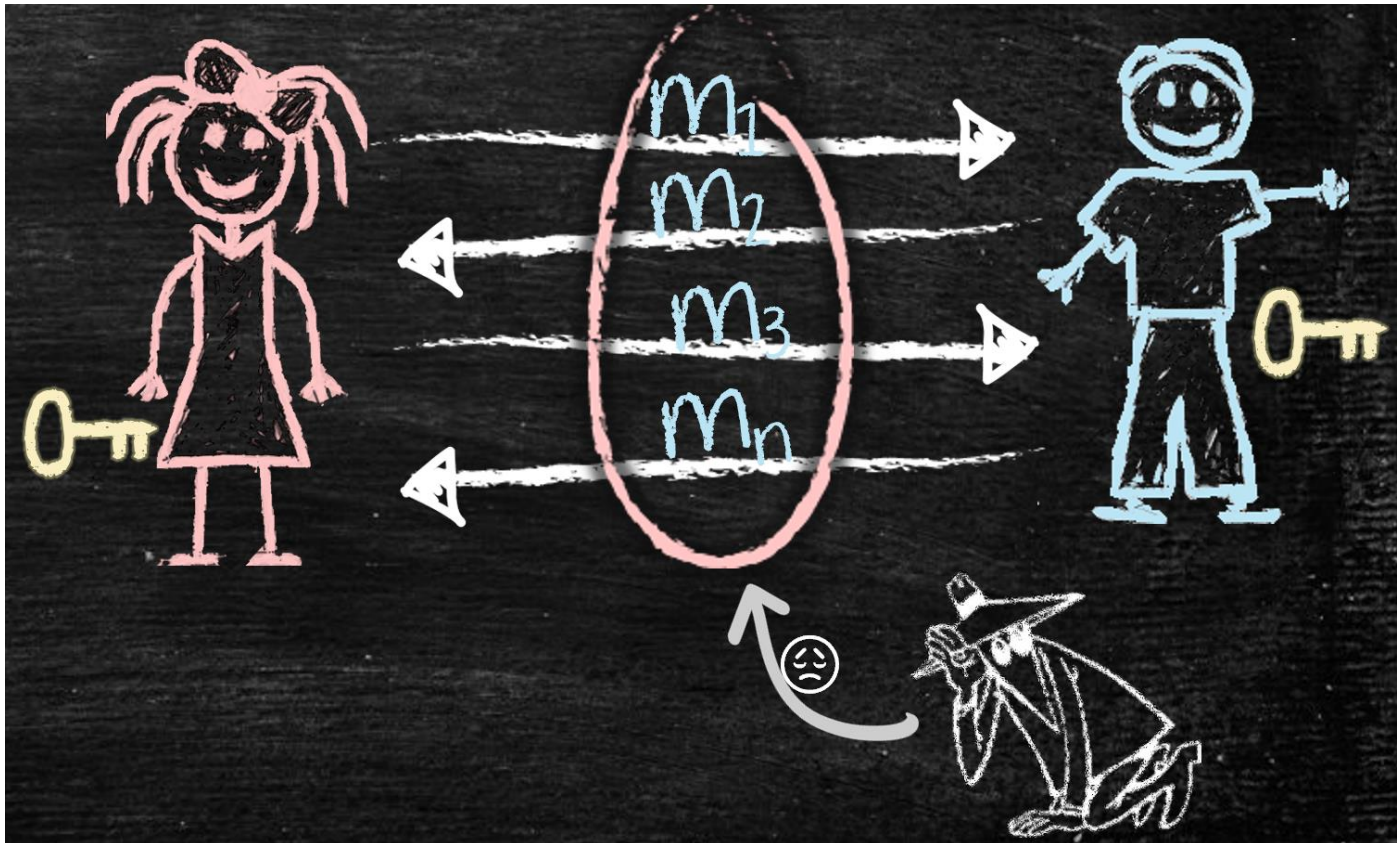
- Intro to stream ciphers
- Random Number Generators (RNGs)
- One-Time Pad (OTP)
- Linear Feedback Shift Registers (LFSRs)
- A5/1 Stream Cipher
- RC4 Cipher

# Intro to stream ciphers

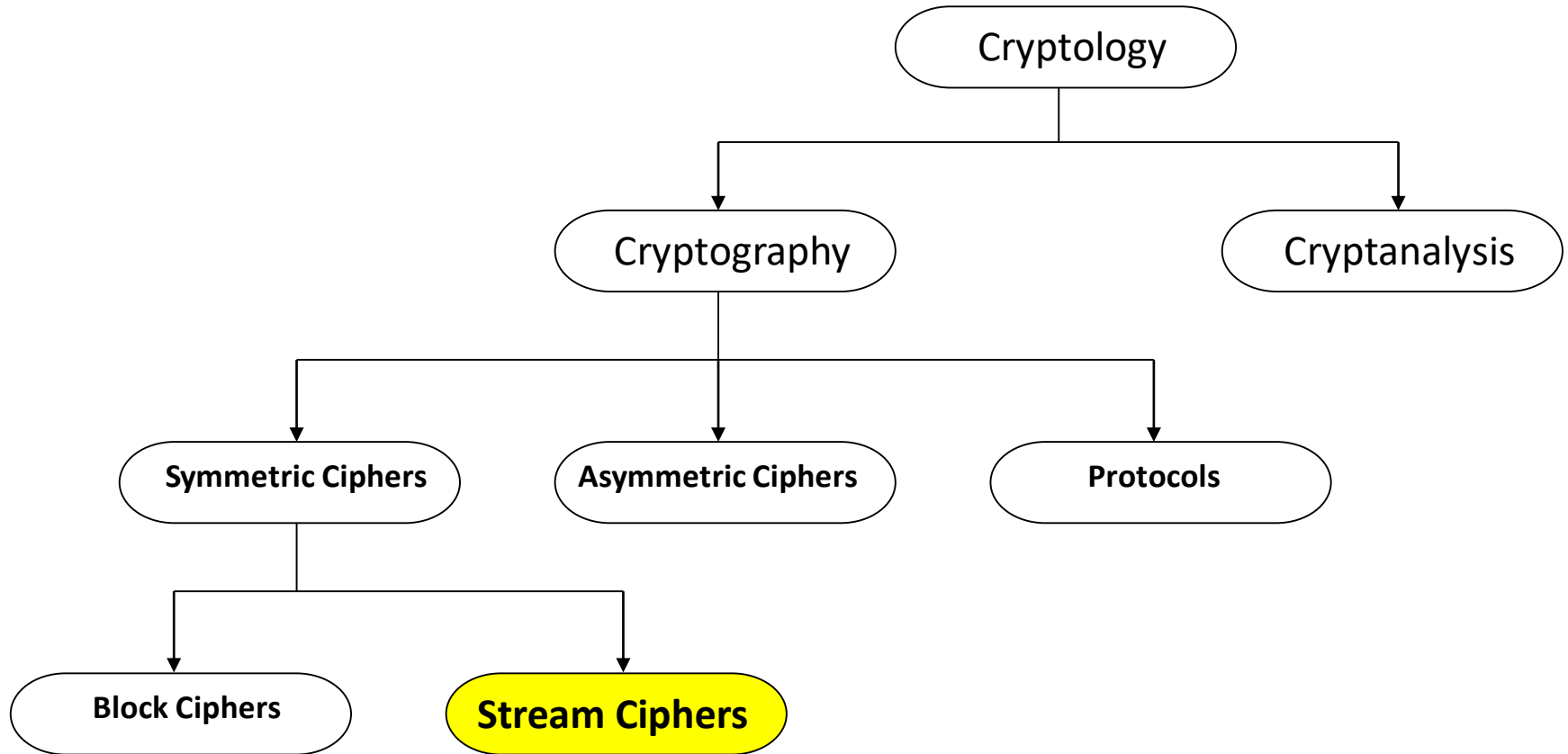
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# Symmetric Key Cryptography

- A cryptographic technique where both parties in the communication share the same key

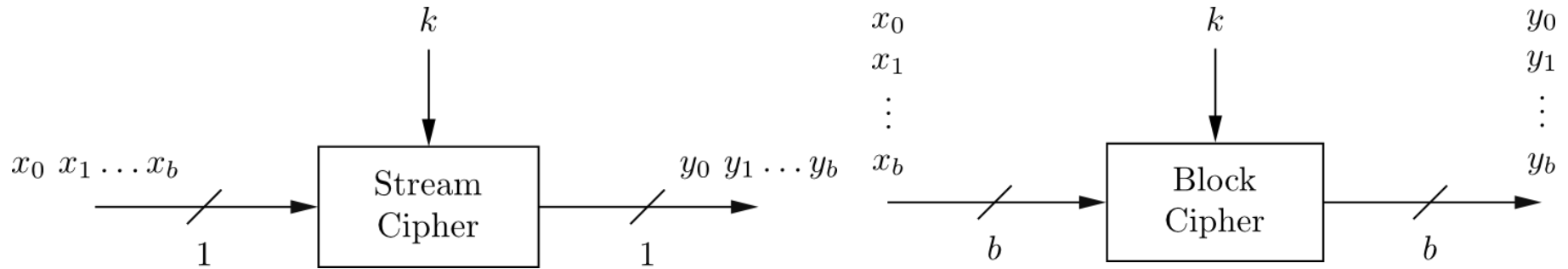


# Stream Ciphers in the Field of Cryptology



Stream Ciphers were invented in 1917 by Gilbert Vernam

# Stream Cipher vs. Block Cipher



- **Stream Ciphers**

- Encrypt bits individually
- Usually small and fast → common in embedded devices (e.g., A5/1 for GSM phones)

- **Block Ciphers:**

- Always encrypt a full block (several bits)
- Are common for Internet applications

# Stream Ciphers

- Type of symmetric key crypto
- Use a fixed length key to produce a **pseudo-random stream of bits**
  - Same key gets you the same stream
- XOR those bits with your PT in order to encrypt
- XOR those same bits with your CT in order to decrypt
  - Inverting XOR is simple, since it is the same XOR operation
- Tries to approximate a one-time-pad

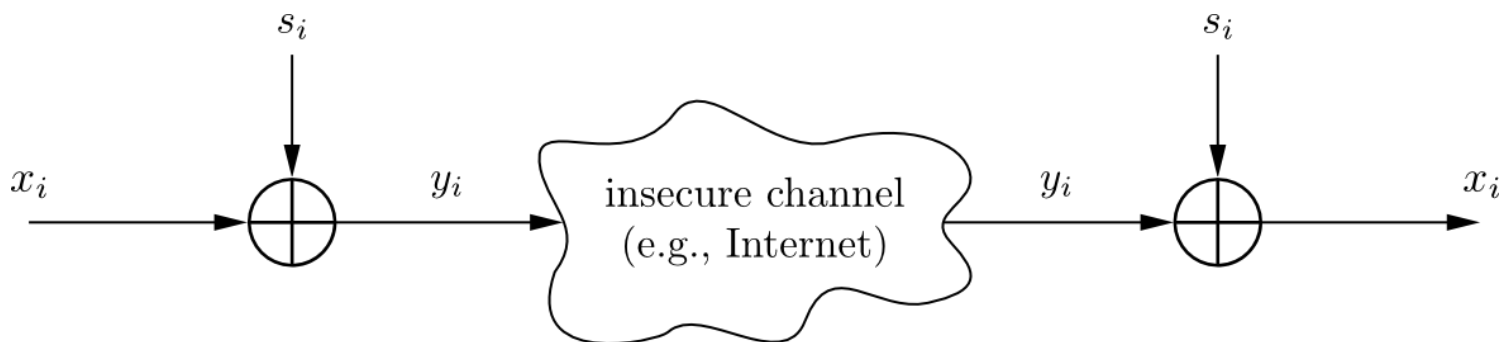
# Real-Word Stream Ciphers

- RC4
  - Used in WEP for wireless network security
  - One option in TLS/HTTPS for encrypting web traffic
  - Not recommended for use anymore
- A5/1
  - Use for encrypting GSM phone data and conversations
  - NSA is known to be routinely breaking it



# Encryption and Decryption with Stream Ciphers

Plaintext  $x_i$ , ciphertext  $y_i$  and key stream  $s_i$  consist of individual bits



- Encryption and decryption are simple additions modulo 2 (aka XOR)
- Encryption and decryption are the same functions

• **Encryption:**  $y_i = e_{s_i}(x_i) = x_i + s_i \bmod 2 \quad x_i, y_i, s_i \in \{0,1\}$

• **Decryption:**  $x_i = e_{s_i}(y_i) = y_i + s_i \bmod 2$

| $x_i \text{ XOR } s_i$ |   | $y_i$ |
|------------------------|---|-------|
| 0                      | 0 | 0     |
| 0                      | 1 | 1     |
| 1                      | 0 | 1     |
| 1                      | 1 | 0     |

# Stream Cipher Encryption Example

Key (128-bits)

Key Stream  
Generator

Keystream

01001010...1001010

Plaintext Data

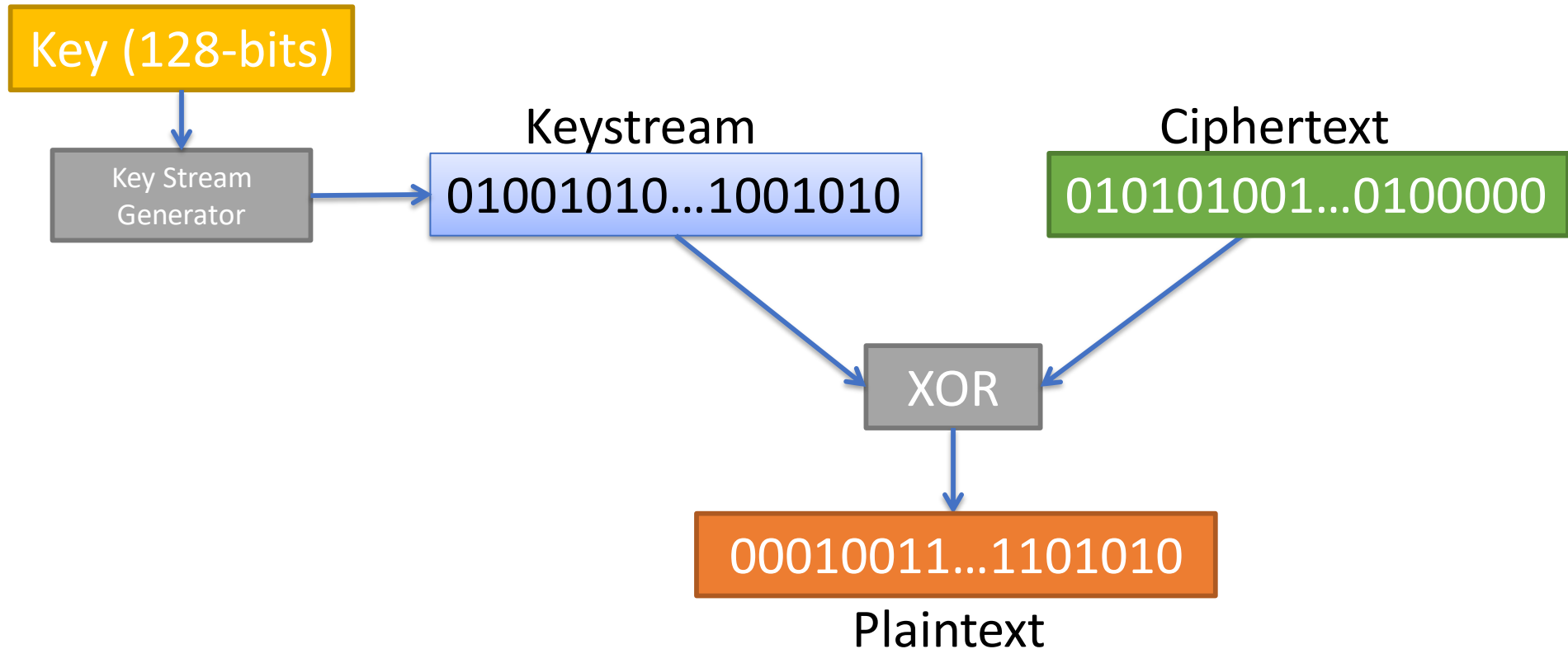
00010011...1101010

XOR

010101001...0100000

Ciphertext

# Stream Cipher Decryption Example



# Why Does XOR Work Here?

- A few properties of XOR:

$$A \oplus A = 0$$

$$A \oplus 0 = A$$

$$(A \oplus B) \oplus C = A \oplus (B \oplus C)$$

- Using XOR for encryption:

$$PT \oplus KEY = CT$$

$$CT \oplus KEY = PT$$

$$(PT \oplus KEY) \oplus KEY = PT$$

$$PT \oplus (KEY \oplus KEY) = PT$$

$$PT \oplus (0) = PT$$

$$PT = PT$$

# XOR Example

- Encrypt

Plaintext: 0110

Key: 1100

Ciphertext: 1010

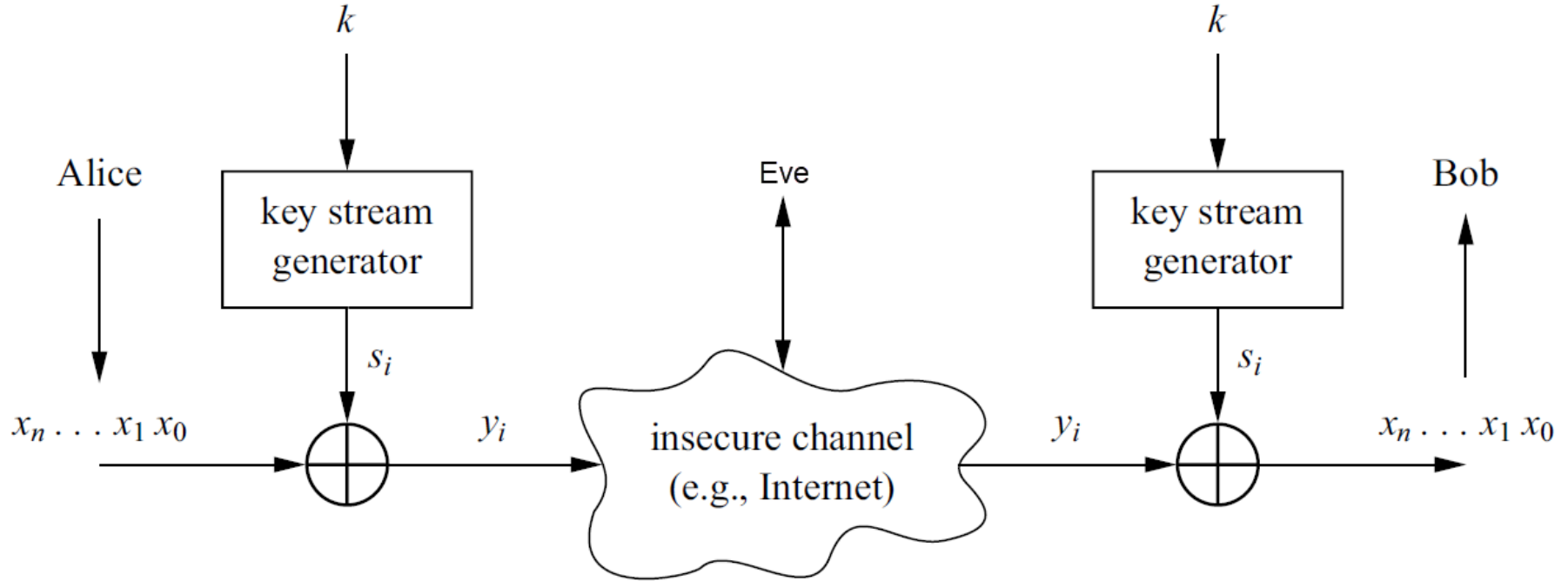
- Decrypt

Ciphertext: 1010

Key: 1100

Plaintext: 0110

# Key Stream Generator



- Security of stream cipher depends entirely on the key stream  $s_i$  :
  - Should be **random** , i.e.,  $\Pr(s_i = 0) = \Pr(s_i = 1) = 0.5$
  - Must be **reproducible** by sender and receiver
- For perfectly random key stream  $s_i$  , each ciphertext output bit has a 50% chance to be 0 or 1  
→ Good statistic property for the keystream

# Stream Cipher: Throughput

Performance comparison of symmetric ciphers (Pentium4):

| Cipher              | Key length  | Mbit/s |
|---------------------|-------------|--------|
| DES                 | 56          | 36.95  |
| 3DES                | 112         | 13.32  |
| AES                 | 128         | 51.19  |
| RC4 (stream cipher) | (choosable) | 211.34 |

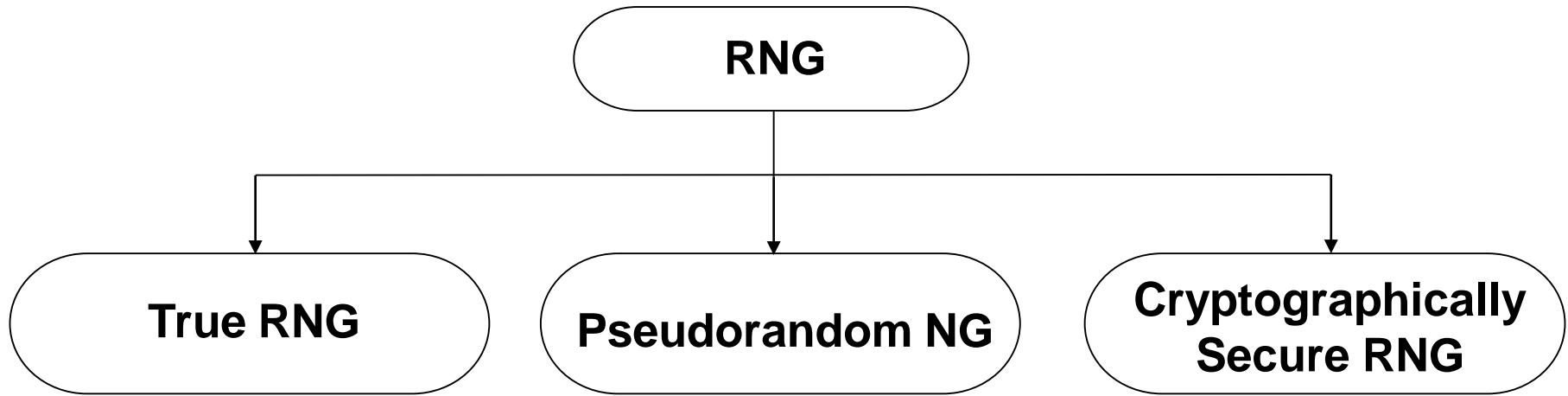
Source: Zhao et al., Anatomy and Performance of SSL Processing, ISPASS 2005

# Random number generators (RNGs)

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# Random number generators (RNGs)



- Randomness is found everywhere in cryptography: **generation of secret keys, encryption schemes, attacks on cryptosystems**
- Without randomness, cryptography would be impossible because all operations would become predictable, and therefore insecure

# True Random Number Generators (TRNGs)

- Based on physical random processes: coin flipping, dice rolling, semiconductor noise, radioactive decay, mouse movement, clock jitter of digital circuits
- Output stream  $s_i$  should have good statistical properties:  
 $\Pr(s_i = 0) = \Pr(s_i = 1) = 50\%$
- Output can neither be **predicted** nor be **reproduced**

Typically used for **generation of keys, nonces** (used only once values), **one-time pads** and for many other purposes

# Generating Keys

- Secret keys are the crux of cryptographic security and should be randomly generated so that they are unpredictable and secret
- e.g., Use the OpenSSL toolkit to generate a random symmetric key by dumping pseudorandom bytes

```
openssl rand -hex 16
```

```
4d138b893c8b59c2363f5f3ddfc0ed55
```

- Generating asymmetric keys (e.g., web site's public key and its associated private key may be valid for years). It requires constructing a private key and its respective public key, ensuring that both satisfy all the necessary criteria

```
openssl genrsa 2048 -out example.key
```

```
-----BEGIN RSA PRIVATE KEY-----
```

```
MIIEowIBAAKQAQEAA6OQ6RBPoQNdWw1n9xjZ6d0uMUz2lcnEIVdXKKpp+u+ScLeWa/iQ9gLX0NmHb5+6BwnkqblexJ/ccZWIIISrDqtbwfmITtRlxyvSEm6hME1rgXVdYGsxbEX1wMUFDHMT3fWpAOHwtBBMZft6QH/3Ki5zw4GFMrrqrfc6/HHY19nmXMsbdD2eTQxT9KX5qKqXF3pLRn79x1J+vzoAITEyBJaslTKEZnY/aKZS5PvXsMndtKsrNHo/RMDKVHj+YrZ8du3W+rIayMmUXgj5XwIr3Vcj6261Pr53RVrUZuw2RvjtfYcgvLp6/kR5wy/GVatzJLUBQWGYanqV8U3sjFKZ7hwIDAQABAoIBAQQCC7awEEd0EdwUED9en04DOCF7/fSzBXAM6FjLt+KU/JKRY8rhMIDuJXKfK2wEyRtfZ8Tx2FCq4eSgxqP77BnuyU9wnh0R1D8T1gLn8eIUsbG6uiygcCKo6Dvc//UQnsAuhKlaoascz/zjZPI7yLU/tZXHV812U7cTzmYo/kHyPL4GYQUV1t6ByQHefV5jOpaG0mL+/LM9BXC46PGt93kFWRNnIqEAc+nfOK4uk2phXQiZTf1joe+u8TGw1SDEsdj16h7VY0lt+VihZ0PWwpZIU02t9mgIR7UdsirRdA022BUCjL1lnmegZOH8fd2xen4e+a5ev5CONn0X4H99DaYhAoGBAP4/JyNEHvgh5ukL3klBD9prAPQDEpfRVIFDvFvuXF6Oyr6wtRAR9rTB1YzaOTzA2t5R/YGwsx/8suF77tSvqUBFGenzvnc/m/oPIECc+36JyCCGaFyDymjokQQi4AaclsmzC4GVmndGull4Z49nCwbcUj/WbGyS6wel4Ah7sdcjAoGBAOp/X9FsZuLSdtrkProAdNj165s+Qk3BUwWykrfUWewVaohUFxxho8neV+kMc43UUIF3k+2D/jyUsHmB4OHdjCLU3C3huSNmRHDpbDEWFGajncIroyQSaEdcYlow/69kt674vofeuQoYQQtmpI+vWPbL8kFgWWs+aOjNm+9+RNAoGAbpxLCqy4THtzWjAvp08JVp27THpBOOtQA+YAuBQinLWod9+5l7LgGRT49OM0GT9uT0xUwq4d+ucyrX30/97iwr+hZw49x4n3G1NmcVgxuWXgGLx+nd70eWg5SKI43i8i5WMcdSbMsDuCzLVBHG8muvtcq1JHwbp1bulUWKsECgYAqNCB3naCifar+IsPICtqWatl7jrn6KogZp2j2gtZoKu0QeinqrwGseiafe4yIXsXwgCyp6XTG2OjireBjxEQ18TE0TIz58z7kVcygmoWac3I7Jeib5AA2j6730ofB9kjNEgGrHobEgBq399QKNvFX+Tmadd9DMHahzHcaZpQKBgBAJsZ+LVDPV66p+fNW0m4NmaH4IR197qyH+aIni1lufqwcraNHOe9y5mMbNCxGTZ8qUU4kR2XFtKwpN1yrpUJHAGrffu6befPbEnqJIKr+gDwvEBU1E4w96whZS/EVbhYX/w8gOP4j+ibTQewAnAuCiV64sdVbBPY0AmJ5YJrP-----END RSA PRIVATE KEY-----
```

```
openssl rsa -in example.key -pubout
```

# Pseudorandom Number Generator (PRNG)

- Generate sequences from initial seed value
- Typically, output stream has good statistical properties
- Output can be **reproduced** and can be **predicted**

Often computed in a recursive way:

$$S_0 = seed$$

$$S_{i+1} = AS_i + B \bmod m$$

Example: *rand()* function in ANSI C:

$$s_0 = 12345$$

$$s_{i+1} = 1103515245s_i + 12345 \bmod 2^{31}$$

**Most PRNGs have bad cryptographic properties but they are more efficient and practical!**

# Cryptanalyzing a Simple PRNG

Simple PRNG: **Linear Congruential Generator (LCG)**

$$S_0 = \textit{seed}$$

$$S_{i+1} = AS_i + B \bmod m$$

- Unknown  $A$ ,  $B$  and  $S_0$  as the key
- Can be **cracked** if 3 output are known, i.e.  $S_1$ ,  $S_2$  and  $S_3$  by solving:

$$S_2 = AS_1 + B \bmod m$$

$$S_3 = AS_2 + B \bmod m$$

$$A \equiv (S_2 - S_3) / (S_1 - S_2) \bmod m$$

$$B \equiv S_2 - S_1(S_2 - S_3) / (S_1 - S_2) \bmod m$$

...directly reveals  $A$  and  $B$ . All  $S_i$  can be computed easily!

**Bad cryptographic properties due to the linearity of most PRNGs**

# Cryptographically Secure Pseudorandom Number Generator (CSPRNG)

- Special PRNG with additional property:

Output must be **unpredictable**

**More precisely:** Given  $n$  consecutive bits of output  $s_i$ , the following output bits  $s_{n+1}$  cannot be predicted (in polynomial time)

- Needed in cryptography, in particular for stream ciphers
- E.g.,
  - The Blum Blum Shub algorithm
  - Other [https://en.wikipedia.org/wiki/Cryptographically\\_secure\\_pseudorandom\\_number\\_generator](https://en.wikipedia.org/wiki/Cryptographically_secure_pseudorandom_number_generator)

# One-Time Pad (OTP)

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# One-Time Pad (OTP)

## Unconditionally secure cryptosystem:

- A cryptosystem is unconditionally secure if it cannot be broken even with *infinite* computational resources

## One-Time Pad

- A cryptosystem developed by Mauborgne that is based on Vernam's stream cipher. Has these properties:

Let the plaintext, ciphertext and key consist of individual bits

$$x_i, y_i, k_i \in \{0,1\}$$

$$\text{Encryption: } e_{k_i}(x_i) = x_i \oplus k_i$$

$$\text{Decryption: } d_{k_i}(y_i) = y_i \oplus k_i$$

**OTP is unconditionally secure if and only if the keystream  $k_i$  truly random and is used only once!**



# One-Time Pad (OTP)

Unconditionally secure cryptosystem:

$$y_0 = x_0 \oplus k_0$$

$$y_1 = x_1 \oplus k_1$$

Every equation is a linear equation with two unknowns

⇒ for every  $y_i$  are  $y_i = 0$  and  $y_i = 1$  equiprobable!

⇒ This is true if  $k_0, k_1, \dots$  are independent, i.e., all  $k_i$  have to be generated truly random

⇒ It can be shown that this systems can *provably* not be solved

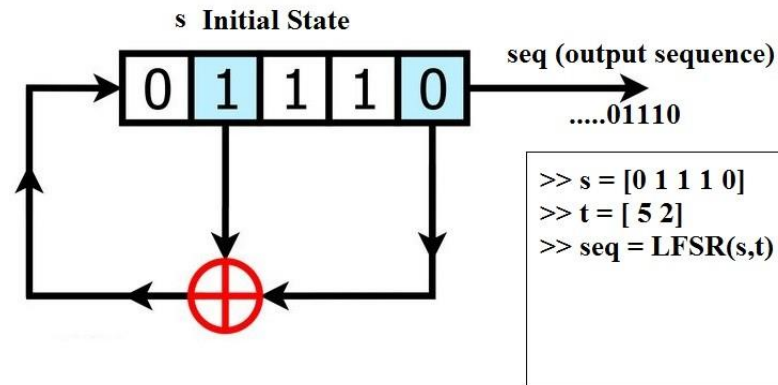
**Disadvantage:** For almost all applications the OTP is **impractical** since the key must be as long as the message! (Imagine you have to encrypt a 1GByte message)

# Linear feedback shift registers (LFSRs)

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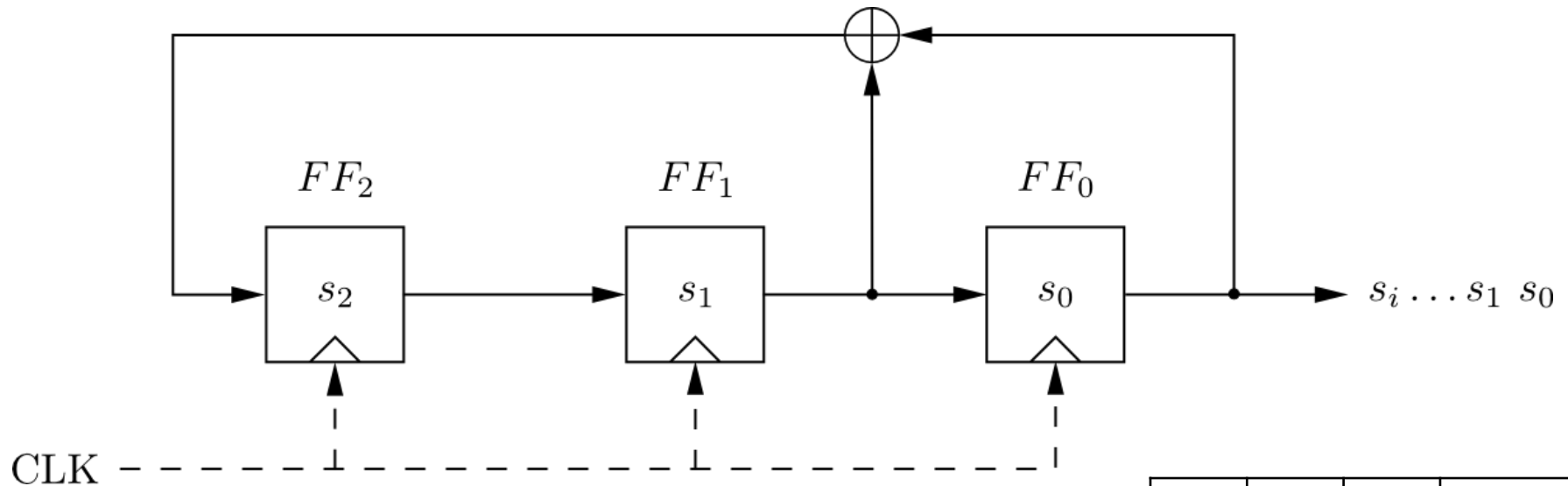
# Linear Feedback Shift Registers (LFSRs)

## LFSR



- Is a shift register whose input bit is a linear function (e.g., XOR) of its previous state:
  - shift all the bits one position to the right and
  - replace the vacated bit by *XOR* of certain bits of the new state
- Output sequence repeats periodically
- Feedback computes fresh input by XOR of certain state bits
- *Degree*  $m$  given by number of storage elements
- Maximum output length:  $2^m - 1$

# Linear Feedback Shift Registers (LFSRs): Example with m=3



- LFSR output described by recursive equation:

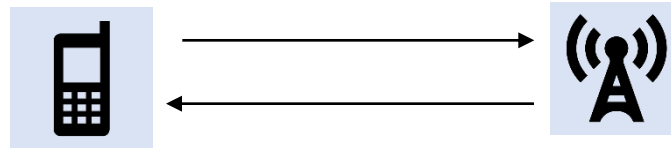
$$s_{i+3} = s_{i+1} + s_i \bmod 2$$

- Maximum output length (of  $2^3-1=7$ )
- Vulnerable to attacks => many stream ciphers use **combinations** of LFSRs

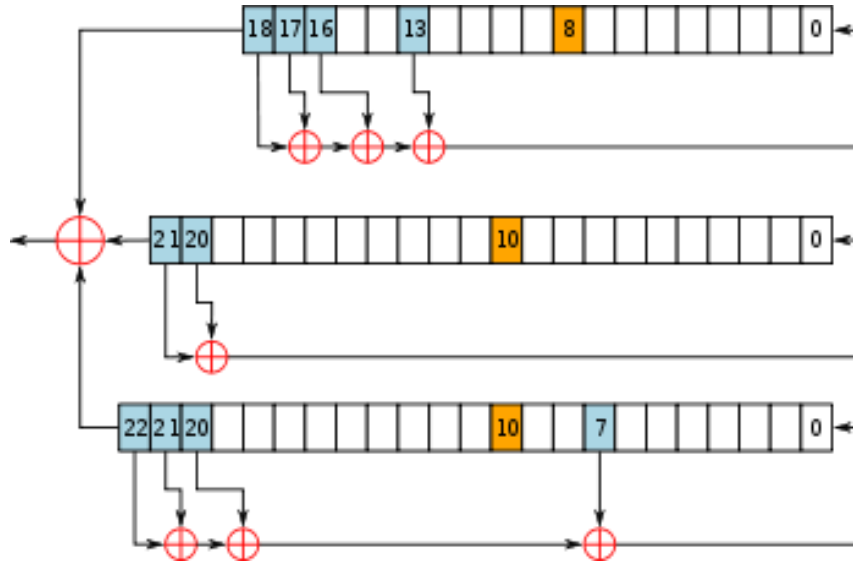
| <i>clk</i> | $FF_2$ | $FF_1$ | $FF_0=s_i$ |
|------------|--------|--------|------------|
| 0          | 1      | 0      | 0          |
| 1          | 0      | 1      | 0          |
| 2          | 1      | 0      | 1          |
| 3          | 1      | 1      | 0          |
| 4          | 1      | 1      | 1          |
| 5          | 0      | 1      | 1          |
| 6          | 0      | 0      | 1          |
| 7          | 1      | 0      | 0          |
| 8          | 0      | 1      | 0          |

# A5/1 Stream Cipher

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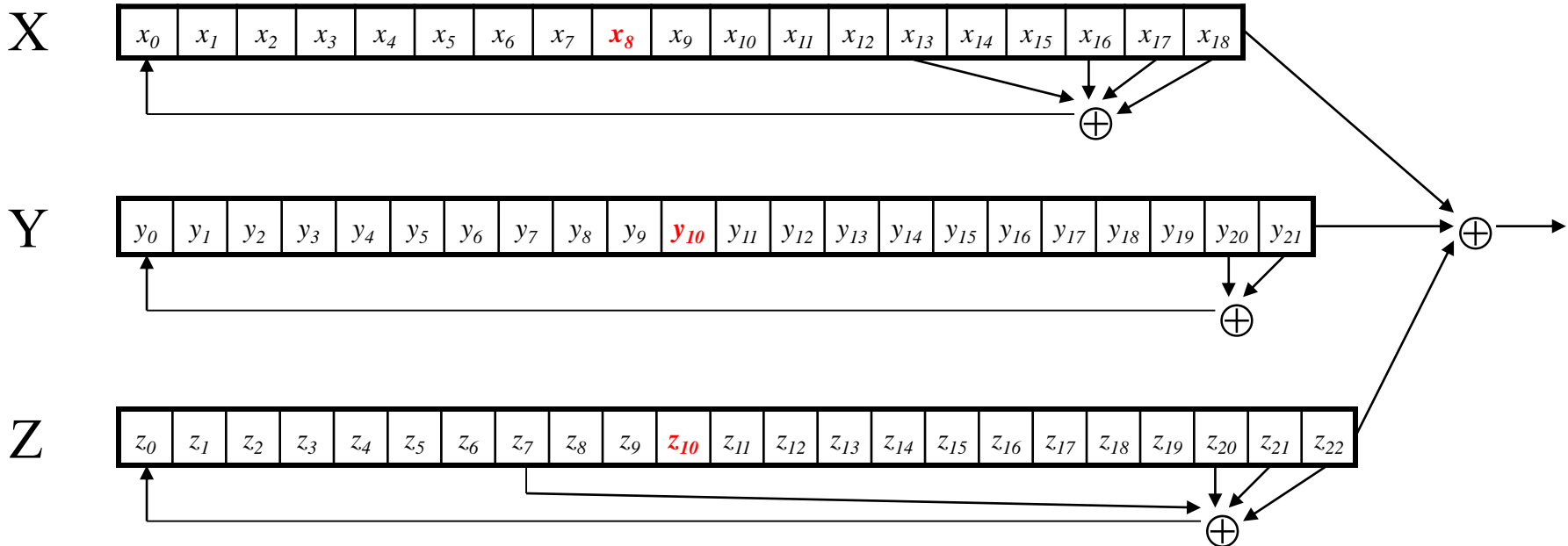


# A5/1 Stream Cipher



- A5/1 uses 3 LFSRs (X, Y, Z)
  - X: 19 bits ( $x_0, x_1, x_2, \dots, x_{18}$ )
  - Y: 22 bits ( $y_0, y_1, y_2, \dots, y_{21}$ )
  - Z: 23 bits ( $z_0, z_1, z_2, \dots, z_{22}$ )
- XOR-Sum of all three NLFSR outputs generates key stream  $S_i$

# A5/1 Keystream



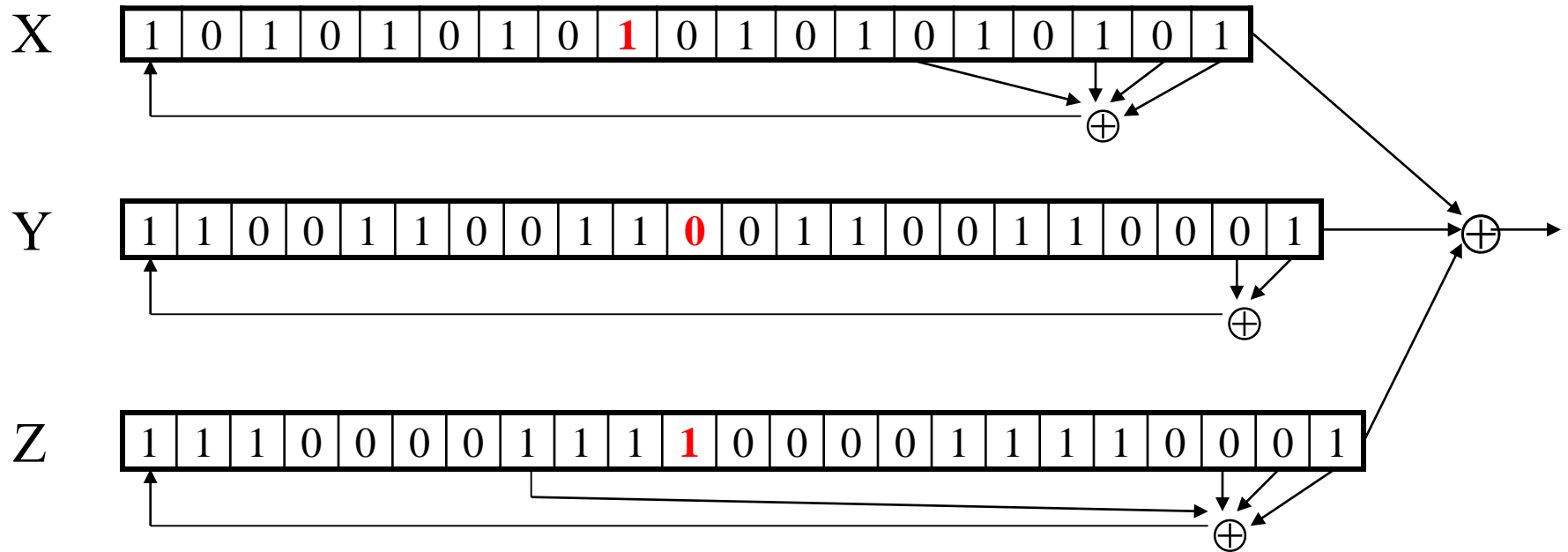
- Each variable here is a single bit
- Key is used as **initial fill** of registers
- Each register steps (or not) based on  $\text{maj}(x_8, y_{10}, z_{10})$
- Keystream bit is XOR of rightmost bits of registers

## A5/1 Keystream

- At each step:  $m = \text{maj}(x_8, y_{10}, z_{10})$ 
  - Examples:  $\text{maj}(0,1,0) = 0$  and  $\text{maj}(1,1,0) = 1$
- If  $x_8 = m$  then *X steps*
  - $t = x_{13} \oplus x_{16} \oplus x_{17} \oplus x_{18}$
  - $x_i = x_{i-1}$  for  $i = 18, 17, \dots, 1$  and  $x_0 = t$
- If  $y_{10} = m$  then *Y steps*
  - $t = y_{20} \oplus y_{21}$
  - $y_i = y_{i-1}$  for  $i = 21, 20, \dots, 1$  and  $y_0 = t$
- If  $z_{10} = m$  then *Z steps*
  - $t = z_7 \oplus z_{20} \oplus z_{21} \oplus z_{22}$
  - $z_i = z_{i-1}$  for  $i = 22, 21, \dots, 1$  and  $z_0 = t$
- Keystream **bit** is  $(x_{18} \oplus y_{21} \oplus z_{22})$



# A5/1



- In this example,  $m = \text{maj}(x_8, y_{10}, z_{10}) = \text{maj}(\mathbf{1}, \mathbf{0}, \mathbf{1}) = \mathbf{1}$
- Register X steps, Y does not step, and Z steps
- Keystream bit is XOR of right bits of registers
- Here, keystream bit will be  $0 \oplus 1 \oplus 0 = 1$

# RC4 Cipher

---

# RC4

- Rivest Cipher 4 (RC4)
- A self-modifying lookup table
- Table always contains a permutation of the byte values  $0, 1, \dots, 255$
- Initialize the permutation using key
- At each step, RC4 does the following
  - Swaps elements in current lookup table
  - Selects a keystream **byte** from table
- Each step of RC4 produces a **byte**
  - Efficient in software
- Each step of A5/1 produces only a bit
  - Efficient in hardware

# RC4 Initialization

- $S[]$  is permutation of  $0, 1, \dots, 255$
- $key[]$  contains  $N$  bytes of key

```
for i = 0 to 255
    S[i] = i
    K[i] = key[i mod N]
next i
j = 0
for i = 0 to 255
    j = (j + S[i] + K[i]) mod 256
    swap(S[i], S[j])
next i
i = j = 0
```

## RC4 Keystream

- For each keystream byte, swap elements in table and select byte

```
i = (i + 1) mod 256
```

```
j = (j + S[i]) mod 256
```

```
swap(S[i], S[j])
```

```
t = (S[i] + S[j]) mod 256
```

```
keystreamByte = S[t]
```

- Use keystream bytes like a one-time pad
- **Note:** first 256 bytes should be discarded
  - Otherwise, related key attack exists

# Summary

- Stream ciphers produce a pseudo-random stream of bits that you XOR with your PT to produce CT and vice-versa
- Stream ciphers require fewer resources and suitable for use in constrained environments such as cell phones (e.g., RC4 and A5/1)
- The requirements for a *cryptographically secure pseudorandom number generator* are far more demanding than the requirements for pseudorandom number generators used in other applications such as testing or simulation
- The One-Time Pad is a provable secure symmetric cipher. However, it is highly impractical for most applications because the key length has to equal the message length
- Single LFSRs make poor stream ciphers despite their good statistical properties. However, careful combinations of several LFSR can yield strong ciphers