Method: 2P Weibull distribution and Weighted Least Squares Regression

Contents

Statistical evaluation of failure stresses.

The procedure is as follows:

- Sort the stress at failure for each specimen in increasing order of magnitude: σ_i, where i is the rank number.
- Calculate the probability of failure $P_{f,i}$ for each measured value. Use the following estimator: $P_{f,i} = \frac{i-0.5}{n}$ (*Hazen's probability estimator*) or $P_{f,i} = \frac{i}{n+1}$ (*Mean rank probability estimator*) where n is the number of tested samples.
- Put the Weibull distribution into linearized form. Enter the measured data into the Weibull mesh.
- Fit the data by Weighted Least Squares Regression . Find the slope, the intercept, and the coefficient of determination R^2, coefficient of variation COV and the Anderson Darling goodness of fit metric p_{AD}.
- Calculate the confidence interval CI.
- Determine the Weibull parameters β (shape parameter) and θ (scale parameter).
- Determine the desired fractile value of the bending tensile strength f_y using the regression line and using the confidence interval. In the case of the confidence interval, the target goal seek is used.
- Plot the cumulative distribution function $F(x)$ and the density function $f(x)$.

> **Note:** You can use the 🚀 **`Live Code`** button in the top right to activate the interactive features and use Python interactively!

> Once the "Live Code" is enabled it is advised to **"Run All"** cells first to load all the necessary packages and functions.
>
> Afterwards, any changes can be made in the input form and when the **"Evaluate"** button is clicked the changes are recorded.
>
> Finally, the last two cells can be run individually by clicking on the **"Run"** button to produce the Weibull plots.

▶ Show code cell source

Definition of useful functions:

- Convert failure stress to equivalent failure stress for a constant load given a reference time period.
- Calculation of standard error.
- Confidence interval calculation
- Weibull pdf and cdf distributions
- Calculationf of Anderson Darling goodness of fit metric p_{AD}

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Interactive Data Input

This widget allows users to input datasets, set analysis parameters, and optionally convert failure stress values.

What can be done:

Choose the number of datasets (default: 3).
Enter dataset names and values (comma-separated).
Set target values:

- stress fractile (default: 5%)
- confidence interval: Enter the alpha value. (default: 95%)
- x-limits (default: lower limit = min_stress_level - 20, upper limit =max_stress_level + 20)

Select a probability estimator:

- *(i - 0.5) / n* (*Hazen's probability estimator*)
- *i / (n + 1)* (*Mean rank probability estimator*)

Convert failure stress (optional):

- Toggle conversion of failure stress to equivalent failure stress for a selected reference period ON/OFF, by clicking Yes/No respectively.
- Enter time-to-failure values.
- Choose a reference time period (1s, 3s, 5s, 60s).

How It Works:

1️⃣ Click **"Confirm"** to generate input fields for the failure stress datasets.
2️⃣ *(Optional)* Click **"Yes"** and enter the time-to-failure values corresponding to each failure stress dataset.
3️⃣ Enter values and click **"Evaluate"** to process the data.

> **Note:** The script checks for errors in the input or mismatch in the dimensions between the time-to-failure datasets and the failure stress datasets.

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Enter the data separated by commas, with decimal point (e.g. "1.44, 2.33, 4.22, 3.01,...")

Data protection declaration: The data entered will not be saved or transmitted over the network.

Number of Datasets: 3

Confirm

Values 1: 93.00592096, 168.4571941, 130.88936, 235.4164091, 120.0878419, 69.56877085, 74.69715591, 192.0145377, 171.8313357,

me 2: SC-air Values 2: 35.16144405, 30.60514126, 138.8818397 37.18030706, 54.2010631, 23.63721413

-HT600 Values 3: 60.33015595, 66.41931354, 50.66136465, 58.10095432, 64.12264846, 60.14501919

Target stress fractile: 0,05

Target confidence interval: 0,1

Lower x limit: 0

Upper x limit: 0

Probability Estimator:

(i-0.5)/n | i/(n+1)

Convert to equivalent constant failure stress for a reference time period?

No | Yes

Select reference time period[s]: 5

Time to failure 1: 4.525933315, 2.697913735, 3.072665231, 7.005076121, 6.608416492, 6.378624526

Time to failure 2: 2.872084933, 3.447791426, 3.732858824, 2.774454034, 1.47491471, 3.588400669, 1.45993247, 1.631301631,

Time to failure 3: 4.2400681, 2.903586143, 2.570186317, 3.285288571, 4.915063035, 2.809045658

Evaluate

```
Selected Probility Estimator: (i-0.5)/n
AR-air; n = 30 samples: [194.2578912   53.26219681 105.2809725  106.74093
 105.5981283  187.2480902   56.87943945  72.95842129  65.25731384
  45.61614573  83.11089449  67.22053889 129.8572468  109.5988269
 109.1843532   56.43452966  83.14540165  73.16139321 135.3082157
  93.00592096 168.4571941  130.88936    235.4164091  120.0878419
  69.56877085  74.69715591 192.0145377  171.8313357  165.9220807 ]
SC-air; n = 28 samples: [ 44.05905838  95.52746419  97.88635848  62.79741
  77.78845508  33.67569736  35.82483461  49.36540736  35.90489898
 107.0092387  119.8728506   35.49953673  85.3009363   25.4379841
 118.124423    59.3273157  152.3074532   33.79298294 193.2203499
  91.20844662  35.16144405  30.60514126 138.8818397   30.45567483
  37.18030706  54.2010631   23.63721413]
SC-air-HT600; n = 30 samples: [ 32.75757414  47.86841675  60.14501919  49
  74.92940077  50.53147813  53.14626824  46.74926022 115.9896314
  91.14776829  51.08320505  61.95877782  62.48239497  66.61511262
  57.78360472  35.67187869  58.26554187 100.4862213   44.83139852
  39.15729621  83.78029471  53.32171198  70.81975251  60.33015595
  66.41931354  50.66136465  58.10095432  64.12264846  60.14501919]

Target stress fractile: 5.0%
Target confidence interval: 95%
Default values for Lower x limit and Upper x limit.

Conversion of failure stress: Yes, Reference time period [s]: 5
AR-air; n = 30; Time to failure values[s]: [7.44896787, 2.176724145, 3.93
The equivalent failure stress for 5 seconds is [166.84  42.36  86.89  88.
  36.2   68.83  54.27 108.66  90.86  90.67  44.96  68.    59.28 113.85
  77.01 143.14 109.6  203.73  99.98  56.07  60.7  164.28 146.48 141.13]
SC-air; n = 28; Time to failure values[s]: [2.872084933, 3.447791426, 3.7
The equivalent failure stress for 5 seconds is [ 35.65  78.19  80.52  50.
  88.49  99.79  27.99  69.58  17.35 100.42  49.4  128.33  27.   165.76
  74.45  27.21  23.39 116.68  23.96  29.11  45.11  18.43]
SC-air-HT600; n = 30; Time to failure values[s]: [3.767937519, 2.41830675
```

```
The equivalent failure stress for 5 seconds is [26.96 38.32 48.6  40.09 4
 50.78 50.62 54.39 47.29 28.76 48.41 84.1  36.8  32.04 68.77 43.5  57.72
 50.02 53.78 40.71 47.41 53.66 48.6 ]
```

Start of Statistical evaluation

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```
plt.ylim(bottom=0.01, top=0.99)
plt.xscale('function', functions=(forwardX,inverseX))
plt.yscale('function', functions=(forwardY,inverseY))
plt.yticks(probs)

# Set x-ticks and their visible labels
plt.xticks(vertical_lines, [str(v) for v in vertical_lines])
if lower_x_limit != 0 or upper_x_limit != 0:
    plt.xlim(left=lower_x_limit, right=upper_x_limit)
else:
    plt.xlim(left=0.01, right=max_stress_data+20)

# Get handles and labels, then filter to make them unique
handles, labels = plt.gca().get_legend_handles_labels()
```

run   run all   add cell   clear

AR-air (n=30)

Fractile [%]	Stress [MPa]	95% CI lower [MPa]	95% CI upper [MPa]
0.8%	10.25	7.56	13.02
5%	24.89	20.73	28.77
50%	86.53	81.73	91.71
Selected 5.0%	24.89	20.73	28.77

AR-air (n=30)

Min Stress [MPa]	Max Stress [MPa]	Mean Stress [MPa]	Coeff. of variation [%]	Goodness of fit, p_{AD}
36.2	203.73	93.05	50.25	14.12

Regression line for "**AR-air**" (**n=30**) is: **y = 2.09x - 9.69; R^2= 0.866**

SC-air (n=28)

Fractile [%]	Stress [MPa]	95% CI lower [MPa]	95% CI upper [MPa]
0.8%	1.97	0.97	3.23
5%	7.37	4.81	9.95
50%	47.11	41.94	52.98
Selected 5.0%	7.37	4.81	9.95

SC-air (n=28)

Min Stress [MPa]	Max Stress [MPa]	Mean Stress [MPa]	Coeff. of variation [%]	Goodness of fit, p_{AD}
17.35	165.76	56.73	72.22	2.08

Regression line for "**SC-air**" (**n=28**) is: **y = 1.40x - 5.77; R^2= 0.788**

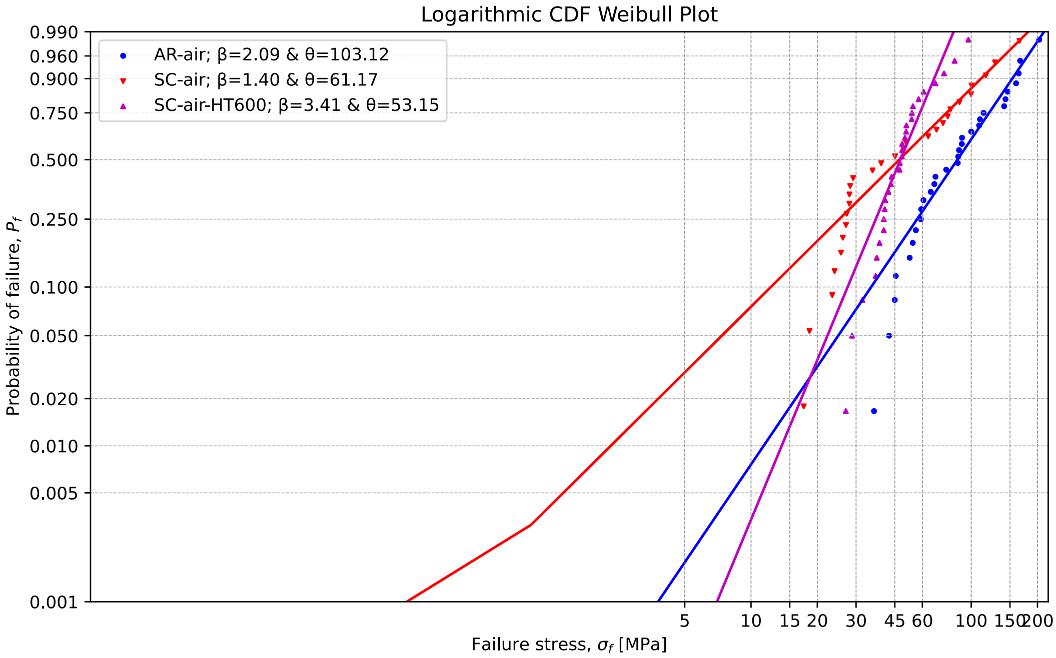
SC-air-HT600 (n=30)

Fractile [%]	Stress [MPa]	95% CI lower [MPa]	95% CI upper [MPa]
0.8%	12.91	10.46	15.16
5%	22.24	19.58	24.53
50%	47.73	45.91	49.62
Selected 5.0%	22.24	19.58	24.53

SC-air-HT600 (n=30)

Min Stress [MPa]	Max Stress [MPa]	Mean Stress [MPa]	Coeff. of variation [%]	Goodness of fit, p_{AD}
26.96	97.09	49.59	32.41	0.52

Regression line for "**SC-air-HT600**" (**n=30**) is: **y = 3.41x - 13.54; R^2= 0.840**



Plot the cumulative distribution function $F(x)$ and the density function $f(x)$.

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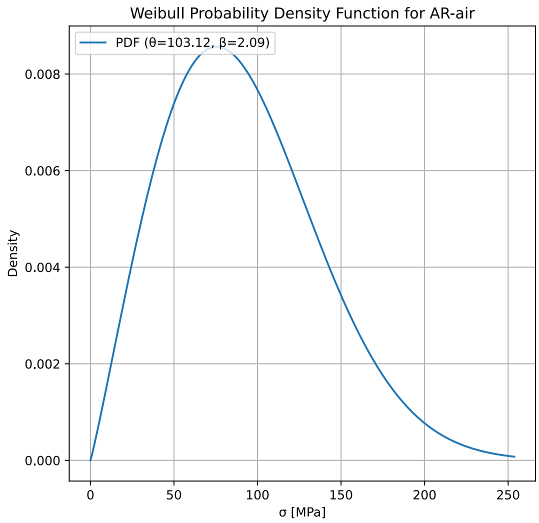
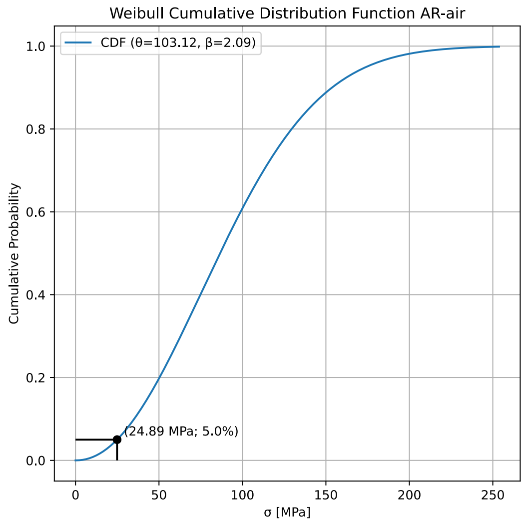
```
            # Plot PDF
            plt.subplot(1, 2, 1)
            plt.plot(x, pdf_values, label=f"PDF (\u03B8={Weibull_
            plt.title(f'Weibull Probability Density Function for
            plt.xlabel('\u03C3 [MPa]')
            plt.ylabel('Density')
            plt.grid(True)
            plt.legend(loc='upper left')
            # Plot CDF
            plt.subplot(1, 2, 2)
            plt.plot(x, cdf_values, label=f"CDF (\u03B8={Weibull_
            plt.hlines(target_P_f, 0, x_percentile, color='black'
            plt.vlines(x_percentile, 0, target_P_f, color='black'
            # Add a circular marker at the 5% fractile
            plt.scatter([x_percentile], [target_P_f], color='blac
```

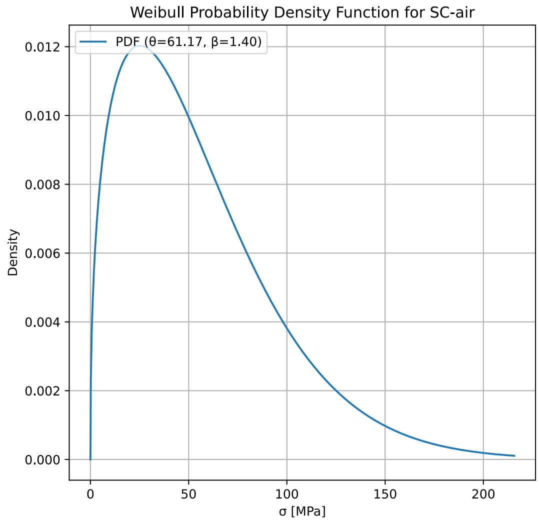
run run all add cell clear



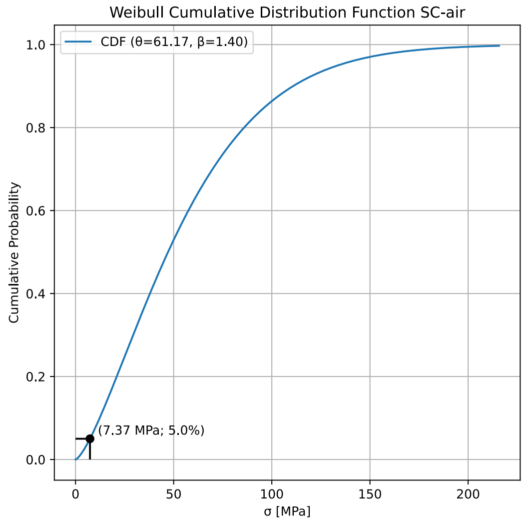
Weibull Probability Density Function for AR-air
PDF (θ=103.12, β=2.09)
Density
σ [MPa]



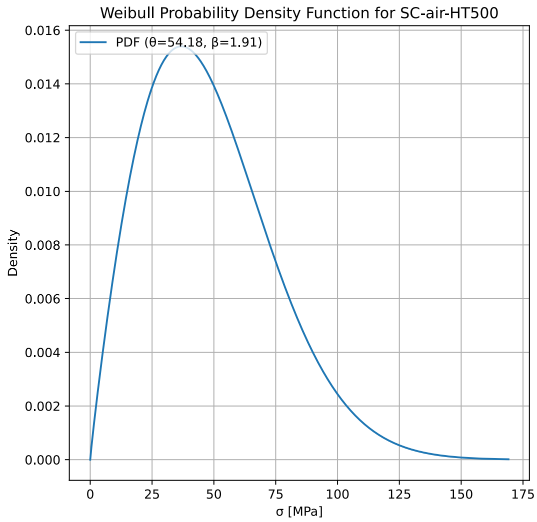
Weibull Cumulative Distribution Function AR-air
CDF (θ=103.12, β=2.09)
Cumulative Probability
(24.89 MPa; 5.0%)
σ [MPa]



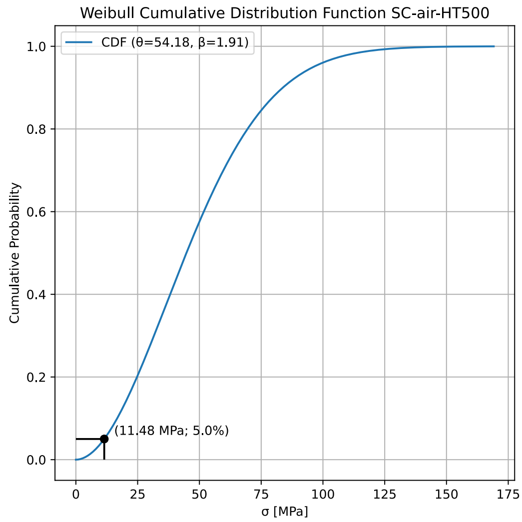
Weibull Probability Density Function for SC-air
PDF (θ=61.17, β=1.40)
Density
σ [MPa]



Weibull Cumulative Distribution Function SC-air
CDF (θ=61.17, β=1.40)
Cumulative Probability
(7.37 MPa; 5.0%)
σ [MPa]



Weibull Probability Density Function for SC-air-HT500
PDF (θ=54.18, β=1.91)
Density
σ [MPa]



Weibull Cumulative Distribution Function SC-air-HT500
CDF (θ=54.18, β=1.91)
Cumulative Probability
(11.48 MPa; 5.0%)
σ [MPa]