Method: 2P Weibull distribution and Weighted Least Squares Regression

Contents

Statistical evaluation of failure stresses.

The procedure is as follows:

- Sort the stress at failure for each specimen in increasing order of magnitude: σ_i, where i is the rank number.
- Calculate the probability of failure $P_{f,i}$ for each measured value. Use the following estimator: $P_{f,i} = \frac{i-0.5}{n}$ (*Hazen's probability estimator*) or $P_{f,i} = \frac{i}{n+1}$ (*Mean rank probability estimator*) where n is the number of tested samples.
- Put the Weibull distribution into linearized form. Enter the measured data into the Weibull mesh.
- Fit the data by Weighted Least Squares Regression . Find the slope, the intercept, and the coefficient of determination R^2, coefficient of variation COV and the Anderson Darling goodness of fit metric p_{AD}.
- Calculate the confidence interval CI.
- Determine the Weibull parameters β (shape parameter) and θ (scale parameter).
- Determine the desired fractile value of the bending tensile strength f_y using the regression line and using the confidence interval. In the case of the confidence interval, the target goal seek is used.
- Plot the cumulative distribution function $F(x)$ and the density function $f(x)$.

Back to top

> **Note:** You can use the 🚀 **`Live Code`** button in the top right to activate the interactive features and use Python interactively!

> Once the "Live Code" is enabled it is advised to **"Run All"** cells first to load all the necessary packages and functions.
>
> Afterwards, any changes can be made in the input form and when the **"Evaluate"** button is clicked the changes are recorded.
>
> Finally, the last two cells can be run individually by clicking on the **"Run"** button to produce the Weibull plots.

▶ Show code cell source

Definition of useful functions:

- Convert failure stress to equivalent failure stress for a constant load given a reference time period.
- Calculation of standard error.
- Confidence interval calculation
- Weibull pdf and cdf distributions
- Calculationf of Anderson Darling goodness of fit metric p_{AD}

▶ Show code cell source

Interactive Data Input

This widget allows users to input datasets, set analysis parameters, and optionally convert failure stress values.

What can be done:

Choose the number of datasets (default: 3).
Enter dataset names and values (comma-separated).
Set target values:

- stress fractile (default: 5%)
- confidence interval: Enter the alpha value. (default: 95%)
- x-limits (default: lower limit = min_stress_level - 20, upper limit =max_stress_level + 20)

Select a probability estimator:

- *(i - 0.5) / n* (*Hazen's probability estimator*)
- *i / (n + 1)* (*Mean rank probability estimator*)

Convert failure stress (optional):

- Toggle conversion of failure stress to equivalent failure stress for a selected reference period ON/OFF, by clicking Yes/No respectively.
- Enter time-to-failure values.
- Choose a reference time period (1s, 3s, 5s, 60s).

How It Works:

1️⃣ Click **"Confirm"** to generate input fields for the failure stress datasets.
2️⃣ *(Optional)* Click **"Yes"** and enter the time-to-failure values corresponding to each failure stress dataset.
3️⃣ Enter values and click **"Evaluate"** to process the data.

> **Note:** The script checks for errors in the input or mismatch in the dimensions between the time-to-failure datasets and the failure stress datasets.

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Enter the data separated by commas, with decimal point (e.g. "1.44, 2.33, 4.22, 3.01,...")

Data protection declaration: The data entered will not be saved or transmitted over the network.

Number of Datasets: 2

Confirm

Name 1: AR-air Values 1: 93.00592096, 168.4571941, 130.8893 69.56877085, 74.69715591, 192.0145

Name 2: SC-air Values 2: 35.16144405, 30.60514126, 138.8818 37.18030706, 54.2010631, 23.637214

Target stress fractile: 0,05

Target confidence interval: 0,1

Lower x limit: 0

Upper x limit: 0

Probability Estimator:

(i-0.5)/n   i/(n+1)

Convert to equivalent constant failure stress for a reference time period?

No   Yes

Select reference time period[s]: 5

Time to failure 1: 4.525933315, 2.697913735, 3.072665231, 7.005076121, 6.608416492, 6.378624526

Time to failure 2: 3.299593792, 1.406979209, 1.150396824, 5.236106571, 1.834370555, 1.697329354, 4.498399073, 1.589122662

Evaluate

```
Selected Probility Estimator: (i-0.5)/n
AR-air; n = 30 samples: [194.2578912   53.26219681 105.2809725  106.74093
 105.5981283  187.2480902   56.87943945  72.95842129  65.25731384
  45.61614573  83.11089449  67.22053889 129.8572468  109.5988269
 109.1843532   56.43452966  83.14540165  73.16139321 135.3082157
  93.00592096 168.4571941  130.88936    235.4164091  120.0878419
  69.56877085  74.69715591 192.0145377  171.8313357  165.9220807 ]
SC-air; n = 28 samples: [ 44.05905838  95.52746419  97.88635848  62.79741
  77.78845508  33.67569736  35.82483461  49.36540736  35.90489898
 107.0092387  119.8728506   35.49953673  85.3009363   25.4379841
 118.124423    59.3273157  152.3074532   33.79298294 193.2203499
  91.20844662  35.16144405  30.60514126 138.8818397   30.45567483
  37.18030706  54.2010631   23.63721413]

Target stress fractile: 5.0%
Target confidence interval: 95%
Default values for Lower x limit and Upper x limit.

Conversion of failure stress: Yes, Reference time period [s]: 5
AR-air; n = 30; Time to failure values[s]: [7.44896787, 2.176724145, 3.93
The equivalent failure stress for 5 seconds is [166.84  42.36  86.89  88.
  36.2   68.83  54.27 108.66  90.86  90.67  44.96  68.    59.28 113.85
  77.01 143.14 109.6  203.73  99.98  56.07  60.7  164.28 146.48 141.13]
SC-air; n = 28; Time to failure values[s]: [2.872084933, 3.447791426, 3.7
The equivalent failure stress for 5 seconds is [ 35.65  78.19  80.52  50.
  88.49  99.79  27.99  69.58  17.35 100.42  49.4  128.33  27.   165.76
  74.45  27.21  23.39 116.68  23.96  29.11  45.11  18.43]
```

Start of Statistical evaluation

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```
        # '',      # No line
    ]

    # List of colors to cycle through
    colors = [
        'b',     # Blue
        'r',     # Green
        'g',     # Red
        'c',     # Cyan
        'm',     # Magenta
        'y',     # Yellow
        'k',     # Black
        # Uncomment more colors for larger number of datasets
        # 'w',    # White
```

Print to PDF

run run all add cell clear

AR-air (n=30)

Fractile [%]	Stress [MPa]	95% CI lower [MPa]	95% CI upper [MPa]
0.8%	10.25	7.56	13.02
5%	24.89	20.73	28.77
50%	86.53	81.73	91.71
Selected 5.0%	24.89	20.73	28.77

AR-air (n=30)

Min Stress [MPa]	Max Stress [MPa]	Mean Stress [MPa]	Coeff. of variation [%]	Goodness of fit, p_{AD}
36.2	203.73	93.05	50.25	14.12

Regression line for "**AR-air**" (**n=30**) is: **$y = 2.09x - 9.69$; $R^2 = 0.866$**

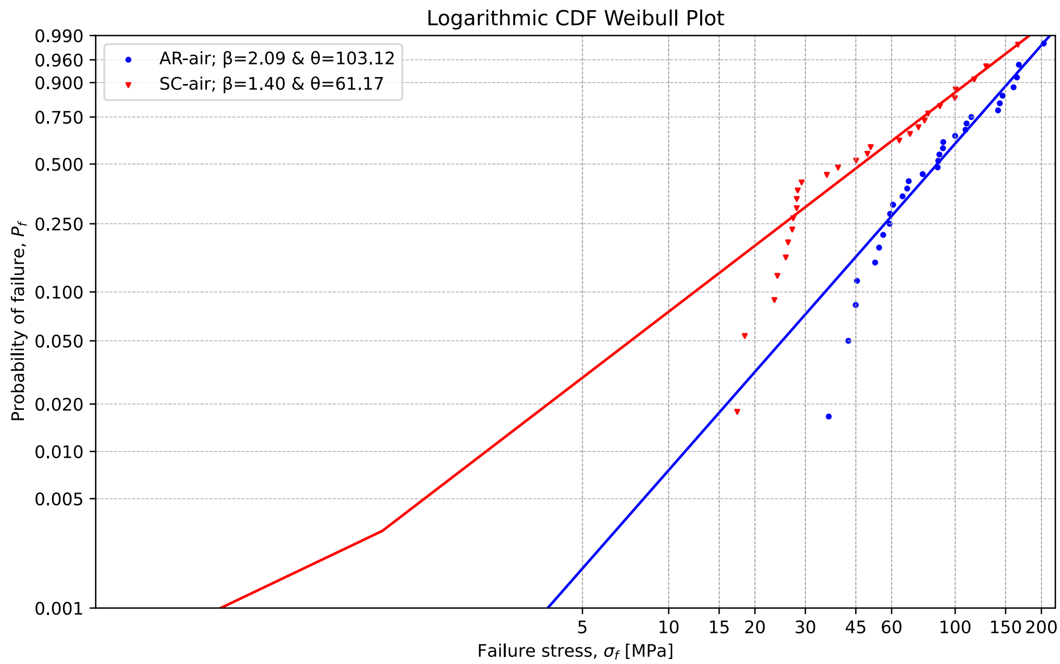
SC-air (n=28)

Fractile [%]	Stress [MPa]	95% CI lower [MPa]	95% CI upper [MPa]
0.8%	1.97	0.97	3.23
5%	7.37	4.81	9.95
50%	47.11	41.94	52.98
Selected 5.0%	7.37	4.81	9.95

SC-air (n=28)

Min Stress [MPa]	Max Stress [MPa]	Mean Stress [MPa]	Coeff. of variation [%]	Goodness of fit, p_{AD}
17.35	165.76	56.73	72.22	2.08

Regression line for "**SC-air**" (**n=28**) is: **y = 1.40x - 5.77; R^2= 0.788**



Plot the cumulative distribution function $F(x)$ and the density function $f(x)$.

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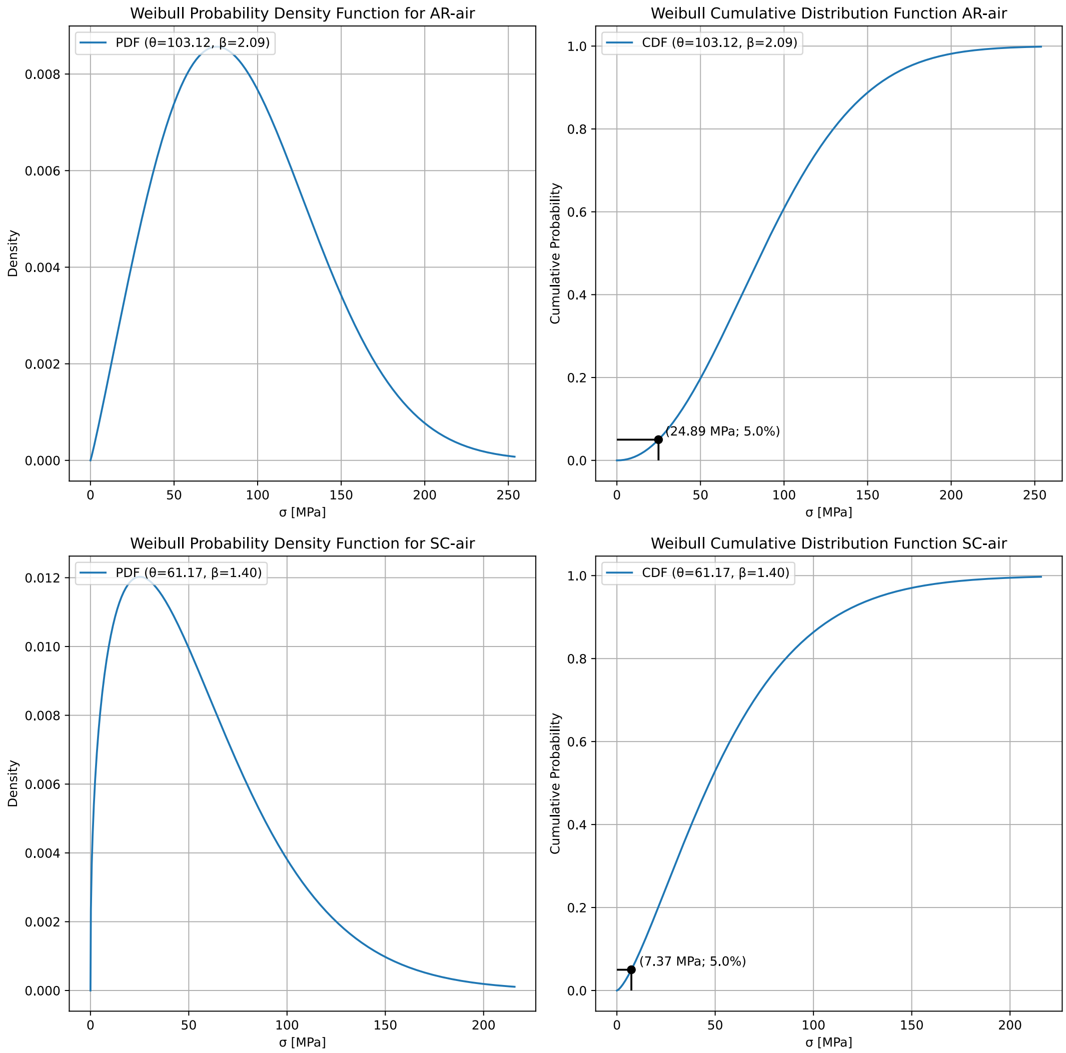
```
if 'stress_values' not in globals() or 'stress_values' not in locals(
    print("Please enter all the necessary input data in the above inp
else:
    for dataset, stress_data in stress_values.items():
            if stress_data.size > 0 and np.any(stress_data != 0):
                if convert_to_equivalent_stress == "Yes" and len(time
                    stress_data = equivalent_stress_values[dataset]
                # Creating a range of x values
                x = np.linspace(0,max(stress_data)+50,1000)
                # Calculate PDF and CDF
                pdf_values = weibull_pdf(x, Weibull_distribution_para
                cdf_values = weibull_cdf(x, Weibull_distribution_para
                # Calculate the x% fractile
                target_percentile = 1 - target_P_f
```

run   run all   add cell   clear



Weibull Probability Density Function for AR-air
PDF (θ=103.12, β=2.09)
Density
σ [MPa]
Weibull Cumulative Distribution Function AR-air
CDF (θ=103.12, β=2.09)
Cumulative Probability
(24.89 MPa; 5.0%)
σ [MPa]
Weibull Probability Density Function for SC-air
PDF (θ=61.17, β=1.40)
Density
σ [MPa]
Weibull Cumulative Distribution Function SC-air
CDF (θ=61.17, β=1.40)
Cumulative Probability
(7.37 MPa; 5.0%)
σ [MPa]