Method: 2P Weibull distribution and Weighted Least Squares Regression

Contents

Statistical evaluation of failure stresses.

The procedure is as follows:

- Sort the stress at failure for each specimen in increasing order of magnitude: σ_i, where i is the rank number.
- Calculate the probability of failure $P_{f,i}$ for each measured value. Use the following estimator: $P_{f,i} = \frac{i-0.5}{n}$ (*Hazen's probability estimator*) or $P_{f,i} = \frac{i}{n+1}$ (*Mean rank probability estimator*) where n is the number of tested samples.
- Put the Weibull distribution into linearized form. Enter the measured data into the Weibull mesh.
- Fit the data by Weighted Least Squares Regression . Find the slope, the intercept, and the coefficient of determination R^2, coefficient of variation COV and the Anderson Darling goodness of fit metric p_{AD}.
- Calculate the confidence interval CI.
- Determine the Weibull parameters β (shape parameter) and θ (scale parameter).
- Determine the desired fractile value of the bending tensile strength f_y using the regression line and using the confidence interval. In the case of the confidence interval, the target goal seek is used.
- Plot the cumulative distribution function $F(x)$ and the density function $f(x)$.

> **Note:** You can use the 🚀 **`Live Code`** button in the top right to activate the interactive features and use Python interactively!

> Once the "Live Code" is enabled it is advised to **"Run All"** cells first to load all the necessary packages and functions.
>
> Afterwards, any changes can be made in the input form and when the **"Evaluate"** button is clicked the changes are recorded.
>
> Finally, the last two cells can be run individually by clicking on the **"Run"** button to produce the Weibull plots.

▶ Show code cell source

Definition of useful functions:

- Convert failure stress to equivalent failure stress for a constant load given a reference time period.
- Calculation of standard error.
- Confidence interval calculation
- Weibull pdf and cdf distributions
- Calculationf of Anderson Darling goodness of fit metric p_{AD}

▶ Show code cell source

Interactive Data Input

This widget allows users to input datasets, set analysis parameters, and optionally convert failure stress values.

What can be done:

Choose the number of datasets (default: 3).
Enter dataset names and values (comma-separated).
Set target values:

- stress fractile (default: 5%)
- confidence interval: Enter the alpha value. (default: 95%)
- x-limits (default: lower limit = min_stress_level - 20, upper limit =max_stress_level + 20)

Select a probability estimator:

- *(i - 0.5) / n* (*Hazen's probability estimator*)
- *i / (n + 1)* (*Mean rank probability estimator*)

Convert failure stress (optional):

- Toggle conversion of failure stress to equivalent failure stress for a selected reference period ON/OFF, by clicking Yes/No respectively.
- Enter time-to-failure values.
- Choose a reference time period (1s, 3s, 5s, 60s).

How It Works:

1️⃣ Click **"Confirm"** to generate input fields for the failure stress datasets.
2️⃣ *(Optional)* Click **"Yes"** and enter the time-to-failure values corresponding to each failure stress dataset.
3️⃣ Enter values and click **"Evaluate"** to process the data.

> **Note:** The script checks for errors in the input or mismatch in the dimensions between the time-to-failure datasets and the failure stress datasets.

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Enter the data separated by commas, with decimal point (e.g. "1.44, 2.33, 4.22, 3.01,...")

Data protection declaration: The data entered will not be saved or transmitted over the network.

Number of Datasets: 3

Confirm

Name 1: At scratch Values 1: 50.53147813, 53.14626824, 46.74926 35.67187869, 50.66136465

Name 2: Not at scratch Values 2: 49.82311194, 74.92940077, 115.9896 61.95877782, 62.48239497, 66.61511

Name 3: SC-air-HT600 Values 3: 32.75757414, 47.86841675, 60.14501 50.07416126, 74.92940077, 50.53147

Target stress fractile: 0,05

Target confidence interval: 0,1

Lower x limit: 0

Upper x limit: 0

Probability Estimator:

(i-0.5)/n i/(n+1)

Convert to equivalent constant failure stress for a reference time period?

No Yes

Select reference time period[s]: 5

Time to failure 1: 2.450329985, 3.058800345, 2.250515649, 3.891416742, 2.715857897, 2.570186317

Time to failure 2: 2.625922186, 2.983272823, 4.936526389, 3.995906808, 3.520593473, 2.927314211, 3.315677399, 3.436962913,

Time to failure 3: 4.2400681, 2.903586143, 2.570186317, 3.285288571, 4.915063035, 2.809045658

Evaluate

```
Selected Probility Estimator: (i-0.5)/n
At scratch; n = 10 samples: [32.75757414 47.86841675 60.14501919 50.07416
 46.74926022 51.08320505 35.67187869 50.66136465]
Not at scratch; n = 20 samples: [ 49.82311194  74.92940077 115.9896314
  62.48239497  66.61511262  57.78360472  58.26554187 100.4862213
  44.83139852  39.15729621  83.78029471  53.32171198  70.81975251
  60.33015595  66.41931354  58.10095432  64.12264846  60.14501919]
SC-air-HT600; n = 30 samples: [ 32.75757414  47.86841675  60.14501919  49
  74.92940077  50.53147813  53.14626824  46.74926022 115.9896314
  91.14776829  51.08320505  61.95877782  62.48239497  66.61511262
  57.78360472  35.67187869  58.26554187 100.4862213   44.83139852
  39.15729621  83.78029471  53.32171198  70.81975251  60.33015595
  66.41931354  50.66136465  58.10095432  64.12264846  60.14501919]

Target stress fractile: 5.0%
Target confidence interval: 95%
Default values for Lower x limit and Upper x limit.

Conversion of failure stress: Yes, Reference time period [s]: 5
At scratch; n = 10; Time to failure values[s]: [3.767937519, 2.418306754,
The equivalent failure stress for 5 seconds is [26.96 38.32 48.6  40.08 4
Not at scratch; n = 20; Time to failure values[s]: [2.625922186, 2.983272
The equivalent failure stress for 5 seconds is [40.09 60.78 97.09 75.29 5
 68.77 43.5  57.72 50.02 53.78 47.41 53.66 48.6 ]
SC-air-HT600; n = 30; Time to failure values[s]: [3.767937519, 2.41830675
The equivalent failure stress for 5 seconds is [26.96 38.32 48.6  40.09 4
 50.78 50.62 54.39 47.29 28.76 48.41 84.1  36.8  32.04 68.77 43.5  57.72
 50.02 53.78 40.71 47.41 53.66 48.6 ]
```

Start of Statistical evaluation

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```
            # Apply Hazen’s probability estimator
            P_f = np.array([(i-0.5) / (n) for i in range(1,n+1)]
        else:
            # Apply mean rank probability estimator
            P_f = np.array([(i) / (n+1) for i in range(1,n+1)])


        # Linearize the Weibull distribution
        ln_stress = np.log(sorted_stress)

        ln_ln_Pf = np.log(np.log(1 / (1 - P_f)))

        # Calculate the weight function W based on the Faucher a
        W = 3.3 * P_f - 27.5 * (1 - (1 - P_f) ** 0.025)
```

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At scratch (n=10)

Fractile [%]	Stress [MPa]	95% CI lower [MPa]	95% CI upper [MPa]
0.8%	20.43	17.7	22.59
5%	26.75	24.54	28.45
50%	39.05	37.85	40.36
Selected 5.0%	26.75	24.54	28.45

At scratch (n=10)

Min Stress [MPa]	Max Stress [MPa]	Mean Stress [MPa]	Coeff. of variation [%]	Goodness of fit, p_{AD}
26.96	48.6	38.65	17.07	29.69

Regression line for "**At scratch**" (**n=10**) is: **y = 6.88x - 25.58; R^2= 0.916**

Not at scratch (n=20)

Fractile [%]	Stress [MPa]	95% CI lower [MPa]	95% CI upper [MPa]
0.8%	14.69	10.81	18.12
5%	25.05	20.83	28.48
50%	53.02	50.42	55.74
Selected 5.0%	25.05	20.83	28.48

Not at scratch (n=20)

Min Stress [MPa]	Max Stress [MPa]	Mean Stress [MPa]	Coeff. of variation [%]	Goodness of fit, p_{AD}
32.04	97.09	55.06	31.87	1.24

Regression line for "**Not at scratch**" (**n=20**) is: **y = 3.47x - 14.16; R^2= 0.814**

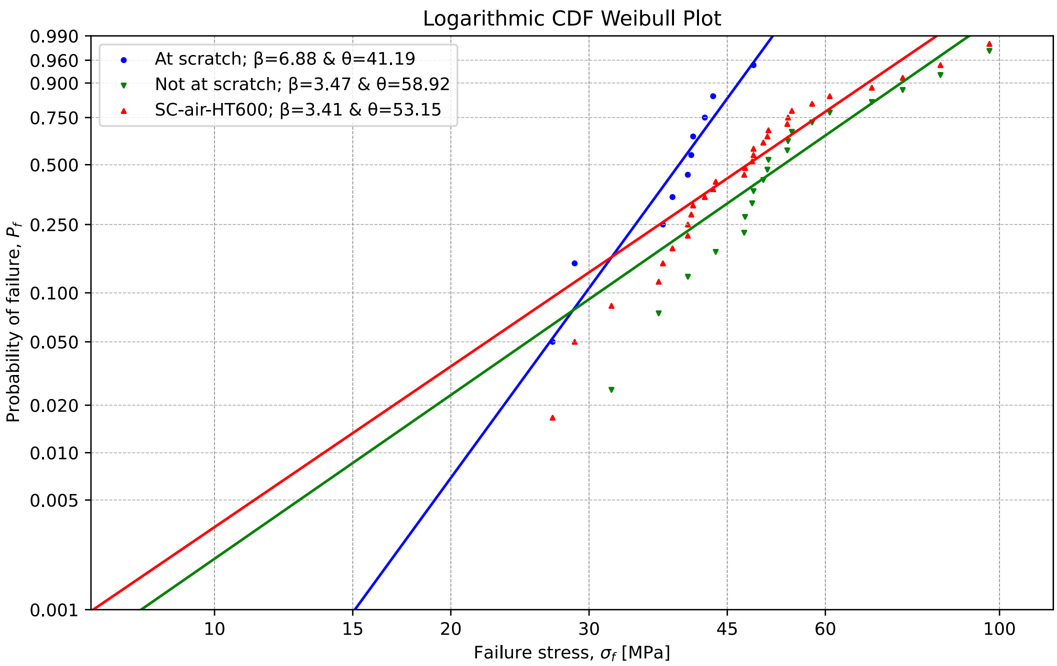
SC-air-HT600 (n=30)

Fractile [%]	Stress [MPa]	95% CI lower [MPa]	95% CI upper [MPa]
0.8%	12.91	10.46	15.16
5%	22.24	19.58	24.53
50%	47.73	45.91	49.62
Selected 5.0%	22.24	19.58	24.53

SC-air-HT600 (n=30)

Min Stress [MPa]	Max Stress [MPa]	Mean Stress [MPa]	Coeff. of variation [%]	Goodness of fit, p_{AD}
26.96	97.09	49.59	32.41	0.52

Regression line for "**SC-air-HT600**" (**n=30**) is: **y = 3.41x - 13.54; R^2= 0.840**



Plot the cumulative distribution function $F(x)$ and the density function $f(x)$.

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