

Method: 2P Weibull distribution and Weighted Least Squares Regression

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- Definition of useful functions:
- Interactive Data Input
- Plot the cumulative distribution function $F(x)$ and the density function $f(x)$.

Statistical evaluation of failure stresses.

The procedure is as follows:

- Sort the stress at failure for each specimen in increasing order of magnitude: σ_i , where i is the rank number.
- Calculate the probability of failure $P_{f,i}$ for each measured value. Use the following estimator: $P_{f,i} = \frac{i-0.5}{n}$ (*Hazen's probability estimator*) or $P_{f,i} = \frac{i}{n+1}$ (*Mean rank probability estimator*) where n is the number of tested samples.
- Put the Weibull distribution into linearized form. Enter the measured data into the Weibull mesh.
- Fit the data by Weighted Least Squares Regression . Find the slope, the intercept, and the coefficient of determination R^2 , coefficient of variation COV and the Anderson Darling goodness of fit metric p_{AD} .
- Calculate the confidence interval CI .
- Determine the Weibull parameters β (shape parameter) and θ (scale parameter).
- Determine the desired fractile value of the bending tensile strength f_y using the regression line and using the confidence interval. In the case of the confidence interval, the target goal seek is used.
- Plot the cumulative distribution function $F(x)$ and the density function $f(x)$.

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Note: You can use the  **Live Code** button in the top right to activate the interactive features and use Python interactively!

Once the "Live Code" is enabled it is advised to **"Run All"** cells first to load all the necessary packages and functions.

Afterwards, any changes can be made in the input form and when the **"Evaluate"** button is clicked the changes are recorded.

Finally, the last two cells can be run individually by clicking on the **"Run"** button to produce the Weibull plots.

► Show code cell source

Definition of useful functions:

- Convert failure stress to equivalent failure stress for a constant load given a reference time period.
- Calculation of standard error.
- Confidence interval calculation
- Weibull pdf and cdf distributions
- Calculation of Anderson Darling goodness of fit metric p_{AD}

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Interactive Data Input

This widget allows users to input datasets, set analysis parameters, and optionally convert failure stress values.

What can be done:

Choose the number of datasets (default: 3).

Enter dataset names and values (comma-separated).

Set target values:

- stress fractile (default: 5%)
- confidence interval: Enter the alpha value. (default: 95%)
- x-limits (default: lower limit = min_stress_level - 20, upper limit = max_stress_level + 20)

Select a probability estimator:

- $(i - 0.5) / n$ (Hazen's probability estimator)
- $i / (n + 1)$ (Mean rank probability estimator)

Convert failure stress (optional):

- Toggle conversion of failure stress to equivalent failure stress for a selected reference period ON/OFF, by clicking Yes/No respectively.
- Enter time-to-failure values.
- Choose a reference time period (1s, 3s, 5s, 60s).

How It Works:

- 1 Click "**Confirm**" to generate input fields for the failure stress datasets.
- 2 (Optional) Click "**Yes**" and enter the time-to-failure values corresponding to each failure stress dataset.
- 3 Enter values and click "**Evaluate**" to process the data.

Note: The script checks for errors in the input or mismatch in the dimensions between the time-to-failure datasets and the failure stress datasets.

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Enter the data separated by commas, with decimal point (e.g. "1.44, 2.33, 4.22, 3.01,...")

Data protection declaration: The data entered will not be saved or transmitted over the network.

Number of Datasets:

Confirm

<input type="text" value="SC-air"/>	Values 1:	30.43432900, 63.14340103, 73.10139321, 133.3002137, 93.00592096, 168.4571941, 130.88936, 235.4164091, 120.0878419, 69.56877085, 74.69715591, 192.0145377, 171.8313357,
<hr/>		
me 2:	<input type="text" value="SC-air"/>	Values 2: 132.3074002, 33.73230234, 133.2203433, 35.16144405, 30.60514126, 138.8818397 37.18030706, 54.2010631, 23.63721413
<hr/>		
HT500	Values 3:	33.20313330, 37.73474003, 33.01203030, 30.03040041, 42.89434463, 88.71811493, 50.50840873, 62.87593174, 141.0013163, 117.6306604, 41.14746787, 58.94676758,
<hr/>		
Target stress fractile:	<input type="text" value="0,05"/>	

Target confidence interval: 0,1

Lower x limit: 0

Upper x limit: 0

Probability Estimator:

(i-0.5)/n

i/(n+1)

Convert to equivalent constant failure stress for a reference time period?

No

Yes

Select reference time period[s]: 5

Time to failure 1: 4.525933315, 2.697913735, 3.072665231, 7.005076121, 6.608416492, 6.378624526

Time to failure 2: 3.299593792, 1.406979209, 1.150396824, 5.236106571, 1.834370555, 1.697329354, 4.498399073, 1.589122662

Time to failure 3: 5.733701498, 4.358551369, 1.612755329, 2.368135054, 2.909189821

Evaluate

Selected Probability Estimator: (i-0.5)/n

AR-air; n = 30 samples: [194.2578912 53.26219681 105.2809725 106.74093 105.5981283 187.2480902 56.87943945 72.95842129 65.25731384 45.61614573 83.11089449 67.22053889 129.8572468 109.5988269 109.1843532 56.43452966 83.14540165 73.16139321 135.3082157 93.00592096 168.4571941 130.88936 235.4164091 120.0878419 69.56877085 74.69715591 192.0145377 171.8313357 165.9220807]

SC-air; n = 28 samples: [44.05905838 95.52746419 97.88635848 62.79741 77.78845508 33.67569736 35.82483461 49.36540736 35.90489898 107.0092387 119.8728506 35.49953673 85.3009363 25.4379841 118.124423 59.3273157 152.3074532 33.79298294 193.2203499 91.20844662 35.16144405 30.60514126 138.8818397 30.45567483 37.18030706 54.2010631 23.63721413]

SC-air-HT500; n = 29 samples: [92.37645599 75.78975356 59.94805695 36 75.16784915 37.41138875 34.92163864 74.98678347 33.31065808 38.22288871 34.1066492 61.57474021 35.29675867 36.84531616 112.1285123 33.28913956 37.73474803 59.01209856 96.03848041 42.89434463 88.71811493 50.50840873 62.87593174 141.0013163 117.6306604 41.14746787 58.94676758 76.46729593]

Target stress fractile: 5.0%

Target confidence interval: 95%

Default values for Lower x limit and Upper x limit.

Conversion of failure stress: Yes, Reference time period [s]: 5

AR-air; n = 30; Time to failure values[s]: [7.44896787, 2.176724145, 3.93

The equivalent failure stress for 5 seconds is [166.84 42.36 86.89 88. 36.2 68.83 54.27 108.66 90.86 90.67 44.96 68. 59.28 113.85 77.01 143.14 109.6 203.73 99.98 56.07 60.7 164.28 146.48 141.13]

SC-air; n = 28; Time to failure values[s]: [2.872084933, 3.447791426, 3.7

The equivalent failure stress for 5 seconds is [35.65 78.19 80.52 50. 88.49 99.79 27.99 69.58 17.35 100.42 49.4 128.33 27. 165.76 74.45 27.21 23.39 116.68 23.96 29.11 45.11 18.43]

SC-air-HT500; n = 29; Time to failure values[s]: [3.784500028, 2.90465348

The equivalent failure stress for 5 seconds is [76.05 61.37 49.76 28.30.24 29.49 49.27 27.41 29.16 93.39 25.52 29.25 47.1 78.78 34.1 72.51 40.47 50.89 119.13 97.7 32.12 47.13 61.93]

Start of Statistical evaluation

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```
127 # List of colors to cycle through
128 colors = [
129     'b',    # Blue
130     'r',    # Green
131     'c',    # Red
132     'g',    # Cyan
133     'm',    # Magenta
134     'y',    # Yellow
135     'k',    # Black
136     # Uncomment more colors for larger number of datasets
137     # 'w',    # White
138     # '#1f77b4', # Light blue
139     # '#ff7f0e', # Orange
140     # '#2ca02c', # Light green
141     # '#d62728' # Light red
```

run run all add cell clear

AR-air (n=30)

Fractile [%]	Stress [MPa]	95% CI lower [MPa]	95% CI upper [MPa]
0.8%	10.25	7.56	13.02
5%	24.89	20.73	28.77
50%	86.53	81.73	91.71
Selected 5.0%	24.89	20.73	28.77

AR-air (n=30)

Min Stress [MPa]	Max Stress [MPa]	Mean Stress [MPa]	Coeff. of variation [%]	Goodness of fit, pAD
36.2	203.73	93.05	50.25	14.12

Regression line for "AR-air" (n=30) is: $y = 2.09x - 9.69$; $R^2 = 0.866$

SC-air (n=28)

Fractile [%]	Stress [MPa]	95% CI lower [MPa]	95% CI upper [MPa]
0.8%	1.97	0.97	3.23
5%	7.37	4.81	9.95
50%	47.11	41.94	52.98
Selected 5.0%	7.37	4.81	9.95

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Min Stress [MPa]	Max Stress [MPa]	Mean Stress [MPa]	Coeff. of variation [%]	Goodness of fit, p_{AD}
17.35	165.76	56.73	72.22	2.08

Regression line for "SC-air" (n=28) is: $y = 1.40x - 5.77$; $R^2 = 0.788$

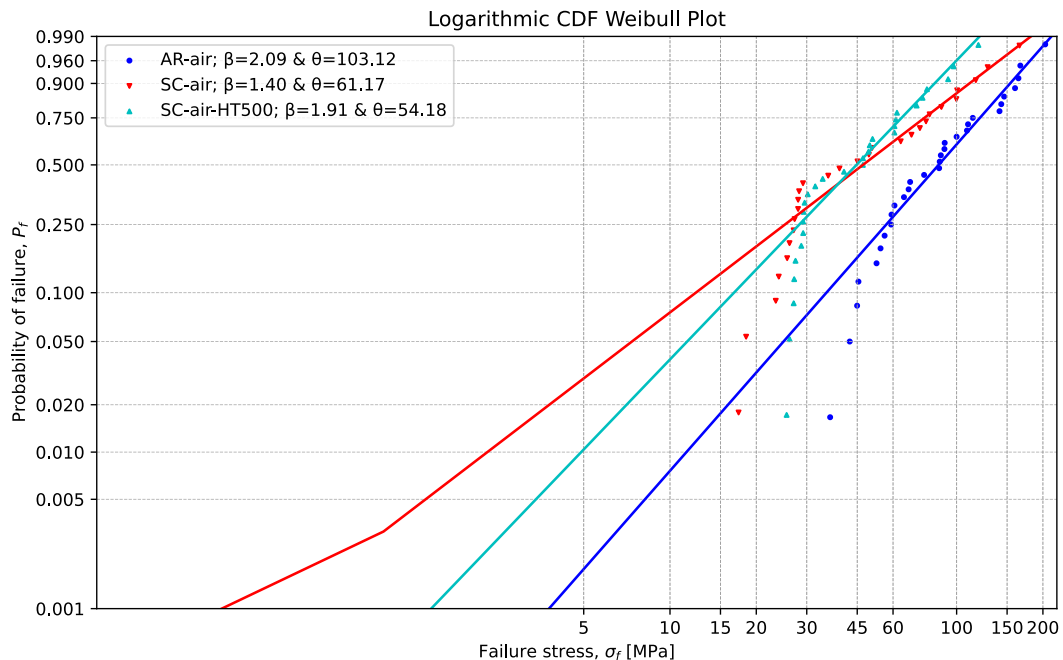
SC-air-HT500 (n=29)

Fractile [%]	Stress [MPa]	95% CI lower [MPa]	95% CI upper [MPa]
0.8%	4.36	2.31	6.6
5%	11.48	7.85	14.78
50%	44.74	40.55	49.37
Selected 5.0%	11.48	7.85	14.78

SC-air-HT500 (n=29)

Min Stress [MPa]	Max Stress [MPa]	Mean Stress [MPa]	Coeff. of variation [%]	Goodness of fit, p_{AD}
25.52	119.13	49.73	54.38	0.59

Regression line for "SC-air-HT500" (n=29) is: $y = 1.91x - 7.64$; $R^2 = 0.733$



Plot the cumulative distribution function $F(x)$ and the density function $f(x)$.

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```

18 # Plot PDF
19 plt.subplot(1, 2, 1)
20 plt.plot(x, pdf_values, label=f"PDF (\u03B8={Weibull_
21 plt.title(f'Weibull Probability Density Function for
22 plt.xlabel('\u03C3 [MPa]')
23 plt.ylabel('Density')
24 plt.grid(True)
25 plt.legend(loc='upper left')
26 # Plot CDF
27 plt.subplot(1, 2, 2)
28 plt.plot(x, cdf_values, label=f"CDF (\u03B8={Weibull_
29 plt.hlines(target_P_f, 0, x_percentile, color='black')
30 plt.vlines(x_percentile, 0, target_P_f, color='black')
31 # Add a circular marker at the 5% fractile
32 plt.scatter([x_percentile], [target_P_f], color='black',

```

run

run all

add cell

clear

