If X is symmetric about 0, then $X \stackrel{d}{=} -X$ and hence for any $x \in \mathbb{R}$, $F_X(x) = F_{-X}(x)$ and hence $F_X(x) = \mathbb{P}(X \leq x) = \mathbb{P}(-X \leq x) = \mathbb{P}(X \geq -x) = 1 - F_X(-x)$. This implies $f_X(x) = f_X(-x), \forall x \in \mathbb{R}$.

Assume that the moments in question exist. Then $\mathbb{E}X^n = \int_{-\infty}^{\infty} x^n f_X(x)\, dx = 0$, since the function $x \mapsto x^n f_X(x)$ is odd. $\square$

Remark 1.226. Let X be a continuous RV, which is symmetric about $\mu \in \mathbb{R}$. As argued in the above proposition, we have for all $x \in \mathbb{R}$

$$F_X(\mu + x) = \mathbb{P}(X \leq \mu + x) = \mathbb{P}(X - \mu \leq x) = \mathbb{P}(\mu - X \leq x) = \mathbb{P}(X - \mu \geq -x) = 1 - F_X(\mu - x)$$

and hence $f_X(\mu+x) = f_X(\mu-x), \forall x$. Conversely, given a continuous RV X such that $f_X(\mu+x) = f_X(\mu - x), \forall x$ for some $\mu \in \mathbb{R}$, we have $F_{X-\mu} = F_{\mu-X}$ and hence X is symmetric about μ.

We now look at some special examples of discrete RVs.

Example 1.227 (Degenerate RV)**.** We have already mentioned this example earlier in Example 1.179. Fix $c \in \mathbb{R}$. Say that X is degenerate at c if its distribution is given by the p.m.f.

$$f_X(x) = \mathbb{P}(X = x) = \begin{cases} 1, & \text{if } x = c \\ 0, & \text{otherwise.} \end{cases}$$

This is a discrete RV with support $S_X = \{c\}$. As computed earlier, $\mathbb{E}X = c$. We also have $\mathbb{E}X^n = c^n, \forall n \geq 1$ and $M_X(t) = e^{tc}, \forall t \in \mathbb{R}$. Note that $Var(X) = \mathbb{E}X^2 - (\mathbb{E}X)^2 = 0$.

Remark 1.228 (Bernoulli Trial). Suppose that a random experiment has exactly two outcomes, identified as a 'success' and a 'failure'. For example, while tossing a coin, we may think of obtaining a head as a success and a tail as a failure. Here, the sample space is $\Omega = \{Success, Failure\}$. A single trial of such an experiment is referred to as a Bernoulli trial. In this case, $Probability(\{Success\}) = 1 - Probability(\{Failure\})$. If we define an RV $X : \Omega \to \mathbb{R}$ by $X(Success) = 1$ and $X(Failure) =$

0, then X is a discrete RV with p.m.f.

$$f_X(x) = \mathbb{P}(X = x) = \begin{cases} 1 - Probability(\{Success\}), \text{ if } x = 0, \\ Probability(\{Success\}), \text{ if } x = 1, \\ 0, \text{ otherwise.} \end{cases}$$

If $Probability(\{Success\}) = 0$ or $Probability(\{Failure\}) = 0$, then X is degenerate at 0 or 1, respectively. The case when $Probability(\{Success\}) \in (0, 1)$ is therefore of interest.

Example 1.229 (Bernoulli(p) RV)**.** Let $p \in (0, 1)$. An RV X is said to follow Bernoulli(p) distribution or equivalently, X is a Bernoulli(p) RV if its distribution is given by the p.m.f.

$$f_X(x) = \begin{cases} 1 - p, \text{ if } x = 0, \\ p, \text{ if } x = 1, \\ 0, \text{ otherwise.} \end{cases}$$

In relation with the Bernoulli trial described above, p may be treated as the probability of success. Here, $\mathbb{E}X = p$, $\mathbb{E}X^2 = p$, $Var(X) = \mathbb{E}X^2 - (\mathbb{E}X)^2 = p - p^2 = p(1-p)$, $M_X(t) = 1 - p + pe^t, t \in \mathbb{R}$. By standard arguments, we can establish the existence of these moments.

Notation 1.230. We may write $X \sim Bernoulli(p)$ to mean that X is a Bernoulli(p) RV. Similar notations shall be used for other RVs and their distributions.

Example 1.231 (Binomial(n, p) RV)**.** Fix a positive integer n and let $p \in (0, 1)$. By the Binomial theorem, we have

$$1 = [p + (1-p)]^n = \sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k}$$

and hence the function $f : \mathbb{R} \to [0, 1]$ given by

$$f(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x}, \text{ if } x \in \{0, 1, \cdots, n\}, \\ 0, \text{ otherwise.} \end{cases}$$

is a p.m.f.. An RV X is said to follow Binomial(n,p) distribution or equivalently, X is a Binomial(n,p) RV if its distribution is given by the above p.m.f.. Here,

$$\mathbb{E}X = \sum_{k=0}^{n} k \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k} = np \sum_{k=1}^{n} \binom{n-1}{k-1} p^{k-1}(1-p)^{n-k} = np\, [p + (1-p)]^{n-1} = np,$$

and

$$\mathbb{E}X(X-1) = \sum_{k=0}^{n} k(k-1) \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k} = n(n-1)p^2 \sum_{k=2}^{n} \binom{n-2}{k-2} p^{k-2}(1-p)^{n-k} = n(n-1)p^2.$$

Then $\mathbb{E}X^2 = \mathbb{E}X(X-1) + \mathbb{E}X = n(n-1)p^2 + np$ and $Var(X) = \mathbb{E}X^2 - (\mathbb{E}X)^2 = n(n-1)p^2 + np - n^2p^2 = np(1-p)$. Also

$$M_X(t) = \mathbb{E}e^{tX} = \sum_{k=0}^{n} e^{tk} \binom{n}{k} p^k (1-p)^{n-k} = (1-p+pe^t)^n, \forall t \in \mathbb{R}.$$

By standard arguments, we can establish the existence of these moments.

Note 1.232. Observe that Binomial$(1,p)$ distribution is the same as Bernoulli(p) distribution. We shall explore the connection between Binomial and Bernoulli distributions later in the course.

Remark 1.233 (Factorial moments). In the computation for $\mathbb{E}X^2$ for $X \sim Binomial(n,p)$, we first computed $\mathbb{E}X(X-1)$, which is easy to compute. It turns out that expectations of the form $\mathbb{E}X(X-1)$, $\mathbb{E}X(X-1)(X-2)$ etc. are often easy to compute for integer valued RVs X. We refer to such expectations as factorial moments of X.

Remark 1.234 (Symmetry of Binomial$(n,\frac{1}{2})$ distribution). Let $X \sim Binomial(n,p)$ and let $Y := n - X$. Since $M_X(t) = (1-p+pe^t)^n, \forall t \in \mathbb{R}$, we have

$$M_Y(t) = \mathbb{E}e^{tY} = \mathbb{E}e^{t(n-X)} = e^{-nt} M_X(-t) = e^{nt}(1-p+pe^{-t})^n = (p + (1-p)e^t)^n.$$

Since MGFs determine the distribution, we conclude that $Y \sim Binomial(n, 1-p)$. In particular, if $p = \frac{1}{2}$, then $Y = n - X \stackrel{d}{=} X \sim Binomial(n, \frac{1}{2})$. Rewriting the relation, we get $\frac{n}{2} - X \stackrel{d}{=} X - \frac{n}{2}$. Therefore, $X \sim Binomial(n, \frac{1}{2})$ is symmetric about $\frac{n}{2}$.

We now look at more examples of discrete RVs. Later in the course, we shall discuss their motivation through various random experiments.

Example 1.235 (Uniform RVs with support on a finite set)**.** Consider a discrete RV X with support $S_X = \{x_1, x_2, \cdots, x_n\}$ and p.m.f. $f_X : \mathbb{R} \to [0,1]$ given by

$$f_X(x) = \begin{cases} \frac{1}{n}, \text{ if } x \in S_X, \\ 0, \text{ otherwise.} \end{cases}$$

We had considered the case $S_X = \{1, 2, \cdots, 6\}$ in Example 1.180 and computed the expectation. In the general setting, we have

$$\mathbb{E}X = \frac{1}{n}\sum_{x \in S_X} x, \quad \mathbb{E}X^2 = \frac{1}{n}\sum_{x \in S_X} x^2, \quad M_X(t) = \mathbb{E}e^{tX} = \frac{1}{n}\sum_{x \in S_X} e^{tx}, \forall t \in \mathbb{R}$$

and hence $Var(X)$ can be computed by the formula $\mathbb{E}X^2 - (\mathbb{E}X)^2$. By standard arguments, we can establish the existence of these moments.

Example 1.236 (Poisson (λ) RV)**.** Fix $\lambda > 0$. Note that $e^\lambda = \sum_{k=0}^\infty \frac{\lambda^k}{k!}$ and hence the function $f : \mathbb{R} \to [0,1]$ given by

$$f(x) = \begin{cases} e^{-\lambda}\frac{\lambda^x}{x!}, \text{ if } x \in \{0, 1, 2, \cdots\}, \\ 0, \text{ otherwise.} \end{cases}$$

is a p.m.f.. An RV X is said to follow Poisson(λ) distribution or equivalently, X is a Poisson(λ) RV if its distribution is given by the above p.m.f.. Recall that we have already computed the following $\mathbb{E}X = \lambda, Var(X) = \lambda$ and $M_X(t) = e^{\lambda\left(e^t - 1\right)}, \forall t \in \mathbb{R}$ in Example 1.216. As done for the case of Binomial(n, p) RVs, we can compute factorial moments. For example,

$$\mathbb{E}X(X-1) = \sum_{k=0}^\infty k(k-1)e^{-\lambda}\frac{\lambda^k}{k!} = \lambda^2 \sum_{k=2}^\infty e^{-\lambda}\frac{\lambda^{k-2}}{(k-2)!} = \lambda^2.$$

In fact, $\mathbb{E}X(X-1)\cdots(X-(n-1)) = \lambda^n$ for all $n \geq 1$.

Example 1.237 (Geometric (p) RV)**.** Fix $p \in (0,1)$. Note that $\sum_{k=0}^\infty p(1-p)^k = 1$ and hence the function $f : \mathbb{R} \to [0,1]$ given by

$$f(x) = \begin{cases} p(1-p)^x, \text{ if } x \in \{0, 1, 2, \cdots\}, \\ 0, \text{ otherwise.} \end{cases}$$

is a p.m.f.. An RV X is said to follow Geometric(p) distribution or equivalently, X is a Geometric(p) RV if its distribution is given by the above p.m.f.. Let us compute the MGF. Here,

$$M_X(t) = \mathbb{E}e^{tX} = \sum_{k=0}^{\infty} e^{tk} p(1-p)^k = \frac{p}{1-(1-p)e^t},$$

for all t such that $0 < (1-p)e^t < 1$ or equivalently, $t < \ln\left(\frac{1}{1-p}\right)$. Looking at the derivatives of M_X and evaluating at $t = 0$, we have $\mathbb{E}X = \frac{1-p}{p}$ and $Var(X) = \frac{1-p}{p^2}$.

We now look at special examples of continuous RVs.

Example 1.238 (Uniform(a,b) RV)**.** Fix $a, b \in \mathbb{R}$ with $a < b$. An RV X is said to follow Uniform(a,b) distribution or equivalently, X is a Uniform(a,b) RV if its distribution is given by the p.d.f.

$$f_X(x) = \begin{cases} \frac{1}{b-a}, \text{ if } x \in (a,b), \\ 0, \text{ otherwise.} \end{cases}$$

We had considered the case $a = 0, b = 1$ in Example 1.184 and computed the expectation. In the general setting, we have

$$\mathbb{E}X = \int_a^b \frac{x}{b-a}\,dx = \frac{b^2-a^2}{2(b-a)} = \frac{a+b}{2}, \quad \mathbb{E}X^2 = \int_a^b \frac{x^2}{b-a}\,dx = \frac{b^3-a^3}{3(b-a)} = \frac{a^2+ab+b^2}{3}$$

and hence $Var(X)$ can be computed by the formula $\mathbb{E}X^2 - (\mathbb{E}X)^2$. The MGF is given by

$$\mathbb{E}e^{tX} = \int_a^b \frac{e^{tx}}{b-a}\,dx = \begin{cases} \frac{e^{tb}-e^{ta}}{t(b-a)}, \text{ if } t \neq 0, \\ 1, \text{ if } t = 0. \end{cases}$$

By standard arguments, we can establish the existence of these moments. Further, observe that $f_X(\frac{a+b}{2} - x) = f_X(\frac{a+b}{2} + x), \forall x \in \mathbb{R}$. Using Remark 1.226, we conclude that X is symmetric about its mean.

Example 1.239 (Cauchy(μ,θ) RV)**.** Let $\theta > 0$ and $\mu \in \mathbb{R}$. An RV X is said to follow Cauchy(μ,θ) distribution if its distribution is given by the p.d.f.

$$f_X(x) = \frac{\theta}{\pi}\,\frac{1}{\theta^2 + (x-\mu)^2}, \forall x \in \mathbb{R}.$$

The fact that f_X is a p.d.f. is easy to check. Set $y = \frac{x-\mu}{\theta}$ and observe that

$$\int_{-\infty}^{\infty} f_X(x)\, dx = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{1}{1+y^2}\; dy = \frac{2}{\pi}\int_{0}^{\infty} \frac{1}{1+y^2}\; dy = \frac{2}{\pi}\tan^{-1}(y)\mid_0^\infty = 1.$$

We have already considered the case $\mu = 0, \theta = 1$ in Example 1.186 and Example 1.218, where we have seen that $\mathbb{E}X$ and the MGF do not exist for this distribution. In the general setting, note that $\frac{X-\mu}{\theta} \sim Cauchy(0,1)$ and by a similar argument, we can show that $\mathbb{E}X$ and MGF do not exist. Moreover, $f_X(\mu+x) = f_X(\mu - x), \forall x \in \mathbb{R}$ and using Remark 1.226, we conclude that X is symmetric about μ.

Example 1.240 (Exponential(λ) RV)**.** Let $\lambda > 0$. Note that $\int_0^\infty \exp(-\frac{x}{\lambda})\, dx = \lambda$ and hence the function $f : \mathbb{R} \to [0,\infty)$ given by

$$f(x) = \begin{cases} \frac{1}{\lambda}\exp(-\frac{x}{\lambda}), & \text{if } x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

is a p.d.f.. An RV X is said to follow Exponential(λ) distribution or equivalently, X is an Exponential(λ) RV if its distribution is given by the above p.d.f.. We have already considered the case $\lambda = 1$ in Example 1.217, where we computed the moments and the MGF. Following similar arguments, in the general setting we have

$$\mathbb{E}X^n = \lambda^n n!, \quad Var(X) = \lambda^2, \quad M_X(t) = (1-\lambda t)^{-1}, \forall t < \frac{1}{\lambda}.$$

By standard arguments, we can establish the existence of these moments.

Definition 1.241 (Gamma function)**.** Recall that the integral $\int_0^\infty x^{\alpha-1}e^{-x}\, dx$ exists if and only if $\alpha > 0$. On $(0,\infty)$, consider the function $\alpha \mapsto \int_0^\infty x^{\alpha-1}e^{-x}\, dx$. It is called the Gamma function and the value at any $\alpha > 0$ is denoted by $\Gamma(\alpha)$.

Remark 1.242. We recall some important properties of the Gamma function.

(a) For $\alpha > 0$, we have $\Gamma(\alpha) > 0$.

(b) $\Gamma(\alpha) = (\alpha - 1)\Gamma(\alpha - 1)$, if $\alpha > 1$.

(c) $\Gamma(1) = \int_0^\infty e^{-x}\, dx = 1$ and hence using (b), $\Gamma(n) = (n-1)!$ for all positive integers n.

(d) $\Gamma(\frac{1}{2}) = \int_0^\infty \frac{1}{\sqrt{x}} e^{-x}\,dx = \sqrt{\pi}$. Putting $x = \frac{y^2}{2}$, this relation may be rewritten as

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{2}\int_0^\infty \exp\left(-\frac{y^2}{2}\right)\,dy = \sqrt{\pi}.$$

(e) Fix $\beta > 0$. Putting $x = \frac{y}{\beta}$, in the integral for $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}\,dx$, we get $\Gamma(\alpha) = \int_0^\infty y^{\alpha-1}\beta^{-\alpha}\exp(-\frac{y}{\beta})\,dy$.

Example 1.243 (Gamma(α,β) RV)**.** Fix $\alpha > 0, \beta > 0$. By the properties of the Gamma function described above, the function $f : \mathbb{R} \to [0,\infty)$ defined by

$$f(x) = \begin{cases} \frac{1}{\Gamma(\alpha)} x^{\alpha-1}\beta^{-\alpha}\exp(-\frac{x}{\beta}), & \text{if } x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

is a p.d.f.. An RV X is said to follow Gamma(α,β) distribution or equivalently, X is a Gamma(α,β) RV if its distribution is given by the above p.d.f.. Note that for $\alpha = 1$, we get back the p.d.f. for an Exponential(β) RV (see Example 1.240), i.e. Gamma$(1,\beta)$ distribution is the same as Exponential(β) distribution. For general $\alpha > 0, \beta > 0$, we have

$$\mathbb{E}X = \alpha\beta, \quad Var(X) = \alpha\beta^2, \quad M_X(t) = (1-\beta t)^{-\alpha}, \forall t < \frac{1}{\beta}.$$

By standard arguments, we can establish the existence of these moments.

Example 1.244 (Normal(μ,σ^2) RV)**.** Fix $\mu \in \mathbb{R}, \sigma > 0$. Note that $\Gamma(\frac{1}{2}) = \int_0^\infty \frac{1}{\sqrt{t}} e^{-t}\,dt = \sqrt{\pi}$ (see Remark 1.242). Putting $t = \frac{y^2}{2}$ and after suitable manipulation, we have $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^\infty \exp\left(-\frac{y^2}{2}\right)\,dy = 1$. Putting $y = \frac{1}{\sigma}(x-\mu)$ (equivalently, $x = \sigma y + \mu$), we have

$$\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^\infty \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)\,dx = 1.$$

Therefore, the function $f : \mathbb{R} \to [0,\infty)$ defined by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), \forall x \in \mathbb{R}$$

is a p.d.f.. An RV X is said to follow Normal(μ, σ^2) distribution or equivalently, X is a Normal(μ, σ^2) RV, denoted by $X \sim N(\mu, \sigma^2)$ if its distribution is given by the above p.d.f.. If $X \sim N(\mu, \sigma^2)$, from our above discussion we conclude that $Y = \frac{X-\mu}{\sigma} \sim N(0,1)$. Now,

$$\begin{aligned} M_Y(t) = \mathbb{E}e^{tY} &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{ty} \exp\left(-\frac{y^2}{2}\right) \, dy \\ &= \exp\left(\frac{t^2}{2}\right) \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-\frac{(y-t)^2}{2}\right) \, dy \\ &= \exp\left(\frac{t^2}{2}\right), \forall t \in \mathbb{R}. \end{aligned}$$

In particular, $\psi_Y(t) = \ln M_Y(t) = \frac{t^2}{2}, \forall t \in \mathbb{R}$ with $\psi'(t) = t, \psi''(t) = 1, \forall t \in \mathbb{R}$. Evaluating at $t = 0$, by Proposition 1.215 we conclude that $\mathbb{E}Y = 0$ and $Var(Y) = 1$. But $X = \sigma Y + \mu$ and hence $\mathbb{E}X = \mu, Var(X) = \sigma^2$. This yields the interpretation of the parameters μ and σ in the distribution of X. Further, $M_X(t) = \mathbb{E}e^{tX} = \mathbb{E}e^{t(\sigma Y + \mu)} = e^{\mu t} M_Y(\sigma t) = \exp(\mu t + \frac{1}{2}\sigma^2 t^2), \forall t \in \mathbb{R}$.

Definition 1.245 (Standard Normal RV)**.** We say X is a Standard Normal RV if $X \sim N(0,1)$, i.e. $\mathbb{E}X = 0$ and $Var(X) = 1$.

Notation 1.246. Normal RVs are also referred to as Gaussian RVs and Normal distribution as Gaussian distribution.

Remark 1.247 (Symmetry of Gaussian Distribution). If $X \sim N(\mu, \sigma^2)$, note that $f_X(\mu + x) = f_X(\mu - x), \forall x \in \mathbb{R}$ and using Remark 1.226, we conclude that X is symmetric about its mean μ.

Remark 1.248 (Moments of a Standard Normal RV). Let $X \sim N(0,1)$. Then X is symmetric about 0 and using Proposition 1.225, we conclude $\mathbb{E}X^n = 0$ for all odd positive integers n. If n is an even positive integer, then $n = 2m$ for some positive integer m and

$$\begin{aligned} \mathbb{E}X^n &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x^n \exp\left(-\frac{x^2}{2}\right) \, dx \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x^{2m} \exp\left(-\frac{x^2}{2}\right) \, dx \end{aligned}$$

$$
\begin{aligned}
&= \frac{\sqrt{2}}{\sqrt{\pi}} \int_0^\infty x^{2m} \exp\left(-\frac{x^2}{2}\right) dx \\
&= \frac{2^m}{\sqrt{\pi}} \int_0^\infty y^{m-\frac{1}{2}} \exp(-y)\, dy, \text{ (putting } y = \frac{x^2}{2}) \\
&= \frac{2^m}{\sqrt{\pi}} \Gamma(m + \frac{1}{2}) \\
&= 2^m \left(m - \frac{1}{2}\right) \times \cdots \times \frac{3}{2} \times \frac{1}{2} \\
&= (2m-1) \times \cdots \times 3 \times 1 =: (2m-1)!!,
\end{aligned}
$$

where we have used the properties of the Gamma function. In particular, $\mathbb{E}X^4 = 3$.

Definition 1.249 (Beta function)**.** Recall that the integral $\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\,dx$ exists if and only if $\alpha > 0$ and $\beta > 0$. On $(0,\infty) \times (0,\infty)$, consider the function $(\alpha,\beta) \mapsto \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\,dx$. It is called the Beta function and the value at any (α,β) is denoted by $B(\alpha,\beta)$.

Remark 1.250. Note that for $\alpha > 0, \beta > 0$, we have $B(\alpha,\beta) > 0$ and $B(\alpha,\beta) = B(\beta,\alpha)$. Moreover,

$$B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}.$$

Example 1.251 (Beta(α,β) RV)**.** Fix $\alpha > 0, \beta > 0$. By the properties of the Beta function described above, the function $f : \mathbb{R} \to [0,\infty)$ defined by

$$f(x) = \begin{cases} \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \text{ if } x \in (0,1) \\ 0, \text{ otherwise.} \end{cases}$$

is a p.d.f.. An RV X is said to follow Beta(α,β) distribution or equivalently, X is a Beta(α,β) RV if its distribution is given by the above p.d.f.. If $\alpha = \beta$, then $f(1-x) = f(x), \forall x \in \mathbb{R}$ and hence $X \stackrel{d}{=} 1 - X$. Then, $X - \frac{1}{2} \stackrel{d}{=} \frac{1}{2} - X$, i.e., X is symmetric about $\frac{1}{2}$. For all $\alpha, \beta, r > 0$, we have

$$\mathbb{E}X^r = \frac{1}{B(\alpha,\beta)} \int_0^1 x^{\alpha+r-1}(1-x)^{\beta-1}\,dx = \frac{B(\alpha+r,\beta)}{B(\alpha,\beta)}$$