Truncated 1-D Shallow Water Equations

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In this work we trace the interaction of 3 wavenumbers $\alpha = 1,\ \beta = 1,\ \gamma = 2$ which correspond to 2 waves and a vortex respectively. The initial conditions, $t = 0$ are :

$$h_\alpha = 2,\ h_\beta = 3,\ h_\gamma = 5 \tag{1}$$

$$u_\alpha = \frac{\omega}{\alpha} h_\alpha,\ u_\beta = \frac{-\omega}{\beta} h_\beta,\ u_\gamma = 0 \tag{2}$$

$$v_\alpha = \frac{1}{i\alpha} h_\alpha,\ v_\beta = \frac{1}{i\beta} h_\beta,\ v_\gamma = i\gamma h_\gamma \tag{3}$$

$$\omega = \sqrt{1 + \alpha^2} = 2 \tag{4}$$

Below we plot the potential vorticity with no waves in the initial conditions and another set with waves in the initial conditions, the potential vorticity is computed as follows:

$$q_\alpha = i\alpha v_\alpha - h_\alpha \tag{5}$$

$$q_\beta = i\beta v_\beta - h_\beta \tag{6}$$

$$q_\gamma = i\gamma v_\gamma - h_\gamma \tag{7}$$

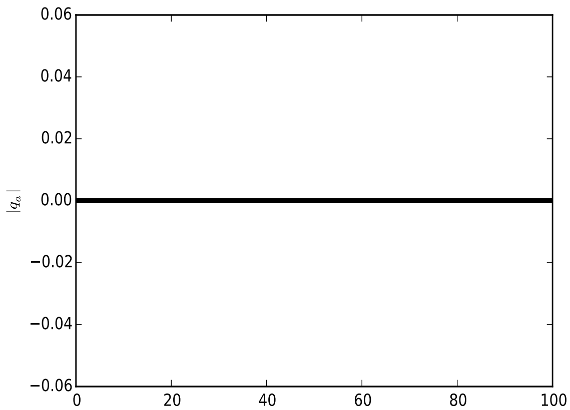

Figure 1: q_α

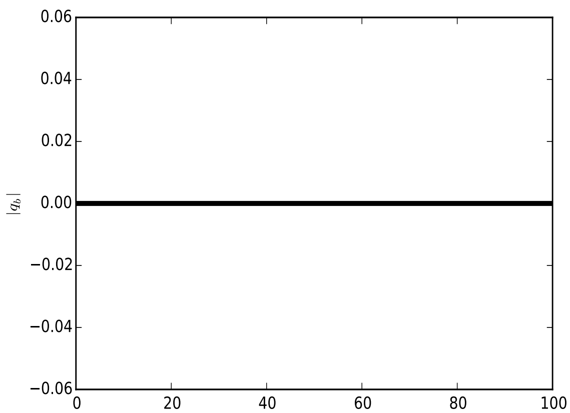

Figure 2: q_β

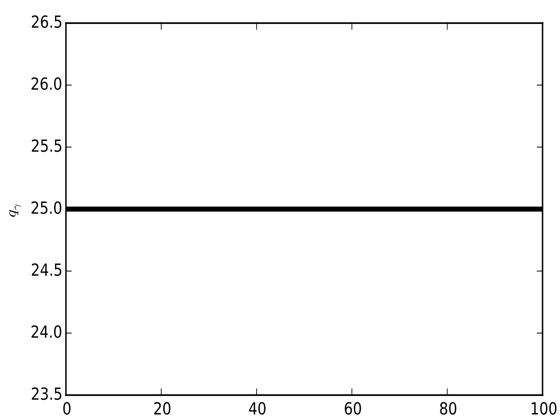

Figure 3: q_γ

Figure 4: $\epsilon = 0.05$, $t = 100$, *with no waves in the initial conditions*

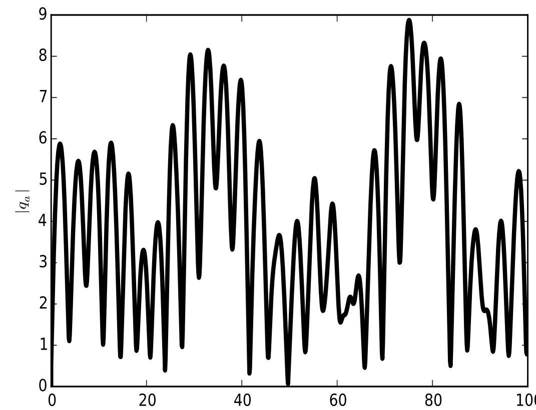

Figure 5: q_α

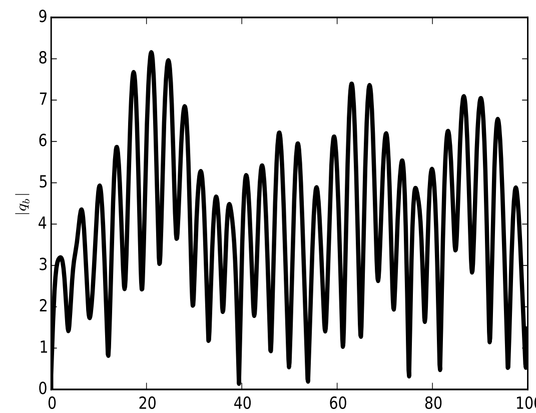

Figure 6: q_β

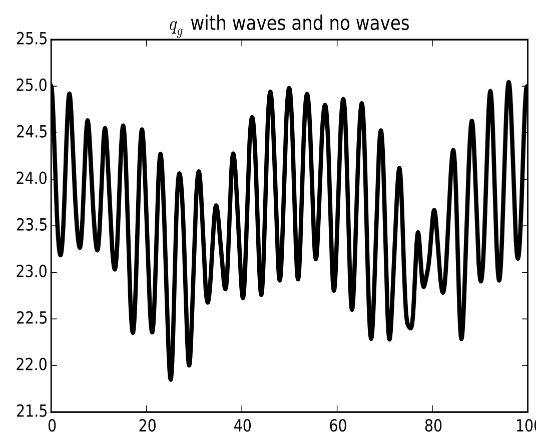

Figure 7: q_γ

Figure 8: $\epsilon = 0.05$, $t = 100$, *with waves in the initial conditions*

Below we plot the time-average of the potential vorticity of q_γ and the evolution of q_γ over time, in case of waves in the initial conditions and no waves in the initial conditions. The time average is calculated as follows:

$$Q_\gamma(t) = \frac{0.01\epsilon}{2\pi} \int_t^{t+\frac{2\pi}{0.01\epsilon}} q_\gamma(t)dt \tag{8}$$

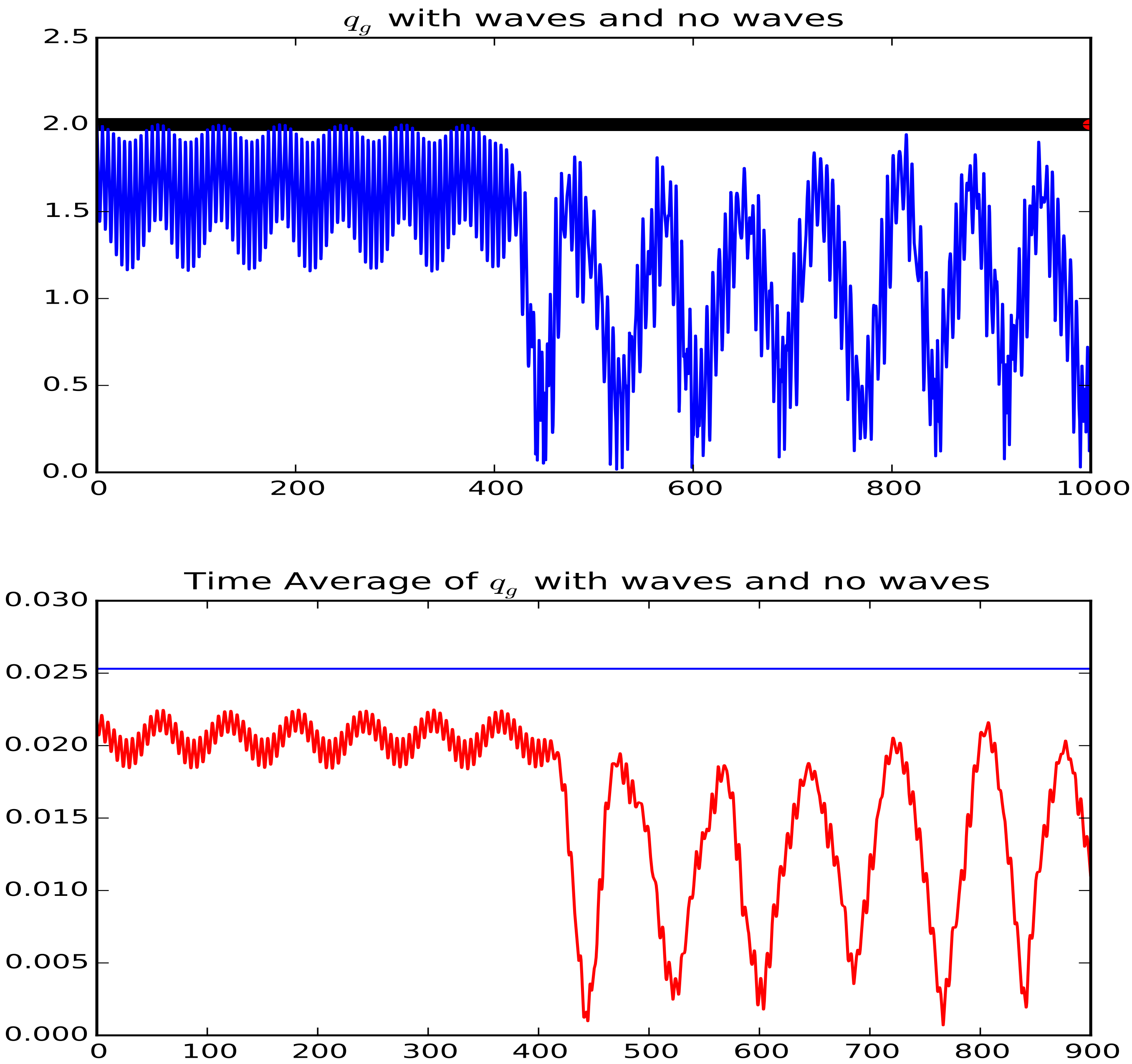

Figure 9: $\epsilon = 0.1,\ t = 1000$