Rare Event Simulation using Interacting Particle Systems for Rare Credit Portfolio Losses

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Outline

The goal of our project is to highlight IPS models for rare event siulation and estimation specifically in the context of Credit portfolio defaults.

To that effect we talk about the following:

- Interacting particle systems.
- Markovian intensity model of credit risk.
- Merton's model of credit risk.
    - We use an adapted model with stochastic volatility which is explained later.

Outline

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Stages of IPS

- Involves 4 stages of operations.
- Initialization of price, volatility.
- *n* loops of:
  - Selection stage to select paths that could lead to the rare event of higher defaults.
  - Mutation stage to evolve the selected paths as per the SDEs.
- Termination stage to collect result of previous stage and estimate default probabilities.

Initialization

Initial Value

$$\hat{X}_0^{(j)} = (\sigma(0), S_1(0), \cdots, S_N(0), S_1(0), \cdots, S_n(0)), \quad \forall 1 \leq j \leq M$$
$$\hat{W}_0^{(j)} = \hat{X}_0^{(j)}$$

- All M portfolios are started with the same price and volatility.
- Initial minimum is same as initial price.
- Initial history is same as current initialization.

Selection

- Resample with replacement M paths from input M paths according to empirical distribution under the given Gibbs measure.
- $\left(\hat{W}_p^{(j)}, \hat{X}_p^{(j)}\right)$ becomes $\left(\breve{W}_p^{(j)}, \breve{X}_p^{(j)}\right)$.

Empirical distribution

$$\eta_p^M(dW, dX) = \frac{1}{M\hat{\eta}_p^M} \sum_{j=1}^{M} \exp\left[\alpha(V(\hat{X}_p^{(j)})) - V(\hat{W}_p^{(j)})\right] \times \delta_{(\hat{W}_p^j, \hat{X}_p^j)}(dW, dX)$$

Where

$$\hat{\eta}_p^M = \frac{1}{M} \sum_{j=1}^{M} \exp\left[\alpha(V(\hat{X}_p^{(j)})) - V(\hat{W}_p^{(j)})\right]$$

Mutation

- Evolve paths from $\check{X}_p^{(j)}$ to $\hat{X}_{p+1}^{(j)}$.
- Set $\hat{W}_{p+1}^{(j)} = \check{X}_p^{(j)}$.
- Evolve based on SDEs using Euler-Maruyama method with time step δt. Evolution is from t_p to t_{p+1} $(t_p + \Delta t)$.
- $\delta t \ll \Delta t$
- True Dynamics.

Termination

- Teminate at maturity time T by running the selection, mutation step n times. $(n\Delta t = T)$
- Estimate default probability $p_k(T) = \mathbb{P}(L(T) = k)$ using formula below.

Estimation

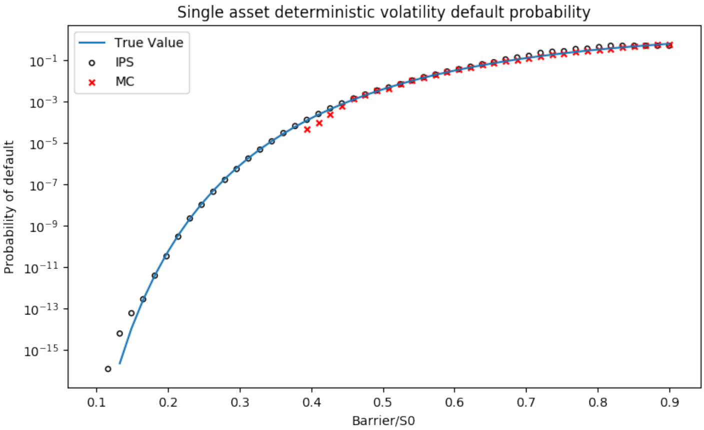
$$\hat{p}_k^M(T) = \left[\frac{1}{M}\sum_{j=1}^{M} \mathbf{1}_{\{f(\hat{X}_n^{(j)})=k\}} \exp\left[\alpha(V(\hat{W}^{(j)}) - V(\hat{X}_0))\right]\right] \times \left[\prod_{p=0}^{n-1} \hat{\eta}_p^M\right]$$

Where

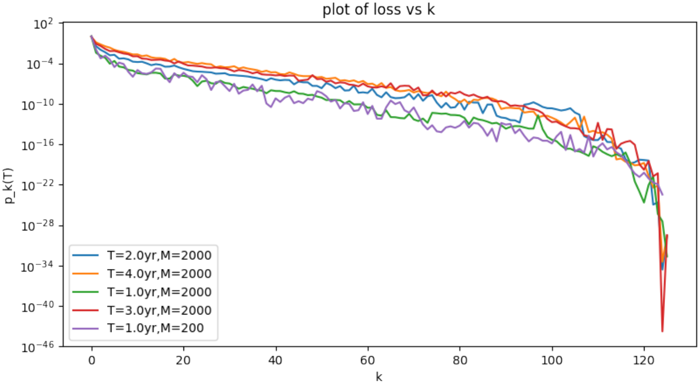
$$f(X_n^{(j)}) = \sum_{i=1}^{N} \mathbf{1}_{\{X_n^{(j)}(N+1+i)\leq B_i\}}$$

Outline

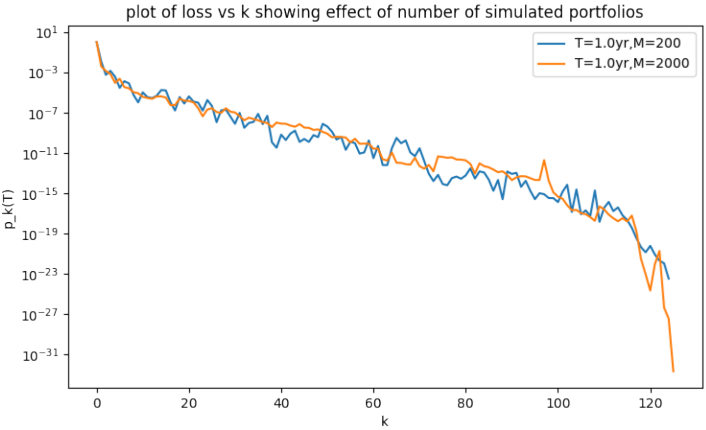
Plots


Single asset deterministic volatility default probability
True Value
IPS
MC
Probability of default
Barrier/S0
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{-13}
10^{-15}
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

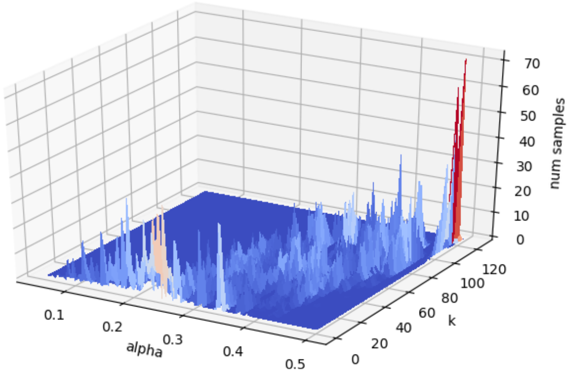
Plots


plot of loss vs k
p_k(T)
k
10^{2}
10^{-4}
10^{-10}
10^{-16}
10^{-22}
10^{-28}
10^{-34}
10^{-40}
10^{-46}
0
20
40
60
80
100
120
T=2.0yr,M=2000
T=4.0yr,M=2000
T=1.0yr,M=2000
T=3.0yr,M=2000
T=1.0yr,M=200

Plots


plot of loss vs k showing effect of number of simulated portfolios
T=1.0yr,M=200
T=1.0yr,M=2000
p_k(T)
k
10^{1}
10^{-3}
10^{-7}
10^{-11}
10^{-15}
10^{-19}
10^{-23}
10^{-27}
10^{-31}
0
20
40
60
80
100
120

Plots


70
60
50
40
30
20
10
0
num samples
0.1
0.2
0.3
0.4
0.5
alpha
0
20
40
60
80
100
120
k

References I

Our code, 2017. URL `https://github.com/alemc2/rare-credit-portfolio`.

R. Carmona and S. CRÉPEY. Importance sampling and interacting particle systems for the estimation of markovian credit portfolios loss distribution. *Forthcoming in International Journal of Theoretical and Applied Finance*, 2009.

R. Carmona, J.-P. Fouque, and D. Vestal. Interacting particle systems for the computation of rare credit portfolio losses. *Finance and Stochastics*, 13(4):613–633, 2009. ISSN 1432-1122. doi: 10.1007/s00780-009-0098-8. URL `http://dx.doi.org/10.1007/s00780-009-0098-8`.

Thank you!

Questions?