Linear Algebra

Matrices :- It is basically arrangement of no. in Rows & Columns.

Let $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{\underbrace{m \times n}_{\text{order of matrix}}}$

Rank of matrix :-

No of diff. rows ↓ यह Rank का सिर्फ यही मतलब है no of diff. rows.

Def :- Let A be any matrix. A number r is called rank of matrix A if (i) There is atleast one minor of A of order r which does not vanish → equal to (0)

(ii) Every Minor of A higher than r vanish.

Note :- No. of non-zero row in upper triangular matrix.

eg :- There is one (3×3) matrix. There is 3 rows. if all 3 rows are diff. then rank = 3

if 2 rows are same/identical then rank = 2

if all 3 rows are same/identical then rank of matrix = 1

& Rank of matrix should equal or less than no. of rows present. (As of course)

ये no of rows से ज्यादा कभी नहीं हो सकती है ∴ Rank ≤ no of rows

Eg :- $A = \begin{bmatrix} 1 & 2 & 3 \\ 7 & 5 & 8 \\ 2 & 4 & 6 \end{bmatrix}_{3\times 3}$ → Submatrices of matrix 'A' / Minors

$M_1 = \begin{vmatrix} 1 & 2 \\ 7 & 5 \end{vmatrix} = 5 - 14 = -9$

$M_2 = \begin{vmatrix} 2 & 3 \\ 5 & 8 \end{vmatrix} = 16 - 15 = 1$

$M_3 = \begin{vmatrix} 7 & 5 \\ 2 & 4 \end{vmatrix} = 28 - 15 = 13$

$\rho(A) = 2$

∴ Rank = 2

$M_4 = \begin{vmatrix} 5 & 8 \\ 4 & 6 \end{vmatrix} = 30 - 32 = -2$

$|A| = 1(-2) + -2(42-16) + 3(13)$

$= -2 - 37 + 39$

$= 0$

(We can also see it by prop. of determinant that $|A| = 0$ as two rows are identical.)

Assignment-1

Q. Test Consistency :-

(1) $2x-3y+7z=5$
$3x+y-3z=13$
$2x+19y-47z=32$

$$AX=B$$

$$\begin{bmatrix}2 & -3 & 7\\3 & 1 & -3\\2 & 19 & -47\end{bmatrix}_{3\times3} \times \begin{bmatrix}x\\y\\z\end{bmatrix}_{3\times1} = \begin{bmatrix}5\\13\\32\end{bmatrix}_{3\times1}$$

$$(A:B) = \begin{bmatrix}2 & -3 & 7 & : 5\\3 & 1 & -3 & : 13\\2 & 19 & -47 & : 32\end{bmatrix}_{3\times4}$$

$R_3 \to R_3 - R_1$
$R_2 \to 2R_2 - 3R_1$

$6-6$ $19+3$
$2+9$ $-47-7$
$-6-21$ $32-5$
$26-15$

$$\begin{bmatrix}2 & -3 & 7 & 5\\0 & 11 & -27 & 11\\0 & 22 & -54 & 27\end{bmatrix}_{3\times4}$$

$R_3 \to R_3 - 2R_2$

$$\begin{bmatrix}2 & -3 & 7 & 5\\0 & 11 & -27 & 11\\0 & 0 & 0 & 5\end{bmatrix}$$

$\rho(A:B)=3$

$\rho(A)=2$

$\therefore$ $\rho(A:B) \neq \rho(A)$

$\therefore$ It is inconsistent & having no soln.

(2) $2x - y + 3z = 8$
$-x + 2y + z = 4$
$3x + y - 4z = 0$

$$AX = B$$

$$\begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{bmatrix}_{3\times3} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times1} = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix}_{3\times1}$$

$$(A:B) = \begin{bmatrix} 2 & -1 & 3 & : & 8 \\ -1 & 2 & 1 & : & 4 \\ 3 & 1 & -4 & : & 0 \end{bmatrix}_{3\times4}$$

$R_2 \to 2R_2 + R_1$

$R_3 \to 2R_3 - 3R_1$

$-2 + 2$
$4 - 1$
$2 + 3$
$8 + 8$

$$\begin{bmatrix} 2 & -1 & 3 & : & 8 \\ 0 & 3 & 5 & : & 16 \\ 0 & 5 & -17 & : & -24 \end{bmatrix}_{3\times4}$$

$6 - 6$
$2 + 3$
$-8 - 9$
$0 - 24$

$R_3 \to 3R_3 - 5R_2$

$$\begin{bmatrix} 2 & -1 & 3 & 8 \\ 0 & 3 & 5 & 16 \\ 0 & 0 & -76 & -152 \end{bmatrix}_{3\times4}$$

$15 - 15$
$-51 - 25 = 76$
$-72 - 80 = -152$

$\rho(A:B) = 3$

$\rho(A) = 3$

$\rho(A:B) = \rho(A) = n$ (no. of unknowns)

$\therefore$ It is consistent & having unique soln.

$2x - y + 3z = 8$

$3y + 5z = 16$

$\cancel{-76}z = \cancel{-152}\ 2$

$z = 2$

$3y + 10 = 16$

$3y = 6$

$y = 2$

$2x - 2 + 6 = 8$

$2x + 4 = 8$

$2x = 4$

$x = 2$

(3.) $4x - y = 12$ $\qquad 4x - y + 0z = 12$

$-x + 5y - 2z = 0$

$-2x + 4z = -8$ $\qquad -2x + 0y + 4z = -8$

$AX = B$

$$\begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -2 & 0 & 4 \end{bmatrix}_{3\times3} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times1} = \begin{bmatrix} 12 \\ 0 \\ -8 \end{bmatrix}_{3\times1}$$

$$(A:B) = \left[\begin{array}{ccc:c} 4 & -1 & 0 & 12 \\ -1 & 5 & -2 & 0 \\ -2 & 0 & 4 & -8 \end{array}\right]_{3\times4}$$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} -1 & 5 & -2 & 0 \\ 4 & -1 & 0 & 12 \\ -2 & 0 & 4 & -8 \end{bmatrix}$$

$R_2 \to R_2 + 4R_1$

$R_3 \to R_3 - 2R_1$

$4-4$
$-1+5\times4$
$0-2\times4$
$12+0$
$-2+2$
$0-10$
$4+4$
$-8+0$

$$\begin{bmatrix} -1 & 5 & -2 & 0 \\ 0 & 19 & -8 & 12 \\ 0 & -10 & 8 & -8 \end{bmatrix}$$

$R_3 \to 19R_3 - 10R_2$

$$\begin{bmatrix} -1 & 5 & -2 & 0 \\ 0 & 19 & -8 & 12 \\ 0 & 0 & 232 & -272 \end{bmatrix}$$

19
$\times 8$
$152 + 80$
120
272

$\rho(A:B) = \rho(A) = 3 = n$

$\therefore$ Consistent & unique solⁿ.

$-x + 5y - 2z = 0$

$19y - 8z = 12$

$232z = 32$

$z = 272/232$

Q-> $x + 3y - 2z = 0$
$2x - y + 4z = 0$
$x - 11y + 14z = 0$

$AX = B$

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$(A:B) = \begin{bmatrix} 1 & 3 & -2 & : 0 \\ 2 & -1 & 4 & : 0 \\ 1 & -11 & 14 & : 0 \end{bmatrix}_{3\times 4}$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - R_1$

$2-2$
$-1-6$
$4+4$
$0-0$
$1-1$
$-11-3$
$14+2$
0

$$\begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & -14 & 16 & 0 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore \quad \rho(A:B) = \rho(A) = 2 < n$

$\therefore$ Consitant & having infinite sol^n

$x + y - 2z = 0$
$-7y + 8z = 0$

Let $z = k$

$-7y + 8k = 0$
$-7y = 8k$
$y = \frac{-8}{7}k$

$x - \frac{8}{7}k - 2k = 0$

$7x - 8k - 14k = 0$
$7x - 22k = 0$
$7x = 22k$
$x = \frac{22}{7}k$

$\therefore$ We can put values & find infinite sol^n for x, y, z

Eg - $k = 7$
$z = 7$
$y = -8$
$x = 22$

Q-> $x+y+z=6$
$x+2y+3z=10$
$x+2y+\lambda z=\mu$
$AX=B$

$$\begin{bmatrix}1&1&1\\1&2&3\\1&2&\lambda\end{bmatrix}_{3\times3}\begin{bmatrix}x\\y\\z\end{bmatrix}_{3\times1}=\begin{bmatrix}6\\10\\\mu\end{bmatrix}_{3\times1}$$

$$(A:B)=\begin{bmatrix}1&1&1&:&6\\1&2&3&:&10\\1&2&\lambda&:&\mu\end{bmatrix}$$

$R_2\to R_2-R_1$
$R_3\to R_3-R_1$

$1-1$
$2-1$
$3-1$
$10-6$

$$\begin{bmatrix}1&1&1&6\\0&1&2&4\\0&1&(\lambda-1)&(\mu-6)\end{bmatrix}$$

$R_3\to R_3-R_2$

$1-1$
$2-1$
$\lambda-1$
$\mu-6$
$\lambda-1-2$

$$\begin{bmatrix}1&1&1&6\\0&1&2&4\\0&0&(\lambda-3)&(\mu-10)\end{bmatrix}_{3\times4}$$

(1) for no solⁿ $\rho(A)\neq\rho(A:B)$
$\lambda-3=0$ & $\mu-10\neq0$
$\lambda=3$ & $\mu\neq10$ $\therefore \rho(A)=2$ & $\rho(A:B)=3$
$\therefore$ no solⁿ

(2) for unique solⁿ $\rho(A)=\rho(A:B)=n$
$\lambda-3\neq0$ $\mu-10\neq0$
$\lambda\neq3$ $\mu\neq10$
If in place of $(\lambda-3)$ & $(\mu-10)$
if there is some values then $\rho(A)=\rho(A:B)=3=n$
$\therefore$ unique solⁿ.

(3) for infinite solⁿ $\rho(A)=\rho(A:B)<n$
$(\lambda-3)=0$ & $(\mu-10)=0$
$\lambda=3$ $\mu=10$
so that
$(\lambda-3)=0$ & $(\mu-10)=0$
$\therefore \rho(A)=\rho(A:B)=2<n$
$\therefore$ infinite solⁿ.

Q-10 $x+y+z=1$
$x+2y+4z=\lambda$
$x+4y+10z=\lambda^2$

To find λ for unique solⁿ.
$\therefore \rho(A) = \rho(A:B) = n$

$AX = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{bmatrix}_{3\times3} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times1} = \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix}_{3\times1}$$

$$(A:B) = \begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 1 & 2 & 4 & : & \lambda \\ 1 & 4 & 10 & : & \lambda^2 \end{bmatrix}_{3\times4}$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$

$1-1$
$2-1$
$4-1$
$\lambda-1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & (\lambda-1) \\ 0 & 3 & 9 & (\lambda^2-1) \end{bmatrix}_{3\times4}$$

$1-1$
$4-1$
$10-1$
λ^2-1

$R_3 \to R_3 - 3R_2$

$\lambda^2 - \cancel{\lambda} - 3\lambda + \cancel{\lambda}$
$(\lambda^2-3\lambda)$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & (\lambda-1) \\ 0 & 0 & 0 & (\lambda^2-1)-3(\lambda-1) \end{bmatrix}_{3\times4} \Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & (\lambda-1) \\ 0 & 0 & 0 & \lambda^2-3\lambda \end{bmatrix}$$

for unique solⁿ.
It is not possible
as if $\lambda^2-3\lambda=0 \to$ ~~$\lambda^2 \times 3\lambda$~~ $\lambda(\lambda-3)=0$
$(\lambda=0)$ $(\lambda=3)$
$\therefore \rho(A) = \rho(A:B) = 2 < n$
$\therefore$ infinite solⁿ
& If $\lambda^2-3\lambda \neq 0 \to (\lambda \neq 0, 3)$
$\therefore \rho(A) \neq \rho(A:B)$
& It is inconsistent &
having no solⁿ.

Assingment -2

(1) $[1,0,0]$, $[1,1,0]$ $[1,1,1]$

first find rank R

There are 3 vectors, if rank R = 3 then Linearly independent

& if rank R < 3 then Linearly dependent

(Put it Column wise)

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

upper triangular matrix

$\rho = 3$

$\therefore$ Linearly Independent

V=3
R=3

(2) $[7,-3,11,-6]$ $[-56, 24, -88, 48]$

V=2
R<2 → L.D
if R=2 → L.I

$$\begin{bmatrix} 7 & -56 \\ -3 & 24 \\ 11 & -88 \\ -6 & 48 \end{bmatrix}_{4\times2}$$

$R_2 \to 7R_2 + 3R_1$

$R_3 \to 7R_3 - 11R_1$

$R_4 \to 7R_4 + 6R_1$

-21+21
168 -168
-616+616
336 -

56
×8
616

$$\begin{bmatrix} 7 & -56 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\rho = 1$

$\therefore$ Linearly dependent.

(3) $[-1\ 5\ 0]$ $[16\ 8\ -3]$ $[-64\ 56\ 9]$

$$\begin{bmatrix} -1 & 16 & -64 \\ 5 & 8 & 56 \\ 0 & -3 & 9 \end{bmatrix}$$

64
×5
320

8+80
56-320

$R_2 \to R_2 + 5R_1$

$$\begin{bmatrix} -1 & 16 & -64 \\ 0 & 88 & -264 \\ 0 & -3 & 9 \end{bmatrix} \quad R_2 \leftrightarrow R_3 \quad \begin{bmatrix} -1 & 16 & 64 \\ 0 & -3 & 9 \\ 0 & 88 & -264 \end{bmatrix}$$

$R_3 \to \frac{R_3}{88}$

$$\begin{bmatrix} -1 & 16 & 64 \\ 0 & -3 & 9 \\ 0 & 1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 16 & -64 \\ 0 & -3 & 9 \\ 0 & 1 & -3 \end{bmatrix}$$

$R_3 \to R_3 + \frac{1}{3} R_2$

~~-3+3=0~~

$$\begin{bmatrix} -1 & 16 & -64 \\ 0 & -3 & 9 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho = 2$

$\therefore$ vectors = 3

$\therefore$ linearly dependent.

How?

$a [-1\ 5\ 0] + b [16\ 8\ -3] + c [-64\ 56\ 9] = 0$

$-a + 16b - 64c = 0$

$5a + 8b + 56c = 0$

$0a - 3b + 9c = 0$

$Ax = b$

$$\begin{bmatrix} -1 & 16 & -64 \\ 5 & 8 & 56 \\ 0 & -3 & 9 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$(A:B) = \left[\begin{array}{ccc:c} -1 & 16 & -64 & 0 \\ 5 & 8 & 56 & 0 \\ 0 & -3 & 9 & 0 \end{array}\right]$$

$\downarrow$

$$\begin{bmatrix} -1 & 16 & -64 & 0 \\ 0 & -3 & 9 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$-a + 16b - 64c = 0$

$-3b + 9c = 0$

Let $c = k$

$-3b + 9k = 0$

$3b = 9k$

$b = 3k$

$16 \times 3 = 48$

$-a + 48k - 64k = 0$

$-a - 16k = 0$

$-a = 16k$

$a = -16k$

It has infinite solⁿ

We can find by putting values.

$k = 1$

$a = -16$

$b = 3$

$c = 1$

(4.) $[1\ -1\ 1]\ [1\ 1\ -1]\ [-1\ 1\ 1]\ [0\ 1\ 0]$

$$\begin{bmatrix} 1 & 1 & -1 & 0 \\ -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 0 \end{bmatrix} \qquad \begin{matrix} R_2 \to R_2 + R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & -2 & 2 & 0 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

$\rho = 3$ Linearly dependent

(5.) $[2\ -4]\ [1\ 9]\ [3\ 5]$

$$\begin{bmatrix} 2 & 1 & 3 \\ -4 & 9 & 5 \end{bmatrix}_{2\times 3}$$

$\rho = 2$ $\therefore$ Linearly dependent

(6.) $[3\ -2\ 0\ 4]\ [5\ 0\ 0\ 1]\ [-6\ 1\ 0\ 1]\ [2\ 0\ 0\ 3]$

$$\begin{bmatrix} 3 & 5 & -6 & 2 \\ -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 4 & 1 & 1 & 3 \end{bmatrix}_{4\times 4}$$

$R_2 \to 3R_2 + 2R_1$

$R_4 \to 3R_4 - 4R_1$

Rank = 3

vector = 4

$\therefore$ Linearly dependent

3-20

3+24

270-153

10-68

$$\begin{bmatrix} 3 & 5 & -6 & 2 \\ 0 & 10 & -9 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 17 & 27 & 1 \end{bmatrix}$$

$R_3 \to 10R_3 - 17R_2$

$$\begin{bmatrix} 3 & 5 & -6 & 2 \\ 0 & 10 & -9 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 117 & -58 \end{bmatrix} \quad R_3 \leftrightarrow R_4 \quad \begin{bmatrix} 3 & 5 & -6 & 2 \\ 0 & 10 & -9 & 4 \\ 0 & 0 & 117 & -58 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(7) [3 4 7] [2 0 3] [8 2 3] [5 5 6]

$$\begin{bmatrix} 3 & 2 & 8 & 5 \\ 4 & 0 & 2 & 5 \\ 7 & 3 & 3 & 6 \end{bmatrix}_{3\times4}$$

$\int = 3$ & vectors = 4

$\therefore$ Linearly dependent

(8) [6 0 3 1 4 2] [0 -1 2 7 0 5]
[12 3 0 -19 8 -11]

$$\begin{bmatrix} 6 & 0 & 12 \\ 0 & -1 & 3 \\ 3 & 2 & 0 \\ 1 & 7 & -19 \\ 4 & 0 & 8 \\ 2 & 5 & -11 \end{bmatrix}_{6\times3}$$

$R_1 \leftrightarrow R_4$

$$\begin{bmatrix} 1 & 7 & -19 \\ 0 & -1 & 3 \\ 3 & 2 & 0 \\ 6 & 0 & 12 \\ 4 & 0 & 8 \\ 2 & 5 & -11 \end{bmatrix}$$

$R_3 \to R_3 - 3R_1$
$R_4 \to R_4 - 6R_1$
$R_5 \to R_5 - 4R_1$
$R_6 \to R_6 - 2R_1$

$$\begin{bmatrix} 1 & 7 & -19 \\ 0 & -1 & 3 \\ 3 & 2 & 0 \\ 6 & 0 & 12 \\ 4 & 0 & 8 \\ 2 & 5 & -11 \end{bmatrix}$$

$R_3 \to R_3 - 3R_1$
$R_4 \to R_4 - 6R_1$
$R_5 \to R_5 - 4R_1$
$R_6 \to R_6 - 2R_1$

$$\begin{bmatrix} 1 & 7 & -19 \\ 0 & -1 & 3 \\ 0 & -19 & 57 \\ 0 & -42 & -102 \\ 0 & -28 & 84 \\ 0 & -9 & 27 \end{bmatrix}_{6\times 3}$$

$R_3 \to R_3 - 19R_2$
$R_4 \to R_4 - 42R_2$
$R_5 \to R_5 - 28R_2$
$R_6 \to R_6 - 9R_2$

$$\begin{bmatrix} 1 & 7 & -19 \\ 0 & -1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & -228 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{6\times 3}$$

$R_4 \leftrightarrow R_3$

$3-3$ $4-4$
$2-21$ $0-28$
$0+57$ $8+76$
$6-6$ $5-14$
$0-42$ $-11+38$
$12+$

$12-114$
-102

$-102 - 3\times 42$
-102
-126
$84 - 228$

$42 \times 3 = \boxed{126}$

$28 \times 3 = 84$

$$\begin{bmatrix} 1 & 7 & -19 \\ 0 & -1 & 3 \\ 0 & 0 & -228 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{6\times 3}$$

Rank = 3

$\therefore$ linearly independent

Eigen Values & Eigen Vectors:-

Let $A = [a_{ij}]_{n \times n}$ be a square matrix of order $n \times n$ & I be unit matrix.

(i) Characteristic matrix of A :- The matrix $(A-\lambda I)$ is called the characteristic matrix of A.

$$A-\lambda I = \begin{bmatrix} a_{11}-\lambda & a_{12}-\lambda & \cdots & a_{1n}-\lambda \\ a_{21}-\lambda & a_{22}-\lambda & \cdots & a_{2n}-\lambda \\ \vdots & & & \\ a_{m1}-\lambda & \cdots & \cdots & a_{mn}-\lambda \end{bmatrix}$$

(ii) Characteristic polynomial :- The determinant $|A-\lambda I|$ is called Characteristic polynomial.

(iii) Characteristic Eq^n :- The equation $|A-\lambda I| = 0$ is called characteristic equation of A. The roots of equation are called characteristic roots or Eigen values or Latent values of matrix A.

(iv) Eigen vectors:- If $\lambda = \lambda_1$ is Eigen values of $n \times n$ matrix A. Then non zero sol^n $X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ of eq^n

$(A-\lambda_1 I)X = 0$ is said to be eigen vector of A corresponding to Eigen values $\lambda = \lambda_1$

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Assignment-4 {Eigen values & Eigen vectors}

① $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ → principal diagonal

(C.M) $A-\lambda I = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} (-2-\lambda) & 2 & -3 \\ 2 & (1-\lambda) & -6 \\ -1 & -2 & -\lambda \end{bmatrix}$

(C.P) $|A-\lambda I| = (-2-\lambda)\,[(-\lambda+\lambda^2)-12] - 2(-2\lambda-6) - 3(-4+1-\lambda)$

$(-2-\lambda)\,[-\lambda+\lambda^2-12] - 2(-2\lambda-6) - 3(-3-\lambda)$

$+2\lambda - 2\lambda^2 + 24 + \lambda^2 - \lambda^3 + 12\lambda + 4\lambda + 12 + 9 + 3\lambda$

$-\lambda^3 - \lambda^2 + 21\lambda + 45$

36
9
45

(C.E) $|A-\lambda I| = 0$

$-\lambda^3 - \lambda^2 + 21\lambda + 45 = 0$

$(\lambda = 1)$ $-1 - 1 + 21 + 45$ ×

$(\lambda = 2)$ $-8 - 4 + 42 + 45$ ×

$(\lambda = -2)$ $8 - 4 - 42 + 45$
$-46 + 53$

$(\lambda = 3)$ × $-27 - 9 + 63 + 45$

$(\lambda = -3)$ ✓ $27 - 9 - 63 + 45 = 0$
$72 - 72 = 0$

27 + 45 = 72; 63 + 9 = 72

multiplication of E.V = constant term in C.E

To check whether all the Eigen values are correct or not
addⁿ of all E.V = addⁿ of principal diagonal = coeff of λ^2

$$\begin{array}{c|cccc} -3 & -1 & -1 & 21 & 45 \\ & & +3 & -6 & -45 \\ \hline & -1 & 2 & 15 & 0 \end{array}$$

$-\lambda^2 + 2\lambda + 15 = 0$

$-\lambda^2 - 3\lambda + 5\lambda + 15 = 0$

$-\lambda(\lambda+3) + 5(\lambda+3) = 0$

$(-\lambda+5)(\lambda+3) = 0$

$\therefore (\lambda+3)(-\lambda+5)(\lambda+3) = 0$

$\lambda = -3, -3, 5$

all 3
When Eigen values $\neq 0$
then Rank of $A = 3$
& when we put it in matrix $(A-\lambda I)$ it become
1 less than 3 that is 2
It gives infinite sol^n

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But if two Eigen values are same then Rank become more less than 1 that is rank = 1
Eigen vectors are infinite sol^n.

$-2+3$

$\lambda = -3$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix}_{3\times3} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times1} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}_{3\times1}$$

$x_1 + 2x_2 - 3x_3 = 0$

$x_2 = k$

$x_3 = 0$

$x_1 + 2k - 0 = 0$

$x_1 = -2k$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -2k \\ k \\ 0 \end{pmatrix} = k\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$$

$x_3 = k$

$x_2 = 0$

$x_1 + 0 - 3k = 0$

$x_1 = 3k$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = k\begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$$

1, 2

Eigen vectors are infinite sol^n.

To check whether Eigen vectors are correct or not u can put in it eq^n if satifies than correct

$\lambda = 5$

$$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}_{3\times1}$$

$20-12$

$-10-6$

$-4-4$

$$\frac{x_1}{\begin{vmatrix} -4 & -6 \\ -2 & -5 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 2 & -6 \\ -1 & -5 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 2 & -4 \\ -1 & -2 \end{vmatrix}}$$

$$\frac{x_1}{8} = \frac{-x_2}{-16} = \frac{x_3}{-8}$$

$$\frac{x_1}{1} = \frac{x_2}{2} = \frac{x_3}{-1}$$

$$\begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

(2)
$$\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

(C.M) $A-\lambda I = \begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix}_{3\times3}$

(C.P) $|A-\lambda I| = (4-\lambda)(1-\lambda)^2 - 0 + 1(0+2(1-\lambda))$

$= (4-\lambda)(1+\lambda^2-2\lambda) + 2-2\lambda$

$= 4+4\lambda^2-8\lambda-\lambda-\lambda^3+2\lambda^2+2-2\lambda$

$= -\lambda^3+6\lambda^2-11\lambda+6$

(C.E) $|A-\lambda I| = 0$

$-\lambda^3+6\lambda^2-11\lambda+6=0$

$\lambda=1$ $(\lambda-1)$

$-1+6-11+6=0$

$-12+12=0$

$$\begin{array}{r|rrrr} +1 & -1 & 6 & -11 & 6 \\ & & -1 & +5 & -6 \\ \hline & -1 & 5 & -6 & 0 \end{array}$$

$-\lambda^2+5\lambda-6=0$

$-\lambda^2+2\lambda+3\lambda-6=0$

$-\lambda(\lambda-2)+3(\lambda-2)=0$

$(-\lambda+3)(\lambda-2)=0$

$\therefore$ Roots $(\lambda-1)(-\lambda+3)(\lambda-2)=0$

$1+3+2=6$

$4+1+1=6$

$6=6$

$1\times3\times2$

$\lambda = 1, 3, 2$

$\lambda=1$ $\begin{bmatrix} 3 & 0 & 1 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix}_{3\times3} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times1} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}_{3\times1}$

$\frac{x_1}{0} = \frac{x_2}{0} = \frac{x_3}{0}$

$\begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\lambda = 2$

$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -2 & 0 \\ -2 & -1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -2 & -1 \\ -2 & 0 \end{vmatrix}}$$

$$\frac{x_1}{1} = \frac{-x_2}{2} = \frac{x_3}{-2}$$

$$\cancel{x_1} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$$

$\lambda = 3$

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ -2 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{4} = \frac{-x_2}{4} = \frac{x_3}{-4}$$

$$\frac{x_1}{1} = \frac{x_2}{-1} = \frac{x_3}{-1}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$$

To find Eigen value corresponding to its Eigent vector we can also find when Eigen vectors is given:-

eg:- $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$

$4 + 0 - 2$
$-2 - 2$
$-2 + 0 - 2$

$$\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ -4 \end{bmatrix}$$

$$" \quad \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix} = \underline{2} \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}$$

First make equal both of them & which is common is eigen value

(3.) $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}_{3\times 3}$

(C.M) $(A-\lambda I) = \begin{bmatrix} 5-\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ -1 & 0 & 3-\lambda \end{bmatrix}$

(C.P) $|A-\lambda I| = (5-\lambda)(-\lambda(3-\lambda)) - 0 + 0$

$(5-\lambda)(-3\lambda+\lambda^2)$

$\cancel{(5-\lambda)} = -15\lambda + 5\lambda^2 + 3\lambda^2 - \lambda^3$

$= -\lambda^3 + 8\lambda^2 - 15\lambda$

(C.E) $|A-\lambda I| = 0$

$-\lambda^3 + 8\lambda^2 - 15\lambda = 0$

$-\lambda^3 + 3\lambda^2 + 5\lambda^2 - 15\lambda = 0$

$-\lambda^2(\lambda-3) + 5\lambda(\lambda-3) = 0$

$(-\lambda^2+5\lambda)\ (\lambda-3) = 0$

$-\lambda^2+5\lambda = 0$ | $(\lambda-3) = 0$

$\lambda(-\lambda+5) = 0$ | $\lambda = 3$

$\lambda = 0$, $\lambda = 5$

$\lambda = 0, 5, 3$

$(\lambda = 0)$

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{0} = \frac{-x_2}{0} = \frac{x_3}{0}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$(\lambda = 5)$ $\begin{bmatrix} 0 & 0 & 0 \\ 0 & -5 & 0 \\ -1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\frac{x_1}{10} = \frac{-x_2}{0} = \frac{x_3}{-5} \Rightarrow \frac{x_1}{2} = \frac{x_2}{0} = \frac{x_3}{-1} \quad \therefore \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$$

$$(\lambda-3)\quad \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{0} = \frac{-x_2}{0} = \frac{x_3}{-3}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix}$$

(4.) $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix}_{3\times3}$

(C.M) $A-\lambda I = \begin{bmatrix} -\lambda & 0 & 0 \\ 0 & 3-\lambda & 4 \\ 0 & 0 & -2-\lambda \end{bmatrix}_{3\times3}$

(C.P) $|A-\lambda I| = -\lambda\left((3-\lambda)(-2-\lambda) - 0\right) - 0 + 0$

$= \cancel{+\lambda}\,\cancel{\lambda\times 6}$

$\lambda(3-\lambda)(2+\lambda)$

$\lambda(6+3\lambda-2\lambda-\lambda^2)$

$\lambda(6+\lambda-\lambda^2)$

$-\lambda^3+\lambda^2+6\lambda$

(C.E) $|A-\lambda I| = 0$

$-\lambda^3+\lambda^2+6\lambda = 0$

2×3 $\quad -\lambda^3+3\lambda^2-2\lambda^2+6\lambda = 0$

$-\lambda^2(\lambda-3)-2\lambda(\lambda-3) = 0$

$(-\lambda^2-2\lambda)\quad(\lambda-3) = 0$

$-\lambda^2-2\lambda = 0$ | $(\lambda-3) = 0$

$-\lambda(+\lambda+2) = 0$ | $\lambda = +3$

$\lambda = 0,\ \lambda = -2$

$\therefore \lambda = 0$ $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\frac{x_1}{-6} = \frac{-x_2}{0} = \frac{x_3}{0}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -6 \\ 0 \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$

as we know Eigen ~~vector~~ vectors are infinite.

$\lambda = 2$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\frac{x_1}{0} = \frac{-x_2}{0} = \frac{x_3}{0}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\lambda = 3$

$$\begin{bmatrix} -3 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix}$$

$$\frac{x_1}{0} = \frac{-x_2}{0} = \frac{x_3}{0}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

(5.) $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$

$\rho(A) = 1$ as all Rows are same

$\therefore$ All Eigen values are not diff. nonzero values $\therefore$

sum of Eigen values $= 6$

$|A| = 0$ $\therefore$ A is not invertible

$\therefore$ 0 is eigenvalue of A

Assignment = 3

(1.)

$x + 3y + 2z = 0$
$2x - y + 3z = 0$
$3x - 5y + 4z = 0$
$x + 17y + 4z = 0$

$$AX = B$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix}_{4\times3} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{4\times1}$$

$$(A:B) = \begin{bmatrix} 1 & 3 & 2 & : & 0 \\ 2 & -1 & 3 & : & 0 \\ 3 & -5 & 4 & : & 0 \\ 1 & 17 & 4 & : & 0 \end{bmatrix}_{4\times4}$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$
$R_4 \to R_4 - R_1$

$2-2$
$-1-6$
$3-4$
$0-0$

$3-3$ $\quad$ $1-1$
$-5-9$ $\quad$ $17-3$
$4-6$ $\quad$ $4-2$
$0-0$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$
$R_4 \to R_4 + 2R_2$

$-14+14$
$-2+2=0$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore \rho(A:B) = 2$
$\rho(A) = 2$
$\rho(A) = \rho(A:B) = 2 < n$ (no. of unknowns)
$\therefore$ Consistent & infinite soln

$x + 3y + 2z = 0$
$-7y - z = 0$

let ($z = k$)

$-7y - k = 0$

($y = k/7$)

$x + \frac{3}{7}k + 2k = 0$
$7x + 3k + 14k = 0$
$7x + 17k = 0$
($x = -\frac{17k}{7}$)

for $k = 7$
$x = -17$
$y = 1$
$z = 7$

Rank of Matrix :- 6

· by Row echelon form

Only Row operation are allowed

Reduce Matrix to upper Triangular matrix

all are zeros → P.D.

Rank ≤ no. of rows

ρ (Rank) = no. of non zeros rows.

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}_{4\times 4}$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$R_4 \to R_4 - 6R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix} \quad R_1 \leftrightarrow R_2$$

$$\begin{bmatrix} 0 & 0 & -3 & 2 \\ 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_3 \to R_3 + 2R_2$

$R_4 \to R_4 + 2R_2$

$$\begin{bmatrix} 0 & 0 & -3 & 2 \\ 1 & 2 & 3 & 0 \\ 0 & 0 & -2 & 3 \\ 0 & 0 & -5 & 5 \end{bmatrix} \quad R_1 \leftrightarrow R_2$$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -2 & 3 \\ 0 & 0 & -5 & 5 \end{bmatrix}_{4\times 4}$$

$R_4 \to 2R_4 - 5R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -2 & 3 \\ 0 & 0 & 0 & -5 \end{bmatrix}_{4\times 4}$$

no. of non zero rows = $\rho(A) = 4$

(3) $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

Characteristic Eqn:- $\lambda^2 - \underset{\substack{\downarrow \\ \text{Sum of diagonals}}}{S_1}\lambda + \underset{\substack{\downarrow \\ \text{determinant}}}{S_2} = 0$

$4-1=3$

$\therefore \lambda^2 - 4\lambda + 3 = 0$

$\lambda^2 - 3\lambda - \lambda + 3 = 0$

$\lambda(\lambda-3) - 1(\lambda-3) = 0$

$(\lambda-1)(\lambda-3) = 0$

$\lambda = 1, 3$

$\therefore \lambda = 1$

$(A-\lambda) = \begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix}$

$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\frac{x_1}{1} = \frac{+x_2}{+1}$

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\lambda = 3$

$= \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\frac{x_1}{-1} = \frac{+x_2}{+1}$

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

Ans We know if λ is the eigen value of A then $1/\lambda$ is eigen value of A^{-1}

$\therefore$ $\lambda = 1, 1/3$ for A^{-1}

$$A^{-1} = \frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$$

$\lambda = 1$

$$\begin{bmatrix} 2/3-\lambda & 1/3-\lambda \\ 1/3-\lambda & 2/3-\lambda \end{bmatrix}$$

$2/3-1$
$-1/3$

$$\begin{bmatrix} -1/3 & -2/3 \\ -2/3 & -1/3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{-1/3} = \frac{+x_2}{+1/3}$$

$$\left(\frac{x_1}{x_2}\right) = \left(\frac{-1/3}{1/3}\right) = \left(\frac{-1}{1}\right)$$

$2/3-1/3$
$1/3$

$$\lambda = 1/3 \quad \begin{bmatrix} 1/3 & 0 \\ 0 & 1/3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{1/3} = \frac{-x_2}{0}$$

$$\frac{x_1}{x_2} = \begin{pmatrix} 1/3 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

for $A + 4I = \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}$

$36-1=35$

$$\lambda^2 - 12\lambda + 35 = 0$$

$$\lambda^2 - 7\lambda - 5\lambda + 35 = 0$$

$$\lambda(\lambda-7) - 5(\lambda-7) = 0$$

$(\lambda-5)(\lambda-7)=0$

$\lambda = 5, 7$

We can also do it by

For A = $\lambda = 1, 3$

For $A+4I$ = $\lambda = 1+4, 3+4$

$5, 7$

$\lambda = 5$

$$\begin{bmatrix} 6-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{1} = \frac{+x_2}{+1}$$

$$\begin{matrix} x_1 \\ x_2 \end{matrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$\lambda = 7$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{-1} = \frac{+x_2}{+1}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

SEMESTER

Assignment :-

Q1) Find rank?

$$A \Rightarrow \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$
$R_3 \rightarrow R_3 - 3R_1$
$R_4 \rightarrow R_4 - 6R_1$

$$A \Rightarrow \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$$R_2 \longleftrightarrow R_3$$

$$A \Rightarrow \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$$R_2 \Rightarrow -\frac{R_2}{4}$$

$$A \Rightarrow \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$$R_1 = R_1 - 2R_2$$

$$A \Rightarrow \begin{bmatrix} 1 & 0 & -1 & 3/2 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$$R_4 = R_4 + 4R_2$$

$$A \Rightarrow \begin{bmatrix} 1 & 0 & -1 & 3/2 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$$R_3 = -\frac{R_3}{3}$$

$$A \Rightarrow \begin{bmatrix} 1 & 0 & -1 & 3/2 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$$R_1 = R_1 + R_3$$

$$A \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 5/6 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$$R_2 = R_2 - 2R_3$$

$$A \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 5/6 \\ 0 & 1 & 0 & 7/12 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$$R_4 = R_4 + 3R_3$$

$$A \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 5/6 \\ 0 & 1 & 0 & 7/12 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank Of Matrix is "3"

Q27 Let W be the vector space of all symmetric 2×2 matrices and let $T: W \to P_2$ be the Linear Transformation defined by $T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b)+(b-c)x+(c-a)x^2$

Find rank & nullity of T.

Since the ~~dimension~~ maximum degree of polynomial $T \Rightarrow 2$.
So $\dim(P_2) \Rightarrow 3$

Kernel

So a subset of Kernel T is $T(A) = 0$

$$(a-b) + (b-c)x + (c-a)x^2 \Rightarrow 0$$

$$\hookrightarrow \boxed{a = b = c = t \text{ (let)}}$$

New matrix $\begin{bmatrix} t & t \\ t & d \end{bmatrix}$

dimension of Kernel is 1, because there's only one independent parameter as 't'.

Acc. to rank nullity Theorem $\Rightarrow$

$$\text{rank}(T) + \text{nullity}(T) \Rightarrow \dim(W)$$

$$\text{rank}(T) + 1 = 4$$

So, rank of T is 3, & nullity is 1.

Q3) Let $A \Rightarrow \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$. Find eigen-values and eigen vectors of A^{-1} & $A + 4I$.

$$A - \lambda I \Rightarrow 0$$

$$\det \begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix} = 0$$

$$(2-\lambda)^2 - 1 = 0$$
$$(2-\lambda) \Rightarrow \pm 1$$

$$\lambda \Rightarrow 1, 3$$

for $\lambda = 1$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x - y = 0$$
$$x \Rightarrow y \quad , \text{let } x = t, \ y \Rightarrow t$$

Eigen vector v_1, $t \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ Ans for $\lambda = 3$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-x - y = 0$

$x = -y$, let $x = t$

$y = -t$

So, eigen value $V_2 \Rightarrow t \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

Now, find for A^{-1}

→ eigen values of A^{-1} will be $\frac{1}{\lambda_1}$ & $\frac{1}{\lambda_2}$ $\Rightarrow 1, \frac{1}{3}$

→ and eigen vectors are same as of A.

$V_1 \Rightarrow t \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $V_2 = t \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

Now, for $A + 4I$

→ eigen values for $A + 4I$ will be $\lambda_1 + 4$, $\lambda_2 + 4 \Rightarrow 5, 7$

→ and eigen vectors are same as as of A

$$V_1 = t\begin{bmatrix}1\\1\end{bmatrix} \qquad V_2 = t\begin{bmatrix}-1\\1\end{bmatrix}$$

Q4) Solve by Gauss-Siedel Method (Take three Iterations)

$$3x - 0.1y - 0.2z \Rightarrow 7.85$$
$$0.1x - 7y - 0.3z \Rightarrow -19.3$$
$$0.3x - 0.2y + 10z \Rightarrow 71.4$$

with initial values $x(0) \Rightarrow 0, y(0) = 0, z(0) = 0$.

eqn $\Rightarrow$ $x^{k+1} \Rightarrow \dfrac{7.85 + 0.1y^k + 0.2z^k}{3}$

$$y^{k+1} \Rightarrow \frac{-19.3 - 0.1x^{k+1} - 0.3z^k}{-7}$$

$$z^{k+1} \Rightarrow \frac{71.4 - 0.3x^{k+1} + 0.2y^{k+1}}{10}$$

We know $x(0) = 0$, $y(0) = 0$, $z(0) = 0$

Iterat^n-1 :→

$x(1) \Rightarrow \frac{7.85 + 0.1(0) + 0.2(0)}{3} \Rightarrow 2.6167$

$y(1) \Rightarrow \frac{-19.3 - 0.1(2.6167) - 0.3(0)}{-7} \Rightarrow 2.7956$

$z(1) \Rightarrow \frac{71.4 - 0.3(2.6167) - 0.2(2.7956)}{10} \Rightarrow 7.1373$

Iterat^n-2 :→

$x(2) \Rightarrow \frac{7.85 + 0.1(2.7956) + 0.2(7.1373)}{3} = 3$

$y(2) \Rightarrow \frac{-19.3 - 0.1(3) - 0.3(7.1373)}{-7} = 3$

$z(2) \Rightarrow \frac{71.4 - 0.3(3) - 0.2(3)}{10} = 3$

Iterat^n-3 :→

$x(3) = (7.85 + 0.1(3) + 0.2(3))/3 \rightarrow 3$

$y(3) \rightarrow (-19.3 - 0.1(3) - 0.3(3))/-7 \Rightarrow 3$

$z(3) \Rightarrow (71.4 - 0.3(3) + 0.2(3))/10 \rightarrow 3$

After, three iterat^n $x, y, z \approx 3$

So value of $x = 3$, $y = 3$ & $z \Rightarrow 3$

Q5) Define consistent or inconsistent system of equations.

Consistent
(atleast one solution)

↓ Dependent
(infinite sol^n)

↓ Independent
(unique sol^n)

Inconsistent
(No solution)

$$A \Rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$, $R_4 \to R_4 - R_1$

$$\Rightarrow \begin{array}{cccc} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{array}$$

$R_3 \to R_3 - 2R_2$, $R_4 \to R_4 + 2R_2$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) \Rightarrow 2$
$\rho(A:B) \Rightarrow 2$
$n \Rightarrow 3$

$\rho(A) = \rho(A:B) \neq n$

Consistent, but infinite solⁿ.

Q6) Determine whether fxn $T: P_2 \to P_2$ is linear transformatⁿ or not.

$$T(a+bx+cx^2) = (a+1)+(b+1)x+(c+1)x^2$$

1.) Additive

$$T(u+v) \Rightarrow T(u)+T(v)$$
$$u = a_1 + b_1 x + c_1 x^2$$
$$v = a_2 + b_2 x + c_2 x^2$$

$$T(u+v) \Rightarrow T\left((a_1+a_2)+(b_1+b_2)x+(c_1+c_2)x^2\right)$$

$$\Rightarrow (a_1+a_2+1)+(b_1+b_2+1)x + (c_1+c_2+1)x^2$$

$$\Rightarrow (a_1+1)+(b_1+1)x+(c_1+1)x^2+(a_2+1) + (b_2+1)x + (c_2+1)x^2$$

$$\Rightarrow T(u) + T(v)$$

Hence Proved

2.) Homogenity

$$T(kU) \Rightarrow kT(u)$$

$$T(k(a+bx+cx^2))$$
$$T(ka + kbx + kcx^2)$$

$$\Rightarrow (ka+kb+kc+1) + (ka+kb+kc+1)x + (ka+kb+kc+1)x^2$$

$$\Rightarrow k(a+1) + k(b+1)x + k(c+1)x^2$$
$$\Rightarrow kT(u)$$

Hence Proved

Hence, It's a Linear Transformatⁿ

Q7) Determine whether set $S \Rightarrow \{(1,2,3), (3,1,0), (-2,1,3)\}$ is a basis of $V_3(R)$. In case S is not a basis, determine the dimⁿ & basis of subspace spanned by S.

$$a(1,2,3) + b(3,1,0) + c(-2,1,3) = (0,0,0)$$

$$a + 3b - 2c = 0$$
$$2a + b + c = 0$$
$$3a + 3c = 0$$

$$c \Rightarrow -a \quad , b \Rightarrow -a$$

only one sol^n is possible is $a = b = c = 0$. So linearly Independent.

Since dim^n of $V_3(R)$ is 3 and S also contains 3 vector. And $S \to LI$, then it spans $V_3(R)$ making it a basis for $V_3(R)$.

Q8) Using Jacobi's method (perform 3 iterat^{n}s)

$$3x - 6y + 2z \Rightarrow 23$$
$$-4x + y - z \Rightarrow -15$$
$$x - 3y + 7z \Rightarrow 16$$

$x_0 \Rightarrow 1,\ y_0 \Rightarrow 1,\ z_0 \Rightarrow 1$

$\Rightarrow$ first eq^n $x = \frac{1}{3}(23 + 6y - 2z)$

second eq^n $y \Rightarrow (-15 + 4x + z)$

third eq^n $z \Rightarrow \frac{1}{7}(16 - x + 3y)$

$x(0) = 1,\ y(0) = 1,\ z(0) = 1$

Iterat^n-1 :-

$x(1) \to (23 + 6 - 2)/3 \Rightarrow 9$

$y(1) \to (-15 + 4 + 1) \Rightarrow -10$

$z(1) \Rightarrow (16 - 1 + 3)/7 \Rightarrow 18/7$

Q9b

Affine Transformatⁿ

Rotatⁿ

Suppose we have a 2-D image represented as grid on pixels. We can use AT matrix to rotate around centre.

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\longrightarrow$ rotatⁿ of image by θ

to rotate it around centre.

1.) Translatⁿ to origin

translate the image, so that it centre alligns with origin.

2.) Rotatⁿ

Apply rotatⁿ matrix.

3.) Translatⁿ Back

translate it back with its original posⁿ by adding coordinates of centre.

10) Brief Description of Linear Transformatⁿ for computer vision for rotating 2-D image.

⇒ Linear Transformation for rotatⁿ 2D images involves applying a rotatⁿ matrix to each pixel coordinate. This matrix rotates points counterclockwise by an angle θ around the origin. It preserves geometric properties like parallelism and distances. Rotatⁿ is essential in tasks like image allignment and object detection in computer vision.

× ——— × COMPLETE ——— × ——— *