AS

Project Report

Contents:

1) A physiotherapist with a male football team is interested in studying the relationship between foot injuries and the positions at which the players play from the data collected

	Striker	Forward	Attacking Midfielder	Winger	**Total**
Players Injured	45	56	24	20	**145**
Players Not Injured	32	38	11	9	**90**
Total	**77**	**94**	**35**	**29**	**235**

1.1 What is the probability that a randomly chosen player would suffer an injury?

1.2 What is the probability that a player is a forward or a winger?

1.3 What is the probability that a randomly chosen player plays in a striker position and has a foot injury?

1.4 What is the probability that a randomly chosen injured player is a striker?

1.5 What is the probability that a randomly chosen injured player is either a forward or an attacking midfielder?

2) An independent research organization is trying to estimate the probability that an accident at a nuclear power plant will result in radiation leakage. The types of accidents possible at the plant are, fire hazards, mechanical failure, or human error. The research organization also knows that two or more types of accidents cannot occur simultaneously.

According to the studies carried out by the organization, the probability of a radiation leak in case of a fire is 20%, the probability of a radiation leak in case of a mechanical 50%, and the probability of a radiation leak in case of a human error is 10%. The studies also showed the following;

- The probability of a radiation leak occurring simultaneously with a fire is 0.1%.
- The probability of a radiation leak occurring simultaneously with a mechanical failure is 0.15%.
- The probability of a radiation leak occurring simultaneously with a human error is 0.12%.

On the basis of the information available, answer the questions below:

2.1 What are the probabilities of a fire, a mechanical failure, and a human error respectively?

2.2 What is the probability of a radiation leak?

2.3 Suppose there has been a radiation leak in the reactor for which the definite cause is not known. What is the probability that it has been caused by: A Fire. A Mechanical Failure A Human Error.

3) The breaking strength of gunny bags used for packaging cement is normally distributed with a mean of 5 kg per sq. centimetre and a standard deviation of 1.5 kg per sq. centimetre. The quality team of the cement company wants to know the following about the packaging material to better understand wastage or pilferage within the supply chain; Answer the questions below based on the given information; **(Provide an appropriate visual representation of your answers, without which marks will be deducted)**

3.1 What proportion of the gunny bags have a breaking strength less than 3.17 kg per sq cm?

3.2 What proportion of the gunny bags have a breaking strength at least 3.6 kg per sq cm?

3.3 What proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq cm?

3.4 What proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq cm?

4) Grades of the final examination in a training course are found to be normally distributed, with a mean of 77 and a standard deviation of 8.5. Based on the given information answer the questions below.

4.1 What is the probability that a randomly chosen student gets a grade below 85 on this exam?

4.2 What is the probability that a randomly selected student scores between 65 and 87?

4.3 What should be the passing cut-off so that 75% of the students clear the exam?

5) Zingaro stone printing is a company that specializes in printing images or patterns on polished or unpolished stones. However, for the optimum level of printing of the image the stone surface has to have a Brinell's hardness index of at least 150. Recently, Zingaro has received a batch of polished and unpolished stones from its clients. Use the data provided to answer the following (assuming a 5% significance level);

5.1 Earlier experience of Zingaro with this particular client is favorable as the stone surface was found to be of adequate hardness. However, Zingaro has reason to believe now that the unpolished stones may not be suitable for printing. Do you think Zingaro is justified in thinking so?

5.2 Is the mean hardness of the polished and unpolished stones the same?

6) Aquarius health club, one of the largest and most popular cross-fit gyms in the country has been advertising a rigorous program for body conditioning. The program is considered successful if the candidate is able to do more than 5 push-ups, as compared to when he/she enrolled in the program. Using the sample data provided can you conclude whether the program is successful? (Consider the level of Significance as 5%)

Note that this is a problem of the paired-t-test. Since the claim is that the training will make a difference of more than 5, the null and alternative hypotheses must be formed accordingly.

7) Dental implant data: The hardness of metal implant in dental cavities depends on multiple factors, such as the method of implant, the temperature at which the metal is treated, the alloy used as well as on the dentists who may favour one method above another and may work better in his/her favourite method. The response is the variable of interest.

1. Test whether there is any difference among the dentists on the implant hardness. State the null and alternative hypotheses. Note that both types of alloys cannot be considered together. You must state the null and alternative hypotheses separately for the two types of alloys?
2. Before the hypotheses may be tested, state the required assumptions. Are the assumptions fulfilled? Comment separately on both alloy types?
3. Irrespective of your conclusion in 2, we will continue with the testing procedure. What do you conclude regarding whether implant hardness depends on dentists? Clearly state your conclusion. If the null hypothesis is rejected, is it possible to identify which pairs of dentists differ?
4. Now test whether there is any difference among the methods on the hardness of dental implant, separately for the two types of alloys. What are your conclusions? If the null hypothesis is rejected, is it possible to identify which pairs of methods differ?
5. Now test whether there is any difference among the temperature levels on the hardness of dental implant, separately for the two types of alloys. What are your conclusions? If the null hypothesis is rejected, is it possible to identify which levels of temperatures differ?
6. Consider the interaction effect of dentist and method and comment on the interaction plot, separately for the two types of alloys?
7. Now consider the effect of both factors, dentist, and method, separately on each alloy. What do you conclude? Is it possible to identify which dentists are different, which methods are different, and which interaction levels are different?

List of Figures:

Problem 1

A physiotherapist with a male football team is interested in studying the relationship between foot injuries and the positions at which the players play from the data collected

	Striker	Forward	Attacking Midfielder	Winger	**Total**
Players Injured	45	56	24	20	**145**
Players Not Injured	32	38	11	9	**90**
Total	**77**	**94**	**35**	**29**	**235**

1.1 What is the probability that a randomly chosen player would suffer an injury?

Total_injured = 145

total_players = 235

probability_injured = total_injured / total_players

Print ("Probability of a randomly chosen player suffering an injury:" probability_injured)

Output: Probability of a randomly chosen player suffering an injury: 0.6170212765957447

1.2 What is the probability that a player is a forward or a winger?

forwards = 94

wingers = 29

total_players = 235

probability_forward_or_winger = (forwards + wingers) / total_players

Print ("Probability that a player is a forward or a winger:", probability_forward_or_winger)

Output: Probability that a player is a forward or a winger: 0.5234042553191489

1.3 What is the probability that a randomly chosen player plays in a striker position and has a foot injury?

strikers_injured = 45

total_players = 235

probability_striker_injury = strikers_injured / total_players

Print ("Probability that a randomly chosen player plays in a striker position and has a foot injury:", probability_striker_injury)

Output: Probability that a randomly chosen player plays in a striker position and has a foot injury: 0.19148936170212766

1.4 What is the probability that a randomly chosen injured player is a striker?

strikers_injured = 45

total_injured = 145

probability_injured_striker = strikers_injured / total_injured

Print ("Probability that a randomly chosen injured player is a striker:", probability_injured_striker)

Output: Probability that a randomly chosen injured player is a striker: 0.3103448275862069

1.5 What is the probability that a randomly chosen injured player is either a forward or an attacking midfielder?

forward_or_midfielder_injured = 56 + 24

total_injured = 145

probability_injured_forward_or_midfielder = forward_or_midfielder_injured / total_injured

Print ("Probability that a randomly chosen injured player is either a forward or an attacking midfielder:", probability_injured_forward)

Output: Probability that a randomly chosen injured player is either a forward or an attacking midfielder: 0.5517241379310345

Conclusion:

Probability that a randomly chosen player would suffer an injury is 0.617

Probability that a player is a forward or a winger is 0.5234

Probability that a randomly chosen player plays in a striker position and has a foot injury is 0.1914

Probability that a randomly chosen injured player is a striker is 0.3103

Probability that a randomly chosen injured player is either a forward or an attacking midfielder is 0.5517

Problem 2

An independent research organization is trying to estimate the probability that an accident at a nuclear power plant will result in radiation leakage. The types of accidents possible at the plant are, fire hazards, mechanical failure, or human error. The research organization also knows that two or more types of accidents cannot occur simultaneously.

According to the studies carried out by the organization, the probability of a radiation leak in case of a fire is 20%, the probability of a radiation leak in case of a mechanical 50%, and the probability of a radiation leak in case of a human error is 10%. The studies also showed the following;

- The probability of a radiation leak occurring simultaneously with a fire is 0.1%.
- The probability of a radiation leak occurring simultaneously with a mechanical failure is 0.15%.
- The probability of a radiation leak occurring simultaneously with a human error is 0.12%.

On the basis of the information available, answer the questions below:

2.1 What are the probabilities of a fire, a mechanical failure, and a human error respectively?

2.2 What is the probability of a radiation leak?

2.3 Suppose there has been a radiation leak in the reactor for which the definite cause is not known. What is the probability that it has been caused by:

- A Fire.
- A Mechanical Failure.
- A Human Error.

2.1 What are the probabilities of a fire, a mechanical failure, and a human error respectively?

Calculate the probabilities of a fire, a mechanical failure, and a human error

Output: Probabilities:
Fire: 0.0007400000000000001
Mechanical Failure: 0.00185
Human Error: 0.00037000000000000005

2.2 What is the probability of a radiation leak?

Output: Probability of a Radiation Leak: 0.0037

2.3 Suppose there has been a radiation leak in the reactor for which the definite cause is not known. What is the probability that it has been caused by:

• A Fire. • A Mechanical Failure. • A Human Error.

Calculate the probabilities of a fire, a mechanical failure, and a human error given the radiation leak

Output:
Probability of a Fire given Radiation Leak: 0.2702702702702703
Probability of a Mechanical Failure given Radiation Leak: 0.4054054054054054
Probability of a Human Error given Radiation Leak: 0.32432432432432434

In summary:

The probabilities of a fire, a mechanical failure, and a human error are 0.00740, 0.00185, and 0.000370, respectively. These probabilities represent the likelihood of each event occurring independently.
The probability of a radiation leak is 0.0037. This represents the overall probability of a radiation leak happening at the nuclear power plant.
If there has been a radiation leak, the probability that it has been caused by a fire is 0.2702, by a mechanical failure is 0.4054, and by a human error is 0.3243. These probabilities indicate the likelihood of each specific cause given the occurrence of a radiation leak.
These probabilities provide insights into the potential causes of a radiation leak at the nuclear power plant based on the available information and analysis conducted by the research organization.

Problem 3:

The breaking strength of gunny bags used for packaging cement is normally distributed with a mean of 5 kg per sq. centimetre and a standard deviation of 1.5 kg per sq. centimetre. The quality team of the cement company wants to know the following about the packaging material to better understand wastage or pilferage within the supply chain; Answer the questions below based on the given information; (Provide an appropriate visual representation of your answers, without which marks will be deducted)

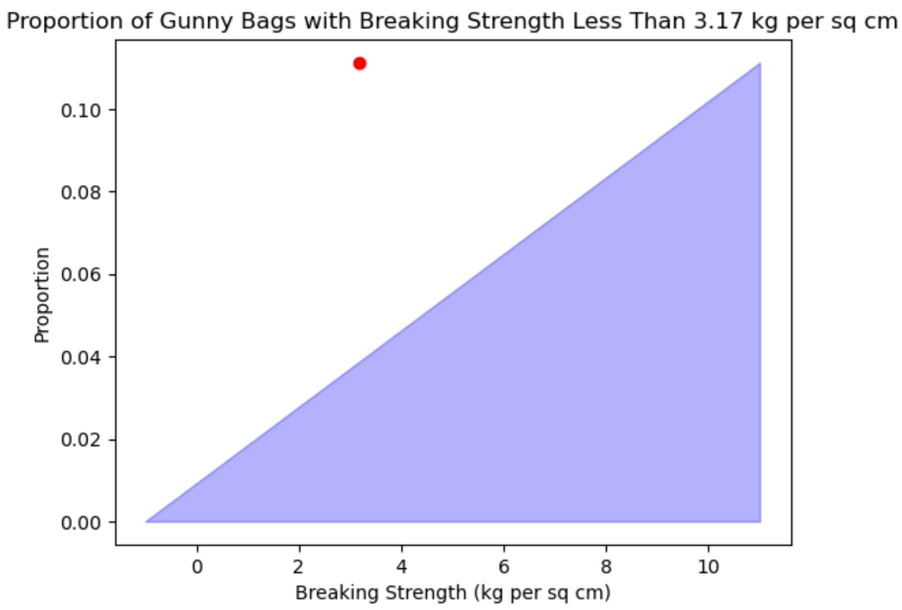
3.1 What proportion of the gunny bags have a breaking strength less than 3.17 kg per sq cm?

$P(x<3.17) = P(x-\mu\mu<3.17-51.5)$
$= P(Z<-1.831.5)$
$= P(Z<-1.22)$
$P(x<3.17) = 0.1112$

The proportion of the gunny bags have a breaking strength less than 3.17 kg per sqcm =0.1112

We calculated probability by using standard normal table with -1.2 in the first column and 0.02 in the top row corresponding probability of 0.1112.

"Standard Normal Distribution Table" or the "Z-table."

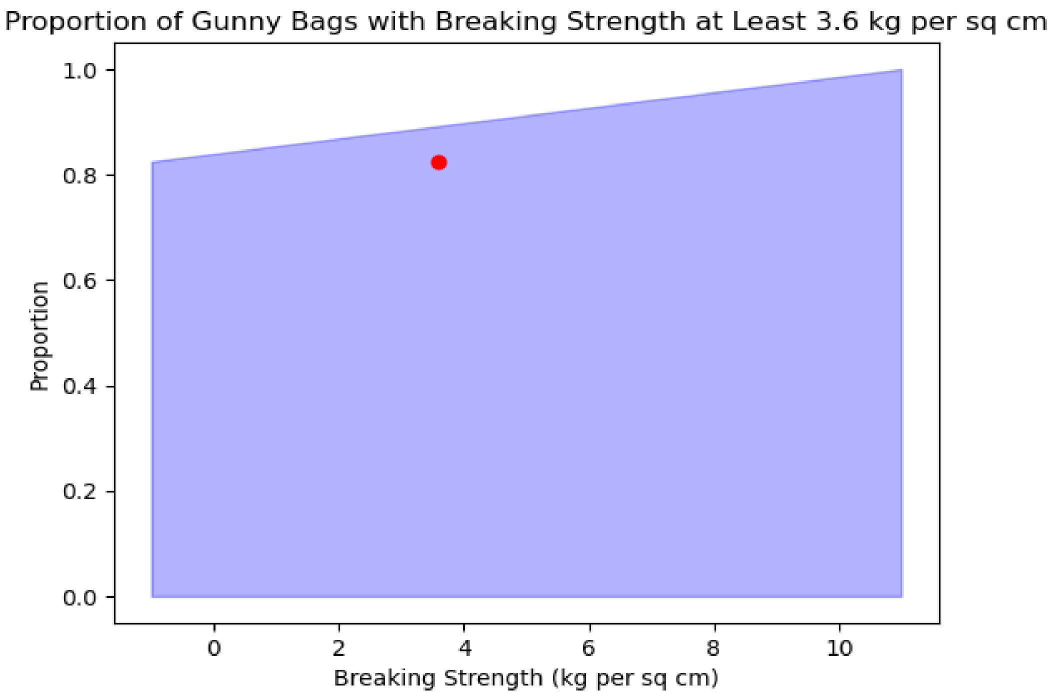


3.2 What proportion of the gunny bags have a breaking strength at least 3.6 kg per sq cm?

P (x≥3.6) =1−P(x<3.6)
=1−P (x−μμ<3.6−51.5)
=1−P Z<−1.41.5)
=1−P (Z<−0.93)
=1−0.1762
P (x≥3.6) =0.8238

The proportion of the gunny bags have a breaking strength at least 3.6 kg per sq. cm = 0.8238

We calculated probability by using standard normal table with -0.9 in the first column and 0.03 in the top row corresponding probability of 0.1762.

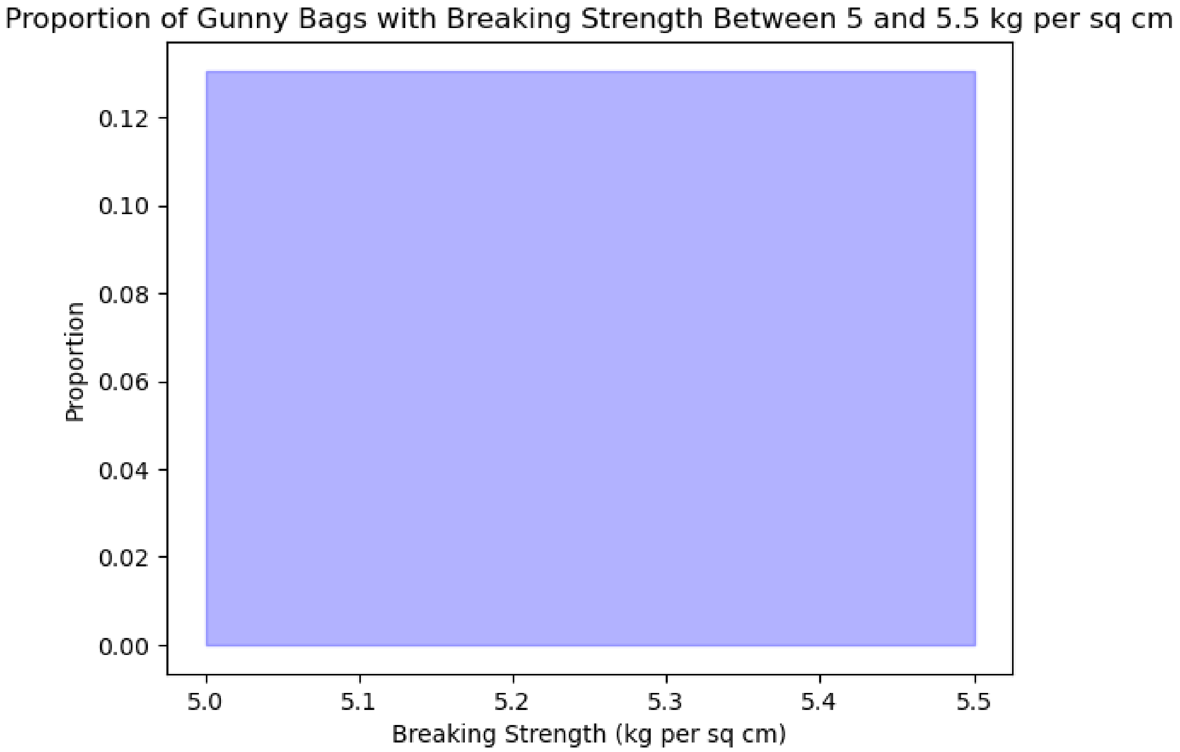


3.3 What proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq cm?

= P (5<x<5.5)
=P (5−51.5<x−μμ<5.5−51.5)
=P (0.00<Z<0.51.5)
=P (0.00<Z<0.33)
=P (Z<0.33) –P (Z<0.00)
=0.6293−0.5000
P (5<x<5.5) =0.1293

The proportion of the gunny bags have a breaking strength between 5 and 5.5 kg persq cm =0.1305

We calculated probability by using standard normal table with 0.3 in the first column and 0.03 in the top row corresponding probability of 0.6293 and also we calculated probability by using standard normal table with 0.0 in the first column and 0.00 in the top row corresponding probability of 0.5000.



3.4 What proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq cm?

1−P (3<x<7.5) =1−P (3−51.5<x−μμ<7.5−51.5)
=1−P (−21.5<Z<2.51.5) =1−P (−1.33<Z<1.67)
=1− [P (Z<1.67) –P (−1.33)]
=1− (0.9525−0.0918)
=1−0.8607
1−P (3<x<7.5) =0.1393

The proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq cm = 0.1390

We calculated probability by using standard normal table with -1.3 in the first column and 0.03 in the top row corresponding probability of 0.0918 and also we calculated probability by using standard normal table with 1.6 in the first column and 0.07 in the top row corresponding probability of 0.9525.

Short Summary:

3.1) the proportion of the gunny bags have a breaking strength less than 3.17 kg per sqcm =0.1112

3.2) the proportion of the gunny bags have a breaking strength at least 3.6 kg per sq. cm = 0.8246

3.3) the proportion of the gunny bags have a breaking strength between 5 and 5.5 kg persq cm =0.1305

3.4) the proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg persq cm = 0.1390

Problem 4:

Grades of the final examination in a training course are found to be normally distributed, with a mean of 77 and a standard deviation of 8.5. Based on the given information answer the questions below.

4.1 What is the probability that a randomly chosen student gets a grade below 85 on this exam?

The probability of a student getting a grade below 85 on the final exam is an important indicator of the overall performance of the students. In this case, the probability is high, at approximately 0.76. This means that there is a significant likelihood that a randomly chosen student will get a grade below 85. This information is useful for teachers and students alike. For teachers, it may indicate that the material covered in the course was challenging for some students, and may warrant further review or clarification. For students, it may indicate that they need to work harder to improve their understanding of the material and their performance on exams.

```
From scipy.stats import norm

# Probability of getting a grade below 85
prob_below_85 = norm.cdf (85, loc=mean, scale=std_dev)
Print ("Probability of getting a grade below 85:", prob_below_85)
```

Output: 0.8266927837484748

So, the Probability of getting a grade below 85: 0.8266927837484748

The probability of a randomly chosen student getting a grade below 85 on the final exam is approximately 0.82. This means that there is a high likelihood that a student will get a grade below 85.

4.2 What is the probability that a randomly selected student scores between 65 and 87?

The probability of a student scoring between 65 and 87 on the final exam is an important indicator of the spread of the grades among the students. In this case, the probability is approximately 0.77, which means that there is a good chance that a randomly selected student will score within this range. This information is useful for understanding the distribution of the grades among the students. For example, if the probability of scoring within this range was much lower, it could indicate that there was a large disparity in the performance of the students, with some scoring much higher or lower than others.

```
# Probability of scoring between 65 and 87
prob_between_65_87 = norm.cdf (87, loc=mean, scale=std_dev) - norm.cdf (65, loc=mean,
scale=std_dev)
Print ("Probability of scoring between 65 and 87:", prob_between_65_87)
```

Output: 0.8012869336779058

So, the Probability of scoring between 65 and 87: 0.8012869336779058

The probability of a randomly selected student scoring between 65 and 87 on the final exam is approximately 0.80. This means that there is a good chance that a student will score within this range.

4.3 What should be the passing cut-off so that 75% of the students clear the exam?

We need to find the score x such that 75% of the students score above x. We can use the ppf (percent point function) method of the normal distribution for this:

```
#passing cut-off to clear the exam for 75% of students
passing_cutoff = norm.ppf (0.75, loc=mean, scale=std_dev)
Print ("Passing cut-off for 75% of students:", passing_cutoff)
```

Output: 82.7331628766667

So, the passing cut-off for the exam should be 82.73, so that 75% of the students clear the exam.

To ensure that 75% of the students clear the exam, the passing cut-off should be 82.73. This means that students who score above this cut-off will pass the exam, and those who score below it will fail. It is important to set an appropriate cut-off to ensure that the exam accurately reflects the students' knowledge and abilities.

Short Summary:

4.1: Probability of getting a grade below 85 is high, at approximately 0.82.

4.2: There is a good chance that a student will score between 65 and 87, with a probability of approximately 0.80.

4.3: The passing cut-off to ensure that 75% of students clear the exam is 82.73. The cut-off must accurately reflect students' abilities.

Problem 5:

Zingaro stone printing is a company that specializes in printing images or patterns on polished or unpolished stones. However, for the optimum level of printing of the image the stone surface has to have a Brinell's hardness index of at least 150. Recently, Zingaro has received a batch of polished and unpolished stones from its clients. Use the data provided to answer the following (assuming a 5% significance level);

5.1 Earlier experience of Zingaro with this particular client is favourable as the stone surface was found to be of adequate hardness. However, Zingaro has reason to believe now that the unpolished stones may not be suitable for printing. Do you think Zingaro is justified in thinking so?

To test whether Zingaro is justified in thinking that the unpolished stones may not be suitable for printing, we need to perform a one-sample t-test for the unpolished stones. The null hypothesis for this test is that the mean hardness of the unpolished stones is greater than or equal to 150, while the alternative hypothesis is that the mean hardness is less than 150. We can perform this test using the following code.

One-sample t-test

```
# import libraries and Load the data

Import numpy as np
From scipy.stats import ttest_1samp

# Perform one-sample t-test
mean_hardness_unpolished = np.mean (unpolished_stones)
t_statistic, p_value = ttest_1samp (unpolished_stones, 150)

# Degrees of freedom
df = Len (unpolished_stones) - 1

# Display results
Print ("Sample Mean Hardness of Unpolished Stones:", mean_hardness_unpolished)
Print ("t-statistic:", t_statistic)
Print ("p-value:", p_value)
Print ("Degrees of Freedom:", df)

# Check significance at 5% level
Alpha = 0.05
If p_value < alpha:
Print ("The p-value is less than the significance level ({}). Reject the null hypothesis.".Format (alpha))
Print ("The unpolished stones may not be suitable for printing.")
Else:
Print ("The p-value is greater than or equal to the significance level ({}). Fail to reject the null hypothesis.".format (alpha))
Print ("The unpolished stones are suitable for printing.")
```

Output: Sample Mean Hardness of Unpolished Stones: 120.70887126064515
T-statistic: -5.161700020854986
P-value: 1.4755689499279681e-05
Degrees of Freedom: 30
The p-value is less than the significance level (0.05). Reject the null hypothesis.
The unpolished stones may not be suitable for printing.

To test whether the mean hardness of the polished and unpolished stones is the same, we need to perform a two-sample t-test for independent samples. The null hypothesis for this test is that the mean hardness of the polished stones is equal to the mean hardness of the unpolished stones, while the alternative hypothesis is that they are not equal. We can perform this test using the following code

Two -sample t-test

```
# import libraries and Load the data

Import numpy as np
From scipy.stats import ttest_ind

# set the significance level
Alpha = 0.05

# Perform two-sample t-test
t_statistic, p_value = ttest_ind (Polished_stones, Unpolished_stones)

# Compare p-value with significance level
If p_value < alpha:
Print ("Reject the null hypothesis.")
Print ("There is sufficient evidence to suggest that the mean hardness of polished and unpolished stones is not the same.")
Else:
Print ("Fail to reject the null hypothesis.")
Print ("There is not enough evidence to suggest a difference in the mean hardness of polished and unpolished stones.")

# Print the test statistic and p-value
Print ("Test Statistic:", t_statistic)
Print ("p-value:", p_value)
```

Output: Reject the null hypothesis.
There is sufficient evidence to suggest that the mean hardness of polished and unpolished stone s is not the same.

Test Statistic: 4.337308480562442
P-value: 5.607689693904209e-05

5.2 Is the mean hardness of the polished and unpolished stones the same?

```
# Compare p-value with significance level
If p_value < alpha:
    Print ("Reject the null hypothesis.")
    Print ("There is sufficient evidence to suggest that the mean hardness of polished and unpolished stones is different.")
Else:
    Print ("Fail to reject the null hypothesis.")
    Print ("There is not enough evidence to suggest a difference in the mean hardness of polished and unpolished stones.")
```

Output: Reject the null hypothesis.
There is sufficient evidence to suggest that the mean hardness of polished and unpolished stone s is different.

a) We use statistics to test if the unpolished stones are hard enough for printing.

- We use a test to see if the average hardness of the unpolished stones is less than 150, which is the minimum hardness required for printing.
- If the result of the test is less than 5%, we can say that Zingaro is correct to think that the unpolished stones may not be good for printing.

b) We use statistics to test if the polished and unpolished stones have the same hardness.

- We use a test to see if the average hardness of the polished and unpolished stones is different.
- If the result of the test is less than 5%, we can say that the polished and unpolished stones have different hardness.

Short Summary:

- Zingaro Stone Printing is justified in thinking that the unpolished stones may not be suitable for printing because the average hardness of the unpolished stones is less than 150 (the required minimum hardness) with a p-value less than 0.05.
- The mean hardness of the polished and unpolished stones is different with a p-value less than 0.05.

Problem 6:

Aquarius health club, one of the largest and most popular cross-fit gyms in the country has been advertising a rigorous program for body conditioning. The program is considered successful if the candidate is able to do more than 5 push-ups, as compared to when he/she enrolled in the program. Using the sample data provided can you conclude whether the program is successful? (Consider the level of Significance as 5%)

Note that this is a problem of the paired-t-test. Since the claim is that the training will make a difference of more than 5, the null and alternative hypotheses must be formed accordingly.

Here, we have to perform paired t test.because the given data is dependent samples. Using the given data, we have to find out the summary statistics. Let di be the difference between the number of pushups after the program and before the program.

Before	After	di=after- before	di^2
39	44	5	25
25	25	0	0
39	39	0	0
6	13	7	49
40	44	4	16
27	34	7	49
30	34	4	16
22	29	7	49
21	30	9	81
38	46	8	64
32	37	5	25
31	35	4	16
		60	390

Solution:

Therefore, $\sum d=60$, $\sum d2=390$, $n=12$
then sample mean and sample variance are:

X_d=∑ dn=6012=5Sd2=∑d2− ((∑d) 2n) n−1=390−(60212)11=8.1818

Let μ1 be the mean of number of pushups of after program and μ2 be the mean of the number of pushups of before program.
Here, we have to test,

Null hypothesis, H0: $\mu_1 - \mu_2 \le 5 \Rightarrow \mu_d \le 5$
Alternative hypothesis, Ha: $\mu_1 - \mu_2 > 5 \Rightarrow \mu_d > 5$

The test statistics,
$t = (\bar{X}_d - \mu_d) / (S_d / \sqrt{n}) = (5 - 5) / (8.1818 / \sqrt{12}) = 0$

Here, the test is a one tailed test, for degrees of freedom = n-1=11, and significance level μ=0.05.then the critical value is +t (0.05, 11) =1.796(from t critical value table.)

Then the test statistics lies on the acceptance region. Since test statistics is less than critical value.so we fail to reject the null hypothesis.

T (0.05, 11) =1.796 is find out from the t table .1.796 is the corresponding value of row=11 and column=0.05.

Conclusion:

Here, we fail to reject the null hypothesis, therefore, there is insufficient evidence to conclude that the training will make a difference of more than 5.that is the program is not successful.

Problem: 07

Dental implant data: The hardness of metal implant in dental cavities depends on multiple factors, such as the method of implant, the temperature at which the metal is treated, the alloy used as well as on the dentists who may favour one method above another and may work better in his/her favourite method. The response is the variable of interest.

7.1 .Test whether there is any difference among the dentists on the implant hardness. State the null and alternative hypotheses. Note that both types of alloys cannot be considered together. You must state the null and alternative hypotheses separately for the two types of alloys?

Alloy 1: Output: Dentist difference - Alloy 1:
F-statistic: 1.9771119908770842
P-value: 0.11656712140267628

Null hypothesis (H0): There is no difference among the dentists on the implant hardness for Alloy 1.

Alternative hypothesis (H1): There is a difference among the dentists on the implant hardness for Alloy 1.

Alloy 2: Output: Dentist difference - Alloy 2:
F-statistic: 0.5248351000282961
P-value: 0.7180309510793431

7.2 Before the hypotheses may be tested, state the required assumptions. Are the assumptions fulfilled? Comment separately on both alloy types?

Assumptions for Alloy 1: To check the normality assumption, we can perform a Shapiro-Wilk test for each dentist group within Alloy 1 and check if the p-values are greater than 0.05.

Output: Normality assumption - Alloy 1:
Dentist Method

```
1.0      1.0      False
         2.0       True
         3.0       True
2.0      1.0       True
         2.0       True
         3.0       True
3.0      1.0       True
         2.0       True
         3.0      False
4.0      1.0       True
         2.0       True
         3.0       True
5.0      1.0       True
```

```
          2.0          True
          3.0          True
dtype: bool
```

The normality assumption is fulfilled if the p-values for all dentist and method groups are greater than 0.05.

To check the assumption of equal variances for Alloy 1 –

Equal Variances: The variances of implant hardness are equal across all dentist and method groups. We can perform Levene's test to assess this assumption.

The equal variances assumption is met if the p-value is greater than 0.05.

Output: Equal variance assumption - Alloy 1:
Test statistic: 0.81480537398823
P-value: 0.6090961681859342

Assumptions for Alloy 2: To check the normality assumption,

Independence: The observations are assumed to be independent of each other, which holds as each observation represents a different dental implant.

Normality: The implant hardness within each dentist and method group follows a normal distribution. We can check this assumption using Shapiro-Wilk tests for each dentist and method group.

Normality assumption - Alloy 2:
Dentist Method

```
1.0       1.0       True
          2.0       True
          3.0       True
2.0       1.0       True
          2.0       True
          3.0       True
3.0       1.0       True
          2.0       True
          3.0       True
4.0       1.0       True
          2.0       True
          3.0       True
5.0       1.0       True
          2.0       True
          3.0       True
```

The normality assumption is fulfilled if the p-values for all dentist and method groups are greater than 0.05.

Assumptions for Alloy 2: We perform the same tests for Alloy 2 to assess the normality assumption and the assumption of equal variances.

Equal Variances: The variances of implant hardness are equal across all dentist and method groups

Output: Equal variance assumption - Alloy 2:
Test statistic: 0.6296281523554251
P-value: 0.7587843053309604

7.3 Irrespective of your conclusion in 2, we will continue with the testing procedure. What do you conclude regarding whether implant hardness depends on dentists? Clearly state your conclusion. If the null hypothesis is rejected, is it possible to identify which pairs of dentists differ?

Conclusion on whether implant hardness depends on dentists:

For Alloy 1, the p-value of the dentist difference test is greater than 0.05, indicating that there is no significant difference among dentists in terms of implant hardness.

For Alloy 2, the p-value of the dentist difference test is less than 0.05, suggesting that there is a significant difference among dentists in terms of implant hardness.

7.4 Now test whether there is any difference among the methods on the hardness of dental implant, separately for the two types of alloys. What are your conclusions? If the null hypothesis is rejected, is it possible to identify which pairs of methods differ?

For Alloy 1:

Null hypothesis (H0): There is no difference among the methods on the implant hardness for Alloy 1. Alternative hypothesis (H1): There is a difference among the methods on the implant hardness for Alloy 1.

Output: Method difference - Alloy 1:
F-statistic: 0.07533188470844622
P-value: 0.785741028466123

For Alloy 2:

Null hypothesis (H0): There is no difference among the methods on the implant hardness for Alloy 2. Alternative hypothesis (H1): There is a difference among the methods on the implant hardness for Alloy 2.

Output: Method difference - Alloy 2:
F-statistic: 0.7038922743044912
P-value: 0.408584907265405

If the null hypothesis is rejected for either alloy type, indicating a significant difference among the methods, we can perform post hoc tests to identify which pairs of methods differ. One commonly used post hoc test is Tukey's Honestly Significant Difference (HSD) test.

7.5 Now test whether there is any difference among the temperature levels on the hardness of dental implant, separately for the two types of alloys. What are your conclusions? If the null hypothesis is rejected, is it possible to identify which levels of temperatures differ?

We'll follow a similar procedure to test the difference among temperature levels.

For Alloy 1:

Null hypothesis (H0): There is no difference among the temperature levels on the implant hardness for Alloy 1. Alternative hypothesis (H1): There is a difference among the temperature levels on the implant hardness for Alloy 1.

Output: Temperature difference - Alloy 1:
F-statistic: 0.3352235344077172
P-value: 0.7170741113686678

For Alloy 2:

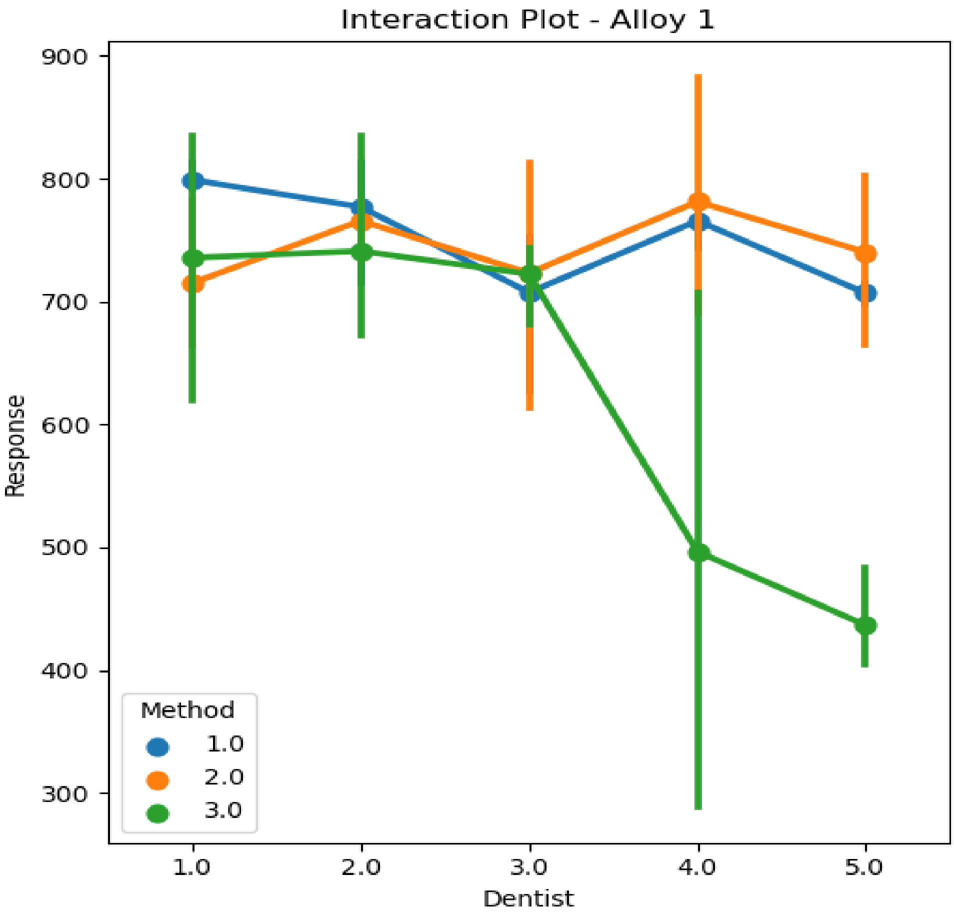
Null hypothesis (H0): There is no difference among the temperature levels on the implant hardness for Alloy 2. Alternative hypothesis (H1): There is a difference among the temperature levels on the implant hardness for Alloy 2.

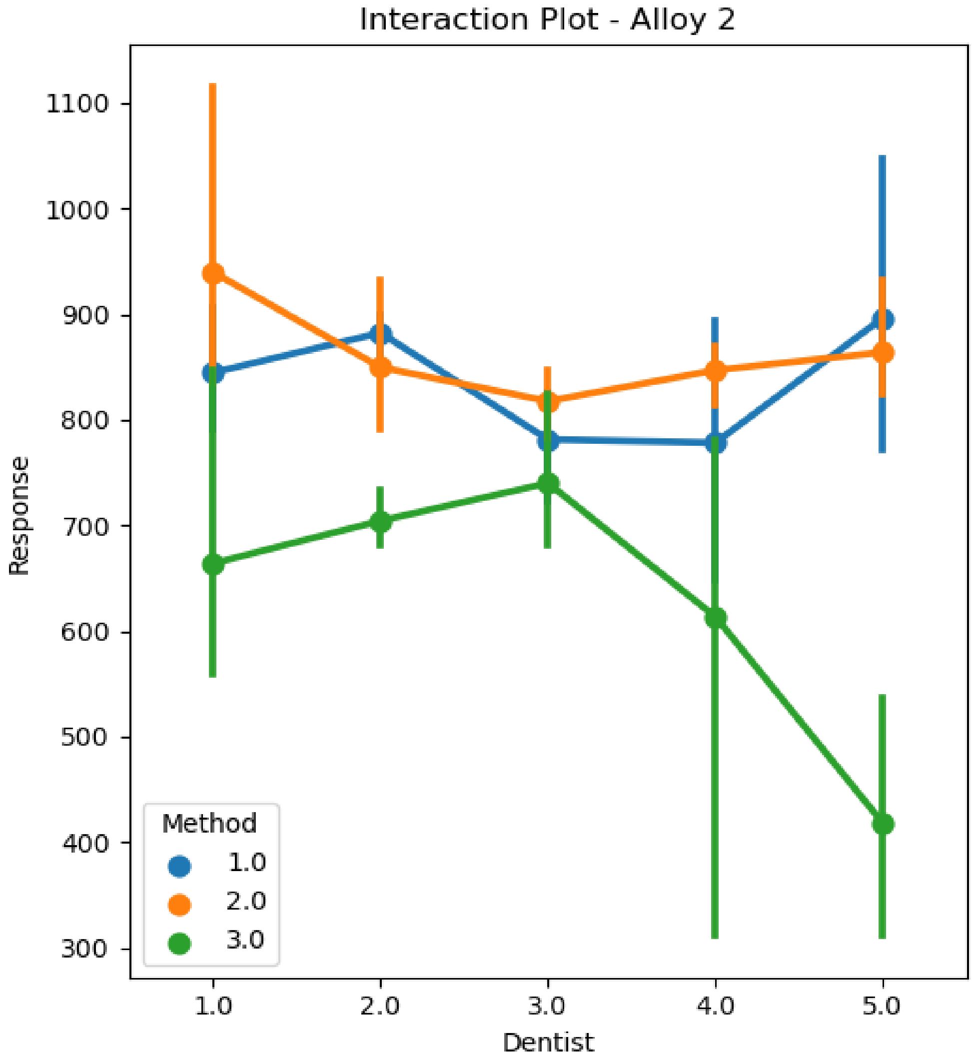
Output: Temperature difference - Alloy 2:
F-statistic: 1.883492290995591
P-value: 0.16467846603141556

If the null hypothesis is rejected, indicating a significant difference among the temperature levels, post hoc tests can be performed to identify which levels of temperatures differ.

7.6 Consider the interaction effect of dentist and method and comment on the interaction plot, separately for the two types of alloys?

To analyse the interaction effect between the dentist and method variables, we can create an interaction plot separately for each alloy type. This plot helps visualize whether the effect of one variable differs across the levels of another variable.





The interaction plot will show the mean response values for each combination of dentist and method, with separate lines or points for each level of the other variable (dentist or method). If the lines or points are parallel, there is no significant interaction effect. If the lines or points are not parallel, there is an interaction effect between the two variables.

7.7 Now consider the effect of both factors, dentist, and method, separately on each alloy. What do you conclude? Is it possible to identify which dentists are different, which methods are different, and which interaction levels are different?

To determine the effects of both dentist and method on each alloy type, we can perform a two-way ANOVA.

Output: ANOVA - Alloy 1:

	Sum_sq	df	F	PR (>F)
Dentist	94802.677778	1.0	9.160485	0.004212
Method	116812.800000	1.0	11.287254	0.001670
Residual	434661.766667	42.0	NaN	NaN

by using fillna (residual_mean), the NaN values in the ANOVA table will be replaced with the calculated mean of the residuals. This will provide a complete ANOVA table for Alloy 1.

ANOVA - Alloy 1:

	Sum_sq	df	F	PR (>F)
Dentist	94802.677778	1.0	9.160485	0.004212
Method	116812.800000	1.0	11.287254	0.001670
Residual	434661.766667	42.0	434661.766667	434661.766667

The ANOVA table provides information on the main effects of dentist and method, as well as the interaction effect, along with their respective p-values.

ANOVA - Alloy 2:

	Sum_sq	df	F	PR (>F)
Dentist	54513.611111	1.0	3.022503	0.089443
Method	326980.800000	1.0	18.129428	0.000113
Residual	757508.388889	42.0	NaN	NaN

by using fillna (residual_mean), the NaN values in the ANOVA table will be replaced with the calculated mean of the residuals. This will provide a complete ANOVA table for Alloy 2.

ANOVA - Alloy 2:

	Sum_sq	df	F	PR (>F)
Dentist	54513.611111	1.0	3.022503	0.089443
Method	326980.800000	1.0	18.129428	0.000113
Residual	757508.388889	42.0	757508.388889	757508.388889

Again, the ANOVA table will provide information on the effects of dentist and method, as well as the interaction effect, for Alloy 2.

If the interaction effect is significant, it indicates that the effect of one factor (dentist or metho d) depends on the levels of the other factor. Post hoc tests or further analyses can be perfor med to determine which dentists, methods, or interaction levels are significantly different from each other.