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TUTORIAL-1

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Class R.No.:-32

SECTION D

SEMESTER IV

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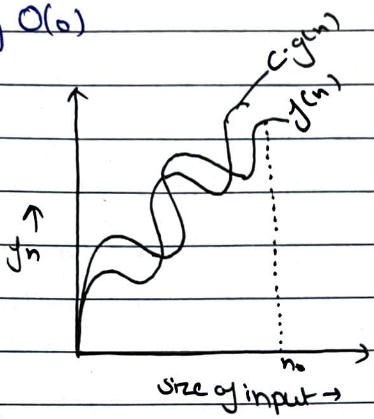
Q1) Asymptotic Notations
↳ Tending to Infinity

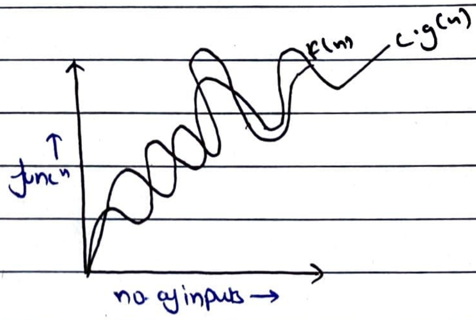
They help you find the complexity of an algorithm when input is very large.

① Big O(o)

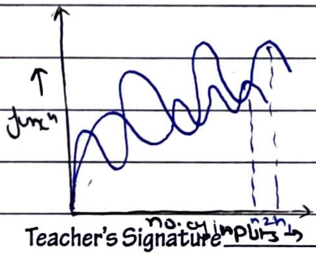


$f(n) = O(g(n))$
if $f(n) \leq c\,g(n)$
$\forall n \geq n_0$
for some constant $c > 0$
→ g(n) is tight upper bound of f(n)

② Big Omega(Ω)
$f(n) = \Omega(g(n))$
g(n) is tight lower bound of f(n)
$f(n) = \Omega(g(n))$
if $f(n) \geq c \cdot g(n)$
$\forall n \geq n_0$ for some constant $c > 0$



③ Theta(θ)
$f(n) = \Theta(g(n))$
g(n) is both 'tight' upper and lower bound of f(n)
$f(n) = \Theta(g(n))$



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if $c_1 g(n) \le f(n) \le c_2 \cdot g(n)$

$\forall n \ge c\max(n_1, n_2)$

for some constant $c_1 > 0$ and $c_2 > 0$

④ Small o(o)

$f(n) = o(g(n))$

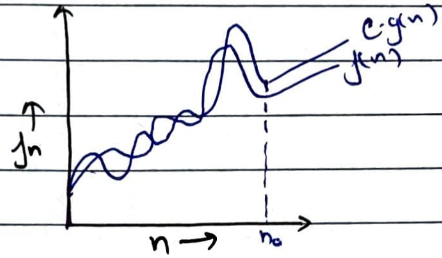
$g(n)$ is upper bound of fn $f(n)$.

$f(n) = o(g(n))$

when $f(n) < c \cdot g(n)$

$\forall n > n_0$

& $\forall c > 0$



⑤ Small omega(ω)

$f(n) = \omega(g(n))$

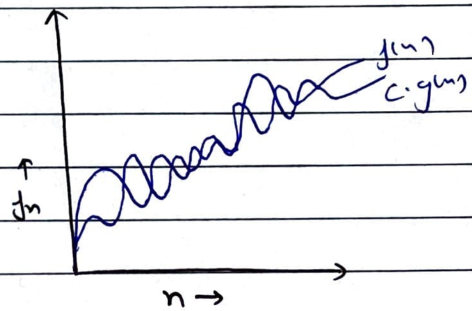
$g(n)$ is lower bound of fn $f(n)$

$f(n) = \omega(g(n))$

when $f(n) > c\, g(n)$

$\forall n > n_0$

$\forall c > 0$



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Q2) What should be T.C of

for (i=1 to n) { i=i*2 }

$$\Rightarrow \sum_{i=1}^{n} 1+2+4+8+\dots+n$$

GP kth value $\Rightarrow T_k = ar^{k-1}$

$\Rightarrow 1 \times 2^{k-1}$

$n = 2^{k}$

$2n = 2^{k}$

$\log 2n = k \log 2$

$\Rightarrow \log_2 + \log n = k \log 2$

$\Rightarrow \log_{n+1} = k$

$\Rightarrow O(k) = O(1+\log n)$

$= O(\log n)$

Q3) $T(n) = \{3T(n-1)$ if $n>0$, otherwise $1\}$

$T(n) = 3T(n-1)$ — ①

put $n = n-1$

$T(n-1) = 3T(n-2)$ — ②

from ① and ②

$\Rightarrow T(n) = 3(3T(n-2))$

$= 9T(n-2)$ — ③

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putting $n=n-2$ in eq ①

$T(n-2) = 3(T(n-3))$ — ④

$T(n) = 27(T(n-3))$

$T(n) = 3^k(T(n-k))$

putting $n-k=0$

$\Rightarrow n=k$

$T(n) = 3^n[T(n-n)]$

$T(n) = 3^n T(0)$

$T(n) = 3^n \times 1$ $[T(0)=1]$

$\Rightarrow T(n) = O(3^n)$ //

Q4. $T(n) = \{2T(n-1)-1$ if $n>0$, Otherwise $1\}$

$T(n) = 2T(n-1)-1$ — ①

let $n=n-1$

$T(n-1) = 2T(n-2)-1$ — ②

from ① & ②

$\Rightarrow T(n) = 2[2T(n-2)-1]-1$

$\Rightarrow T(n) = 4T(n-2)-3$ — ③

let $n=n-2$

$\Rightarrow T(n-2) = 2T(n-3)-1$ — ④

from ③ and ④

$T(n) = 4[2T(n-3)-1]-2-1$

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$\Rightarrow T(n) = 8T(n-3) - 4 - 2 - 1$

$\Rightarrow T(n) = 2^k T(n-k) \cdot 2^{k-1} \cdot 2^{k-2} \cdots 1$

$GP = 2^{k-1} + 2^{k-2} + 2^{k-3} + \cdots + 1$

$a = 2^{k-1}$

$r = \frac{1}{2}$

$S_k = \frac{a(1-r^n)}{1-r}$

$= \frac{2^{k-1}(1-(1/2)^n)}{1-\frac{1}{2}}$

$= 2^k (1-(1/2)^k)$

$= 2^k - 1$

let $n-k=0$

$\Rightarrow n=k$

$\Rightarrow T(n) = 2^n T(n-n) - (2^n-1)$

$T(n) = 2^n \cdot 1 - (2^n - 1)$

$T(n) = 2^n - 2^n + 1$

$\Rightarrow T(n) = O(1)$

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Q5) $i = 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$

$S = 1 + 3 + 6 + 10 + 15 + 21 + \cdots + n$

sum of $S = 1 + 3 + 6 + 10 + \cdots + n$ — ①

also $S = 1 + 3 + 6 + 10 + \cdots + T_{n-1} + T_n$ — ②

from ① − ②

$0 = 1 + 2 + 3 + 4 + \cdots + n - T_n$

$T_k = 1 + 2 + 3 + 4 + \cdots + k$

$T_k = \frac{1}{2} k(k+1)$

for k iterations

$1 + 2 + 3 \cdots + k <= n$

$\Rightarrow \frac{k(k+1)}{2} <= n$

$\Rightarrow \frac{k^2 + k}{2} <= n$

$\Rightarrow O(k^2) <= n$

$\Rightarrow k = O(\sqrt{n})$

$\Rightarrow T(n) = O(\sqrt{n})$

Q6) $O(1)$

as $i^2 <= n$

$\Rightarrow i <= \sqrt{n}$

$i = 1, 2, 3, 4, \ldots, \sqrt{n}$

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$$\sum_{i=1}^{n} 1+2+3+4+\cdots+\sqrt{n}$$

$$\Rightarrow T(n) = \frac{\sqrt{n}\times(\sqrt{n}+1)}{2}$$

$$\Rightarrow T(n) = \frac{n+\sqrt{n}}{2}$$

$$T(n) = O(n) \ //$$

Q7)

for $k = k^2$

$k = 1, 2, 4, 8, \ldots n$

$GP \Rightarrow a = 1, \ r = 2$

$$= \frac{a(r^n - 1)}{r-1}$$

$$= \frac{1(2^k - 1)}{1}$$

$n \Rightarrow 2^k$

$\log n \Rightarrow k$

$\Rightarrow$

i	j	k
1	logn	logn * logn
2	logn	logn * logn
⋮	⋮	⋮
n	logn	logn * logn

$\Rightarrow O(n * \log n * \log n)$

$\Rightarrow O(n \log^2 n)$

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Q8) Time Complexity of

```
function(int n)
 { int(n==1)
   return;                    //O(1)
   for(i=1 to n)              //i=1, 2,3,4..... n => O(n)
 { for(j=1 to n)              //j=1,2,34.... n*n => O(n^2)
     { print('*');
     }
   }
   function(n-3);             T(n/3)
 }
```

$\Rightarrow T(n) = T(n/3) + n^2$

$\Rightarrow a=1, \quad b=3, \quad f(n) = n^2$

$c = \log_3 1 = 0$

$\Rightarrow n^0 = 1 \quad > (f(n) = n^2)$

$\Rightarrow T(n) = \Theta(n^2)$ //

Q9) Time complexity of

```
void function(int n)
 { for(i=1 to n)              //O(n)
  { for(j=1;j<=n;j=j+i)
      print("*")              //O(n)
    }
  }
```

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for $i=1 \Rightarrow j=1,2,3,4 \ldots\ldots n \quad = n$
for $i=2 \Rightarrow j=1,3,5 \ldots\ldots n \quad = n/2$
for $i=3 \Rightarrow j=1,4,7 \ldots\ldots n \quad = n/3$

$\vdots \qquad\qquad\qquad \vdots$

for $i=n \Rightarrow j=1 \ldots \qquad 1$

$$\Rightarrow \sum_{j=n}^{1} n + \frac{n}{2} + \frac{n}{3} + \frac{n}{4} + \cdots + 1$$

$$\Rightarrow \sum_{j=n}^{1} n\left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n}\right]$$

$$\sum_{j=n} n[\log n]$$

$$\Rightarrow T(n) = [n \log n]$$

$$T(n) = O(n \log n) \quad //$$

Q10 for functions, n^k and c^n, what is the asymptotic relation between these functions?
assume that $k>=1$ and $c>1$ are constant.
Find out the value of c and no for which relation holds

as given n^k and c^n
relation b/w n^k and c^n is

$$n^k = O(c^n)$$

$\forall\, n \geq n_0$ some constant $a>0$ as $n^k \leq a c^n$

for $n_0 = 1$

$c = 2$

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$\Rightarrow 1^k \leq a2^1$

$\Rightarrow n_0 = 1$ and $c = 2$

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