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Cheat Sheet
Probability For Dummies

From Probability For Dummies by Deborah Rumsey

Successfully working your way through probability problems means understanding some basic rules of probability along with discrete and continuous probability distributions. Use some helpful study tips so you're well-prepared to take a probability exam.

Principles of Probability

The mathematics field of probability has its own rules, definitions, and laws, which you can use to find the probability of outcomes, events, or combinations of outcomes and events. To determine probability, you need to add or subtract, multiply or divide the probabilities of the original outcomes and events. You use some combinations so often that they have their own rules and formulas. The better you understand the ideas behind the formulas, the more likely it is that you'll remember them and be able to use them successfully.

Probability rules

Addition rule : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

If A and B are mutually exclusive: $P(A \cup B) = P(A) + P(B)$

Multiplication rule : $P(A \cap B) = P(A) * P(B \mid A)$ or $P(B) * P(A \mid B)$

If A and B are independent: $P(A \cap B) = P(A) * P(B)$

Complement rule : $P(A^C) = 1 - P(A)$

Probability definitions

A and B are mutually exclusive if $P(A \cap B) = 0$

A and B are independent if $P(A \mid B) = P(A)$ or $P(B \mid A) = P(B)$

Probability laws

Law of Total Probability : $P(B) = P(A) * P(B \mid A) + P\left(A^C\right) * P\left(B \mid A^C\right)$

Bayes' Law (or Bayes' Theorem) : $P(A \mid B) = \dfrac{P(A) * P(B \mid A)}{P(A) * P(B \mid A) + P\left(A^C\right) * P\left(B \mid A^C\right)}$

Counting rules

$$_nP_k = \frac{n!}{(n-k)!} \qquad _nC_k = \binom{n}{k} = \frac{n!}{(n-k)!k!}$$

$0! = 1$ (by definition)

$1! = 1$

$2! = 2 * 1 = 2$

$n! = n * (n-1) * (n-2) * \ldots (3) * (2) * (1)$

Discrete Probability Distributions

In probability, a discrete distribution has either a finite or a countably infinite number of possible values. That means you can enumerate or make a listing of all possible values, such as 1, 2, 3, 4, 5, 6 or 1, 2, 3, . . .

There are several kinds of discrete probability distributions, including discrete uniform, binomial, Poisson, geometric, negative binomial, and hypergeometric.

X	*X Counts*	*p(x)*	*Values of X*	*E(x)*	*V(x)*
Discrete uniform	Outcomes that are equally likely (finite)	$\frac{1}{b-a+1}$	$a \le x \le b$	$\frac{b+a}{2}$	$\frac{(b-a+2)(b-a)}{12}$
Binomial	Number of sucesses in n fixed trials	$\binom{n}{x} p^x (1-p)^{n-x}$	$x = 0,1,\ldots,n$	np	$np(1-p)$
Poisson	Number of arrivals in a fixed time period	$\frac{e^{-\lambda}\lambda^x}{x!}$	$x = 0,1,2,\ldots$	λ	λ
Geometric	Number of trials up through 1st success	$(1-p)^{x-1}p$	$x = 1,2,3,\ldots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Negative Binomial	Number of trials up through kth success	$\binom{x-1}{k-1}(1-p)^{x-k}p^k$	$x = k, k+1,\ldots$	$\frac{k}{p}$	$\frac{k(1-p)}{p^2}$
Hyper - geometric	Number of marked individuals in sample taken without replacement	$\frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$	$\max(0, M+n-N) \le x \le \min(M,n)$	$n * \frac{M}{N}$	$\frac{nM(N-M)(N-n)}{N^2(N-1)}$

Continuous Probability Distributions

When you work with continuous probability distributions, the functions can take many forms. These include continuous uniform, exponential, normal, standard normal (Z), binomial approximation, Poisson approximation, and distributions for the sample mean and sample proportion.

When you work with the normal distribution, you need to keep in mind that it's a *continuous distribution,* not a discrete one. A continuous distribution's probability function takes the form of a continuous curve, and its random variable takes on an uncountably infinite number of possible values. This means the set of possible values is written as an interval, such as negative infinity to positive infinity, zero to infinity, or an interval like [0, 10], which represents all real numbers from 0 to 10, including 0 and 10.

X	*X Measures*	*f(x)*	*Values of X*	*E(x)*	*V(x)*
Continuous uniform	Outcomes with equal density (continuous)	$\frac{1}{b-a}$	$a \le x \le b$	$\frac{b+a}{2}$	$\frac{(b-a)^2}{12}$
Exponential	Time between events; time until an event	$\lambda e^{-\lambda x}$	$x \ge 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Normal	Values with a bell-shaped distribution (continuous)	$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	$-\infty < x < \infty$	μ	σ
Standard normal (Z)	Standard scores	$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$	$Z = \frac{x-\mu}{\sigma}$	0	1
Binomial approximation	Number of successes in large number of trials	Approx. normal if $np \ge 5$ and $n(1-p) \ge 5$ by CLT	$Z = \frac{x-np}{\sqrt{np(1-p)}}$	np	np(1−p)
Poisson approximation	Number of occurrences in a fixed time period (large average)	Approx. normal if $\lambda > 30$	$z = \frac{x-\lambda}{\sqrt{\lambda}}$	λ	λ
$\bar{x}$	Average of $x_1, x_2, \ldots, x_n$	Exactly normal if x is normal. Approx. normal if $n \ge 30$ by CLT	$Z = \frac{\bar{x}-\mu_x}{\sigma_x / \sqrt{n}}$	μ_x	$\frac{\sigma_x^2}{n}$
$\hat{p}$	Proportion or percentage of successes in binomial with $np \ge 5$, $n(1-p) \ge 5$	Approx. normal if $np \ge 5$ and $n(1-p) \ge 5$ by CLT	$Z = \frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}}$	p	$\frac{p(1-p)}{n}$